

CBSE Class 12 Maths Question Paper Solution 2016

65/1/C

QUESTION PAPER CODE 65/1/C

EXPECTED ANSWER/VALUE POINTS

- | | SECTION A | |
|---|------------------|-----------------------------|
| 1. $(x + 3)2x - (-2)(-3x) = 8$ | | $\frac{1}{2}$ |
| $x = 2$ | | $\frac{1}{2}$ |
| 2. $\begin{pmatrix} 2 & 5 \\ 2 & 4 \end{pmatrix} = \begin{pmatrix} 3 & 1 \\ 2 & 0 \end{pmatrix} \begin{pmatrix} 1 & 2 \\ -1 & -1 \end{pmatrix}$ | | $\frac{1}{2} + \frac{1}{2}$ |
| 3. No. of possible matrices = 3^4
or 81 | } | 1 |
| 4. $\frac{2(2\vec{a} + 3\vec{b}) + 1(3\vec{a} - 2\vec{b})}{2+1}$ | | $\frac{1}{2}$ |
| $= \frac{7}{3}\vec{a} + \frac{4}{3}\vec{b}$ (or external division may also be considered) | | $\frac{1}{2}$ |
| 5. 2 | | 1 |
| 6. $\frac{x}{3} + \frac{y}{-4} + \frac{z}{2} = 1$ | | $\frac{1}{2}$ |
| $\Rightarrow \vec{r} \cdot (4\hat{i} - 3\hat{j} + 6\hat{k}) = 12$ or $\vec{r} \cdot \left(\frac{\hat{i}}{3} - \frac{\hat{j}}{4} + \frac{\hat{k}}{2}\right) = 1$ | | $\frac{1}{2}$ |
| SECTION B | | |
| 7. Equation of line through A(3, 4, 1) and B(5, 1, 6) | | |
| $\frac{x-3}{2} = \frac{y-4}{-3} = \frac{z-1}{5} = k(\text{say})$ | | 1 |
| General point on the line: | | |
| $x = 2k + 3, y = -3k + 4, z = 5k + 1$ | | $\frac{1}{2}$ |
| line crosses xz plane i.e. $y = 0$ if $-3k + 4 = 0$ | | |
| $\therefore k = \frac{4}{3}$ | | 1 |
| Co-ordinate of required point $\left(\frac{17}{3}, 0, \frac{23}{3}\right)$ | | $\frac{1}{2}$ |
| Angle, which line makes with xz plane: | | |
| $\sin \theta = \left \frac{2(0) + (-3)(1) + 5(0)}{\sqrt{4+9+25}\sqrt{1}} \right = \frac{3}{\sqrt{38}} \Rightarrow \theta = \sin^{-1}\left(\frac{3}{\sqrt{38}}\right)$ | | 1 |

8. let \vec{d}_1 & \vec{d}_2 be the two diagonal vectors:

$$\therefore \vec{d}_1 = 4\hat{i} - 2\hat{j} - 2\hat{k}, \vec{d}_2 = -6\hat{j} - 8\hat{k} \quad \frac{1}{2} + \frac{1}{2}$$

$$\text{or } \vec{d}_2 = 6\hat{j} + 8\hat{k}$$

Unit vectors parallel to the diagonals are:

$$\hat{d}_1 = \frac{2}{\sqrt{6}}\hat{i} - \frac{1}{\sqrt{6}}\hat{j} - \frac{1}{\sqrt{6}}\hat{k} \quad \frac{1}{2}$$

$$\hat{d}_2 = -\frac{3}{5}\hat{j} - \frac{4}{5}\hat{k} \quad \left(\text{or } \hat{d}_2 = \frac{3}{5}\hat{j} + \frac{4}{5}\hat{k} \right) \quad \frac{1}{2}$$

$$\vec{d}_1 \times \vec{d}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & -2 & -2 \\ 0 & -6 & -8 \end{vmatrix} = 4\hat{i} + 32\hat{j} - 24\hat{k} \quad 1$$

$$\text{Area of parallelogram} = \frac{1}{2} |\vec{d}_1 \times \vec{d}_2| = \sqrt{404} \text{ or } 2\sqrt{101} \text{ sq. units} \quad 1$$

9. let X = Amount he wins then $x = ₹ 5, 4, 3, -3$ 1

$$P = \text{Probability of getting a no. } > 4 = \frac{1}{3}, q = 1 - p = \frac{2}{3} \quad \frac{1}{2}$$

X:	5	4	3	-3
P(x)	$\frac{1}{3}$	$\frac{2}{3} \cdot \frac{1}{3} = \frac{2}{9}$	$\left(\frac{2}{3}\right)^2 \cdot \frac{1}{3} = \frac{4}{27}$	$\left(\frac{2}{3}\right)^3 = \frac{8}{27}$

2

$$\begin{aligned} \text{Expected amount he wins} &= \sum XP(X) = \frac{5}{3} + \frac{8}{9} + \frac{12}{27} - \frac{24}{27} \\ &= ₹ \frac{19}{9} \text{ or } ₹ 2\frac{1}{9} \quad \frac{1}{2} \end{aligned}$$

OR

E_1 = Event that all balls are white,
 E_2 = Event that 3 balls are white and 1 ball is non white
 E_3 = Event that 2 balls are white and 2 balls are non-white
 A = Event that 2 balls drawn without replacement are white
 } 1

$$P(E_1) = P(E_2) = P(E_3) = \frac{1}{3} \quad \frac{1}{2}$$

$$P(A/E_1) = 1, P(A/E_2) = \frac{3}{4} \cdot \frac{2}{3} = \frac{1}{2}, P(A/E_3) = \frac{2}{4} \cdot \frac{1}{3} = \frac{1}{6} \quad \frac{1}{2}$$

$$P(E_1/A) = \frac{1 \cdot \frac{1}{3}}{1 \cdot \frac{1}{3} + \frac{1}{3} \cdot \frac{1}{2} + \frac{1}{3} \cdot \frac{1}{6}} = \frac{3}{5} \quad 1$$

10. let $y = u + v$, $u = x^{\sin x}$, $v = (\sin x)^{\cos x}$

$$\log u = \sin x \cdot \log x \Rightarrow \frac{du}{dx} = x^{\sin x} \cdot \left\{ \cos x \cdot \log x + \frac{\sin x}{x} \right\} \quad \frac{1}{2} + 1$$

$$\log v = \cos x \cdot \log (\sin x) \Rightarrow \frac{dv}{dx} = (\sin x)^{\cos x} \cdot \{ \cos x \cdot \cot x - \sin x \cdot \log (\sin x) \} \quad \frac{1}{2} + 1$$

$$\frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx} = x^{\sin x} \cdot \left\{ \cos x \cdot \log x + \frac{\sin x}{x} \right\} + (\sin x)^{\cos x} \{ \cos x \cdot \cot x - \sin x \cdot \log (\sin x) \} \quad \frac{1}{2} + \frac{1}{2}$$

OR

$$\frac{dy}{dx} = \frac{-2 \sin (\log x)}{x} + \frac{3 \cos (\log x)}{x} \quad 1$$

$$\Rightarrow x \frac{dy}{dx} = -2 \sin (\log x) + 3 \cos (\log x), \text{ differentiate w.r.t 'x'} \quad \frac{1}{2}$$

$$\Rightarrow x \frac{d^2 y}{dx^2} + \frac{dy}{dx} = \frac{-2 \cos (\log x)}{x} - \frac{3 \sin (\log x)}{x} \quad 2$$

$$\Rightarrow x^2 \frac{d^2 y}{dx^2} + x \frac{dy}{dx} = -y \Rightarrow x^2 \frac{d^2 y}{dx^2} + x \frac{dy}{dx} + y = 0 \quad \frac{1}{2}$$

11. $\frac{dx}{dt} = 2a \cos 2t (1 + \cos 2t) - 2a \sin 2t \cdot \sin 2t \quad \frac{1}{2}$

$$\frac{dy}{dt} = -2b \sin 2t (1 - \cos 2t) + 2b \cos 2t \cdot \sin 2t \quad 1$$

$$\left. \frac{dy}{dx} \right|_{t=\frac{\pi}{4}} = \frac{2b \cos 2t \cdot \sin 2t - 2b \sin 2t (1 - \cos 2t)}{2a \cos 2t (1 + \cos 2t) - 2a \sin 2t \cdot \sin 2t} \Big|_{t=\frac{\pi}{4}} = \frac{b}{a} \quad \frac{1}{2} + 1$$

12. $y^2 = ax^3 + b \Rightarrow 2y \frac{dy}{dx} = 3ax^2 \therefore \frac{dy}{dx} = \frac{3a}{2} \frac{x^2}{y} \quad 1$

$$\text{Slope of tangent at } (2, 3) = \left. \frac{dy}{dx} \right|_{(2,3)} = \frac{3a}{2} \cdot \frac{4}{3} = 2a \quad 1$$

Comparing with slope of tangent $y = 4x - 5$, we get, $2a = 4 \therefore \boxed{a = 2}$ 1

Also $(2, 3)$ lies on the curve $\therefore 9 = 8a + b$, put $a = 2$, we get $b = -7$ 1

13. Let $x^2 = t \therefore \frac{x^2}{x^4 + x^2 - 2} = \frac{x^2}{(x^2 - 1)(x^2 + 2)} = \frac{t}{(t - 1)(t + 2)} = \frac{A}{t - 1} + \frac{B}{t + 2} \quad 1$

Solving for A and B to get, $A = \frac{1}{3}, B = \frac{2}{3} \quad 1$

$$\int \frac{x^2}{x^4 + x^2 - 2} dx = \frac{1}{3} \int \frac{1}{x^2 - 1} dx + \frac{2}{3} \int \frac{1}{x^2 + 2} dx = \frac{1}{6} \log \left| \frac{x-1}{x+1} \right| + \frac{\sqrt{2}}{3} \tan^{-1} \frac{x}{\sqrt{2}} + C \quad 1 + 1$$

$$14. \text{ Let } I = \int_0^{\pi/2} \frac{\sin^2 x}{\sin x + \cos x} dx, \text{ Also } I = \int_0^{\pi/2} \frac{\sin^2\left(\frac{\pi}{2} - x\right)}{\sin\left(\frac{\pi}{2} - x\right) + \cos\left(\frac{\pi}{2} - x\right)} dx = \int_0^{\pi/2} \frac{\cos^2 x}{\cos x + \sin x} dx \quad 1$$

$$\text{Adding to get, } 2I = \int_0^{\pi/2} \frac{1}{\sin x + \cos x} dx = \frac{1}{\sqrt{2}} \int_0^{\pi/2} \frac{1}{\cos\left(x - \frac{\pi}{4}\right)} dx \quad \frac{1}{2} + 1$$

$$\Rightarrow 2I = \frac{1}{\sqrt{2}} \int_0^{\pi/2} \sec\left(x - \frac{\pi}{4}\right) dx = \frac{1}{\sqrt{2}} \log \left| \sec\left(x - \frac{\pi}{4}\right) + \tan\left(x - \frac{\pi}{4}\right) \right|_0^{\pi/2} \quad 1$$

$$\Rightarrow 2I = \frac{1}{\sqrt{2}} \left\{ \log |\sqrt{2} + 1| - \log |\sqrt{2} - 1| \right\}$$

$$\Rightarrow I = \frac{1}{2\sqrt{2}} \left\{ \log |\sqrt{2} + 1| - \log |\sqrt{2} - 1| \right\} \quad \text{or} \quad \frac{1}{2\sqrt{2}} \log \left| \frac{\sqrt{2} + 1}{\sqrt{2} - 1} \right| \quad \frac{1}{2}$$

OR

$$\int_0^{3/2} |x \cos \pi x| dx = \int_0^{1/2} x \cos \pi x dx - \int_{1/2}^{3/2} x \cos \pi x dx \quad \frac{1}{2}$$

$$= \left\{ \frac{x \sin \pi x}{\pi} + \frac{\cos \pi x}{\pi^2} \right\}_0^{1/2} - \left\{ \frac{x \sin \pi x}{\pi} + \frac{\cos \pi x}{\pi^2} \right\}_{1/2}^{3/2} \quad \frac{1}{2}$$

$$= \frac{1}{2\pi} - \frac{1}{\pi^2} - \left(-\frac{3}{2\pi} - \frac{1}{\pi^2} \right) = \frac{5}{2\pi} - \frac{1}{\pi^2} \quad 1$$

$$15. \int (3x + 1) \sqrt{4 - 3x - 2x^2} dx = -\frac{3}{4} \int (-4x - 3) \sqrt{4 - 3x - 2x^2} dx - \frac{5}{4} \int \sqrt{4 - 3x - 2x^2} dx \quad 1$$

$$= -\frac{1}{2} (4 - 3x - 2x^2)^{3/2} - \frac{5}{4} \sqrt{2} \int \sqrt{\left(\frac{\sqrt{41}}{4}\right)^2 - \left(x + \frac{3}{4}\right)^2} dx \quad 1 + 1$$

$$= -\frac{1}{2} (4 - 3x - 2x^2)^{3/2} - \frac{5}{4} \sqrt{2} \left\{ \frac{4x + 3}{8} \sqrt{\frac{41}{16} - \left(x + \frac{3}{4}\right)^2} + \frac{41}{32} \cdot \sin^{-1} \left(\frac{4x + 3}{\sqrt{41}} \right) \right\} + C$$

$$= -\frac{1}{2} (4 - 3x - 2x^2)^{3/2} - \frac{5}{4} \left\{ \frac{4x + 3}{8} \sqrt{4 - 3x - 2x^2} + \frac{41\sqrt{2}}{32} \cdot \sin^{-1} \left(\frac{4x + 3}{\sqrt{41}} \right) \right\} + C \quad 1$$

16. The differential equation can be re-written as:

$$\frac{dy}{dx} = \frac{x - y}{x + y}, \text{ put } y = vx, \frac{dy}{dx} = v + x \frac{dv}{dx} \quad \frac{1}{2} + \frac{1}{2}$$

$$\Rightarrow v + x \frac{dv}{dx} = \frac{1 - v}{1 + v} \Rightarrow \frac{1 + v}{1 - 2v - v^2} dv = \frac{1}{x} dx \quad 1$$

integrating we get

$$\Rightarrow \frac{1}{2} \int \frac{2V + 2}{V^2 + 2V - 1} dv = -\int \frac{1}{x} dx = \frac{1}{2} \log |V^2 + 2V - 1| = -\log x + \log C \quad \frac{1}{2}$$

\therefore Solution of the differential equation is:

$$\frac{1}{2} \log \left| \frac{y^2}{x^2} + \frac{2y}{x} - 1 \right| = \log C - \log x \text{ or, } y^2 + 2xy - x^2 = C^2 \quad \frac{1}{2}$$

17. Let radius of any of the circle touching co-ordinate axes in the second quadrant be "a" then centre is $(-a, a)$

\therefore Equation of the family of circles is:

$$(x + a)^2 + (y - a)^2 = a^2, a \in \mathbb{R} \quad 1 \frac{1}{2}$$

$$\Rightarrow x^2 + y^2 + 2ax - 2ay + a^2 = 0$$

$$\text{Differentiate w.r.t. "x", } 2x + 2yy' + 2a - 2ay' = 0 \Rightarrow a = \frac{x + yy'}{y' - 1} \quad 1 \frac{1}{2}$$

\therefore The differential equation is:

$$\left. \begin{aligned} \left(x + \frac{x + yy'}{y' - 1}\right)^2 + \left(y - \frac{x + yy'}{y' - 1}\right)^2 &= \left(\frac{x + yy'}{y' - 1}\right)^2 \\ \Rightarrow \left(\frac{xy' + yy'}{y' - 1}\right)^2 + \left(\frac{x + y}{y' - 1}\right)^2 &= \left(\frac{x + yy'}{y' - 1}\right)^2 \end{aligned} \right\} \quad 1$$

18. $\sin^{-1} x + \sin^{-1}(1-x) = \cos^{-1} x \Rightarrow \sin^{-1}(1-x) = \frac{\pi}{2} - 2 \sin^{-1} x \quad 1$

$$\Rightarrow 1 - x = \sin\left(\frac{\pi}{2} - 2 \sin^{-1} x\right) \Rightarrow 1 - x = \cos(2 \sin^{-1} x) \Rightarrow 1 - x = 1 - 2 \sin^2(\sin^{-1} x) \quad 1$$

$$\Rightarrow 1 - x = 1 - 2x^2 \quad 1$$

$$\text{Solving we get, } x = 0 \text{ or } x = \frac{1}{2} \quad 1$$

OR

$$\text{From the equation: } \cos^{-1} \frac{x}{a} = \alpha - \cos^{-1} \frac{y}{b}$$

$$\frac{x}{a} = \cos\left(\alpha - \cos^{-1} \frac{y}{b}\right) \Rightarrow \frac{x}{a} = \cos \alpha \cdot \cos\left(\cos^{-1} \frac{y}{b}\right) + \sin \alpha \cdot \sin\left(\cos^{-1} \frac{y}{b}\right) \quad 1 + 1$$

$$\Rightarrow \frac{x}{a} = \frac{y \cdot \cos \alpha}{b} + \sin \alpha \sqrt{1 - \frac{y^2}{b^2}} \Rightarrow \frac{x}{a} - \frac{y}{b} \cos \alpha = \sin \alpha \sqrt{1 - \frac{y^2}{b^2}} \quad 1$$

Squaring both sides,

$$\Rightarrow \left(\frac{x}{a} - y \frac{\cos \alpha}{b}\right)^2 = \left(\sin \alpha \sqrt{1 - \frac{y^2}{b^2}}\right)^2 \quad 1 \frac{1}{2}$$

$$\Rightarrow \frac{x^2}{a^2} - \frac{2xy}{ab} \cos \alpha + \frac{y^2}{b^2} = \sin^2 \alpha. \quad 1 \frac{1}{2}$$

19. let ₹ x be invested in first bond
and ₹ y be invested in second bond
then the system of equations is:

$$\left. \begin{aligned} \frac{10x}{100} + \frac{12y}{100} &= 2800 \\ \frac{12x}{100} + \frac{10y}{100} &= 2700 \end{aligned} \right\} \Rightarrow \begin{cases} 5x + 6y = 140000 \\ 6x + 5y = 135000 \end{cases} \quad 1$$

$$\text{let } A = \begin{bmatrix} 5 & 6 \\ 6 & 5 \end{bmatrix}; X = \begin{bmatrix} x \\ y \end{bmatrix}; B = \begin{bmatrix} 140000 \\ 135000 \end{bmatrix}$$

$$\therefore A \cdot X = B$$

$$|A| = -11; A^{-1} = \frac{1}{-11} \begin{bmatrix} 5 & -6 \\ -6 & 5 \end{bmatrix}$$

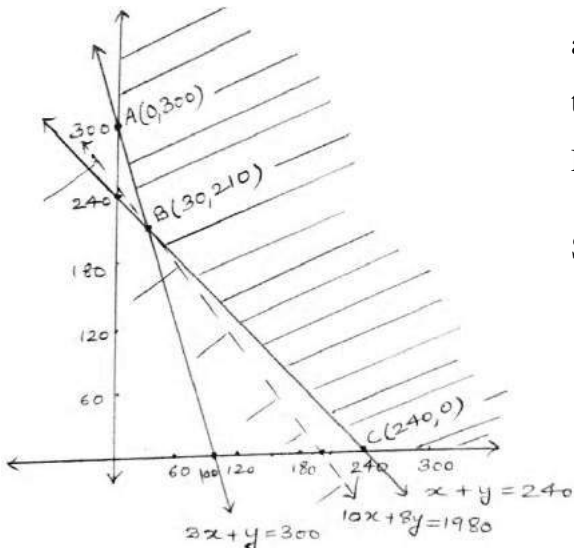
$$\therefore \text{Solution is } X = A^{-1}B \Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{-11} \begin{bmatrix} 5 & -6 \\ -6 & 5 \end{bmatrix} \begin{bmatrix} 140000 \\ 135000 \end{bmatrix} = \begin{bmatrix} 10000 \\ 15000 \end{bmatrix} \quad \left. \vphantom{\begin{bmatrix} x \\ y \end{bmatrix}} \right\} \frac{1}{2} + \frac{1}{2}$$

$$\therefore x = 10000, y = 15000, \therefore \text{Amount invested} = ₹ 25000$$

Value: caring elders

SECTION C

20.



Let x kg of fertilizer A be used

and y kg of fertilizer B be used

then the linear programming problem is:

$$\text{Minimise cost: } z = 10x + 8y$$

$$\text{Subject to } \frac{12x}{100} + \frac{4y}{100} \geq 12 \Rightarrow 3x + y \geq 300$$

$$\frac{5x}{100} + \frac{5y}{100} \geq 12 \Rightarrow x + y \geq 240$$

$$x, y \geq 0$$

Correct Graph

Value of Z at corners of the unbounded region ABC:

Corner	Value of Z
A (0, 300)	₹ 2400
B(30, 210)	₹ 1980 (Minimum)
C(240, 0)	₹ 2400

The region of $10x + 8y < 1980$ or $5x + 4y < 990$ has no point in common to the feasible region. Hence, minimum cost = ₹ 1980 at $x = 30$ and $y = 210$

21. Let X = Number of bad oranges out of 4 drawn = 0, 1, 2, 3, 4

$$P = \text{Probability of a bad orange} = \frac{1}{5}, q = 1 - p = \frac{4}{5}$$

\therefore Probability distribution is:

X:	0	1	2	3	4
P(X):	${}^4C_0 \left(\frac{4}{5}\right)^4 = \frac{256}{625}$	${}^4C_1 \frac{1}{5} \left(\frac{4}{5}\right)^3$	${}^4C_2 \left(\frac{1}{5}\right)^2 \left(\frac{4}{5}\right)^2$	${}^4C_3 \left(\frac{1}{5}\right)^2 \left(\frac{4}{5}\right)$	${}^4C_4 \left(\frac{1}{5}\right)^4$
		$= \frac{256}{625}$	$= \frac{96}{625}$	$= \frac{16}{625}$	$= \frac{1}{625}$

$$\text{Mean } (\mu) = \sum X.P(X) = 0 \times \frac{256}{625} + 1 \times \frac{256}{625} + 2 \times \frac{96}{625} + 3 \times \frac{16}{625} + 4 \times \frac{1}{625} = \frac{4}{5} \quad 1$$

$$\begin{aligned} \text{Variance } (\sigma^2) &= \sum x^2.P(x) - [\sum x.P(x)]^2 \\ &= 0 \times \frac{256}{625} + \frac{1 \times 256}{625} + \frac{4 \times 96}{625} + \frac{9 \times 16}{625} + \frac{16}{625} - \left(\frac{4}{5}\right)^2 = \frac{16}{25} \quad 1 \end{aligned}$$

22. Line through 'P' and perpendicular to plane is:

$$\vec{r} = (2\hat{i} + 3\hat{j} + 4\hat{k}) + \lambda(2\hat{i} + \hat{j} + 3\hat{k}) \quad 1$$

$$\text{General point on line is: } \vec{r} = (2 + 2\lambda)\hat{i} + (3 + \lambda)\hat{j} + (4 + 3\lambda)\hat{k}$$

For some $\lambda \in \mathbb{R}$, \vec{r} is the foot of perpendicular, say Q, from P to the plane, since it lies on plane

$$\therefore [(2 + 2\lambda)\hat{i} + (3 + \lambda)\hat{j} + (4 + 3\lambda)\hat{k}] \cdot (2\hat{i} + \hat{j} + 3\hat{k}) - 26 = 0$$

$$\Rightarrow 4 + 4\lambda + 3 + \lambda + 12 + 9\lambda - 26 = 0 \Rightarrow \lambda = \frac{1}{2} \quad 1 \frac{1}{2}$$

$$\therefore \text{Foot of perpendicular is } Q\left(3\hat{i} + \frac{7}{2}\hat{j} + \frac{11}{2}\hat{k}\right) \quad 1$$

let $P'(a\hat{i} + b\hat{j} + c\hat{k})$ be the image of P in the plane then Q is mid point of PP'

$$\therefore Q\left(\frac{a+2}{2}\hat{i} + \frac{b+3}{2}\hat{j} + \frac{c+4}{2}\hat{k}\right) = Q\left(3\hat{i} + \frac{7}{2}\hat{j} + \frac{11}{2}\hat{k}\right) \quad 1$$

$$\Rightarrow \frac{a+2}{2} = 3, \frac{b+3}{2} = \frac{7}{2}, \frac{c+4}{2} = \frac{11}{2} \Rightarrow a = 4, b = 4, c = 7 \therefore P'(4\hat{i} + 4\hat{j} + 7\hat{k}) \quad 1$$

$$\text{Perpendicular distance of P from plane} = PQ = \sqrt{(2-3)^2 + \left(3-\frac{7}{2}\right)^2 + \left(4-\frac{11}{2}\right)^2} = \sqrt{\frac{7}{2}} \quad 1 \frac{1}{2}$$

23. Commutative: For any elements $a, b \in A$

$$a * b = a + b + ab = b + a + ba = b * a. \text{ Hence } * \text{ is commutative} \quad 1 \frac{1}{2}$$

Associative: For any three elements $a, b, c \in A$

$$\begin{aligned} a * (b * c) &= a * (b + c + bc) = a + b + c + bc + ab + ac + abc \\ (a * b) * c &= (a + b + ab) * c = a + b + ab + c + ac + bc + abc \end{aligned} \quad 1 \frac{1}{2}$$

$$\therefore a * (b * c) = (a * b) * c, \text{ Hence } * \text{ is Associative.}$$

Identity element: let $e \in A$ be the identity element then $a * e = e * a = a$

$$\Rightarrow a + e + ae = e + a + ea = a \Rightarrow e(1 + a) = 0, \text{ as } a \neq -1$$

$$e = 0 \text{ is the identity element} \quad 1 \frac{1}{2}$$

Invertible: let $a, b \in A$ so that 'b' is inverse of a

$$\therefore a * b = b * a = e$$

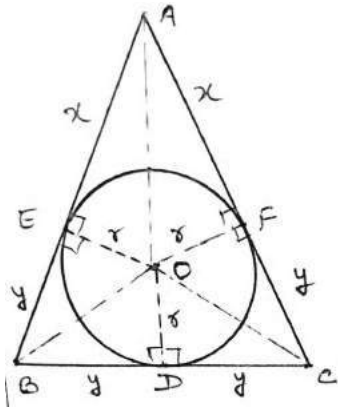
$$\Rightarrow a + b + ab = b + a + ba = 0$$

$$\text{As } a \neq -1, b = \frac{-a}{1+a} \in A. \text{ Hence every element of } A \text{ is invertible} \quad 1 \frac{1}{2}$$

24.

Correct Figure

1



Let ΔABC be isosceles with inscribed circle of radius 'r' touching sides AB, AC and BC at E, F and D respectively.

let $AE = AF = x$, $BE = BD = y$, $CF = CD = y$ then

area (ΔABC) = ar(ΔAOB) + ar(ΔAOC) + ar(ΔBOC)

$$\Rightarrow \frac{1}{2} \cdot 2y (r + \sqrt{r^2 + x^2}) = \frac{1}{2} \{2yr + 2(x+y)r\} \Rightarrow x = \frac{2r^2y}{y^2 - r^2} \quad 1$$

Then,

$$P(\text{Perimeter of } \Delta ABC) = 2x + 4y = \frac{4r^2y}{y^2 - r^2} + 4y \quad 1$$

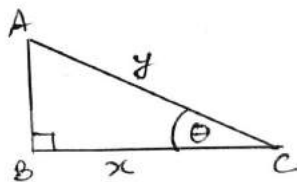
$$\frac{dP}{dy} = \frac{-4r^2(r^2 + y^2)}{(y^2 - r^2)^2} + 4 \text{ and } \frac{dP}{dy} = 0 \Rightarrow y = \sqrt{3}r \quad 1 + \frac{1}{2}$$

$$\left. \frac{d^2P}{dy^2} \right|_{y=\sqrt{3}r} = \frac{4r^2y(2y^2 + 6r^2)}{(y^2 - r^2)^3} = \frac{6\sqrt{3}}{r} > 0 \quad \frac{1}{2}$$

\therefore Perimeter is least iff $y = \sqrt{3}r$ and least perimeter is

$$P = 4y + \frac{4r^2y}{y^2 - r^2} = 4\sqrt{3}r + \frac{4r^2\sqrt{3}r}{2r^2} = 6\sqrt{3}r \quad 1$$

OR



let ABC be the right triangle with $\angle B = 90^\circ$

$\angle ACB = \theta$, $AC = y$, $BC = x$, $x + y = k$ (constant)

$$A (\text{Area of triangle}) = \frac{1}{2} \cdot BC \cdot AB = \frac{1}{2} \cdot x \sqrt{y^2 - x^2} \quad \frac{1}{2}$$

$$\text{let } z = A^2 = \frac{1}{4} x^2 (y^2 - x^2) = \frac{1}{4} x^2 \{(k-x)^2 - x^2\} = \frac{1}{4} (x^2k^2 - 2kx^3) \quad 1$$

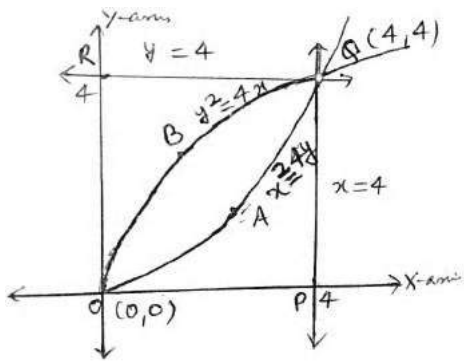
$$\frac{dz}{dx} = \frac{1}{4} (2xk^2 - 6kx^2) \text{ and } \frac{dz}{dx} = 0 \Rightarrow x = \frac{k}{3}, y = k - x = \frac{2k}{3} \quad 1+1$$

$$\left. \frac{d^2z}{dx^2} \right|_{x=\frac{k}{3}} = \frac{1}{4} (2k^2 - 12kx) \Big|_{x=\frac{k}{3}} = -\frac{k^2}{2} < 0 \quad \frac{1}{2}$$

$\therefore z$ and area of ΔABC is max at $x = \frac{k}{3}$

$$\text{and, } \cos \theta = \frac{x}{y} = \frac{k}{3} \cdot \frac{3}{2k} = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{3} \quad 1$$

25.

Point of intersection of $y^2 = 4x$ and $x^2 = 4y$ are $(0, 0)$ and $(4, 4)$;

Correct Graph

 $1\frac{1}{2}$

$$\text{area (OAQBO)} = \int_0^4 \left(2\sqrt{x} - \frac{x^2}{4} \right) dx$$

1

$$= \left\{ \frac{4}{3} x^{3/2} - \frac{x^3}{12} \right\} \Big|_0^4$$

$$= \frac{32}{3} - \frac{16}{3} = \frac{16}{3}$$

 $\frac{1}{2}$

$$\text{area (OPQAO)} = \int_0^4 \frac{x^2}{4} dx = \frac{1}{12} x^3 \Big|_0^4 = \frac{16}{3}$$

 $1\frac{1}{2}$

$$\text{area (OBQRO)} = \int_0^4 \frac{y^2}{4} dy = \frac{1}{12} y^3 \Big|_0^4 = \frac{16}{3}$$

 $1\frac{1}{2}$

Hence the areas of the three regions are equal.

26.

$$\begin{vmatrix} 1 & 1 & 1 \\ 1 + \cos A & 1 + \cos B & 1 + \cos C \\ \cos^2 A + \cos A & \cos^2 B + \cos B & \cos^2 C + \cos C \end{vmatrix} = 0$$

Apply $C_2 \rightarrow C_2 - C_1$, $C_3 \rightarrow C_3 - C_1$

$$\Leftrightarrow \begin{vmatrix} 1 & 0 & 0 \\ 1 + \cos A & \cos B - \cos A & \cos C - \cos A \\ \cos^2 A + \cos A & (\cos B - \cos A)(\cos B + \cos A + 1) & (\cos C - \cos A)(\cos C + \cos A + 1) \end{vmatrix} = 0 \quad 3$$

Taking $(\cos B - \cos A)$, $(\cos C - \cos A)$ common from C_2 & C_3

$$\Leftrightarrow (\cos B - \cos A)(\cos C - \cos A) \begin{vmatrix} 1 & 0 & 0 \\ 1 + \cos A & 1 & 1 \\ \cos^2 A + \cos A & \cos B + \cos A + 1 & \cos C + \cos A + 1 \end{vmatrix} = 0 \quad 1$$

Expand along R_1

$$\Leftrightarrow (\cos B - \cos A)(\cos C - \cos A)(\cos C - \cos B) = 0 \quad 1$$

$$\Leftrightarrow \cos A = \cos B \quad \Leftrightarrow A = B \quad \Leftrightarrow \Delta ABC \text{ is an isosceles triangle} \quad 1$$

or

or

$$\cos B = \cos C$$

$$B = C$$

or

or

$$\cos C = \cos A$$

$$C = A$$

OR

let the cost of one pen of variety 'A', 'B' and 'C' be ₹ x, ₹ y and ₹ z respectively then the system of equations is:

$$\left. \begin{array}{l} x + y + z = 21 \\ 4x + 3y + 2z = 60 \\ 6x + 2y + 3z = 70 \end{array} \right\} \quad \frac{1}{2}$$

Matrix form of the system is:

$$A \cdot X = B, \text{ where } A = \begin{bmatrix} 1 & 1 & 1 \\ 4 & 3 & 2 \\ 6 & 2 & 3 \end{bmatrix}; X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}; B = \begin{bmatrix} 21 \\ 60 \\ 70 \end{bmatrix} \quad \frac{1}{2}$$

$$|A| = (5) - 1(0) + 1(-10) = -5 \quad 1$$

co-factors of the matrix A are:

$$\left. \begin{array}{l} C_{11} = 5; \quad C_{21} = -1; \quad C_{31} = -1 \\ C_{12} = 0; \quad C_{22} = -3 \quad C_{32} = 2 \\ C_{13} = -10; \quad C_{23} = 4; \quad C_{33} = -1 \end{array} \right\} \quad 2$$

$$\therefore A^{-1} = \frac{1}{|A|} \text{Adj } A = \frac{1}{-5} \begin{bmatrix} 5 & -1 & -1 \\ 0 & -3 & 2 \\ -10 & 4 & -1 \end{bmatrix} \quad \frac{1}{2}$$

Solution of the matrix equation is $X = A^{-1} B$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = -\frac{1}{5} \begin{bmatrix} 5 & -1 & -1 \\ 0 & -3 & 2 \\ -10 & 4 & -1 \end{bmatrix} \begin{bmatrix} 21 \\ 60 \\ 70 \end{bmatrix} = \begin{bmatrix} 5 \\ 8 \\ 8 \end{bmatrix} \therefore x = 5, y = 8, z = 8 \quad \frac{1}{2}$$