QUESTION PAPER CODE 65/2/1
EXPECTED ANSWER/VALUE POINTS

SECTION A

1. $|A'|\,|A| = |I| \Rightarrow |A|^2 = 1$ $\frac{1}{2}$

$\therefore\ |A| = 1$ or $|A| = -1$ $\frac{1}{2}$

2. For $x < 0$, $y = x\,|x| = -x^2$ $\frac{1}{2}$

$\therefore\ \frac{dy}{dx} = -2x$ $\frac{1}{2}$

3. Order = 2, Degree not defined $\frac{1}{2}+\frac{1}{2}$

4. D. Rs are 1, 1, 1

$\therefore$ Direction cosines of the line are:

$\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}$ 1

OR

Equation of the line is:

$\frac{x-2}{1} = \frac{y+1}{1} = \frac{z-4}{-2}$ 1

SECTION B

5. $\left.\begin{array}{l}\forall\, a, b \in \mathbb{R}, \sqrt{a^2+b^2} \in \mathbb{R} \\ \therefore * \text{ is a binary operation on } \mathbb{R}\end{array}\right\}$ 1

Also,

$\left.\begin{array}{l} a*(b*c) = a*\sqrt{b^2+c^2} = \sqrt{a^2+b^2+c^2} \\ (a*b)*c = \sqrt{a^2+b^2}*c = \sqrt{a^2+b^2+c^2}\end{array}\right\} \left.\begin{array}{l}\Rightarrow\ a*(b*c) = (a*b)*c \\ \therefore\ * \text{ is Associative}\end{array}\right\}$ 1

6. $(A - 2I)(A - 3I) = \begin{bmatrix} 2 & 2 \\ -1 & -1 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ -1 & -2 \end{bmatrix}$ 1

$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = O$ 1

7. $\int \sqrt{3-2x-x^2}\,dx = \int \sqrt{2^2-(x+1)^2}\,dx$ 1

$= \frac{x+1}{2}\sqrt{3-2x-x^2} + 2\sin^{-1}\left(\frac{x+1}{2}\right) + c$ 1

8. $\int \frac{\sin^3 x + \cos^3 x}{\sin^2 x \cos^2 x}dx = \int (\sec x \cdot \tan x + \text{cosec}\, x \cdot \cot x)\,dx$ 1

$= \sec x - \text{cosec}\, x + c$ 1

OR

$\int \frac{x-3}{(x-1)^3} e^x dx = \int e^x \{(x-1)^{-2} - 2(x-1)^{-3}\}\,dx$ 1

$= e^x (x-1)^{-2} + c$ or $\frac{e^x}{(x-1)^2} + c$ 1

9. Differentiating $y = Ae^{2x} + Be^{-2x}$, we get

$\frac{dy}{dx} = 2Ae^{2x} - 2Be^{-2x}$, differentiate again to get, 1

$\frac{d^2y}{dx^2} = 4Ae^{2x} + 4Be^{-2x} = 4y$ or $\frac{d^2y}{dx^2} - 4y = 0$ 1

10. Let θ be the angle between $\vec{a}$ & $\vec{b}$, then

$\sin\theta = \frac{|\vec{a}\times\vec{b}|}{|\vec{a}||\vec{b}|} = \frac{7}{2.7} = \frac{1}{2}$ $1\frac{1}{2}$

$\Rightarrow \quad \theta = \frac{\pi}{6}$ or $\frac{5\pi}{6}$ $\frac{1}{2}$

OR

$(-3\hat{i}+7\hat{j}+5\hat{k})\cdot\{(-5\hat{i}+7\hat{j}-3\hat{k})\times(7\hat{i}-5\hat{j}-3\hat{k})\} = \begin{vmatrix} -3 & 7 & 5 \\ -5 & 7 & -3 \\ 7 & -5 & -3 \end{vmatrix}$ 1

$= -264$ $\frac{1}{2}$

∴ Volume of cuboid = 264 cubic units $\frac{1}{2}$

11. $P(\bar{A}) = 0.7 \Rightarrow 1 - P(A) = 0.7 \Rightarrow P(A) = 0.3$ $\frac{1}{2}$

$P(A \cap B) = P(A) \cdot P(B|A) = 0.3 \times 0.5 = 0.15$ $\frac{1}{2}$

$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{0.15}{0.7} = \frac{15}{70}$ or $\frac{3}{14}$ 1

12. (i) $P(3 \text{ heads}) = {}^5C_3\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right)^2 = \frac{5}{16}$ 1

(ii) $P(\text{At most 3 heads}) = P\ (r \leq 3)$

$$\left.\begin{aligned} &= 1 - P(4 \text{ heads or } 5 \text{ heads}) \\ &= 1 - \left\{{}^5C_4\left(\frac{1}{2}\right)^4\left(\frac{1}{2}\right) + {}^5C_5\left(\frac{1}{2}\right)^5\right\} \\ &= \frac{26}{32} \text{ or } \frac{13}{16} \end{aligned}\right\}$$ 1

OR

X = No. of heads in simultaneous toss of two coins.

X:	0	1	2
P(x):	1/4	1/2	1/4

$\frac{1}{2}$

$1\frac{1}{2}$

SECTION C

13. R = {(1, 2), (2, 3), (3, 4), (4, 5), (5, 6)}

For $1 \in A$, $(1, 1) \notin R \Rightarrow$ R is not reflexive 1

For $1, 2 \in A$, $(1, 2) \in R$ but $(2, 1) \notin R \Rightarrow$ R is not symmetric $1\frac{1}{2}$

For $1, 2, 3 \in A$, $(1, 2), (2, 3) \in R$ but $(1, 3) \notin R \Rightarrow$ R is not transitive $1\frac{1}{2}$

OR

One-One: Let for $x_1, x_2 \in N$, $f(x_1) = f(x_2) \Rightarrow 4x_1 + 3 = 4x_2 + 3 \Rightarrow x_1 = x_2$ 2

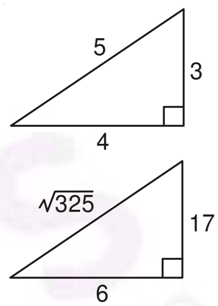
$\therefore$ 'f ' is one-one

Onto: co-domain of f = Range of f = Y 1

$\therefore$ 'f ' is onto

$\therefore$ f is invertible with, $f^{-1}: Y \to N$ and $f^{-1}(y) = \frac{y-3}{4}$ or $f^{-1}(x) = \frac{x-3}{4}, x \in Y$ 1

14. $\sin\left[\cos^{-1}\left(\frac{4}{5}\right) + \tan^{-1}\left(\frac{2}{3}\right)\right]$



$= \sin\left[\tan^{-1}\left(\frac{3}{4}\right) + \tan^{-1}\left(\frac{2}{3}\right)\right]$ 1

$= \sin\left[\tan^{-1}\left(\frac{3/4 + 2/3}{1 - 3/4 \cdot 2/3}\right)\right] = \sin\left[\tan^{-1}\left(\frac{17}{6}\right)\right]$ $1\frac{1}{2}$

$= \sin\left[\sin^{-1}\left(\frac{17}{\sqrt{325}}\right)\right] = \frac{17}{\sqrt{325}}$ $1\frac{1}{2}$

15. $\begin{vmatrix} 3a & -a+b & -a+c \\ -b+a & 3b & -b+c \\ -c+a & -c+b & 3c \end{vmatrix}$

$= \begin{vmatrix} a+b+c & -a+b & -a+c \\ a+b+c & 3b & -b+c \\ a+b+c & -c+b & 3c \end{vmatrix} \begin{pmatrix} \text{By applying,} \\ C_1 \to C_1 + C_2 + C_3 \end{pmatrix}$ 1

$= \begin{vmatrix} a+b+c & -a+b & -a+c \\ 0 & 2b+a & a-b \\ 0 & a-c & 2c+a \end{vmatrix} \begin{cases} \text{By applying,} \\ R_2 \to R_2 - R_1; \\ R_3 \to R_3 - R_1 \end{cases}$ 2

$= (a + b + c)\{4bc + 2ab + 2ac + a^2 - (a^2 - ac - ba + bc)\}$

$= 3(a + b + c)(ab + bc + ca)$ 1

16. $x\sqrt{1+y}+y\sqrt{1+x}=0 \Rightarrow x\sqrt{1+y}=-y\sqrt{1+x}$ $\frac{1}{2}$

Squaring to get: $x^2(1+y)=y^2(1+x)$ $\frac{1}{2}$

Simplifying to get: $(x-y)(x+y+xy)=0$ 1

As, $x \neq y \quad \therefore \quad y=-\frac{x}{1+x}$ 1

Differentiating w.r.t. 'x', we get:

$$\frac{dy}{dx}=\frac{-1(1+x)-(-x)\cdot 1}{(1+x)^2}=\frac{-1}{(1+x)^2}$$ 1

OR

$(\cos x)^y=(\sin y)^x \Rightarrow y\cdot\log(\cos x)=x\cdot\log(\sin y)$ 1

Differentiating w.r.t 'x'

$$\Rightarrow \frac{dy}{dx}\cdot\log(\cos x)+y(-\tan x)=\log(\sin y)+x\cdot\cot y\cdot\frac{dy}{dx}$$ 2

$$\Rightarrow \frac{dy}{dx}=\frac{y\cdot\tan x+\log(\sin y)}{\log(\cos x)-x\cot y}$$ 1

17. $(x-a)^2+(y-b)^2=c^2,\ c>0$

Differentiating both sides with respect to 'x', we get

$$2(x-a)+2(y-b)\cdot\frac{dy}{dx}=0 \Rightarrow \frac{dy}{dx}=-\frac{x-a}{y-b}$$ $1\frac{1}{2}$

Differentiating again with respect to 'x', we get;

$$\frac{d^2y}{dx^2}=-\frac{(y-b)-(x-a)\cdot\frac{dy}{dx}}{(y-b)^2}=\frac{-c^2}{(y-b)^3}\quad \text{(By substituting '}\tfrac{dy}{dx}\text{')}$$ $1+\frac{1}{2}$

$$\frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\frac{d^2y}{dx^2}}=\frac{\left[1+\frac{(x-a)^2}{(y-b)^2}\right]^{3/2}}{-\frac{c^2}{(y-b)^3}}=\frac{\frac{c^3}{(y-b)^3}}{-\frac{c^2}{(y-b)^3}}=-c$$ 1

Which is a constant independent of 'a' & 'b'.

18. Let (α, β) be the point on the curve where normal

passes through $(-1, 4)$ $\therefore$ $\alpha^2 = 4\beta$, also $\frac{dy}{dx} = \frac{x}{2}$

Slope of normal at (α, β) = $\frac{-1}{\left.\frac{dy}{dx}\right]_{(\alpha,\beta)}} = \frac{-1}{\frac{\alpha}{2}} = \frac{-2}{\alpha}$ 1

Equation of normal: $y - 4 = \frac{-2}{\alpha}(x+1)$ $\frac{1}{2}$

(α, β) lies on the normal $\Rightarrow$ $\beta - 4 = \frac{-2}{\alpha}(\alpha+1)$

Putting $\beta = \frac{\alpha^2}{4}$, we get; $\alpha^3 - 8\alpha + 8 = 0 \Rightarrow (\alpha - 2)(\alpha^2 + 2\alpha - 4) = 0$ 1

For $\alpha = 2$ Equation of normal is: $x + y - 3 = 0$ 1

For $\alpha = \pm\sqrt{5} - 1$; Equation of normal is: $y - 4 = \frac{-2}{\pm\sqrt{5}-1}(x+1)$ $\frac{1}{2}$

19. $\int \frac{x^2+x+1}{(x+2)(x^2+1)}dx = \frac{3}{5}\int\frac{1}{x+2}dx + \frac{1}{5}\int\frac{2x}{x^2+1}dx + \frac{1}{5}\int\frac{1}{x^2+1}dx$ 2

$= \frac{3}{5}\log|x+2| + \frac{1}{5}\log|x^2+1| + \frac{1}{5}\tan^{-1}x + c$ 2

20. $\int_0^a f(x)\,dx = -\int_a^0 f(a-t)dx$ Put $x = a - t, dx = -dt$

Upper limit = $t = a - x = a - a = 0$

Lower limit = $t = a - x = a - 0 = a$

$= \int_0^a f(a-t)\,dt = \int_0^a f(a-x)\,dx$ 1

Let $I = \int_0^{\pi/2} \frac{x}{\sin x + \cos x}dx$ ...(i)

$\Rightarrow I = \int_0^{\pi/2} \frac{\pi/2 - x}{\sin\left(\frac{\pi}{2}-x\right)+\cos\left(\frac{\pi}{2}-x\right)}dx$ $\therefore$ $I = \int_0^{\pi/2}\frac{\pi/2 - x}{\cos x + \sin x}dx$ ...(ii) 1

Adding, (i) and (ii) we get

$$2I = \frac{\pi}{2}\int_0^{\pi/2}\frac{1}{\cos x+\sin x}dx = \frac{\pi}{2\sqrt{2}}\int_0^{\pi/2}\frac{1}{\cos\left(x-\frac{\pi}{4}\right)}dx$$

$$= \frac{\pi}{2\sqrt{2}}\int_0^{\pi/2}\sec\left(x-\frac{\pi}{4}\right)dx$$ $\frac{1}{2}$

$$\Rightarrow\ 2I = \frac{\pi}{2\sqrt{2}}\left\{\log\left|\sec\left(x-\frac{\pi}{4}\right)+\tan\left(x-\frac{\pi}{4}\right)\right|\right\}_0^{\frac{\pi}{2}}$$ $\frac{1}{2}$

$$= \frac{\pi}{2\sqrt{2}}\{\log(\sqrt{2}+1)-\log(\sqrt{2}-1)\}$$ $\frac{1}{2}$

$$\Rightarrow\ I = \frac{\pi}{4\sqrt{2}}\{\log(\sqrt{2}+1)-\log(\sqrt{2}-1)\}\ \text{or}\ \frac{\pi}{4\sqrt{2}}\log\left(\frac{\sqrt{2}+1}{\sqrt{2}-1}\right)$$ $\frac{1}{2}$

21. The given differential equation can be written as:

$$\frac{dy}{dx} = \frac{y}{x} - \tan\left(\frac{y}{x}\right)$$ $\frac{1}{2}$

Put $\frac{y}{x} = v$ and $\frac{dy}{dx} = v + x\frac{dv}{dx}$, to get 1

$$v + x\frac{dv}{dx} = v - \tan v \Rightarrow x\frac{dv}{dx} = -\tan v$$

$$\Rightarrow\ \cot v\ dv = -\frac{1}{x}dx,$$ 1

Integrating both sides we get,

$$\left.\begin{aligned}&\log|\sin v| = -\log|x| + \log c\\ \Rightarrow\ &\log|\sin v| = \log\left|\frac{c}{x}\right|\end{aligned}\right\}$$ 1

$\therefore$ Solution of differential equation is

$$\sin\left(\frac{y}{x}\right) = \frac{c}{x}\ \text{or}\ x\cdot\sin\left(\frac{y}{x}\right) = c$$ $\frac{1}{2}$

OR

The given differential equation can be written as:

$$\frac{dy}{dx} + \frac{\cos x}{1+\sin x} \cdot y = \frac{-x}{1+\sin x};$$ $1\frac{1}{2}$

I.F. $= e^{\int \frac{\cos x}{1+\sin x} dx} = e^{\log(1+\sin x)} = 1 + \sin x$ 1

$\therefore$ Solution of the given differential equation is:

$$y(1 + \sin x) = \int \frac{-x}{1+\sin x} \times (1+\sin x)\, dx + c$$ 1

$$\Rightarrow y(1 + \sin x) = \frac{-x^2}{2} + c \text{ or } y = \frac{-x^2}{2(1+\sin x)} + \frac{c}{1+\sin x}$$ $\frac{1}{2}$

22. $\vec{a} \cdot \frac{(\vec{b}+\vec{c})}{|\vec{b}+\vec{c}|} = 1$ 1

$\Rightarrow (\hat{i}+\hat{j}+\hat{k}) \cdot \{(2+\lambda)\hat{i} + 6\hat{j} - 2\hat{k}\} = \sqrt{(2+\lambda)^2 + 36 + 4}$ $1\frac{1}{2}$

$\Rightarrow \lambda + 6 = \sqrt{(2+\lambda)^2 + 40}$

Squaring to get

$\lambda^2 + 12\lambda + 36 = \lambda^2 + 4\lambda + 44 \Rightarrow \lambda = 1$ $\frac{1}{2}$

$\therefore$ Unit vector along $(\vec{b}+\vec{c})$ is $\frac{3}{7}\hat{i} + \frac{6}{7}\hat{j} - \frac{2}{7}\hat{k}$ 1

23. Lines are perpendicular

$\therefore$ $-3(3\lambda) + 2\lambda(2) + 2(-5) = 0 \Rightarrow \lambda = -2$ 2

$$\begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_3 \end{vmatrix} = \begin{vmatrix} 1-1 & 1-2 & 6-3 \\ -3 & 2(-2) & 2 \\ 3(-2) & 2 & -5 \end{vmatrix} = -63 \neq 0$$ $1\frac{1}{2}$

$\therefore$ Lines are not intersecting $\frac{1}{2}$

SECTION D

24. $|A| = 11$; $\text{Adj}(A) = \begin{bmatrix} -1 & 1 & 2 \\ 8 & -19 & 6 \\ -3 & 14 & -5 \end{bmatrix}$ 1+2

$\therefore \quad A^{-1} = \frac{1}{|A|} \cdot \text{Adj}\,A = \frac{1}{11}\begin{bmatrix} -1 & 1 & 2 \\ 8 & -19 & 6 \\ -3 & 14 & -5 \end{bmatrix}$ $\frac{1}{2}$

Taking; $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$; $B = \begin{bmatrix} 8 \\ 5 \\ 7 \end{bmatrix}$

The system of equations in matrix form is

$A \cdot X = B \qquad \therefore X = A^{-1} \cdot B$ 1

$\therefore$ Solution is:

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{11}\begin{bmatrix} -1 & 1 & 2 \\ 8 & -19 & 6 \\ -3 & 14 & -5 \end{bmatrix}\begin{bmatrix} 8 \\ 5 \\ 7 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$ 1

$\therefore \quad x = 1,\ y = 1,\ z = 1$ $\frac{1}{2}$

OR

We know: A = IA

$\Rightarrow \begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$ 1

$R_2 \to R_2 - 2R_1$

$$\Rightarrow \begin{bmatrix} 2 & 0 & -1 \\ 1 & 1 & 2 \\ 0 & 1 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

$R_1 \leftrightarrow R_2$

$$\Rightarrow \begin{bmatrix} 1 & 1 & 2 \\ 2 & 0 & -1 \\ 0 & 1 & 3 \end{bmatrix} = \begin{bmatrix} -2 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

$R_2 \to R_2 - 2R_1$

$$\Rightarrow \begin{bmatrix} 1 & 1 & 2 \\ 0 & -2 & -5 \\ 0 & 1 & 3 \end{bmatrix} = \begin{bmatrix} -2 & 1 & 0 \\ 5 & -2 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

$R_2 \to R_2 + 2R_3$

$$\Rightarrow \begin{bmatrix} 1 & 1 & 2 \\ 0 & 0 & 1 \\ 0 & 1 & 3 \end{bmatrix} = \begin{bmatrix} -2 & 1 & 0 \\ 5 & -2 & 2 \\ 0 & 0 & 1 \end{bmatrix} A$$

4

$R_2 \leftrightarrow R_3$

$$\Rightarrow \begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -2 & 1 & 0 \\ 0 & 0 & 1 \\ 5 & -2 & 2 \end{bmatrix} A$$

$R_2 \to R_2 - 3R_3$

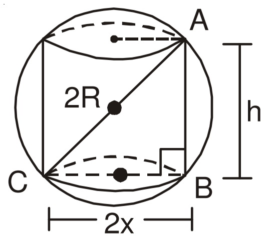
$$\Rightarrow \begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -2 & 1 & 0 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix} A$$

$R_1 \to R_1 - R_2 - 2R_3$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix} A$$

$$\therefore \quad A^{-1} = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$$ 1

25. Correct Figure 1



In rt. ΔABC; $4x^2 + h^2 = 4R^2$, $x^2 = \dfrac{4R^2 - h^2}{4}$ 1

V(Volume of cylinder) = $\pi x^2 h = \dfrac{\pi}{4}(4R^2h - h^3)$ 1

$V'(h) = \dfrac{\pi}{4}(4R^2 - 3h^2)$; $V''(h) = \dfrac{\pi}{4}(-6h)$ $\frac{1}{2}+\frac{1}{2}$

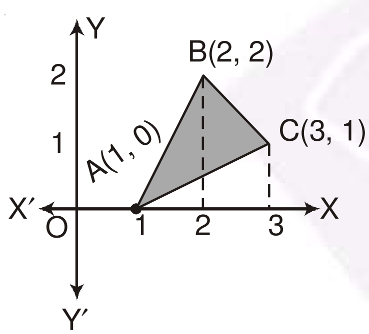
$V'(h) = 0 \Rightarrow h = \dfrac{2R}{\sqrt{3}}$ 1

$V''\left(\dfrac{2R}{\sqrt{3}}\right) = \dfrac{-6\pi}{4}\left(\dfrac{2R}{\sqrt{3}}\right) < 0 \Rightarrow$ Volume 'V' is max. $\frac{1}{2}$

for $h = \dfrac{2R}{\sqrt{3}}$

Max. Volume: $V = \dfrac{4}{3\sqrt{3}}\pi R^3$ $\frac{1}{2}$

26. Correct Figure 1



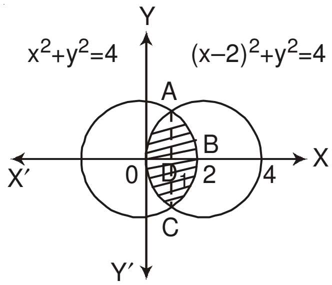
Equation of line AB : $y = 2(x-1)$
Equation of line BC : $y = 4 - x$
Equation of line AC : $y = \dfrac{1}{2}(x-1)$ $1\frac{1}{2}$

$\text{ar}(\Delta ABC) = 2\int_1^2 (x-1)\,dx + \int_2^3 (4-x)\,dx - \dfrac{1}{2}\int_1^3 (x-1)\,dx$ $1\frac{1}{2}$

$= (x-1)^2\Big]_1^2 - \dfrac{1}{2}(4-x)^2\Big]_2^3 - \dfrac{1}{4}(x-1)^2\Big]_1^3$ $1\frac{1}{2}$

$= 1 + \dfrac{3}{2} - 1 = \dfrac{3}{2}$ $\frac{1}{2}$

OR



Correct Figure 1

Getting the point of intersection as x = 1 1

Area (OABCO) = 4 × ar(ABD)

$$= 4\int_1^2 \sqrt{2^2 - x^2}\,dx$$ 2

$$= 4\left\{\frac{x\sqrt{4-x^2}}{2} + 2\sin^{-1}\left(\frac{x}{2}\right)\right\}_1^2$$ 1

$$= \left(\frac{8\pi}{3} - 2\sqrt{3}\right)$$ 1

27. Let $\vec{a} = \hat{i} + \hat{j} - 2\hat{k},\ \vec{b} = 2\hat{i} - \hat{j} + \hat{k},\ \vec{c} = \hat{i} + 2\hat{j} + \hat{k}$

$$(\vec{b} - \vec{a}) \times (\vec{c} - \vec{a}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & 3 \\ 0 & 1 & 3 \end{vmatrix} = -9\hat{i} - 3\hat{j} + \hat{k}$$ 2

Vector equation of plane is:

$$\{\vec{r} - (\hat{i} + \hat{j} - 2\hat{k})\} \cdot (-9\hat{i} - 3\hat{j} + \hat{k}) = 0$$

$\Rightarrow \quad \vec{r} \cdot (-9\hat{i} - 3\hat{j} + \hat{k}) + 14 = 0$ 1

Cartesian Equation of plane is: $-9x - 3y + z + 14 = 0$ 1

Equation of plane through (2, 3, 7) and parallel to above plane is:

$$-9(x - 2) - 3(y - 3) + (z - 7) = 0$$

$\Rightarrow \quad -9x - 3y + z + 20 = 0$ 1

Distance between parallel planes = $\left|\frac{-14 + 20}{\sqrt{91}}\right| = \frac{6}{\sqrt{91}}$ 1

OR

Equation of line: $\frac{x-2}{3}=\frac{y+1}{4}=\frac{z-2}{2}$ 1

Equation of plane: $\begin{vmatrix} x-2 & y & z-3 \\ -1 & 1 & 2 \\ 1 & 2 & 1 \end{vmatrix}=0$ 1

$\Rightarrow\ -3(x-2)+3y-3(z-3)=0$

$\Rightarrow\ x-y+z-5=0$ 1

General point on line: $\frac{x-2}{3}=\frac{y+1}{4}=\frac{z-2}{2}$

is: P(3k + 2, 4k – 1, 2k + 2); Putting in the equation of plane 1

we get, $3k+2-4k+1+2k+2=5 \Rightarrow k=0$ 1

$\therefore$ Point of intersection is: (2, –1, 2) 1

28. Let E_1 = Event that two-headed coin is chosen
E_2 = Event that biased coin is chosen
E_3 = Event that unbiased coin is chosen
A = Event that coin tossed shows head 1

Then, $P(E_1)=P(E_2)=P(E_3)=1/3$ 1

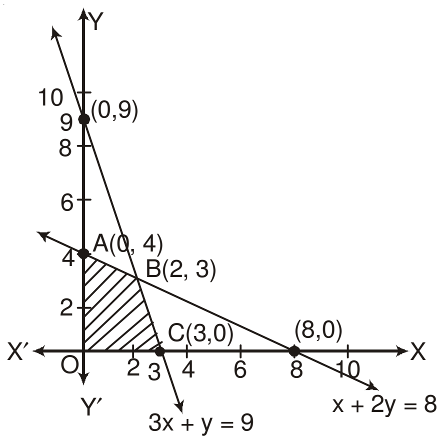
$P(A|E_1)=1,\ P(A|E_2)=\frac{75}{100}=\frac{3}{4},\ P(A|E_3)=\frac{1}{2}$ $1\frac{1}{2}$

$$P(E_1|A)=\frac{P(E_1)\cdot P(A|E_1)}{P(E_1)\cdot P(A|E_1)+P(E_2)\cdot P(A|E_2)+P(E_3)\cdot P(A|E_3)}$$

$$=\frac{\frac{1}{3}\cdot 1}{\frac{1}{3}\cdot 1+\frac{1}{3}\cdot\frac{3}{4}+\frac{1}{3}\cdot\frac{1}{2}}=\frac{4}{9}$$ $2+\frac{1}{2}$

29. let the company produce: Goods A = x units

Goods B = y units



then, the linear programming problem is:

Maximize profit: z = 40x + 50y (In ₹) $\frac{1}{2}$

Subject to constraints:

$$\left.\begin{array}{l} 3x + y \le 9 \\ x + 2y \le 8 \\ x, y \ge 0 \end{array}\right\}$$ $2\frac{1}{2}$

Correct graph: 2

Corner point	Value of z (₹)
A(0, 4)	200
B(2, 3)	230 (Max)
C(3, 0)	120

$\frac{1}{2}$

∴ Maximum profit = ₹ 230 at:

Goods A produced = 2 units, Goods B produced = 3 units $\frac{1}{2}$