QUESTION PAPER CODE 65/1/E

EXPECTED ANSWER/VALUE POINTS

SECTION A

1. $R_1 \to R_1 + R_2 + R_3$ or $C_1 \to C_1 + C_2 + C_3$ $\frac{1}{2}$

Ans. 0 $\frac{1}{2}$

2. $b_{21} = -16,\ b_{23} = -2$ [For any one correct value] $\frac{1}{2}$

$b_{21} + b_{23} = -16 + (-2) = -18$ $\frac{1}{2}$

3. 2^6 or 64 1

4. $(\alpha, -\beta, \gamma)$ 1

5. $\frac{1(\vec{a} - \vec{b}) + 3(\vec{a} + 3\vec{b})}{4}$ (i.e., using correct formula) $\frac{1}{2}$

$= \vec{a} + 2\vec{b}$ $\frac{1}{2}$

6. Finding $\cos\theta = \frac{\sqrt{3}}{2}$ $\frac{1}{2}$

$|\vec{a} \times \vec{b}| = 6$ $\frac{1}{2}$

SECTION B

7. $\tan^{-1}\left(\frac{2-x}{2+x}\right) = \frac{1}{2}\tan^{-1}\frac{x}{2}$

$\Rightarrow \quad 2\tan^{-1}\left(\frac{2-x}{2+x}\right) = \tan^{-1}\frac{x}{2}$ $\frac{1}{2}$

$\Rightarrow \quad \tan^{-1}\frac{2\left(\frac{2-x}{2+x}\right)}{1-\left(\frac{2-x}{2+x}\right)^2} = \tan^{-1}\frac{x}{2}$ $1\frac{1}{2}$

$\Rightarrow \quad \tan^{-1}\frac{4-x^2}{4x} = \tan^{-1}\frac{x}{2}$ 1

$\Rightarrow \quad \frac{4-x^2}{4x} = \frac{x}{2}$ $\frac{1}{2}$

$\Rightarrow \quad x = \frac{2}{\sqrt{3}} \quad (\because x > 0)$ $\frac{1}{2}$

OR

$2\sin^{-1}\left(\frac{3}{5}\right)-\tan^{-1}\left(\frac{17}{31}\right)$

$= 2\tan^{-1}\left(\frac{3}{4}\right)-\tan^{-1}\left(\frac{17}{31}\right)$ 1

$= \tan^{-1}\frac{2\times\frac{3}{4}}{1-\left(\frac{3}{4}\right)^2}-\tan^{-1}\left(\frac{17}{31}\right)$ 1

$= \tan^{-1}\frac{24}{7}-\tan^{-1}\frac{17}{31}$

$= \tan^{-1}\left(\frac{\frac{24}{7}-\frac{17}{31}}{1+\frac{24}{7}\times\frac{17}{31}}\right)$ 1

$= \tan^{-1}(1)$ 1

$= \frac{\pi}{4}$

8. Let the number of children be x and the amount distributed by Seema for one student be ₹ y.

So, $(x-8)(y+10)=xy$

$\Rightarrow\quad 5x-4y=40$ ...(i) $\frac{1}{2}$

and $(x+16)(y-10)=xy$

$\Rightarrow\quad 5x-8y=-80$ ...(ii) $\frac{1}{2}$

Here $A=\begin{bmatrix}5 & -4\\ 5 & -8\end{bmatrix}, X=\begin{bmatrix}x\\ y\end{bmatrix}, B=\begin{bmatrix}40\\ -80\end{bmatrix}$

$AX=B\Rightarrow X=A^{-1}B$

$A^{-1}=-\frac{1}{20}\begin{bmatrix}-8 & 4\\ -5 & 5\end{bmatrix}$ 1

$\Rightarrow\quad \begin{bmatrix}x\\ y\end{bmatrix}=\begin{bmatrix}32\\ 30\end{bmatrix}$

$\Rightarrow\quad x=32, y=30$ 1

No. of students = 32

Amount given to each student = ₹ 30.

Value reflected: To help needy people. 1

9. $\frac{dx}{dt}=e^{\cos 2t}(-2\sin 2t)$ or $-2x\sin 2t$ 1

$\frac{dy}{dt}=e^{\sin 2t}\,2\cos 2t$ or $2y\cos 2t$ 1

$$\frac{dy}{dx}=\frac{-e^{\sin 2t}2\cos 2t}{e^{\cos 2t}2\sin 2t} \text{ or } -\frac{y}{x}\frac{\cos 2t}{\sin 2t}$$ 1

$$=\frac{-y\log x}{x\log y}$$ 1

OR

$f(x) = 2\sin x + \sin 2x$ on $[0, \pi]$

$f(x)$ is continuous in $[0, \pi]$
$f(x)$ is differentiable in $(0, \pi)$ } 1

$\therefore$ Mean value theorem is applicable

$f(0) = 0, f(\pi) = 0$

$f'(x) = 2\cos x + 2\cos 2x$ 1

$f'(c) = 2\cos c + 2\cos 2c$

$$f'(c)=\frac{f(\pi)-f(0)}{\pi-0}=0$$ 1

$\therefore$ $2\cos c + 2\cos 2c = 0$

$\Rightarrow$ $\cos c + 2\cos 2\, c - 1 = 0$

$\Rightarrow$ $(2\cos c - 1)(\cos c + 1) = 0$

$\Rightarrow$ $\cos c = \frac{1}{2}$

$\Rightarrow$ $c = \frac{\pi}{3} \in (0, \pi)$

Hence mean value theorem is verified. $\frac{1}{2}+\frac{1}{2}$

10. $$f(x)=\begin{cases}\dfrac{e^{\frac{1}{x}}-1}{e^{\frac{1}{x}}+1} & x\neq 0\\ -1 & x=0\end{cases}$$

LHL: $$\lim_{x\to 0^-}\frac{e^{\frac{1}{x}}-1}{e^{\frac{1}{x}}+1}$$

$$=\lim_{h\to 0}\frac{e^{-\frac{1}{h}}-1}{e^{-\frac{1}{h}}+1}=\frac{0-1}{0+1}=-1$$ 2

RHL: $$\lim_{h\to 0}\frac{e^{\frac{1}{h}}-1}{e^{\frac{1}{h}}+1}=\lim_{h\to 0}\frac{1-e^{-\frac{1}{h}}}{1+e^{-\frac{1}{h}}}=1$$ 2

$LHL \neq RHL$

$\therefore$ $f(x)$ is discontinuous at $x = 0$

11. $y = \sqrt{5x-3} - 5$ ...(i)

$\dfrac{dy}{dx} = \dfrac{5}{2\sqrt{5x-3}}$ 1

Slope of line 4x – 2y + 5 = 0 is $\dfrac{-4}{-2} = 2.$ $\dfrac{1}{2}$

$\therefore \quad \dfrac{5}{2\sqrt{5x-3}} = 2 \qquad x \quad \dfrac{73}{80}$ 1

Putting $x = \dfrac{73}{80}$ in eqn. (i), we get $y = \dfrac{-15}{4}$ $\dfrac{1}{2}$

Equation of tangent

$y + \dfrac{15}{4} = 2\left(x - \dfrac{73}{80}\right)$ 1

or $\quad 80x - 40y - 223 = 0$

12. $\int_1^5 \{|x-1| + |x-2| + |x-3|\}\, dx$

$= \int_1^5 (x-1)\, dx + \int_1^2 (2-x)\, dx + \int_2^5 (x-2)\, dx + \int_1^3 (3-x)\, dx + \int_3^5 (x-3)\, dx$ $2\dfrac{1}{2}$

$= \left[\dfrac{x^2}{2} - x\right]_1^5 + \left[2x - \dfrac{x^2}{2}\right]_1^2 + \left[\dfrac{x^2}{2} - 2x\right]_2^5 + \left[3x - \dfrac{x^2}{2}\right]_1^3 + \left[\dfrac{x^2}{3} - 3x\right]_3^5$ 1

$= 17$ $\dfrac{1}{2}$

OR

Let $\quad I = \int_0^{\pi} \dfrac{x \sin x}{1 + 3\cos^2 x}\, dx$ ...(i)

$I = \int_0^{\pi} \dfrac{(\pi - x)\sin(\pi - x)}{1 + 3\cos^2(\pi - x)}\, dx$ 1

$= \int_0^{\pi} \dfrac{\pi \sin x}{1 + 3\cos^2 x}\, dx - \int_0^{\pi} \dfrac{x \sin x}{1 + 3\cos^2 x}\, dx$ ...(ii)

Adding (i) & (ii), we have

$2I = \int_0^{\pi} \dfrac{\pi \sin x}{1 + 3\cos^2 x}\, dx$ 1

Put cos x = t

–sin x dx = dt, when x = 0 ⇒ t = 1, for x = π ⇒ t = –1

$2I = -\pi \int_1^{-1} \dfrac{dt}{1 + 3t^2}$ 1

$$= \frac{\pi}{3}\int_{-1}^{1}\frac{dt}{\left(\frac{1}{\sqrt{3}}\right)^2+(t)^2}$$

$$= \frac{\pi}{3}\times\sqrt{3}\left[\tan^{-1}(\sqrt{3}t)\right]_{-1}^{1}$$

$$= \frac{\sqrt{3}\pi}{3}[\tan^{-1}\sqrt{3}-(-\tan^{-1}\sqrt{3})]$$

$$I = \frac{\sqrt{3}\pi}{3}\cdot\frac{\pi}{3} = \frac{\sqrt{3}\pi^2}{9}$$ 1

13. Let $I = \int \frac{2x+1}{(x^2+1)(x^2+4)}dx$

Let $\frac{2x+1}{(x^2+1)(x^2+4)} = \frac{Ax+B}{x^2+1}+\frac{Cx+D}{x^2+4}$ 1

Getting $A = \frac{2}{3}, B = \frac{1}{3}, C = \frac{-2}{3}, D = \frac{-1}{3}$ 1

$$\therefore \quad I = \frac{2}{3}\int\frac{x}{x^2+1}dx+\frac{1}{3}\int\frac{1}{x^2+1}dx+\frac{-2}{3}\int\frac{xdx}{x^2+4}+\frac{-1}{3}\int\frac{dx}{x^2+4}$$

$$= \frac{1}{3}\log|x^2+1|+\frac{1}{3}\tan^{-1}x-\frac{1}{3}\log|x^2+4|-\frac{1}{6}\tan^{-1}\frac{x}{2}+C$$ 2

14. $(3x+5)\sqrt{5+4x-2x^2}\,dx$

let $3x + 5 = A(4 - 4x) + B$

$\Rightarrow \quad A = -\frac{3}{4}, B = 8$

$$I = -\frac{3}{4}\quad(4-4x)\sqrt{5+4x-2x^2}\,dx + 8\quad\sqrt{5+4x-2x^2}\,dx$$ 1

$$= -\frac{3}{4}I_1+8I_2 \text{ (let)}$$

For I_1, put $5 + 4x - 2x^2 = t$

$\Rightarrow \quad (4 - 4x)\,dx = dt$

$$-\frac{3}{4}I_1 = -\frac{3}{4}\quad\sqrt{t}\,dt = -\frac{3}{4}\times\frac{2}{3}t^{3/2}$$

$$= -\frac{1}{2}(5+4x-2x^2)^{3/2}$$ 1

$$8I_2 = 8\sqrt{2}\quad\sqrt{\frac{7}{2}-(x-1)^2}\,dx$$ $\frac{1}{2}$

$$= 8\sqrt{2}\left[\frac{x-1}{2}\sqrt{\frac{5}{2}+2x-x^2}+\frac{7}{4}\sin^{-1}\frac{\sqrt{2}(x-1)}{\sqrt{7}}\right]$$ 1

$$I = -\frac{1}{2}(5+4x-2x^2)^{3/2}+4\sqrt{2}(x-1)\sqrt{\frac{5}{2}+2x-x^2}+14\sqrt{2}\sin^{-1}\frac{\sqrt{2}(x-1)}{\sqrt{7}}+C$$ $\frac{1}{2}$

15. $x\dfrac{dy}{dx} + y - x + xy\cot x = 0$

$\Rightarrow \quad \dfrac{dy}{dx} + \left(\dfrac{1}{x} + \cot x\right)y = 1$ 1

$\text{I·F} = e^{\int\left(\frac{1}{x}+\cot x\right)dx} = e^{\log(x\sin x)}$ $\frac{1}{2}$

$= x\sin x$ 1

$\therefore \quad y \times x\sin x = \quad x\sin x\, dx$ $\frac{1}{2}$

$\Rightarrow \quad xy\sin x = -x\cos x + \sin x + C$ 1

16. $(x^2 + 3xy + y^2)\, dx - x^2\, dy = 0$

$\dfrac{dy}{dx} = \dfrac{x^2 + 3xy + y^2}{x^2}$ $\frac{1}{2}$

let $\quad y = vx$

$\dfrac{dy}{dx} = v + x\dfrac{dv}{dx}$ $\frac{1}{2}$

$\therefore \quad v + x\dfrac{dv}{dx} = 1 + 3v + v^2$

$\Rightarrow \quad x\dfrac{dv}{dx} = v^2 + 2v + 1$

$\Rightarrow \quad \dfrac{dv}{(v+1)^2} = \dfrac{dx}{x}$ 1

Integrating both sides

$\Rightarrow \quad -\dfrac{1}{v+1} = \log|x| + C$ 1

$\Rightarrow \quad \dfrac{-x}{x+y} = \log|x| + C$

When $x = 1,\ y = 0 \Rightarrow C = -1$ $\frac{1}{2}$

$\Rightarrow \quad \dfrac{-x}{x+y} = \log|x| - 1$

$\Rightarrow \quad y = (x + y)\log|x|$ $\frac{1}{2}$

or $\quad y = \dfrac{x\log|x|}{1 - \log|x|}$

17. $\vec{a} + \vec{b} = 5\hat{i} + \hat{k}$ $\frac{1}{2}$

$\vec{a} - \vec{b} = -\hat{i} - 2\hat{j} + 5\hat{k}$ $\frac{1}{2}$

Getting $\cos\theta = 0$ $1\frac{1}{2}$

$\Rightarrow \quad \theta = \dfrac{\pi}{2}$ $\frac{1}{2}$

a vector perpendicular to both $\vec{a} + \vec{b}$ & $\vec{a} - \vec{b}$ is $(\vec{a} + \vec{b}) \times (\vec{a} - \vec{b}) = 2\hat{i} - 26\hat{j} - 10\hat{k}$ 1

18. $\frac{x-1}{3} = \frac{y-1}{-1} = \frac{z+1}{0} = \lambda$ (let) ...(i)

$\Rightarrow \quad x = 3\lambda + 1,\ y = -\lambda + 1,\ z = -1$ 1

$\frac{x-4}{2} = \frac{y}{0} = \frac{z+1}{3} = \mu$ ...(ii)

$\Rightarrow \quad x = 2\mu + 4,\ y = 0,\ z = 3\mu - 1$ 1

If the lines intersect, then they have a common point for some value of λ and μ.

So $\quad 3\lambda + 1 = 2\mu + 4$ ...(iii)

$-\lambda + 1 = 0 \Rightarrow \lambda = 1$

$3\mu - 1 = -1 \Rightarrow \mu = 0$ 1

Since $\lambda = 1$ & $\mu = 0$ satisfy equation (iii) so the given lines intersect and $\frac{1}{2}$

the point of intersection is (4, 0, –1). $\frac{1}{2}$

19. Let A = exactly 2 boys in the committee

B = at least one girl must be there in the committee.

$$P(B) = \frac{{}^4C_1 \times {}^7C_3 + {}^4C_2 \times {}^7C_2 + {}^4C_3 \times {}^7C_1 + {}^4C_4}{{}^{11}C_4}$$ 2

$$= \frac{59}{66}$$

$$P(A \cap B) = \frac{{}^4C_2 \times {}^7C_2}{{}^{11}C_4} = \frac{21}{55}$$ 1

$$P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{21/55}{59/66} = \frac{126}{295}$$ 1

OR

$C + 2C + 2C + 3C + C^2 + 2C^2 + 7C^2 + C = 1$ 1

$\Rightarrow \quad 10C^2 + 9C = 1$

$\Rightarrow \quad 10C^2 + 9C - 1 = 0$

$\Rightarrow \quad C = \frac{1}{10}$ or $C = -1$ (not possible)

$\therefore \quad C = \frac{1}{10}$ 1

Mean $= 0 \times C + 1 \times 2C + 2 \times 2C + 3 \times 3C + 4 \times C^2 + 5 \times 2C^2 + 6(7C^2 + C)$ 1

$= 56C^2 + 21C$

$= 56 \times \frac{1}{100} + 21 \times \frac{1}{10}$

$= 0.56 + 2.1 = 2.66$ 1

SECTION C

20. (a, b) R(c, d) $\Rightarrow$ a + d = b + c

$\because$ a + b = b + a

$\Rightarrow$ (a, b) R(a, b) $\forall$ (a, b) $\in$ A $\times$ A

$\Rightarrow$ R is reflexive $\frac{1}{2}$

(a, b) R(c, d) $\Rightarrow$ a + d = b + c

$\Rightarrow$ b + c = a + d

$\Rightarrow$ c + b = d + a

$\Rightarrow$ (c, d) R(a, b)

$\Rightarrow$ R is symmetric $1\frac{1}{2}$

For (a, b), (c, d) & (e, f) $\in$ A $\times$ A

(a, b) R(c, d) $\Rightarrow$ a + d = b + c ...(1)

(c, d) R(e, f) $\Rightarrow$ c + f = d + e ...(2)

adding (1) & (2), we get

a + d + c + f = b + c + d + e

$\Rightarrow$ a + f = b + e

$\Rightarrow$ (a, b) R(e, f)

$\therefore$ R is transitive. $1\frac{1}{2}$

Hence R is an equivalence relation. $\frac{1}{2}$

Now [3, 4] = {(1, 2), (2, 3), (3, 4), (4, 5), (5, 6), (6, 7), (7, 8), (8, 9), (9, 10)} 1

21. $\begin{vmatrix} a+x & a-x & a-x \\ a-x & a+x & a-x \\ a+x & a-x & a+x \end{vmatrix} = 0$

Applying $C_1 \to C_1 + C_2 + C_3$ 1

$\Rightarrow \begin{vmatrix} 3a-x & a-x & a-x \\ 3a-x & a+x & a-x \\ 3a-x & a-x & a+x \end{vmatrix} = 0$ 1

$\Rightarrow (3a-x)\begin{vmatrix} 1 & a-x & a-x \\ 1 & a+x & a-x \\ 1 & a-x & a+x \end{vmatrix} = 0$ 1

Applying $R_2 \to R_2 - R_1$ & $R_3 \to R_3 - R_1$

$\Rightarrow (3a-x)\begin{vmatrix} 1 & a-x & a-x \\ 0 & 2x & 0 \\ 0 & 0 & 2x \end{vmatrix} = 0$ 2

$\Rightarrow (3a-x)(4x^2) = 0$ 1

$\Rightarrow$ x = 0 or 3a

OR

A = I·A 1

$$\Rightarrow \begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

Applying $R_1 \to R_1 - R_2$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

Applying $R_2 \to R_2 - 2R_1$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 0 \\ -2 & 3 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

Applying $R_2 \to R_2 - 4R_3$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 0 \\ -2 & 3 & -4 \\ 0 & 0 & 1 \end{bmatrix} A$$

Applying $R_3 \to R_3 + R_2$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 0 \\ -2 & 3 & -4 \\ -2 & 3 & -3 \end{bmatrix} A$$

$\left[2\frac{1}{2} \text{ for correct operations to get } A^{-1}\right]$

$$\therefore \quad A^{-1} = \begin{bmatrix} 1 & -1 & 0 \\ -2 & 3 & -4 \\ -2 & 3 & -3 \end{bmatrix}$$ $\frac{1}{2}$

The matrix form of given equations

$$\begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 21 \\ 20 \\ 5 \end{bmatrix}$$ 1

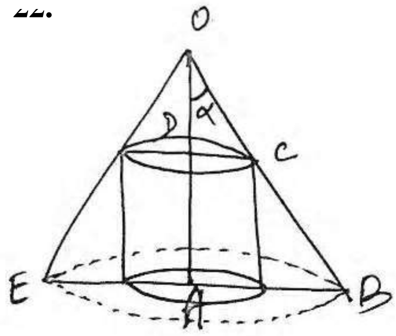
$$\Rightarrow \quad AX = B, \text{ where } X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, B = \begin{bmatrix} 21 \\ 20 \\ 5 \end{bmatrix}$$

$$\Rightarrow \quad X = A^{-1}B$$

$$\Rightarrow \begin{bmatrix} 1 & -1 & 0 \\ -2 & 3 & -4 \\ -2 & 3 & -3 \end{bmatrix} \begin{bmatrix} 21 \\ 20 \\ 5 \end{bmatrix} = \begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix}$$

$\therefore \quad x = 1, y = -2, z = 3$ 1

22.



Correct Figure 1

Let CD = R, AD = x

$\Rightarrow$ OD = h – x

$\because$ ODC ~ ΔOAB

$\Rightarrow \dfrac{h-x}{h} = \dfrac{R}{AB} \Rightarrow \dfrac{h-x}{h} = \dfrac{R}{h\tan\alpha}$

$\Rightarrow R = (h-x)\tan\alpha$ 1

$V = \pi R^2 x$ $\frac{1}{2}$

$= \pi(h-x)^2 \tan^2\alpha \cdot x$

$= \pi\tan^2\alpha (h-x)^2 x$ $\frac{1}{2}$

$\dfrac{dV}{dx} = \pi\tan^2\alpha(h^2 - 4hx + 3x^2)$ $\frac{1}{2}$

$\dfrac{dV}{dx} = 0 \Rightarrow h^2 - 4hx + 3x^2 = 0$

$\Rightarrow (h-x)(h-3x) = 0$

$\Rightarrow x = h$ (not possible) or $x = \dfrac{h}{3}$ 1

$\dfrac{d^2V}{dx^2} = \pi\tan^2\alpha(-4h + 6x)$

$\left(\dfrac{d^2V}{dx^2}\right)_{x=h/3} = \pi\tan^2\alpha(-2h) < 0$ 1

$\Rightarrow$ V is maximum for $x = \dfrac{h}{3}$

So $V_{max} = \pi\tan^2\alpha(h-x)^2 x$

$= \pi\tan^2\alpha\left(h - \dfrac{h}{3}\right)^2 \dfrac{h}{3}$

$= \dfrac{4\pi h^3}{27}\tan^2\alpha$ $\frac{1}{2}$

OR

$y = \dfrac{4\sin x}{2+\cos x} - x, \quad x \in [0, 2\pi]$

$\dfrac{dy}{dx} = \dfrac{(2+\cos x)4\cos x - 4\sin x(-\sin x)}{(2+\cos x)^2} - 1$ 2

$\dfrac{dy}{dx} = \dfrac{\cos x(4-\cos x)}{(2+\cos x)^2}$ 1

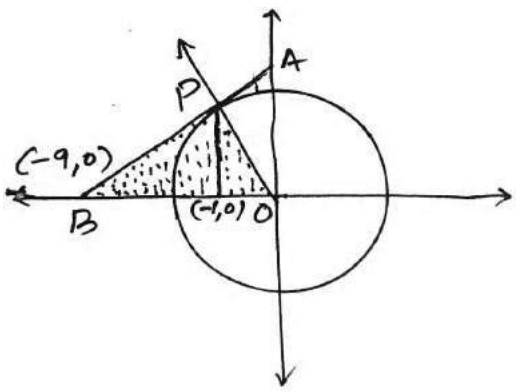
f(x) is strictly increasing for $f'(x) > 0$

i.e., $\cos x > 0 \Rightarrow x \in \left[0, \frac{\pi}{2}\right) \cup \left(\frac{3\pi}{2}, 2\pi\right]$ 2

and f(x) is strictly decreasing for $f'(x) < 0$

i.e., $\cos x < 0 \Rightarrow x \in \left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$ 1

23. Correct Figure 1



Equation of circle $x^2 + y^2 = 9$

Diff. w.r.t x, we have

$$\frac{dy}{dx} = -\frac{x}{y}$$ $\frac{1}{2}$

Slope of tangent at $\left(-1, 2\sqrt{2}\right)$

$$m_T = \left(-\frac{x}{y}\right)_{(-1, 2\sqrt{2})} = \frac{1}{2\sqrt{2}}$$ $\frac{1}{2}$

eqn. of tangent

$$y - 2\sqrt{2} = \frac{1}{2\sqrt{2}}(x+1)$$ 1

$\Rightarrow \quad x - 2\sqrt{2}y + 9 = 0$

It cuts x-axis at (–9, 0)

eqn. of normal

$$y - 2\sqrt{2} = -2\sqrt{2}(x+1)$$ 1

$\Rightarrow \quad 2\sqrt{2}x + y = 0$

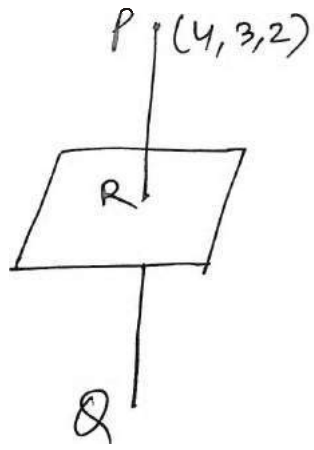
Area of ΔOPB

$$A = \int_{-9}^{-1} \frac{x+9}{2\sqrt{2}}\,dx + \int_{-1}^{0} -2\sqrt{2}x\,dx$$ 1

$$= \frac{1}{2\sqrt{2}}\left[\frac{x^2}{2} + 9x\right]_{-9}^{-1} - 2\sqrt{2}\left[\frac{x^2}{2}\right]_{-1}^{0}$$

$$= 9\sqrt{2} \text{ sq. unit}$$ 1

24.



Eqn. of plane $x + 2y + 3z = 2$

eqn. of PR is $\frac{x-4}{1} = \frac{y-3}{2} = \frac{z-3}{3} = \lambda$ (let) 1

$\Rightarrow \quad x = \lambda + 4,\ y = 2\lambda + 3,\ z = 3\lambda + 2$ 1

Let the co-ordinate of R be $(\lambda + 4, 2\lambda + 3, 3\lambda + 2)$

R also lies on the plane

So, $\lambda + 4 + 2(2\lambda + 3) + 3(3\lambda + 2) = 2$

$\Rightarrow \lambda = -1$ 1

So point R is (3, 1, –1) i.e., foot of perpendicular 1

let Q(α, β, γ) be the image of P

$$\therefore \quad \frac{4+\alpha}{2} = 3, \frac{3+\beta}{2} = 1, \frac{2+\gamma}{2} = -1$$

$\Rightarrow \alpha = 2, \beta = -1, \gamma = -4$

So Image point Q is (2, –1, –4) 1

Perpendicular distance PR = $\sqrt{14}$ 1

25. P (winning) = $\frac{1}{9}$

P (not winning) = $\frac{8}{9}$ 1

P(A winning) $= P(A) + P(\bar{A}\bar{B}\bar{C}A) + P(\bar{A}\bar{B}\bar{C}\bar{A}\bar{B}\bar{C}A) + ...$ 1

$$= \frac{1}{9} + \left(\frac{8}{9}\right)^3 \frac{1}{9} + \left(\frac{8}{9}\right)^6 \frac{1}{9} + ...$$

$$= \frac{\frac{1}{9}}{1 - \frac{512}{729}} = \frac{81}{217}$$ 1

P(B winning) $= P(\bar{A}B) + P(\bar{A}\bar{B}\bar{C}\bar{A}B) + P(\bar{A}\bar{B}\bar{C}\bar{A}\bar{B}\bar{C}\bar{A}B) + ...$ 1

$$= \frac{8}{9} \times \frac{1}{9} + \left(\frac{8}{9}\right)^4 \times \frac{1}{9} + \left(\frac{8}{9}\right)^7 \times \frac{1}{9} + ...$$

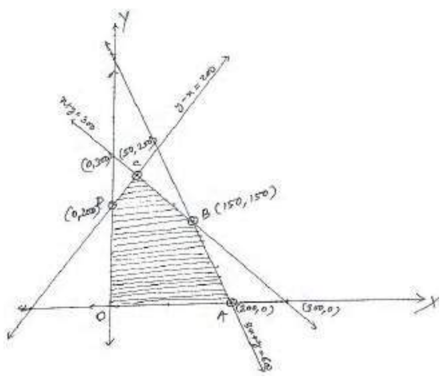
$$= \frac{\frac{8}{9} \times \frac{1}{9}}{1 - \frac{512}{729}} = \frac{72}{217}$$ 1

P(C winning) = 1 – [P(A winning) + P(B winning)]

$$= 1 - \left[\frac{81}{217} + \frac{72}{217}\right]$$

$$= 1 - \frac{153}{217} = \frac{64}{217}$$ 1

26.



Let no. of cardigans of type A be x and that of type B by y.

Then, max. $Z = 100x + 50y$ 1

subject to constraint,

$x + y \leq 300$ ...(1)

$360x + 120y \leq 72,000$

$\Rightarrow$ $3x + y \leq 600$ ...(2)

$y - x \leq 200$ ...(3) 2

$x, y \geq 0$

Correct Figure 2

Corner points A(200, 0), B(150, 150), C(50, 250), D(0, 200), O(0, 0)

Corner points	**Z = 100x + 50y**
O(0, 0)	0
A(200, 0)	20,000
B(150, 150)	22,500 ← maximum
C(50, 250)	17,500
D(0, 200)	10,000

Hence no. of cardigans of type A = 150 and of type B = 150.

and max. profit is ₹ 22,500. 1