QUESTION PAPER CODE 65/1/S

EXPECTED ANSWER/VALUE POINTS

SECTION A

1. Getting $\sin\theta = \dfrac{\frac{1}{\sqrt{3}}}{\frac{1}{2}\cdot\frac{4}{\sqrt{3}}} = \dfrac{1}{2}$ $\quad \frac{1}{2}$

Hence $|\vec{a}.\vec{b}| = \dfrac{1}{2}\cdot\dfrac{4}{\sqrt{3}}\cdot\dfrac{\sqrt{3}}{2} = 1$ $\quad \frac{1}{2}$

2. $|\vec{a} - \sqrt{2}\vec{b}|^2 = 1 \Rightarrow \vec{a}.\vec{b} = \dfrac{1}{\sqrt{2}}$. $\quad \frac{1}{2}$

$\therefore$ Angle between $\vec{a}$ and $\vec{b}$ = $\dfrac{\pi}{4}$ $\quad \frac{1}{2}$

3. Writing or using, that given planes are parallel $\quad \frac{1}{2}$

$d = \dfrac{|4+10|}{\sqrt{4+9+36}} = 2$ units $\quad \frac{1}{2}$

4. $|AA^T| = |A||A^T| = |A|^2$ $\quad \frac{1}{2}$

$= 25$ $\quad \frac{1}{2}$

5. Getting AB = $\begin{pmatrix} 7 & -8 \\ 0 & -10 \end{pmatrix}$ $\quad \frac{1}{2}$

$|AB| = -70$ $\quad \frac{1}{2}$

6. $k(2) = -8 \Rightarrow k = -4$ $\quad \frac{1}{2}$

$-4(3) = 4a \Rightarrow a = -3$ $\quad \frac{1}{2}$

SECTION B

7. $y = (\sin 2x)^x + \sin^{-1}(\sqrt{3x}) = u + v$

$\therefore \quad \dfrac{dy}{dx} = \dfrac{du}{dx} + \dfrac{dv}{dx}$ $\quad \frac{1}{2}$

$u = (\sin 2x)^x \Rightarrow \log u = x \log \sin 2x$ $\quad \frac{1}{2}$

$\dfrac{1}{u}\dfrac{du}{dx} = 2x \cdot \cot 2x + \log \sin 2x$ $\quad 1$

$\therefore \quad \frac{du}{dx} = (\sin 2x)^x [2x \cot 2x + \log \sin 2x]$ $\frac{1}{2}$

$\frac{dv}{dx} = \frac{1}{\sqrt{1-3x}} \frac{\sqrt{3}}{2\sqrt{x}}$ 1

$\therefore \quad \frac{dy}{dx} = (\sin 2x)^x [2x \cot 2x + \log \sin 2x] + \frac{\sqrt{3}}{2\sqrt{x}\sqrt{1-3x}}$ $\frac{1}{2}$

OR

Let $y = \tan^{-1}\left(\frac{\sqrt{1+x^2} - \sqrt{1-x^2}}{\sqrt{1+x^2} + \sqrt{1-x^2}}\right)$ and $z = \cos^{-1} x^2$

$z = \cos^{-1} x^2 \Rightarrow x^2 = \cos z \Rightarrow y = \tan^{-1}\left(\frac{\sqrt{1+\cos z} - \sqrt{1-\cos z}}{\sqrt{1+\cos z} + \sqrt{1-\cos z}}\right)$ 1

$\therefore \quad y = \tan^{-1}\left(\frac{\cos\frac{z}{2} - \sin\frac{z}{2}}{\cos\frac{z}{2} + \sin\frac{z}{2}}\right) = \tan^{-1}\left(\frac{1-\tan\frac{z}{2}}{1+\tan\frac{z}{2}}\right)$ $\frac{1}{2} + \frac{1}{2}$

$\therefore \quad y = \tan^{-1}\left[\tan\left(\frac{\pi}{4} - \frac{z}{2}\right)\right] = \frac{\pi}{4} - \frac{z}{2}$ $\frac{1}{2} + \frac{1}{2}$

$\Rightarrow \quad \frac{dy}{dz} = -\frac{1}{2}$ 1

8. LHL $= \lim_{x \to 0^-} k.\sin\frac{\pi}{2}(x+1) = k$ 1

RHL $= \lim_{x \to 0^+} \frac{\tan x(1-\cos x)}{x^3}$ 1

$= \lim_{x \to 0^+} \frac{\tan x}{x}.2\left(\frac{\sin x/2}{2x/2}\right)^2 = \frac{1}{2}$ 1

$\Rightarrow \quad k = \frac{1}{2}$ 1

9. When $x = am^2$, we get $y = \pm am^3$ 1

$ay^2 = x^3 \Rightarrow 2ay\frac{dy}{dx} = 3x^2 \Rightarrow \frac{dy}{dx} = \frac{3x^2}{2ay}$ 1

slope of normal $= \mp\frac{2a}{3}\frac{am^3}{a^2m^4} = \mp\frac{2}{3m}$ 1

$\therefore$ Equation of normal is $y \mp am^3 = \mp\frac{2}{3m}(x - am^2)$ 1

[Full marks may be given, if only one value for point, slope and equation is derived]

10. Writing $\int \frac{1-\sin x}{\sin x(1+\sin x)}dx = \int \frac{(1+\sin x)-2\sin x}{\sin x(1+\sin x)}dx$ 1

$= \int \frac{1}{\sin x}dx - 2\int \frac{1}{1+\sin x}dx$ 1

$= \int \text{cosec } x\, dx - 2\int \frac{(1-\sin x)}{\cos^2 x}dx$ 1

$= \log|\text{cosec } x - \cot x| - 2\int (\sec^2 x - \sec x \tan x)\, dx$ $\frac{1}{2}$

$= \log|\text{cosec } x - \cot x| - 2(\tan x - \sec x) + C$ $\frac{1}{2}$

11. $I = \int\left[\log(\log x) + \frac{1}{(\log x)^2}\right]dx$

$= \int \log(\log x).1\, dx + \int \frac{1}{(\log x)^2}dx$ 1

$= \log(\log x)\cdot x - \int \frac{1}{\log x}\cdot\frac{1}{\cancel{x}}\cdot\cancel{x}\, dx + \int \frac{1}{(\log x)^2}dx$ 2

$= x\cdot\log(\log x) - \left[\frac{1}{\log x}\cdot x - \int \frac{-1}{(\log x)^2}\cdot\frac{1}{x}\cdot x\, dx\right] + \int \frac{1}{(\log x)^2}dx$ $\frac{1}{2}$

$= x\log(\log x) - \frac{x}{\log x} + C$ $\frac{1}{2}$

12. $I = \int_0^{\pi/2} \frac{\sin^2 x}{\sin x + \cos x}dx$ ...(i)

$I = \int_0^{\pi/2} \frac{\sin^2(\pi/2 - x)}{\sin(\pi/2 - x) + \cos(\pi/2 - x)}dx = \int_0^{\pi/2} \frac{\cos^2 x}{\cos x + \sin x}dx$ ...(ii) 1

$2I = \int_0^{\pi/2} \frac{1}{\sin x + \cos x}dx$ 1

$\Rightarrow I = \frac{1}{2\sqrt{2}}\int_0^{\pi/2} \frac{1}{\cos x\frac{1}{\sqrt{2}} + \sin x\frac{1}{\sqrt{2}}}dx = \frac{1}{2\sqrt{2}}\int_0^{\pi/2} \sec\left(x - \frac{\pi}{4}\right)dx$ 1

$= \frac{1}{2\sqrt{2}}\left[\log\left|\sec\left(x - \frac{\pi}{4}\right) + \tan\left(x - \frac{\pi}{4}\right)\right|\right]_0^{\pi/2}$ $\frac{1}{2}$

$= \frac{1}{2\sqrt{2}}\log\left|\frac{\sqrt{2}+1}{\sqrt{2}-1}\right|$ or $\frac{1}{\sqrt{2}}\log|\sqrt{2}+1|$ $\frac{1}{2}$

OR

$$I = \int_0^1 \cot^{-1}(1-x+x^2)\,dx = \int_0^1 \tan^{-1}\left(\frac{1}{1-x+x^2}\right)dx$$ $\frac{1}{2}$

$$= \int_0^1 \tan^{-1}\left(\frac{x+(1-x)}{1-x(1-x)}\right)dx = \int_0^1 \tan^{-1} x\,dx + \int_0^1 \tan^{-1}(1-x)\,dx$$ 1

$$= 2\int_0^1 \tan^{-1} x\,dx$$ $\frac{1}{2}$

$$= 2\left[\left(\tan^{-1} x.x\right)_0^1 - \int_0^1 \frac{x}{1+x^2}\,dx\right]$$ $\frac{1}{2}$

$$= 2\left[x\tan^{-1} x - \frac{1}{2}\log|1+x^2|\right]_0^1$$ 1

$$= 2\left[\frac{\pi}{4} - \frac{1}{2}\log 2\right] \text{ or } \frac{\pi}{2} - \log 2$$ $\frac{1}{2}$

13. The given differential equation can be written as

$$\frac{dy}{dx} - \frac{1}{x+1}y = (x+1)^2.e^{3x}$$ $\frac{1}{2}$

Here, integrating factor $= e^{\int -\frac{1}{x+1}dx} = \frac{1}{x+1}$ 1

$\therefore$ Solution is $y\frac{1}{x+1} = \int (x+1)e^{3x}dx$ 1

$\therefore$ $$\frac{y}{x+1} = (x+1)\frac{e^{3x}}{3} - \frac{e^{3x}}{9} + C$$ $1\frac{1}{2}$

or $$y = \left[\frac{1}{3}(x+1)^2 - \frac{x+1}{9}\right]e^{3x} + C(x+1)$$

14. From the given differential equation, we can write

$$\frac{dx}{dy} = \frac{2xe^{x/y} - y}{2ye^{x/y}} = \frac{2\,x/y\;e^{x/y} - 1}{2e^{x/y}}$$ 1

Putting $\frac{x}{y} = v \Rightarrow \frac{dx}{dy} = v + y\frac{dv}{dy}$ $\frac{1}{2}$

$\therefore$ $$v + y\frac{dv}{dy} = \frac{2ve^v - 1}{2e^v} \Rightarrow y\frac{dv}{dy} = -\frac{1}{2e^v}$$ 1

$\Rightarrow$ $$2\int e^v\,dv = -\int\frac{dy}{y}$$ $\frac{1}{2}$

$\therefore$ $$2e^v + \log|y| = C \Rightarrow 2e^{x/y} + \log|y| = C$$ 1

15. Let length be x m and breadth be y m

$\therefore \quad (x-50)(y+50) = xy \Rightarrow 50x - 50y = 2500$ or $x - y = 50$ $\quad \frac{1}{2}$

and $\quad (x-10)(y-20) = xy - 5300 \Rightarrow 2x + y = 550$ $\quad \frac{1}{2}$

$$\begin{pmatrix} 1 & -1 \\ 2 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 50 \\ 550 \end{pmatrix} \Rightarrow \begin{pmatrix} x \\ y \end{pmatrix} = \frac{1}{3}\begin{pmatrix} 1 & 1 \\ -2 & 1 \end{pmatrix}\begin{pmatrix} 50 \\ 550 \end{pmatrix}$$ $\frac{1}{2} + 1$

$\Rightarrow \quad x = \frac{1}{3}(600) = 200$ m, $\; y = \frac{1}{3}(450) = 150$ m $\quad \frac{1}{2}$

"Helping the children of his village to learn" (or any other relavent value) $\quad 1$

16. LHS $= 2\sin^{-1}\left(\frac{3}{5}\right) - \tan^{-1}\left(\frac{17}{31}\right)$

$= 2\tan^{-1}\left(\frac{3}{4}\right) - \tan^{-1}\left(\frac{17}{31}\right)$ $\quad 1$

$= \tan^{-1}\left(\frac{2\cdot\frac{3}{4}}{1-\frac{9}{16}}\right) - \tan^{-1}\left(\frac{17}{31}\right) = \tan^{-1}\left(\frac{24}{7}\right) - \tan^{-1}\left(\frac{17}{31}\right)$ $\quad 1$

$\tan^{-1}\left(\frac{\frac{24}{7}-\frac{17}{31}}{1+\frac{24}{7}\cdot\frac{17}{31}}\right) = \tan^{-1}\left(\frac{625}{625}\right) = \tan^{-1}(1) = \frac{\pi}{4} = \text{RHS}$ $\quad 1 + 1$

OR

$\cos(\tan^{-1} x) = \sin\left(\cot^{-1}\frac{3}{4}\right)$

$\Rightarrow \quad \cos\left(\cos^{-1}\frac{1}{\sqrt{1+x^2}}\right) = \sin\left(\sin^{-1}\frac{4}{5}\right)$ $\quad 1 + 1$

$\Rightarrow \quad \frac{1}{\sqrt{1+x^2}} = \frac{4}{5}$ or $\sqrt{1+x^2} = \frac{5}{4}$ $\quad 1$

$\Rightarrow \quad x^2 = \frac{9}{16} \Rightarrow x = \pm\frac{3}{4}$ $\quad 1$

17. Let E_1 : selecting bag A, E_2 : selecting bag B $\quad \frac{1}{2}$

A : getting 2 white and 1 red out of 3 drawn (without replacement)

$\therefore P(E_1) = P(E_2) = \frac{1}{2}$ $\quad \frac{1}{2}$

$P\left(\frac{A}{E_1}\right) = \frac{{}^3C_2 \cdot {}^4C_1}{{}^7C_3} = \frac{12}{35}$

1

$P\left(\frac{A}{E_2}\right) = \frac{{}^4C_2 \cdot {}^3C_1}{{}^7C_3} = \frac{18}{35}$ 1

$$P\left(\frac{E_2}{A}\right) = \frac{P(E_2)\,P\left(\frac{A}{E_2}\right)}{P(E_1)P\left(\frac{A}{E_1}\right) + P(E_2)P\left(\frac{A}{E_2}\right)}$$

$$= \frac{\frac{1}{2}\cdot\frac{18}{35}}{\frac{1}{2}\cdot\frac{12}{35} + \frac{1}{2}\cdot\frac{18}{35}} = \frac{3}{5}$$ 1

18. $\vec{a} = \vec{b} + \vec{c} \Rightarrow p\hat{i} + q\hat{j} + r\hat{k} = (s+3)\hat{i} + 4\hat{j} + 2\hat{k}$

$p = s + 3,\ q = 4,\ r = 2$ $1\frac{1}{2}$

area $= \frac{1}{2}|\vec{b}\times\vec{c}| = 5\sqrt{6}$

$$\vec{b}\times\vec{c} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ s & 3 & 4 \\ 3 & 1 & -2 \end{vmatrix} = -10\hat{i} + (2s+12)\hat{j} + (s-9)\hat{k}$$ $\frac{1}{2}$

$\therefore\ 100 + (2s+12)^2 + (s-9)^2 = (10\sqrt{6})^2 = 600$

$\Rightarrow s^2 + 6s + 55 = 0 \Rightarrow s = -11,\ p = -8$, or $s = 5,\ p = 8$ 1 + 1

19. Equation of plane passing through A, B and C is

$$\begin{vmatrix} x-3 & y-2 & z-1 \\ 1 & 0 & -3 \\ 2 & 3 & 1 \end{vmatrix} = 0$$ 2

$\Rightarrow (x-3)9 - (y-2)7 + (z-1)3 = 0 \Rightarrow 9x - 7y + 3z = 16$ …(i) 1

If A, B, C and D are coplanar, D must lie on (i)

$\Rightarrow 9\lambda - 35 + 15 - 16 = 0 \Rightarrow \lambda = 4.$ 1

OR

Equation of plane, perpendicular to $\vec{n} = \hat{i} + \hat{j} + 3\hat{k}$ and at a distance $\frac{4}{\sqrt{11}}$ from origin is

$\vec{r}\cdot\frac{(\hat{i}+\hat{j}+3\hat{k})}{\sqrt{11}} = \frac{4}{\sqrt{11}}$ or $\vec{r}\cdot(\hat{i}+\hat{j}+3\hat{k}) = 4$ ...(i) $1\frac{1}{2}$

Any point on the line $\vec{r} = (-\hat{i} - 2\hat{j} - 3\hat{k}) + \lambda(3\hat{i} + 4\hat{j} + 3\hat{k})$ is

$(-1+3\lambda)\hat{i} + (-2+4\lambda)\hat{j} + (-3+3\lambda)\hat{k}$ ...(ii) 1

If this point is the point of intersection of the plane and the line then,

$(-1+3\lambda)1 + (-2+4\lambda)1 + (-3+3\lambda)3 = 4$

$\Rightarrow \lambda = 1.$ 1

Hence the point of intersection is (2, 2, 0) $\frac{1}{2}$

SECTION C

20. Let $x_1, x_2 \in N$ and $f(x_1) = f(x_2)$

$\Rightarrow 4x_1^2 + 12x_1 + 15 = 4x_2^2 + 12x_2 + 15$

$\Rightarrow 4(x_1^2 - x_2^2) + 12(x_1 - x_2) = 0 \Rightarrow (x_1 - x_2)(4x_1 + 4x_2 + 12) = 0$

$\Rightarrow x_1 - x_2 = 0$ or $x_1 = x_2$ as $4x_1 + 4x_2 + 12 \neq 0$, $x_1, x_2 \in N$

$\therefore f$ is a 1 – 1 function 2

$f : N \to S$ is onto as co-domain = range 1

Hence f is invertible.

$$y = 4x^2 + 12x + 15 = (2x+3)^2 + 6 \Rightarrow x = \frac{\sqrt{y-6}-3}{2}$$

$\therefore \quad f^{-1}(y) = \dfrac{\sqrt{y-6}-3}{2}, \; y \in S.$ 2

$f^{-1}(31) = \dfrac{\sqrt{31-6}-3}{2} = 1$ $\frac{1}{2}$

$f^{-1}(87) = \dfrac{\sqrt{87-6}-3}{2} = 3$ $\frac{1}{2}$

21. Let $\Delta = \begin{vmatrix} (b+c)^2 & a^2 & bc \\ (c+a)^2 & b^2 & ca \\ (a+b)^2 & c^2 & ab \end{vmatrix}$

$C_1 \to C_1 + C_2 - 2C_3 \Rightarrow \Delta = (a^2+b^2+c^2)\begin{vmatrix} 1 & a^2 & bc \\ 1 & b^2 & ca \\ 1 & c^2 & ab \end{vmatrix}$ 1

$R_1 \to R_1 - R_2$, and $R_2 \to R_2 - R_3 \Rightarrow \Delta = (a^2+b^2+c^2)\begin{vmatrix} 0 & a^2-b^2 & c(b-a) \\ 0 & b^2-c^2 & a(c-b) \\ 1 & c^2 & ab \end{vmatrix}$ $1\frac{1}{2}$

$= (a^2+b^2+c^2)(a-b)(b-c)\begin{vmatrix} 0 & a+b & -c \\ 0 & b+c & -a \\ 1 & c^2 & ab \end{vmatrix}$ $\frac{1}{2}$

$R_2 \to R_2 - R_1 \Rightarrow \Delta = (a^2+b^2+c^2)(a-b)(b-c)\begin{vmatrix} 0 & a+b & -c \\ 0 & c-a & c-a \\ 1 & c^2 & ab \end{vmatrix}$ 1

$\therefore \Delta = (a^2+b^2+c^2)(a-b)(b-c)(c-a)\begin{vmatrix} 0 & a+b & -c \\ 0 & 1 & 1 \\ 1 & c^2 & ab \end{vmatrix}$ 1

Expanding by C_1 to get $\Delta = (a^2+b^2+c^2)(a-b)(b-c)(c-a)(a+b+c)$ 1

OR

Let $A = IA \therefore \begin{pmatrix} 2 & -1 & 3 \\ -5 & 3 & 1 \\ -3 & 2 & 3 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} A$ 1

$R_2 \to R_2 - 2R_3 \Rightarrow \begin{pmatrix} 2 & -1 & 3 \\ 1 & -1 & -5 \\ -3 & 2 & 3 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{pmatrix} A$ $\frac{1}{2}$

$\begin{matrix} R_1 \to R_1 - 2R_2 \\ R_3 \to R_3 + 3R_2 \end{matrix} \Rightarrow \begin{pmatrix} 0 & 1 & 13 \\ 1 & -1 & -5 \\ 0 & -1 & -12 \end{pmatrix} = \begin{pmatrix} 1 & -2 & 4 \\ 0 & 1 & -2 \\ 0 & 3 & -5 \end{pmatrix} A$ 1

$R_1 \leftrightarrow R_2 \Rightarrow \begin{pmatrix} 1 & -1 & -5 \\ 0 & 1 & 13 \\ 0 & -1 & -12 \end{pmatrix} = \begin{pmatrix} 0 & 1 & -2 \\ 1 & -2 & 4 \\ 0 & 3 & -5 \end{pmatrix} A$ $\frac{1}{2}$

$\begin{cases} R_1 \to R_1 + R_2 \\ R_3 \to R_3 + R_2 \end{cases} \Rightarrow \begin{pmatrix} 1 & 0 & 8 \\ 0 & 1 & 13 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & -1 & 2 \\ 1 & -2 & 4 \\ 1 & 1 & -1 \end{pmatrix} A$ 1

$\begin{cases} R_1 \to R_1 - 8R_3 \\ R_2 \to R_2 - 13R_3 \end{cases} \Rightarrow \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} -7 & -9 & 10 \\ -12 & -15 & 17 \\ 1 & 1 & -1 \end{pmatrix} A$ 1

$\Rightarrow A^{-1} = \begin{pmatrix} -7 & -9 & 10 \\ -12 & -15 & 17 \\ 1 & 1 & -1 \end{pmatrix}$ 1

22. $f'(x) = 4x^3 - 24x^2 + 44x - 24$ 1

$= 4(x^3 - 6x^2 + 11x - 6) = 4(x-1)(x-2)(x-3)$ $1\frac{1}{2}$

$f'(x) = 0 \Rightarrow x = 1, x = 2, x = 3$ $\frac{1}{2}$

The intervals are $(-\infty, 1), (1, 2), (2, 3), (3, \infty)$ 1

since $f'(x) > 0$ in $(1, 2)$ and $(3, \infty)$

$\therefore$ $f(x)$ is strictly increasing in $(1, 2) \cup (3, \infty)$ 1

and strictly decreasing in $(-\infty, 1) \cup (2, 3)$ 1

OR

$f(x) = \sec x + 2 \log |\cos x|$

$f'(x) = \sec x \tan x - 2 \tan x = \tan x (\sec x - 2)$ 1

$f'(x) = 0 \Rightarrow \tan x = 0$ or $\sec x = 2$

$\Rightarrow x = \pi, x = \frac{\pi}{3}, \frac{5\pi}{3}$ $1\frac{1}{2}$

$f''(x) = \sec x \tan^2 x + (\sec x - 2) \sec^2 x$ 1

$f''(\pi/3) = 6$ (+ve) $\Rightarrow f(x)$ is minimum at $x = \pi/3$

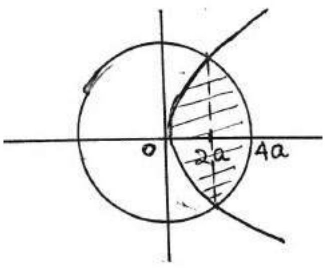
$f''(\pi) = -3$ (–ve) $\Rightarrow f(x)$ is maximum at $x = \pi$ 1 $\frac{1}{2}$

$f''(5\pi/3) = 6$ (+ve) $\Rightarrow f(x)$ is minimum at $x = 5\pi/3$

Maximum value $= f(\pi) = -1$. $\frac{1}{2}$

Minimum value $= f(\pi/3) = f(5\pi/3) = 2 - 2\log 2$ or $2 + \log(1/4)$ $\frac{1}{2}$

23.



Solving $y^2 = 6ax$ and $x^2 + y^2 = 16a^2$

we get $x^2 + 6ax - 16a^2 = 0$

$(x + 8a)(x - 2a) = 0$

$x = -8a,\ x = 2a$ 1

Correct Figure 1

Required area $= 2\left[\int_0^{2a} \sqrt{6a}\sqrt{x}\,dx + \int_{2a}^{4a}\sqrt{16a^2 - x^2}\,dx\right]$ 2

$= 2\left[\left(\sqrt{6}\sqrt{a}\frac{2}{3}x^{3/2}\right)_0^{2a} + \left(\frac{x}{2}\sqrt{16a^2 - x^2} + 8a^2\sin^{-1}\frac{x}{4a}\right)_{2a}^{4a}\right]$ 1

$= 2\left[\frac{8\sqrt{3}a^2}{3} + 8a^2\frac{\pi}{2} - 2a^2\sqrt{3} - 8a^2\frac{\pi}{6}\right]$

$= 2\left[\frac{2\sqrt{3}a^2}{3} + 8a^2\frac{\pi}{3}\right]$ sq. units 1

24. Points on the lines are $a_1 = (1, -1, 0)$, $a_2 = (0, 2, -1)$

and the direction of lines is $2\hat{i} - \hat{j} + 3\hat{k}$

let the equation of plane through a_1 be

$a(x-1) + b(y+1) + c(z) = 0$ ...(i) $\frac{1}{2}$

$(0, 2, -1)$ lies on it, $\therefore -a + 3b - c = 0$ ...(ii) 1

and a, b, c are DR's of a line $\perp$ to the line with DR's 2, –1, 3

$\therefore 2a - b + 3c = 0$ ...(iii) 1

Solving (ii) & (iii) we get $\frac{a}{8} = \frac{b}{1} = \frac{c}{-5}$ 1

$\therefore$ Equation of plane is $8(x - 1) + 1(y + 1) - 5z = 0$

$\Rightarrow 8x + y - 5z = 7$ ...(iv) $\frac{1}{2}$

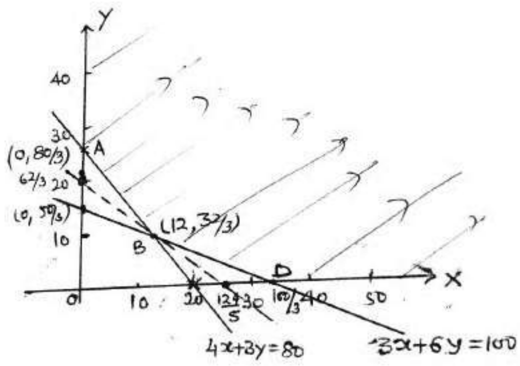
For the line $\frac{x-2}{3} = \frac{y-1}{1} = \frac{z-2}{5}$, since the point (2, 1, 2) lies on plane (iv)

as $8(2) + 1 - 5(2) = 7$ 1

and $3(8) + 1(1) + 5(-5) = 25 - 25 = 0$

$\therefore$ The plane (iv) contains the given line 1

25.



Let x units of F_1 and y units of F_2 be mixed

$\therefore$ We have Minimise cost (C) = $5x + 6y$ 1

subject to
$$\left.\begin{aligned} 4x + 3y &\geq 80 \\ 3x + 6y &\geq 100 \\ x \geq 0,\ y &\geq 0 \end{aligned}\right\}$$ 2

Correct Figure $1\frac{1}{2}$

C(A) = 160

C(B) = 60 + 64 = 124

C(D) = $\frac{500}{3}$

$5x + 6y \leq 124$ passes through B only $\frac{1}{2}$

$\therefore$ Minimum cost = ₹ 124

F_1 = 12 units

$F_2 = \frac{32}{3}$ units 1

26. Total number of ways = ${}^6C_3 = 20$ $\frac{1}{2}$

X :	1	2	3	4
P(X) :	$\frac{10}{20}$	$\frac{6}{20}$	$\frac{3}{20}$	$\frac{1}{20}$
XP(X) :	$\frac{10}{20}$	$\frac{12}{20}$	$\frac{9}{20}$	$\frac{4}{20}$
X^2 P(X) :	$\frac{10}{20}$	$\frac{24}{20}$	$\frac{27}{20}$	$\frac{16}{20}$

2

$1\frac{1}{2}$

Mean = $\sum X\,P(X) = \frac{35}{20} = \frac{7}{4}$ 1

Variance = $\sum X^2\,P(X) - \left[\sum X\,P(X)\right]^2 = \frac{77}{20} - \frac{49}{16} = \frac{63}{80}$ 1