QUESTION PAPER CODE 65/1/1/D

EXPECTED ANSWER/VALUE POINTS

SECTION A

1. $\Delta = \begin{vmatrix} 1 & 0 & 0 \\ 1 & \sin\theta & 0 \\ 1 & 0 & \cos\theta \end{vmatrix} = \sin\theta\cos\theta$ $\dfrac{1}{2}$

    $= \dfrac{1}{2}\sin 2\theta \therefore \text{Max value} = \dfrac{1}{2}$ $\dfrac{1}{2}$

2. $(A-I)^3 + (A+I)^3 - 7A, \quad A^2 = I \Rightarrow A^3 = A$ $\dfrac{1}{2}$

    $= 2A - A = A$ $\dfrac{1}{2}$

3. $\left.\begin{array}{l} 2b = 3 \text{ and } 3a = -2 \\ b = \dfrac{3}{2} \text{ and } a = -\dfrac{2}{3} \end{array}\right\}$ $\dfrac{1}{2}+\dfrac{1}{2}$

4. Getting position vector as $2(2\vec{a}+\vec{b}) - 1(\vec{a}-2\vec{b})$ $\dfrac{1}{2}$

    $= 3\vec{a} + 4\vec{b}$ $\dfrac{1}{2}$

5. $\overrightarrow{AD} = \overrightarrow{AB} + \dfrac{1}{2}[\overrightarrow{AC}-\overrightarrow{AB}] = \dfrac{1}{2}(\overrightarrow{AC}+\overrightarrow{AB})$ $\dfrac{1}{2}$

    $|\overrightarrow{AD}| = \dfrac{1}{2}|3\hat{i}+5\hat{k}| = \dfrac{1}{2}\sqrt{34}$ $\dfrac{1}{2}$

6. $\vec{r}.\dfrac{(2\hat{i}-3\hat{j}+6\hat{k})}{7} = 5$ 1

SECTION B

7. $\text{LHS} = \tan^{-1}\left(\dfrac{\dfrac{1}{5}+\dfrac{1}{7}}{1-\dfrac{1}{5}\cdot\dfrac{1}{7}}\right) + \tan^{-1}\left(\dfrac{\dfrac{1}{3}+\dfrac{1}{8}}{1-\dfrac{1}{3}\cdot\dfrac{1}{8}}\right)$ 1

    $= \tan^{-1}\left(\dfrac{6}{17}\right) + \tan^{-1}\left(\dfrac{11}{23}\right)$ 1

    $= \tan^{-1}\left(\dfrac{\dfrac{6}{17}+\dfrac{11}{23}}{1-\dfrac{6}{17}\cdot\dfrac{11}{23}}\right) = \tan^{-1}\left(\dfrac{325}{325}\right)$ 1

    $= \tan^{-1}(1) = \dfrac{\pi}{4}$ 1

OR

$2\tan^{-1}(\cos x) = \tan^{-1}(2\operatorname{cosec} x)$

$\Rightarrow \quad \tan^{-1}\left(\dfrac{2\cos x}{1 - \cos^2 x}\right) = \tan^{-1}\left(\dfrac{2}{\sin x}\right)$ 2

$\Rightarrow \quad \sin x\,(\sin x - \cos x) = 0$ 1

$\Rightarrow \quad \sin x = \cos x$ $\dfrac{1}{2}$

the solution is $x = \dfrac{\pi}{4}$ $\dfrac{1}{2}$

8. Let the income be 3x, 4x and expenditures, 5y, 7y

$\therefore \quad \begin{aligned}3x - 5y &= 15000 \\ 4x - 7y &= 15000\end{aligned}\Big\}$ 1

$\begin{pmatrix} 3 & -5 \\ 4 & -7 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 15000 \\ 15000 \end{pmatrix}$

$\begin{pmatrix} x \\ y \end{pmatrix} = -1\begin{pmatrix} -7 & 5 \\ -4 & 3 \end{pmatrix}\begin{pmatrix} 15000 \\ 15000 \end{pmatrix}$

$\Rightarrow \quad x = 30000,\ y = 15000$ $1\dfrac{1}{2}$

$\therefore$ Incomes are ₹ 90000 and ₹ 120000 respectively $\dfrac{1}{2}$

"Expenditure must be less than income"

(or any other relevant answer)   1

9. Here $x = a\left(\sin 2t + \dfrac{1}{2}\sin 4t\right)$, $y = b(\cos 2t - \cos^2 2t)$

$\dfrac{dx}{dt} = 2a[\cos 2t + \cos 4t],\ \dfrac{dy}{dt} = 2b[-\sin 2t + 2\cos 2t \sin 2t] = 2b[\sin 4t - \sin 2t]$ 1 + 1

$\dfrac{dy}{dx} = \dfrac{b}{a}\left[\dfrac{\sin 4t - \sin 2t}{\cos 4t + \cos 2t}\right]$ 1

$\dfrac{dy}{dx}\bigg]_{t=\frac{\pi}{4}} = \dfrac{b}{a}$ $\dfrac{1}{2}$

and $\dfrac{dy}{dx}\bigg]_{t=\frac{\pi}{3}} = \sqrt{3}\,\dfrac{b}{a}$ $\dfrac{1}{2}$

OR

$y = x^x \Rightarrow \log y = x.\log x$ $\dfrac{1}{2}$

$\Rightarrow \quad \dfrac{1}{y}\dfrac{dy}{dx} = (1 + \log x)$ $1\dfrac{1}{2}$

$$\Rightarrow \quad \frac{1}{y}\frac{d^2y}{dx^2} - \frac{1}{y^2}\left(\frac{dy}{dx}\right)^2 = \frac{1}{x}$$ $1\frac{1}{2}$

$$\Rightarrow \quad \frac{d^2y}{dx^2} - \frac{1}{y}\left(\frac{dy}{dx}\right)^2 - \frac{y}{x} = 0$$ $\frac{1}{2}$

10. LHL $= \lim\limits_{x \to \frac{\pi^-}{2}} \dfrac{(1 - \sin x)(1 + \sin x + \sin^2 x)}{3(1 - \sin x)(1 + \sin x)}$ 1

$= \dfrac{1}{2}$ $\frac{1}{2}$

$\therefore$ $p = \dfrac{1}{2}$ $\frac{1}{2}$

RHL $= \lim\limits_{x \to \frac{\pi^+}{2}} \dfrac{q(1 - \sin x)}{(\pi - 2x)^2} = \lim\limits_{h \to 0} \dfrac{q(1 - \cos h)}{(2h)^2}$, where $x - \dfrac{\pi}{2} = h$ 1

$= \lim\limits_{h \to 0} \dfrac{2q \sin^2 \dfrac{h}{2}}{4.4.\dfrac{h^2}{4}} = \dfrac{q}{8}$ $\frac{1}{2}$

$\therefore$ $\dfrac{q}{8} = \dfrac{1}{2} \Rightarrow q = 4$ $\frac{1}{2}$

11. $\dfrac{dx}{dt} = -3\sin t + 3\cos^2 t \sin t = -3 \sin t (1 - \cos^2 t) = -3 \sin^3 t$ 1

$\dfrac{dy}{dt} = 3\cos t - 3\sin^2 t \cos t = 3\cos t (1 - \sin^2 t) = 3\cos^3 t$ 1

Slope of normal $= -\dfrac{dx}{dy} = \dfrac{\sin^3 t}{\cos^3 t}$ 1

Eqn. of normal is

$y - (3\sin t - \sin^3 t) = \dfrac{\sin^3 t}{\cos^3 t}[x - (3\cos t - \cos^3 t)]$ $\frac{1}{2}$

$\Rightarrow \quad y \cos^3 t - x \sin^3 t = 3\sin t \cos t (\cos^2 t - \sin^2 t)$

$= \dfrac{3}{4} \sin 4t$ $\frac{1}{2}$

or $4(y \cos^3 t - x \sin^3 t) = 3 \sin 4t$

12. $I = \displaystyle\int \dfrac{(3 \sin \theta - 2) \cos \theta}{5 - (1 - \sin^2 \theta) - 4 \sin \theta} d\theta$ $\frac{1}{2}$

$\sin \theta = t \Rightarrow \cos \theta \, d\theta = dt$

$\therefore \quad I = \displaystyle\int \dfrac{3t - 2}{t^2 - 4t + 4} dt = \int \dfrac{3t - 2}{(t - 2)^2} dt$ 1

$$= \int \frac{3(t-2)}{(t-2)^2}\, dt + 4\int \frac{1}{(t-2)^2}\, dt \qquad\qquad 1$$

$$= 3\log|t-2| - \frac{4}{(t-2)} + C \qquad\qquad 1$$

$$= 3\log|\sin\theta - 2| - \frac{4}{(\sin\theta - 2)} + C \qquad\qquad \frac{1}{2}$$

OR

Let $I = \int_0^\pi \sin\left(\frac{\pi}{4} + x\right) e^{2x}\, dx$

$$= \sin\left(\frac{\pi}{4} + x\right)\frac{e^{2x}}{2}\Big]_0^\pi - \int_0^\pi \cos\left(\frac{\pi}{4} + x\right)\frac{e^{2x}}{2}\, dx \qquad\qquad 1$$

$$I = \left[\sin\left(\frac{\pi}{4} + x\right)\frac{e^{2x}}{2} - \frac{1}{2}\left\{\cos\left(\frac{\pi}{4} + x\right)\frac{e^{2x}}{2}\right\}\right]_0^\pi + \frac{1}{2}\int_0^\pi -\sin\left(\frac{\pi}{4} + x\right)\frac{e^{2x}}{2}\, dx \qquad\qquad 1$$

$$\frac{5}{4}I = \left\{\frac{1}{4}\left[2\sin\left(\frac{\pi}{4} + x\right) - \cos\left(\frac{\pi}{4} + x\right)\right]e^{2x}\right\}_0^\pi \qquad\qquad 1$$

$$I = \frac{1}{5}\left[\left\{2\left(-\frac{1}{\sqrt{2}}\right) + \frac{1}{\sqrt{2}}\right\}e^{2\pi} - \left\{2\left(\frac{1}{\sqrt{2}}\right) - \frac{1}{\sqrt{2}}\right\}\right] = \frac{-1}{5\sqrt{2}}\left(e^{2\pi} + 1\right) \qquad\qquad 1$$

13. $I = \int \frac{\sqrt{x}}{\sqrt{a^3 - x^3}}\, dx$

Put $x^{3/2} = t \Rightarrow \frac{3}{2}\cdot x^{1/2}\, dx = dt$ or $\sqrt{x}\, dx = \frac{2}{3} dt \qquad\qquad 1\frac{1}{2}$

$$I = \frac{2}{3}\int \frac{dt}{\sqrt{(a^{3/2})^2 - t^2}} \qquad\qquad 1$$

$$= \frac{2}{3}\cdot \sin^{-1}\left(\frac{t}{a^{3/2}}\right) + C \qquad\qquad 1$$

$$= \frac{2}{3}\sin^{-1}\left(\frac{x^{3/2}}{a^{3/2}}\right) + C \qquad\qquad \frac{1}{2}$$

14. $I = \int_{-1}^2 |x^3 - x|\, dx$

$$= \int_{-1}^0 (x^3 - x)\, dx + \int_0^1 -(x^3 - x)\, dx + \int_1^2 (x^3 - x)\, dx \qquad\qquad 1\frac{1}{2}$$

$$= \frac{x^4}{4} - \frac{x^2}{2}\Big]_{-1}^0 + \left[\frac{x^2}{2} - \frac{x^4}{4}\right]_0^1 + \left[\frac{x^4}{4} - \frac{x^2}{2}\right]_1^2 \qquad\qquad 1\frac{1}{2}$$

$$= -\left(\frac{1}{4} - \frac{1}{2}\right) + \left(\frac{1}{2} - \frac{1}{4}\right) + (4 - 2) - \left(\frac{1}{4} - \frac{1}{2}\right)$$

$$= \frac{1}{4} + \frac{1}{4} + 2 + \frac{1}{4} = \frac{11}{4} \qquad\qquad 1$$

15. Given differential equation can be written as

$$\frac{(1+\log x)}{x} dx + \frac{2y}{1-y^2} dy = 0$$ 1

integrating to get, $\frac{1}{2}(1+\log x)^2 - \log|1-y^2| = C$ 2

$$x = 1, \ y = 0 \Rightarrow C = \frac{1}{2}$$ $\frac{1}{2}$

$$\Rightarrow \quad (1+\log x)^2 - 2\log|1-y^2| = 1$$ $\frac{1}{2}$

16. Given differential equation can be written as

$$\frac{dx}{dy} + \frac{1}{1+y^2} x = \frac{e^{\tan^{-1}y}}{1+y^2}$$ 1

Integrating factor is $e^{\tan^{-1}y}$ 1

$\therefore$ Solution is $\quad x \cdot e^{\tan^{-1}y} = \int e^{2\tan^{-1}y} \frac{1}{1+y^2} dy$ 1

$\therefore \quad x \ e^{\tan^{-1}y} = \frac{1}{2} e^{2\tan^{-1}y} + C$ 1

17. Given, that $\vec{a}+\vec{b}, \ \vec{b}+\vec{c}, \ \vec{c}+\vec{a}$ are coplanar

$\therefore \quad \left[\vec{a}+\vec{b}, \vec{b}+\vec{c}, \vec{c}+\vec{a}\right] = 0$

i.e. $\quad (\vec{a}+\vec{b}).\{(\vec{b}+\vec{c})\times(\vec{c}+\vec{a})\} = 0$ 1

$(\vec{a}+\vec{b}).\{(\vec{b}\times\vec{c}+\vec{b}\times\vec{a}+\vec{c}\times\vec{a})\} = 0$ 1

$\Rightarrow \quad \vec{a}.(\vec{b}\times\vec{c}) + \vec{a}.(\vec{b}\times\vec{a}) + \vec{a}.(\vec{c}\times\vec{a}) + \vec{b}.(\vec{b}\times\vec{c}) + \vec{b}.(\vec{b}\times\vec{a}) + \vec{b}.(\vec{c}\times\vec{a}) = 0$ $1\frac{1}{2}$

$\Rightarrow \quad 2[\vec{a}, \vec{b}, \vec{c}] = 0$ or $[\vec{a}, \vec{b}, \vec{c}]=0$ $\frac{1}{2}$

$\Rightarrow \quad \vec{a}, \vec{b}, \vec{c}$ are coplanar.

18. Vector equation of the required line is

$$\vec{r} = (\hat{i}+2\hat{j}-4\hat{k}) + \mu[(3\hat{i}-16\hat{j}+7\hat{k})\times(3\hat{i}+8\hat{j}-5\hat{k})]$$ 1

$$\Rightarrow \quad \vec{r} = (\hat{i}+2\hat{j}-4\hat{k}) + \lambda[(2\hat{i}+3\hat{j}+6\hat{k})]$$ 2

in cartesian form, $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z+4}{6}$ 1

19. Let events are:

E_1 : A is selected
E_2 : B is selected
E_3 : C is selected
A : Change is not introduced

$$P(E_1) = \frac{1}{7}, P(E_2) = \frac{2}{7}, P(E_3) = \frac{4}{7}$$ 1

$$P(A/E_1) = 0.2, P(A/E_2) = 0.5, P(A/E_3) = 0.7$$ 1

$$\therefore \quad P(E_3/A) = \frac{\dfrac{4}{7} \times \dfrac{7}{10}}{\dfrac{1}{7} \times \dfrac{2}{10} + \dfrac{2}{7} \times \dfrac{5}{10} + \dfrac{4}{7} \times \dfrac{7}{10}}$$ 1

$$= \frac{28}{40} = \frac{7}{10}$$ 1

OR

$$\left.\begin{array}{l} \text{Prob. of success for A} = \dfrac{1}{6} \\[2mm] \text{Prob. of failure for A} = \dfrac{5}{6} \\[2mm] \text{Prob. of success for B} = \dfrac{1}{12} \\[2mm] \text{Prob. of failure for B} = \dfrac{11}{12} \end{array}\right\}$$ 1

B can win in 2nd or 4th or 6th or....throw   1

$$\therefore \quad P(B) = \left(\frac{5}{6} \cdot \frac{1}{12}\right) + \left(\frac{5}{6} \cdot \frac{11}{12} \cdot \frac{5}{6} \cdot \frac{1}{12}\right) + \left(\frac{5}{6} \cdot \frac{11}{12} \cdot \frac{5}{6} \cdot \frac{11}{12} \cdot \frac{5}{6} \cdot \frac{1}{12}\right) + \dots$$ 1

$$= \frac{5}{72}\left(1 + \frac{55}{72} + \left(\frac{55}{72}\right)^2 + \dots\right)$$

$$= \frac{5}{72} \times \frac{1}{1 - \dfrac{55}{72}} = \frac{5}{72} \times \frac{72}{17} = \frac{5}{17}$$ 1

SECTION C

20.   Let $x_1, x_2 \in N$ and $f(x_1) = f(x_2)$

$$\Rightarrow \quad 9x_1^2 + 6x_1 - 5 = 9x_2^2 + 6x_2 - 5$$

$$(x_1^2 - x_2^2) + 6(x_1 - x_2) = 0 \Rightarrow (x_1 - x_2)(9x_1 + 9x_2 + 6) = 0$$

$$\Rightarrow \quad x_1 - x_2 = 0 \text{ or } x_1 = x_2 \text{ as } (9x_1 + 9x_2 + 6) \neq 0,\ x_1, x_2 \in N$$

$\therefore$ f is a one-one function   2

   f: $N \to S$ is ONTO as co-domain = Range   1

Hence f is invertible

$$y = 9x^2 + 6x - 5 = (3x + 1)^2 - 6 \Rightarrow x = \frac{\sqrt{y + 6} - 1}{3}$$

$\therefore$ $f^{-1}(y) = \dfrac{\sqrt{y + 6} - 1}{3},\ y \in S$ 2

$$f^{-1}(43) = \frac{\sqrt{49} - 1}{3} = 2$$ $\dfrac{1}{2}$

$$f^{-1}(163) = \frac{\sqrt{169} - 1}{3} = 4$$ $\dfrac{1}{2}$

21. Using $C_1 \to C_1 - C_3$ and $C_2 \to C_2 - C_3$ we get

$$\Delta = \begin{vmatrix} y(z-x)+z^2-x^2 & x(z-y)+z^2-y^2 & xy-z^2 \\ z(x-y)+x^2-y^2 & y(x-z)+x^2-z^2 & yz-x^2 \\ x(y-z)+y^2-z^2 & z(y-x)+y^2-x^2 & zx-y^2 \end{vmatrix}$$

2

Taking $(x + y + z)$ common from C_1 & C_2

$$\Rightarrow \quad \Delta = (x+y+z)^2 \begin{vmatrix} z-x & z-y & xy-z^2 \\ x-y & x-z & yz-x^2 \\ y-z & y-x & zx-y^2 \end{vmatrix}$$

1

$R_1 \to R_1 + R_2 + R_3$

$$\Rightarrow \quad \Delta = (x+y+z)^2 \begin{vmatrix} 0 & 0 & xy+yz+zx-x^2-y^2-z^2 \\ x-y & x-z & yz-x^2 \\ y-z & y-x & zx-y^2 \end{vmatrix}$$

1

Expanding to get

$\Delta = (x + y + z)^2 (xy + zy + zx - x^2 - y^2 - z^2)^2$

1

Hence Δ is divisible by $(x + y + z)$ and

the quotient is $(x + y + z) (xy + yz + zx - x^2 - y^2 - z^2)^2$

1

OR

Writing $\begin{pmatrix} 8 & 4 & 3 \\ 2 & 1 & 1 \\ 1 & 2 & 2 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} A$

1

$R_1 \leftrightarrow R_3 \quad \begin{pmatrix} 1 & 2 & 2 \\ 2 & 1 & 1 \\ 8 & 4 & 3 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix} A$

$\begin{array}{l} R_1 \to R_1 - 2R_2 \\ R_3 \to R_3 - 4R_2 \end{array} \quad \begin{pmatrix} -3 & 0 & 0 \\ 2 & 1 & 1 \\ 0 & 0 & -1 \end{pmatrix} = \begin{pmatrix} 0 & -2 & 1 \\ 0 & 1 & 0 \\ 1 & -4 & 0 \end{pmatrix} A$

$\begin{array}{l} R_1 \to \dfrac{1}{3} R_1 \\ R_3 \to - R_3 \end{array} \quad \begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 2/3 & -1/3 \\ 0 & 1 & 0 \\ -1 & 4 & 0 \end{pmatrix} A$

$R_2 \to R_2 - 2R_1 \quad \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 2/3 & -1/3 \\ 0 & -1/3 & 2/3 \\ -1 & 4 & 0 \end{pmatrix} A$ $2\dfrac{1}{2}$ marks for operation to get A^{-1}

$R_2 \to R_2 - R_3 \quad \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 2/3 & -1/3 \\ 1 & -13/3 & 2/3 \\ -1 & 4 & 0 \end{pmatrix} A$
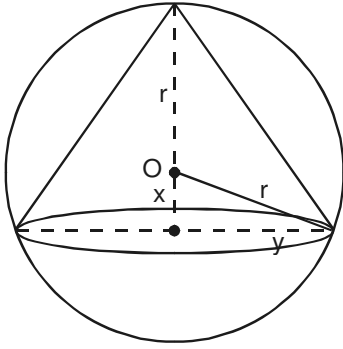
$\therefore \quad A^{-1} = \begin{pmatrix} 0 & 2/3 & -1/3 \\ 1 & -13/3 & 2/3 \\ -1 & 4 & 0 \end{pmatrix}$

$\dfrac{1}{2}$

$AX = B \Rightarrow X = A^{-1}B$

1

$\therefore \quad \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 & 2/3 & -1/3 \\ 1 & -13/3 & 2/3 \\ -1 & 4 & 0 \end{pmatrix} \begin{pmatrix} 19 \\ 5 \\ 7 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$

$\therefore \quad x = 1, \ y = 2, \ z = 1$

1

22.

Correct Figure   1

Let radius of cone be y and the altitude be r + x

$\therefore \qquad x^2 + y^2 = r^2 \qquad\qquad ...(i)$ $\dfrac{1}{2}$

Volume $V = \dfrac{1}{3}\pi\, y^2(r + x)$

$\qquad\qquad = \dfrac{1}{3}\pi(r^2 - x^2)\,(r + x)$ 1

$\dfrac{dV}{dx} = \dfrac{\pi}{3}[(r^2 - x^2)1 + (r + x)\,(-2x)] = \dfrac{\pi}{3}(r + x)(r - 3x)$ 1

$\dfrac{dV}{dx} = 0 \Rightarrow x = \dfrac{r}{3}$ $\dfrac{1}{2}$

$\therefore \qquad$ Altitude $= r + \dfrac{r}{3} = \dfrac{4r}{3}$ $\dfrac{1}{2}$

and $\qquad \dfrac{d^2V}{dx^2} = \dfrac{\pi}{3}[(r + x)(-3) + (r - 3x)] = \dfrac{\pi}{3}[-2r - 6x] < 0$ 1

$\therefore \qquad$ Max. Volume $= \dfrac{\pi}{3}\left(r^2 - \dfrac{r^2}{9}\right)\left(r + \dfrac{r}{3}\right) = \dfrac{8}{27}\left(\dfrac{4}{3}\pi r^3\right)$ $\dfrac{1}{2}$

$\qquad\qquad\qquad = \dfrac{8}{27}$(Vol. of sphere)

OR

$f(x) = \sin 3x - \cos 3x, \ 0 < x < \pi$

$f'(x) = 3\cos 3x + 3\sin 3x$ 1

$f'(x) = 0 \Rightarrow \tan 3x = -1$ $\dfrac{1}{2}$

$\Rightarrow \quad x = \dfrac{n\pi}{3} + \dfrac{\pi}{4}, \ n \in Z$

$\Rightarrow \quad x = \dfrac{\pi}{4}, \dfrac{7\pi}{12}, \dfrac{11\pi}{12}$ $1\dfrac{1}{2}$

Intervals are: $\left(0, \dfrac{\pi}{4}\right), \left(\dfrac{\pi}{4}, \dfrac{7\pi}{12}\right), \left(\dfrac{7\pi}{12}, \dfrac{11\pi}{12}\right), \left(\dfrac{11\pi}{12}, \pi\right)$ 1

$f(x)$ is strictly increasing in $\left(0, \dfrac{\pi}{4}\right) \cup \left(\dfrac{7\pi}{12}, \dfrac{11\pi}{12}\right)$ 1
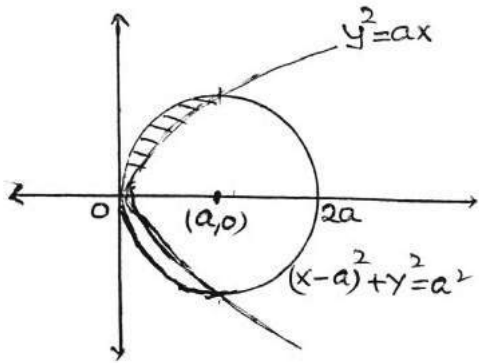
and strictly decreasing in $\left(\dfrac{\pi}{4}, \dfrac{7\pi}{12}\right) \cup \left(\dfrac{11\pi}{12}, \pi\right)$ 1

23.

$y^2 = ax, \ x^2 + y^2 = 2ax \Rightarrow x^2 - ax = 0$

$\Rightarrow \quad x = 0, \ x = a$ 1

Correct Figure   1

Shaded area $= \left[\int_0^a [\sqrt{a^2 - (x-a)^2} - \sqrt{a}\sqrt{x}]dx\right.$ 1

$A = \left[\dfrac{x-a}{2}\sqrt{a^2-(x-a)^2} + \dfrac{a^2}{2}\sin^{-1}\dfrac{x-a}{a} - \sqrt{a}\,\dfrac{2}{3}x^{3/2}\right]_0^a$ 2

$= \left[-\dfrac{2}{3}a^2 + \dfrac{a^2}{2}\dfrac{\pi}{2}\right] = \dfrac{\pi a^2}{4} - \dfrac{2a^2}{3}$ sq. units   1

24. Equation of line $AB : \dfrac{x-3}{-1} = \dfrac{y+4}{1} = \dfrac{z+5}{6} = \lambda$ 1

Eqn. of plane LMN: $\begin{vmatrix} x-2 & y-2 & z-1 \\ 1 & -2 & 0 \\ 2 & -3 & -1 \end{vmatrix} = 0$ $1\frac{1}{2}$

$2(x-2) + 1(y-2) + 1(z-1) = 0$ or $2x + y + z - 7 = 0$ $\frac{1}{2}$

Any point on line AB is $(-\lambda + 3, \ \lambda - 4, \ 6\lambda - 5)$ $\frac{1}{2}$

If this point lies on plane, then $2(-\lambda + 3) + (\lambda - 4) + (6\lambda - 5) - 7 = 0 \Rightarrow 5\lambda = 10 \Rightarrow \lambda = 2$ 1

$\therefore \quad$ P is $(1, -2, 7)$ $\frac{1}{2}$

let P divides AB in K : 1

$\Rightarrow \quad 1 = \dfrac{2K+3}{K+1} \Rightarrow K = -2$ i.e. P divides, AB externally in 2 : 1   1

25. X = No. of red
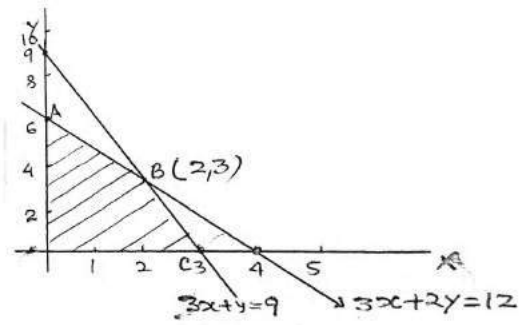
X:	0	1	2	3	4	1
P(X):	${}^4C_0\left(\dfrac{1}{3}\right)^4$ $= \dfrac{1}{81}$	${}^4C_1\left(\dfrac{1}{3}\right)^3\dfrac{2}{3}$ $= \dfrac{8}{81}$	${}^4C_2\left(\dfrac{1}{3}\right)^2\left(\dfrac{2}{3}\right)^2$ $= \dfrac{24}{81}$	${}^4C_3\left(\dfrac{1}{3}\right)\left(\dfrac{2}{3}\right)^3$ $= \dfrac{32}{81}$	${}^4C_4\left(\dfrac{2}{3}\right)^4$ $= \dfrac{16}{81}$	$2\frac{1}{2}$
XP(X):	0	$\dfrac{8}{81}$	$\dfrac{48}{81}$	$\dfrac{96}{81}$	$\dfrac{64}{81}$	
X^2P(X):	0	$\dfrac{8}{81}$	$\dfrac{96}{81}$	$\dfrac{288}{81}$	$\dfrac{256}{81}$	

Mean $= \Sigma XP(X) = \dfrac{216}{81} = \dfrac{8}{3}$ 1

Variance $= \Sigma X^2 P(X) - [\Sigma XP(X)]^2 = \dfrac{648}{81} - \dfrac{64}{9} = \dfrac{8}{9}$ $1\frac{1}{2}$

26.

Let production of A, B (per day) be x, y respectively

Maximise P = 7x + 4y   1

Subject to $\left.\begin{array}{l} 3x + 2y \le 12 \\ 3x + y \le 9 \\ x \ge 0, y \ge 0 \end{array}\right\}$ 2

                                              Correct Graph   2

P(A) = 24

P(B) = 26

P(C) = 21

∴   2 units of product A and 3 units of product B for maximum profit   1