Answers:

Section A

1. **Determine whether the given value of x is a solution of the given quadratic equation or not: $6X^2 - x - 1 = 0$; $x = 1/2$.**

Ans.

When $x = \frac{1}{2}$, L.H.S. $= 6 \times \left(\frac{1}{2}\right)^2 - \frac{1}{2} - 1$

$$= 6 \times \frac{1}{4} - \frac{1}{2} - 1$$

$$= \frac{3}{2} - \frac{3}{2} = 0 = \text{R.H.S.}$$

$\therefore x = \frac{1}{2}$ is a solution of the given quadratic equation.

2. Find 15th term of the AP with second term 11 and common difference 9.

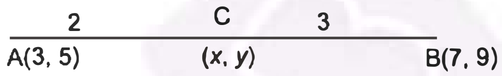
Ans.

$a_2 = 11 \Rightarrow a + d = 11 \Rightarrow a + 9 = 11 \Rightarrow a = 2$

Now, $a_{15} = a + 14d = 2 + 14 \times 9 = 128$.

3. Find the coordinates of the point which divides the line segment joining the points (3, 5) and (7, 9) internally in the ratio 2 : 3.

Ans.



$$x = \frac{2 \times 7 + 3 \times 3}{2 + 3} \Rightarrow x = \frac{14 + 9}{5} \quad \frac{23}{5}$$

$$\text{and } y = \frac{2 \times 9 + 3 \times 5}{2 + 3} = \frac{18 + 15}{5} \quad \frac{33}{5}$$

Coordinates of the point $C\left(\frac{23}{5}, \frac{33}{5}\right)$

4. In a circle of radius 21 cm, an arc subtends an angle of 60° at the centre. Find the length of the arc.

Ans.

Given; $r = 21$ cm and $\theta = 60°$

$$\text{Length of arc, } l = \frac{\theta}{360} \times 2\pi r = \frac{60}{360} \times 2 \times \frac{22}{7} \times 21$$

$$= 22 \text{ cm}$$

Section B

5.Using quadratic formula, solve the following quadratic equation: 2X² - 7x + 3 = 0

Ans.

Given equation is $2x^2 - 7x + 3 = 0$

$$D = b^2 - 4ac = (-7)^2 - 4 \times 2 \times 3 = 25$$

$$x = \frac{-(-7) \pm \sqrt{25}}{2 \times 2} = \frac{7 \pm 5}{4}$$

$$x = 3, \frac{1}{2}$$

6. Find the 31st term of an AP whose 11th term is 38 and the 16th term is 73.

Ans.

Let first term be a and common difference be d.

Now, $a_{11} = 38 \Rightarrow a + 10d = 38$

Also, $a_{16} = 73 \Rightarrow a + 15d = 73$

$$a + 10d = 38$$
$$a + 15d = 73$$

Subtracting we get

$$-5d = -35$$

$\Rightarrow$ $d = 7$

When $d = 7$, equation (i) becomes $a + 10 \times 7 = 38 \Rightarrow a = 38 - 70 = -32$

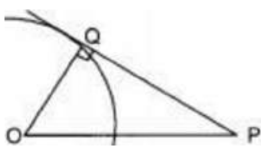
Now, $a_{31} = a + 30d = -32 + 30 \times 7 = 178$

7. A point P is 18 cm from the centre of a circle. The radius of the circle is 12 cm. Find the length of the tangent drawn to the circle from the point P.

Ans.

Given; OP = 18 cm, OQ = 12 cm

OQ $\perp$ PQ [Radius is perpendicular to the tangent at the point of contact]



In right OQP, $OP^2 = OQ^2 + PQ^2$

$\Rightarrow$ $(18)^2 = (12)^2 + PQ^2$

$\Rightarrow$ $324 = 144 \quad PQ^2$

$\Rightarrow$ $PQ^2 = 180$

$\Rightarrow$ $PQ = \sqrt{180}$ cm $= 6\sqrt{5}$ cm

8. In a box, there are 800 bulbs, out of which 20 bulbs are defective. One bulb is taken out at random. Find the probability that the bulb is not

defective.

Ans.

Total no. of bulbs = 800

No. of defective bulbs = 20

No. of non-defective bulbs = 800 − 20 = 780

Probability that the bulb is not defective = $\frac{780}{800} = \frac{39}{40}$

9. The radii of two circles are 8 cm and 6 cm respectively. Find the radius of the circle having area equal to the sum of the areas of these two circles.

Ans.

Area of first circle = $\pi \times 8^2 = 64\pi \text{ cm}^2$

Area of second circle = $\pi \times 6^2 = 36\pi \text{ cm}^2$

Let radius of required circle be x cm.

A.T.Q., $\pi \times x^2 = 64\pi + 36\pi$

$\Rightarrow \quad \pi x^2 = 100\pi$

$\Rightarrow \quad x^2 = 100 \Rightarrow x = 10$ cm.

10. A rectangular solid metallic cuboid 18 cm x 15 cm x 4.5 cm is melted and recast into solid cubes each of side 3 cm. How many solid cubes can be made?

Ans.

Volume of cuboid = $18 \times 15 \times 4.5 \text{ cm}^3$

Volume of one cube = $(3)^3 \text{ cm}^3$

= 27 cm^3

No. of cubes = $\frac{\text{volume of cuboid}}{\text{volume of one cube}}$

= $\frac{18 \times 15 \times 4.5}{27} = 45$

Section C

11. Divide 39 into two parts such that their product is 324.

Ans.

Let one part $= x$

Other part $= 39 - x$

A.T.Q., $x(39 - x) = 324$ $\Rightarrow$ $39x - x^2 = 324$

$\Rightarrow$ $x^2 - 39x + 324 = 0$ $\Rightarrow$ $x^2 - 27x - 12x + 324 = 0$

$(x - 12)(x - 27) = 0$ $\Rightarrow$ $x = 12$ or $x = 27$

If one part is $x = 12$, then other part $= 39 - 12 = 27$

If one part is $x = 27$, then other part $= 39 - 27 = 12$

Parts are 12 and 27.

12. Determine the AP whose third term is 16 and the 7th term exceeds the 5th term by 12.

Ans.

Let the first term of the A.P. be 'a' and common difference be 'd'

$a_3 = 16 \Rightarrow a + 2d = 16$ ...(*i*)

Also, $a_7 = a_5 + 12$ $a + 6d = a + 4d + 12$

$2d = 12$ $\Rightarrow$ $d = 6$

Therefore, eq. (*i*) becomes

$a + 2 \times 6 = 16$ $a = 4$

$a_1 = 4,\ a_2 = 4 + 6 = 10,\ a_3 = 10 + 6 = 16$

AP = 4, 10, 16 ... .

13. Solve for x:(1/ x + 4)-(1/x-7)=11/30, (x not equal to -4, 7)

Ans.

$$\frac{1}{x+4} - \frac{1}{x-7} = \frac{11}{30}$$

$$\frac{(x-7)-(x+4)}{(x+4)(x-7)} - \frac{11}{30} \qquad \frac{-11}{x^2-3x-28} = \frac{11}{30}$$

$$x^2 - 3x - 28 = -30$$

$$x^2 - 3x + 2 = 0$$

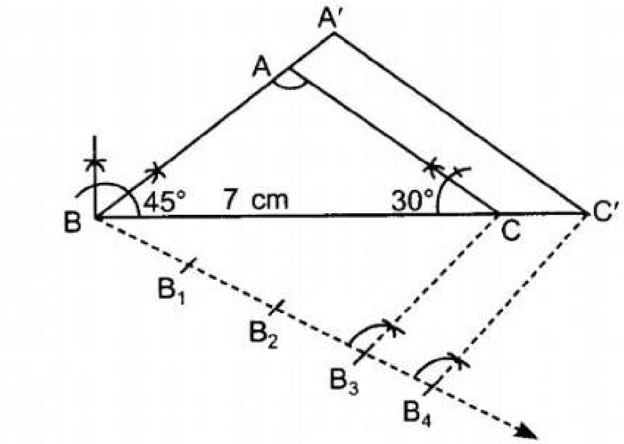
$$(x - 2)(x - 1) = 0$$

$x = 2,\ x = 1$

14. Draw a triangle ABC with side BC = 7 cm, B = 45°, A = 105°. Then construct a triangle whose sides are 4/3 times the corresponding sides of

triangle ABC.

Ans.



A'BC' is the required Δ.

15. 21 cards are numbered 1, 2, ..., 20, 21 and placed in a box. The cards are mixed thoroughly. A card is drawn at random from the box. Find the probability that the number on the card is

(i) an even (ii) a prime number (iii) divisible by 4

Ans.

No. of cards in the box = 21

(*i*) No. of cards with even no. = 10 (*i.e.* 2, 4, 6, 8, 10, 12, 14, 16, 18 and 20)

Required probability = $\frac{10}{21}$

(*ii*) No. of cards with prime numbers = 8 (*i.e.* 2, 3, 5, 7, 11, 13, 17 and 19)

Required probability = $\frac{8}{21}$

(*iii*) No. of cards with a number divisible by 4 = 5 (*i.e.*, 4, 8, 12, 16 and 20)

Required probability = $\frac{5}{21}$

16. A box contains 80 discs which are numbered from 1 to 80. If one disc is drawn at random from the box, find the probability that it bears (i) a two-digit number (ii) a perfect square number (iii) a number divisible by 5.

Ans.

Total no. of discs = 80

(*i*) No. of discs with two-digit numbers = 80 – 9 = 71

∴ Required probability = $\frac{71}{80}$

(*ii*) No. of discs with perfect square numbers = 8 (*i.e.*, 1, 4, 9, 16, 25, 36, 49 and 64)

∴ Required probability = $\frac{8}{80} = \frac{1}{10}$

(*iii*) No. of discs with numbers divisible by 5 = 16.

(*i.e.* 5, 10, 15, 20, 25, 30, 35, 40, 45, 50, 55, 60, 65, 70, 75 and 80)

∴ Required probability = $\frac{16}{80} = \frac{1}{5}$

17. Find the value of k if the points A(2,3), B(4, k) and C(6, -3) are collinear.

Ans.

Points are collinear, if

$$\frac{1}{2}|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)| = 0$$

$$2(k + 3) + 4(-3 - 3) + 6(3 - k) = 0$$

$$\Rightarrow \quad 2k + 6 - 24 + 18 - 6k = 0$$

$$\Rightarrow \quad k = 0$$

18.Find the value of k, if the point P(2,3) is equidistant from the points A{k, 1) and B(7, k).

Ans.

A.T.Q.,

$$AP = BP$$

$$\sqrt{(2-k)^2+(3-1)^2} = \sqrt{(2-7)^2+(3-k)^2}$$

$$(2 - k)^2 + 4 = 25 + (3 - k)^2$$

$$4 + k^2 - 4k + 4 = 25 + 9 + k^2 - 6k$$

$$2k = 26$$

$$k = 13$$

19.The length of the minute hand of a clock is 14 cm. Find the area swept by the minute hand in 5 minutes. (Take pi =22/7)

Ans.

Angle made by minute hand in 5 minutes = $\frac{360}{60} \times 5 = 30°$

Minute hand swept 30° in 5 minutes.

Area swept by minute hand in 5 minu es = Area of the sector of a circle with radius = 14 cm and θ = 30°

$$= \frac{30}{360} \times \pi(14)^2$$

$$= \frac{1}{12} \times \frac{22}{7} \times 14 \times 14 = \frac{154}{3} \text{ cm}^2$$

20. A fez, the cap used by the Turks, is shaped like the frustum of a cone. If its radius on the open side is 10 cm, radius at the upper base is 4 cm and its slant height is 14 cm, find the area of material used for making it. (Take pi =22/7)

Ans.

Here, R_1 = 10 cm

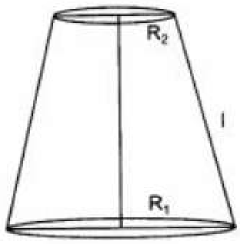
R_2 = 4 cm, l = 14 cm

Area of material used = curved surface area + area of upper circular end.

$$= \pi l(R_1 + R_2) + \pi R_2^2$$

$$= (\frac{22}{7} \times 14\ (10 + 4) + \frac{22}{7} \times (4)^2) \text{ cm}^2$$

$$= (616 + \frac{176}{4}) \text{ cm}^2 = \frac{4488}{7} \text{ cm}^2$$



Section D

21. One year ago, a man was 8 times as old as his son. Now, his age is equal to the square of his son's age in years. Find their present ages.

Ans.

Let present age of the man = x years

and present age of his son = y years.

One year ago,

$$\text{Man's age} = (x - 1) \text{ years}$$

$$\text{Son's age} = (y - 1) \text{ years}$$

A.T.Q., $(x - 1) = 8(y - 1)$

$\Rightarrow$ $x - 1 = 8y - 8 \Rightarrow x = 8y - 7$

Also, $x = y^2$

$8y - 7 = y^2$

$\Rightarrow$ $y^2 - 8y + 7 = 0$

$\Rightarrow$ $(y - 7)(y - 1) = 0$

$\Rightarrow$ $y = 7$ or $y = 1$

Rejecting $y = 1$, we get

$y = 7$ and $x = 8 \times 7 - 7 = 49.$

Present age of man = 49 years and his son's age = 7 years.

22. A manufacturer of TV sets produced 600 units in the third year and 700 units in the 7th year. Assuming that the production increases uniformly by a fixed number every year, find

(i) the production in the first year, (ii) the production in the 10th year,

(iii) the total production in 7 years

Ans.

Let the production in Ist year and a units

Constant increase in the production every year = d units.

Now, production in 3rd year = 600 unit

$\Rightarrow \quad a + 2d = 600$

Production in 7th year = 700

$\Rightarrow \quad a + 6d = 700$

Subtracting (*ii*) from (*i*), we have

$$a + 2d = 600$$
$$a + 6d = 700$$

$$-4d = -100 \qquad d = 25$$

Putting $d = 25$ in (*i*), we get

$$a + 2 \times 25 = 600 \quad \Rightarrow \quad a = 550$$

(*i*) Now production in Ist year = a = 550 units

(*ii*) Production in 10th year = $a + 9d$

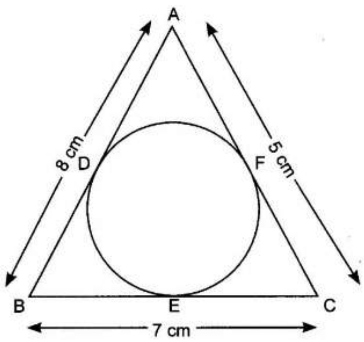
$$= 550 + 9 \times 25 = 550 + 225 = 775 \text{ units}$$

(*iii*) Total production in 7 years

$$= \frac{7}{2}(2 \times 550 + 6 \times 25)$$

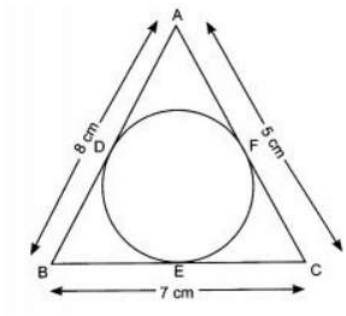
$$= \frac{7}{2}(1100 + 150) = \frac{7}{2} \times 1250 = 7 \times 625 = 4375 \text{ units}$$

23. A circle is inscribed in AABC having sides AB = 8 cm, BC = 7 cm and AC = 5 cm. Find AD, BE and CF



Ans.

Let $AD = x$ cm [Length of the angent from an external point to the circle are equal]



$AF = x$ cm

$BD = AB - AD = (8 - x)$ cm

$BD = BE$

$BE = (8 - x)$ cm

Also, $CE = BC - BE = 7 - (8 - x) = (x - 1)$ cm

$CE = CF$

$CF = (x - 1)$ cm

Now, $AC = AF + CF$

$\Rightarrow$ $x + x - 1 = 5$

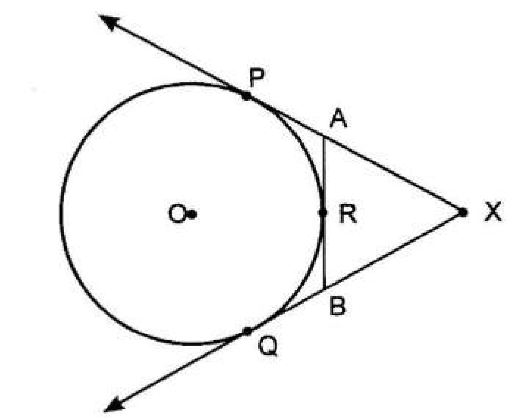
$x = 3$

$AD = 3$ cm

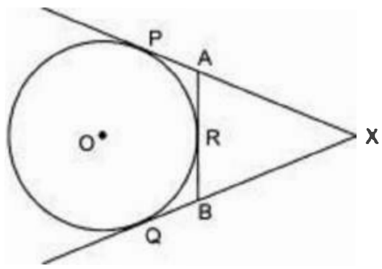
$BE = 8 - x = 8 - 3 = 5$ cm

$CF = x - 1 = 3 - 1 = 2$ cm

24. In figure, given below, XP and XQ are tangents from X to the circle with centre O and ARB is tangent at point R. Prove that XA + AR = XB + BR



Ans.



$XP = XA \quad AP$

$AP = AR$ But $XP = XQ$ [Lengths of the tangents from an exterior point are equal]

$XP = XA + AR$ ...(*i*)

Also, $XQ = XB + BQ$

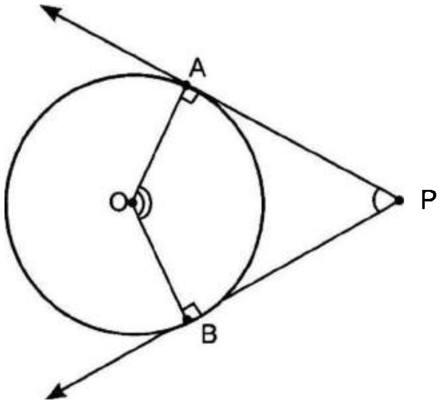
$BQ = BR$

$XQ = XB + BR$ ...(*ii*)

From (*i*) and (*ii*)

$XA + AR = XB + BR$

25. If O be the centre of a circle and tangents drawn to the circle at the points A and B of the circle intersect each other at P, then prove that angle AOB + angle APB = 180°.



Ans.

PA is tangent and OA is radius

$OA \perp PA$ [Radius i perpendicular to the tangent at the point of contact]

$\Rightarrow \angle OAP = 90°$

Similarly,

$\angle OBP = 90°$

In quadrilateral OBPA,

$\angle OBP + \angle APB + \angle OAP + \angle AOB = 360°$

$90° + \angle APB + 90° + \angle AOB = 360°$

$\angle APB + \angle AOB = 180°$

26. An electric pole is 12 m high. A steel wire tied to the top of the pole is affixed at a point on the ground to keep the pole upright. If the wire makes an angle of 60° with the horizontal through the foot of the pole, find the length of the wire. (Take root 3 =73)

Ans.

Let AB be the pole and AC be the length of wire.

A.T.Q., $\angle ACB = 60°$ and AB = 12 m

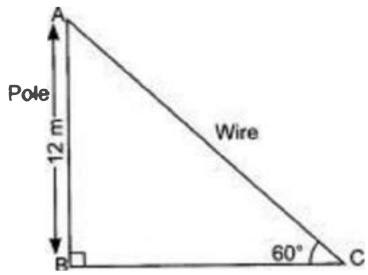
In right ΔACB, $\frac{AB}{AC} = \sin 60°$

$\frac{12}{AC} = \frac{3}{2}$

$AC = \frac{12 \times 2}{\sqrt{3}} m$

$AC = \frac{24 \times 3}{3} = 8 \quad 3 \ m$

$= 8 \times 1.73 = 13.84 \ m$



27. The angle of elevation of the top of a tower from a point on the ground is 45°. On walking 30 metres towards the tower, the angle of elevation becomes 60°. Find the height of the tower and the original distance from the foot of the tower. (Take root 3 = 1.73)

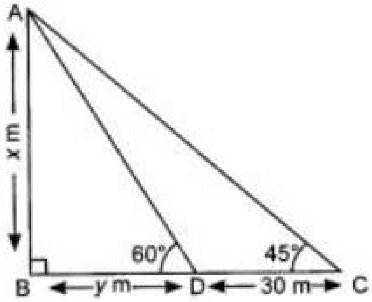
Ans.

Let AB be the height of the tower,

A.T.Q., $\angle ACB = 45°$ and $\angle ADB = 60°$

$DC = 30$ m

Let $AB = x$ m and $BD = y$ m



In right ΔABC, $\frac{AB}{BC} = \tan 45° \Rightarrow \frac{x}{y + 30} = 1$

$x = y + 30$

$y = x - 30$ ...(i)

In right ΔABD,

$\frac{AB}{BD} = \tan 60°$

$\frac{x}{y} = \sqrt{3}$

$\frac{x}{x-30} = \sqrt{3}$ [Using (i)]

$x = \sqrt{3}x - 30\sqrt{3}$

$\sqrt{3}x - x = 30\sqrt{3}$

$x = \frac{30\sqrt{3}}{\sqrt{3}-1} = \frac{30\sqrt{3}}{\sqrt{3}-1} \times \frac{(\sqrt{3}+1)}{(\sqrt{3}+1)}$

$x = \frac{30\sqrt{3}(\sqrt{3}+1)}{2} = 15\sqrt{3}(\sqrt{3}+1)$

$= 45 + 15\sqrt{3}$

$= 45 + 15 \times 1.73 = 70.95$ m

Height of the tower = 70.95 m

Origin distance = BC = y + DC = $(x - 30) + 30 = x$

= 70.95 m

28. Find the length of the median through the vertex A(5, 1) drawn to the triangle ABC where other two vertices are B(l, 5) and C(-3, -1).

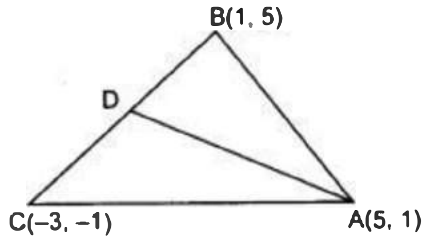
Ans.

Let AD be the median of ΔABC.

D will be the mid-point of BC.

Coordinates of D are

$$\left(\frac{1\ \ (-3)}{2}, \frac{5+(-1)}{2}\right) = (-1\ \ 2)$$

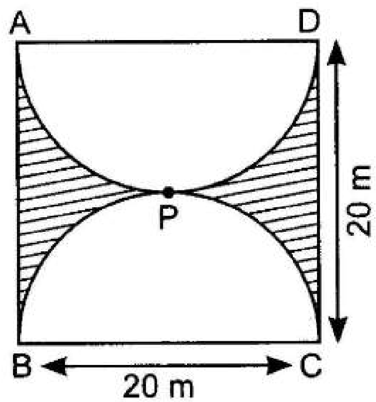


Now, $AD = \sqrt{(-1-5)^2+(2-1)^2}$

$= \sqrt{36+1} = 37$ units

Length of median AD $= \sqrt{37}$ units

29. Find the area of the shaded region in figure if ABCD is a square of side 20 m and APD and BPC are semicircles. (Take pi =22/7)



If shaded region is used by society for planting trees, then what value is reflected?

Ans.

Area of semicircle BPC $= \frac{1}{2}\pi r^2$

$= \frac{1}{2}\pi \times (10)^2 = 50\pi\ m^2$

Similarly,

Area of semicircle APD $= 50\pi\ m^2$

Total area of semicircles $= (50\pi + 50\pi)\ m^2$

$= 100\pi\ m^2$

$= 100 \times \frac{22}{7} = \frac{2200}{7}\ m^2$

Area of the square ABCD $= 20 \times 20 = 400\ m^2$

Area of the shaded part $= \left(400 - \frac{2200}{7}\right)\ m^2$

$= \frac{600}{}\ m^2$

Value reflected : Tree plantation, Eco balance, Eco friendly.

30. A solid is in the form of a cone mounted on a right circular cylinder both having same radii of their bases. Base of the cone is placed on the top base of the cylinder. If the radius of the base and height of the cone be 4 cm and 7 cm respectively and the height of the cylindrical part of the solid is 3.5 cm, find the volume of the solid. (Take pi =22/7)

Ans.

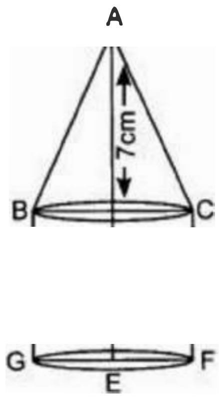
Given; height of conical part = 7 cm

Radius = 4 cm

Volume of conical part $= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \frac{22}{7} \times 4 \times 4 \times 7\ cm^3$

$= \frac{352}{}\ cm^3$



$$\text{Height of cylindrical part} = 3.5 \text{ cm}$$

$$\text{Radius} = 4 \text{ cm}$$

$$\text{Volume of the cylindrical part} = \pi r^2 h = \frac{22}{7} \times 4 \times 4 \times 3.5$$

$$= 176 \text{ cm}^3$$

$$\text{Volume of the solid} = \left(\frac{352}{3} + 176\right) \text{cm}^3$$

$$= \frac{880}{} \text{ cm}^3$$

31. A toy is in the form of a cone of radius 3.5 cm mounted on a hemisphere of same radius. The total height of the toy is 15.5 cm. Find the total surface area of the toy. (Take pi =22/7)

Ans.

Radius of the conical part (r) = 3.5 cm

Height of conical part (h) = 15.5 − 3.5 = 12 cm

Slant height (l) = $\sqrt{r^2 + h^2}$

$= \sqrt{(3.5)^2 + (12)^2}$

$= \sqrt{12.25 + 144} = 12.5$ cm

Curved surface area of conical part = $\pi r l$

$= \frac{22}{7} \times 3.5 \times 12.5 \text{ cm}^2$

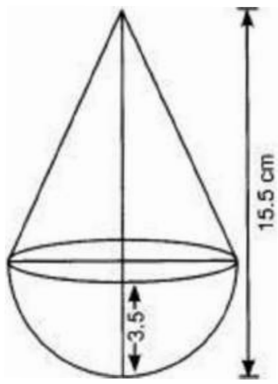
$= 137.5 \text{ cm}^2$

Radius of hemisphere = 3.5 cm

curved surface area of hemisphere = $2\pi r^2$

$= 2 \times \frac{22}{7} \times 3.5 \times 3.5$

$= 77 \text{ cm}^2$



Total surface area of the toy = 137.5 + 77 = 214.5 cm²