

CBSE Class 12 Maths Question Paper Solution 2019

QUESTION PAPER CODE 65/3/1 EXPECTED ANSWER/VALUE POINTS

SECTION A

1. $|-2A| = (-2)^3 \cdot |A|$ $\frac{1}{2}$
 $= -8 \cdot 4 = -32$ $\frac{1}{2}$
2. $y = \frac{\pi}{2} \Rightarrow \frac{dy}{dx} = 0$ $\frac{1}{2} + \frac{1}{2}$
3. order 4, degree 2 $\frac{1}{2} + \frac{1}{2}$
4. $\sqrt{(-18)^2 + (12)^2 + (-4)^2} = 22$ $\frac{1}{2}$
 \therefore DC's are $\frac{-18}{22}, \frac{12}{22}, \frac{-4}{22}$ or $\frac{-9}{11}, \frac{6}{11}, \frac{-2}{11}$ $\frac{1}{2}$
 OR
 D.R's of required line are 3, -5, 6 $\frac{1}{2}$
 Equation of line is $\frac{x+2}{3} = \frac{y-4}{-5} = \frac{z+5}{6}$ $\frac{1}{2}$

SECTION B

5. Let $e \in \mathbb{R}$ be the identity element.
 then $a * e = e * a = a$ 1
 $\Rightarrow a^2 + e^2 = e^2 + a^2 = a^2 \Rightarrow e^2 = 0 \Rightarrow e = 0.$ 1
 \therefore Identity element is $0 \in \mathbb{R}$
6. $kA = k \begin{bmatrix} 0 & 2 \\ 3 & -4 \end{bmatrix} = \begin{bmatrix} 0 & 2k \\ 3k & -4k \end{bmatrix}$
 $\therefore \begin{bmatrix} 0 & 2k \\ 3k & -4k \end{bmatrix} = \begin{bmatrix} 0 & 3a \\ 2b & 24 \end{bmatrix} \Rightarrow 2k = 3a, 3k = 2b \text{ and } -4k = 24$ $\frac{1}{2}$
 $\Rightarrow k = -6, a = \frac{-12}{3} = -4, b = \frac{-18}{2} = -9$ $\frac{1}{2}$
7. $\int \frac{\sin x - \cos x}{\sqrt{1 + \sin 2x}} dx = \int \frac{\sin x - \cos x}{\sin x + \cos x} dx$ 1
 $= -\log |\sin x + \cos x| + c$ 1

$$\begin{aligned}
 8. \int \frac{\sin(x-a)}{\sin(x+a)} dx &= \int \frac{\sin(x+a-2a)}{\sin(x+a)} dx && \frac{1}{2} \\
 &= \int \left[\frac{\sin(x+a)\cos 2a}{\sin(x+a)} - \frac{\cos(x+a)\sin 2a}{\sin(x+a)} \right] dx && \frac{1}{2} \\
 &= x \cdot \cos 2a - \sin 2a \cdot \log |\sin(x+a)| + c && \frac{1}{2} + \frac{1}{2}
 \end{aligned}$$

OR

$$\begin{aligned}
 \int (\log x)^2 \cdot 1 dx &= (\log x)^2 \cdot x - \int 2 \cdot \log x \cdot \frac{1}{x} \cdot x dx && 1 \\
 &= x \cdot (\log x)^2 - \left\{ \log x \cdot 2x - \int \frac{1}{x} \cdot 2x dx \right\} && \frac{1}{2} \\
 &= x(\log x)^2 - 2x \log x + 2x + c && \frac{1}{2}
 \end{aligned}$$

$$9. y^2 = m(a^2 - x^2) \Rightarrow 2y \frac{dy}{dx} = -2mx \quad \frac{1}{2}$$

$$\text{or } y \frac{dy}{dx} = -mx \quad \dots(i)$$

$$y \frac{d^2y}{dx^2} + \left(\frac{dy}{dx} \right)^2 = -m \quad \dots(ii) \quad \frac{1}{2}$$

$$\text{form (i) and (ii) we get } y \frac{d^2y}{dx^2} + \left(\frac{dy}{dx} \right)^2 = \frac{y}{x} \frac{dy}{dx} \quad 1$$

$$\text{or } xy \frac{d^2y}{dx^2} + x \left(\frac{dy}{dx} \right)^2 - y \frac{dy}{dx} = 0$$

$$10. \text{ A vector perpendicular to both } \vec{a} \text{ and } \vec{b} = \vec{a} \times \vec{b} = 19\hat{j} + 19\hat{k} \text{ or } \hat{j} + \hat{k} \quad 1$$

$$\therefore \text{ Unit vector perpendicular to both } \vec{a} \text{ and } \vec{b} = \frac{1}{\sqrt{2}}(\hat{j} + \hat{k}) \quad 1$$

OR

$$\text{Let } \vec{a} = \hat{i} - 2\hat{j} + 3\hat{k}, \vec{b} = -2\hat{i} + 3\hat{j} - 4\hat{k}, \vec{c} = \hat{i} - 3\hat{j} + 5\hat{k}$$

$$\vec{a}, \vec{b}, \vec{c} \text{ are coplanar if } \vec{a} \cdot \vec{b} \times \vec{c} = 0 \quad \frac{1}{2}$$

$$\vec{a} \cdot \vec{b} \times \vec{c} = \left. \begin{array}{l} \begin{vmatrix} 1 & -2 & 3 \\ -2 & 3 & -4 \\ 1 & -3 & 5 \end{vmatrix} = 1(3) + 2(-6) + 3(3) \\ = 3 - 12 + 9 = 0 \end{array} \right\} 1 + \frac{1}{2}$$

Hence $\vec{a}, \vec{b}, \vec{c}$ are coplanar

$$11. \left. \begin{array}{l} A = \{(S, F, M), (S, M, F), (M, F, S), (F, M, S)\} \\ B = \{(S, F, M), (M, F, S)\} \end{array} \right\} 1$$

Total number of possible arrangements = 6

$$P(B|A) = \frac{P(B \cap A)}{P(A)} = \frac{2/6}{4/6} = \frac{1}{2} \quad 1$$

$$12. \text{ Given } 2P(X = x_1) = 3P(X = x_2) = P(X = x_3) = 5P(X = x_4)$$

$$\text{Let } P(X = x_3) = k, \text{ then } P(X = x_1) = \frac{k}{2}, P(X = x_2) = \frac{k}{3} \text{ and } P(X = x_4) = \frac{k}{5} \quad \frac{1}{2}$$

$$\therefore k + \frac{k}{2} + \frac{k}{3} + \frac{k}{5} = 1 \Rightarrow k = \frac{30}{61} \quad 1$$

\therefore Probability distribution is

X	x_1	x_2	x_3	x_4
P(X)	$\frac{15}{61}$	$\frac{10}{61}$	$\frac{30}{61}$	$\frac{6}{61}$

1/2

OR

$$(i) P(\text{at least 4 heads}) = P(r \geq 4) = P(4) + P(5)$$

$$= {}^5C_4 \left(\frac{1}{2}\right)^1 \left(\frac{1}{2}\right)^4 + {}^5C_5 \left(\frac{1}{2}\right)^5 = 6 \left(\frac{1}{2}\right)^5 = \frac{6}{32} \text{ or } \frac{3}{16} \quad 1$$

$$(ii) P(\text{at most 4 heads}) = P(r \leq 4) = 1 - P(5)$$

$$= 1 - \left(\frac{1}{2}\right)^5 = \frac{31}{32} \quad 1$$

SECTION C

13. (i) For $a \in \mathbb{Z}, (a, a) \in R \because a - a = 0$ is divisible by 2

$\therefore R$ is reflexive

...(i)

1

Let $(a, b) \in R$ for $a, b \in \mathbb{Z}$, then $a - b$ is divisible by 2

$\Rightarrow (b - a)$ is also divisible by 2

$\therefore (b, a) \in R \Rightarrow R$ is symmetric

...(ii)

1

For $a, b, c \in \mathbb{Z}$, Let $(a, b) \in R$ and $(b, c) \in R$

$\therefore a - b = 2p, p \in \mathbb{Z}$, and $b - c = 2q, q \in \mathbb{Z}$,

adding, $a - c = 2(p + q) \Rightarrow (a - c)$ is divisible by 2

$\Rightarrow (a, c) \in R$, so R is transitive

...(iii)

$\frac{1}{2}$

(i), (ii), and (iii) $\Rightarrow R$ is an equivalence relation.

$\frac{1}{2}$

OR

$$f \circ f(x) = f\left(\frac{4x+3}{6x-4}\right)$$

1

$$= \frac{4\left[\frac{4x+3}{6x-4}\right] + 3}{6\left[\frac{4x+3}{6x-4}\right] - 4}$$

1

$$\Rightarrow f \circ f(x) = \frac{4(4x+3) + 3(6x-4)}{6(4x+3) - 4(6x-4)} = \frac{34x}{34} = x$$

1

Since $f \circ f(x) = x \Rightarrow f \circ f = I \Rightarrow f^{-1} = f$

1

14. Given $\tan^{-1} x - \cot^{-1} x = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right), x > 0$

$$\Rightarrow \tan^{-1} x - \left(\frac{\pi}{2} - \tan^{-1} x\right) = \frac{\pi}{6}$$

1

$$\Rightarrow 2 \tan^{-1} x = \frac{2\pi}{3} \Rightarrow \tan^{-1} x = \frac{\pi}{3}$$

1

$$\Rightarrow x = \tan\left(\frac{\pi}{3}\right) = \sqrt{3} \quad \therefore \sec^{-1} \frac{2}{x} = \sec^{-1} \frac{2}{\sqrt{3}} = \frac{\pi}{6}$$

1+1

15. Let $\Delta = \begin{vmatrix} b+c & a & a \\ b & c+a & b \\ c & c & a+b \end{vmatrix}$

$$R_1 \rightarrow R_1 - (R_2 + R_3) \Rightarrow \Delta = \begin{vmatrix} 0 & -2c & -2b \\ b & c+a & b \\ c & c & a+b \end{vmatrix} \quad 1$$

$$\therefore \Delta = -2 \begin{vmatrix} 0 & c & b \\ b & c+a & b \\ c & c & a+b \end{vmatrix} = -\frac{2}{b} \begin{vmatrix} 0 & bc & b \\ b & bc+ab & b \\ c & bc & a+b \end{vmatrix} \quad 1 + \frac{1}{2}$$

$$C_2 \rightarrow C_2 - cC_3$$

$$\Rightarrow \Delta = -\frac{2}{b} \begin{vmatrix} 0 & 0 & b \\ b & ab & b \\ c & -ac & a+b \end{vmatrix} \quad 1$$

$$= -\frac{2}{b} \cdot b \cdot (-abc - abc) = 4abc. \quad \frac{1}{2}$$

16. $\sin y = x \cdot \sin(a+y) \Rightarrow x = \frac{\sin y}{\sin(a+y)} \quad \frac{1}{2}$

differentiating w.r.t. y , we get

$$\frac{dx}{dy} = \frac{\sin(a+y)\cos y - \sin y \cos(a+y)}{\sin^2(a+y)} \quad \frac{1}{2}$$

$$\frac{dx}{dy} = \frac{\sin(a+y-y)}{\sin^2(a+y)} = \frac{\sin a}{\sin^2(a+y)} \quad \frac{1}{2}$$

$$\therefore \frac{dy}{dx} = \frac{\sin^2(a+y)}{\sin a} \quad \frac{1}{2}$$

OR

$$(\sin x)^y = (x+y) \Rightarrow y \cdot \log \sin x = \log(x+y) \quad 1$$

differentiating w.r.t. x , we get

$$y \cdot \cot x + \log \sin x \cdot \frac{dy}{dx} = \frac{1}{x+y} \left(1 + \frac{dy}{dx} \right) \quad \frac{1}{2}$$

$$\Rightarrow \frac{dy}{dx} = \frac{\frac{1}{x+y} - y \cot x}{\log \sin x - \frac{1}{x+y}} \quad 1$$

$$= \frac{1 - y(x+y) \cot x}{(x+y) \log \sin x - 1} \quad \frac{1}{2}$$

17. $y = (\sec^{-1} x)^2, x > 0$

$$\frac{dy}{dx} = 2 \sec^{-1} x \cdot \frac{1}{x\sqrt{x^2-1}} \quad 1$$

$$\Rightarrow x\sqrt{x^2-1} \frac{dy}{dx} = 2\sqrt{y} \quad \frac{1}{2}$$

squaring both sides, we get

$$x^2(x^2-1) \left(\frac{dy}{dx} \right)^2 = 4y \quad \text{or} \quad (x^4 - x^2) \left(\frac{dy}{dx} \right)^2 = 4y \quad \frac{1}{2}$$

differentiating w.r.t. x.

$$(x^4 - x^2) 2 \cdot \frac{dy}{dx} \cdot \frac{d^2y}{dx^2} + (4x^3 - 2x) \left(\frac{dy}{dx} \right)^2 = 4 \cdot \frac{dy}{dx} \quad \frac{1}{2}$$

$$\Rightarrow x^2(x^2-1) \frac{d^2y}{dx^2} + (2x^3 - x) \frac{dy}{dx} - 2 = 0 \quad \frac{1}{2}$$

18. Curve $y = \frac{x-7}{(x-2)(x-3)}$ cuts at x-axis at the point $x = 7, y = 0$ i.e. $(7, 0)$ $\frac{1}{2}$

$$\frac{dy}{dx} = \frac{(x^2 - 5x + 6) \cdot 1 - (x-7)(2x-5)}{(x^2 - 5x + 6)^2} \quad \frac{1}{2}$$

$$\text{at } (7, 0) \quad \frac{dy}{dx} = \frac{20}{(20)^2} = \frac{1}{20} \quad \frac{1}{2}$$

$$\therefore \text{Slope of tangent at } (7, 0) \text{ is } \frac{1}{20} \quad \frac{1}{2}$$

$$\text{and slope of Normal at } (7, 0) \text{ is } -20 \quad \frac{1}{2}$$

Equation of tangent at (7, 0) is $y - 0 = \frac{1}{20} (x - 7)$

or $x - 20y - 7 = 0$

Equation of Normal at (7, 0) is $y - 0 = -20 (x - 7)$

or $20x + y = 140.$

1

 $\frac{1}{2}$

$$19. I = \int \frac{\sin 2x}{(\sin^2 x + 1)(\sin^2 x + 3)} dx$$

Put $\sin^2 x = t \Rightarrow \sin 2x dx = dt$

 $\frac{1}{2}$

$$\therefore I = \int \frac{dt}{(t+1)(t+3)} = \int \left(\frac{1/2}{t+1} + \frac{-1/2}{t+3} \right) dt$$

 $\frac{1}{2}$

$$= \frac{1}{2} \log|t+1| - \frac{1}{2} \log|t+3| + c$$

 $\frac{1}{2}$

$$= \frac{1}{2} \log(\sin^2 x + 1) - \frac{1}{2} \log(\sin^2 x + 3) + c.$$

 $\frac{1}{2}$

$$20. \text{ RHS} = \int_a^b f(a+b-x) dx = - \int_b^a f(t) dt, \text{ where } a+b-x = t, dx = -dt$$

 $\frac{1}{2}$

$$= \int_a^b f(t) dt = \int_a^b f(x) dx = \text{LHS}$$

 $\frac{1}{2}$

$$\text{Let } I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{dx}{1 + \sqrt{\tan x}} = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{\cos x}}{\sqrt{\cos x} + \sqrt{\sin x}} dx \quad \dots(i)$$

 $\frac{1}{2}$

$$= \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{\cos(\pi/2-x)}}{\sqrt{\cos(\pi/2-x)} + \sqrt{\sin(\pi/2-x)}} dx = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}} dx \quad \dots(ii)$$

 $\frac{1}{2}$

$$\text{adding (i) and (ii) to get } 2I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} 1 \cdot dx = x \Big|_{\frac{\pi}{6}}^{\frac{\pi}{3}} = \pi/6.$$

 $\frac{1}{2}$

$$\Rightarrow I = \frac{\pi}{12}$$

 $\frac{1}{2}$

$$21. \frac{dy}{dx} = \frac{x+y}{x-y} = \frac{1+\frac{y}{x}}{1-\frac{y}{x}}$$

Put $y/x = v$ so that $\frac{dy}{dx} = v + x \frac{dv}{dx}$ 1/2

$$\therefore v + x \frac{dv}{dx} = \frac{1+v}{1-v} \Rightarrow x \frac{dv}{dx} = \frac{1+v}{1-v} - v = \frac{1+v-v+v^2}{1-v}$$

$$\Rightarrow \int \frac{1-v}{1+v^2} dv = \int \frac{dx}{x} \Rightarrow \int \frac{1}{1+v^2} dv - \frac{1}{2} \int \frac{2v}{1+v^2} dv = \int \frac{dx}{x}$$
 1+1/2

$$\Rightarrow \tan^{-1} v = \frac{1}{2} \log |1+v^2| + \log |x| + c$$
 1

$$\Rightarrow \tan^{-1} \left(\frac{y}{x} \right) = \frac{1}{2} \log \left| \frac{x^2+y^2}{x^2} \right| + \log |x| + c$$
 1

or $\tan^{-1} \left(\frac{y}{x} \right) = \frac{1}{2} \log |x^2+y^2| + c$

OR

$$(1+x^2)dy + 2xy dx = \cot x \cdot dx.$$

$$\Rightarrow \frac{dy}{dx} + \frac{2x}{1+x^2} \cdot y = \frac{\cot x}{1+x^2}$$
 1

$$\text{I.F.} = e^{\int \frac{2x}{1+x^2} dx} = e^{\log(1+x^2)} = (1+x^2)$$
 1

$$\therefore \text{Solution is, } y \cdot (1+x^2) = \int \cot x dx = \log |\sin x| + c$$
 1+1

$$\text{or } y = \frac{1}{1+x^2} \cdot \log |\sin x| + \frac{c}{1+x^2}$$

$$22. \text{ Given } \frac{\vec{b} \cdot \vec{a}}{|\vec{a}|} = \frac{\vec{c} \cdot \vec{a}}{|\vec{a}|} \therefore \vec{b} \cdot \vec{a} = \vec{c} \cdot \vec{a} \quad \dots(i)$$
 1/2

$$\vec{b} \perp \vec{c} \Rightarrow \vec{b} \cdot \vec{c} = 0 \quad \dots(ii)$$
 1/2

$$\begin{aligned}
 (13\vec{a} - 2\vec{b} + 2\vec{c})^2 &= 9|\vec{a}|^2 + 4|\vec{b}|^2 + 4|\vec{c}|^2 - 12\vec{a} \cdot \vec{b} - 8\vec{b} \cdot \vec{c} + 12\vec{a} \cdot \vec{c} & 1 \\
 &= 9(1)^2 + 4(2)^2 + 4(3)^2 & \text{[using (i) and (ii)]} & 1 \\
 &= 9 + 16 + 36 = 61 & &
 \end{aligned}$$

$$\Rightarrow |13\vec{a} - 2\vec{b} + 2\vec{c}| = \sqrt{61} \quad 1$$

23. Writing the equations of given lines in standard form, as

$$\frac{x-5}{5\lambda+2} = \frac{y-2}{-5} = \frac{z-1}{1}; \frac{x}{1} = \frac{y+\frac{1}{2}}{2\lambda} = \frac{z-1}{3} \quad \frac{1}{2}$$

lines are perpendicular to each other,

$$\Rightarrow (5\lambda+2) \cdot 1 + (-5)(2\lambda) + 1(3) = 0 \quad \frac{1}{2}$$

$$-5\lambda + 5 = 0 \Rightarrow \lambda = 1 \quad \frac{1}{2}$$

$$\therefore \text{lines are } \frac{x-5}{7} = \frac{y-2}{-5} = \frac{z-1}{1}; \frac{x}{1} = \frac{y+\frac{1}{2}}{2} = \frac{z-1}{3} \quad \frac{1}{2}$$

$$\begin{aligned}
 \text{Shortest distance between these lines} &= \frac{|(\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2)|}{|\vec{b}_1 \times \vec{b}_2|} = \frac{\left| \left(5\hat{i} + \frac{5}{2}\hat{j} \right) \cdot (-17\hat{i} - 20\hat{j} + 19\hat{k}) \right|}{|\vec{b}_1 \times \vec{b}_2|} & 1 \\
 &= \frac{135}{|\vec{b}_1 \times \vec{b}_2|} \neq 0 & \frac{1}{2}
 \end{aligned}$$

\therefore lines are not intersecting. 1/2

SECTION D

24. $|A| = 1(7) - 1(-3) + 1(-1) = 9$ 1

$$(\text{adj } A) = \begin{bmatrix} 7 & -3 & 2 \\ 3 & 0 & -3 \\ -1 & 3 & 1 \end{bmatrix} \quad 2$$

$$\Rightarrow A^{-1} = \frac{1}{9} \begin{bmatrix} 7 & -3 & 2 \\ 3 & 0 & -3 \\ -1 & 3 & 1 \end{bmatrix} \quad \frac{1}{2}$$

Given equations can be written as
$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 3 \\ 1 & -2 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 11 \\ 0 \end{bmatrix}$$

or $AX = B \Rightarrow X = A^{-1}B$

$$\therefore \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{9} \begin{bmatrix} 7 & -3 & 2 \\ 3 & 0 & -3 \\ -1 & 3 & 1 \end{bmatrix} \begin{bmatrix} 6 \\ 11 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

$\therefore x = 1, y = 2, z = 3$

OR

Let:
$$\begin{bmatrix} 2 & 3 & 1 \\ 2 & 4 & 1 \\ 3 & 7 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

$$\left. \begin{array}{l} R_1 \leftrightarrow R_3 \begin{bmatrix} 3 & 7 & 2 \\ 2 & 4 & 1 \\ 2 & 3 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} A \\ R_1 \rightarrow R_1 - R_3 \\ R_2 \rightarrow R_2 - R_3 \begin{bmatrix} 1 & 4 & 1 \\ 0 & 1 & 0 \\ 2 & 3 & 1 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 1 \\ -1 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} A \\ R_3 \rightarrow R_3 - 2R_1 \begin{bmatrix} 1 & 4 & 1 \\ 0 & 1 & 0 \\ 0 & -5 & -1 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 1 \\ -1 & 1 & 0 \\ 3 & 0 & -2 \end{bmatrix} A \\ R_3 \rightarrow R_3 + 5R_2 \begin{bmatrix} 1 & 4 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 1 \\ -1 & 1 & 0 \\ -2 & 5 & -2 \end{bmatrix} A \\ R_1 \rightarrow R_1 - 4R_2 \\ R_3 \rightarrow -R_3 \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & -4 & 1 \\ -1 & 1 & 0 \\ 2 & -5 & 2 \end{bmatrix} A \\ R_1 \rightarrow R_1 - R_3 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 & -1 \\ -1 & 1 & 0 \\ 2 & -5 & 2 \end{bmatrix} A \end{array} \right\}$$

$$\Rightarrow A^{-1} = \begin{bmatrix} 1 & 1 & -1 \\ -1 & 1 & 0 \\ 2 & -5 & 2 \end{bmatrix}$$

1

25. Let Given surface area of open cylinder be S.

$$\text{Then } S = 2\pi rh + \pi r^2$$

$$\Rightarrow h = \frac{S - \pi r^2}{2\pi r}$$

1

$$\text{Volume } V = \pi r^2 h$$

 $\frac{1}{2}$

$$V = \pi r^2 \left[\frac{S - \pi r^2}{2\pi r} \right] = \frac{1}{2} [Sr - \pi r^3]$$

1

$$\frac{dV}{dr} = \frac{1}{2} [S - 3\pi r^2]$$

 $\frac{1}{2}$

$$\frac{dV}{dr} = 0 \Rightarrow S = 3\pi r^2 \text{ or } 2\pi rh + \pi r^2 = 3\pi r^2$$

1

$$\Rightarrow 2\pi rh = 2\pi r^2 \quad \Rightarrow h = r$$

1

$$\frac{d^2V}{dr^2} = -6\pi r < 0$$

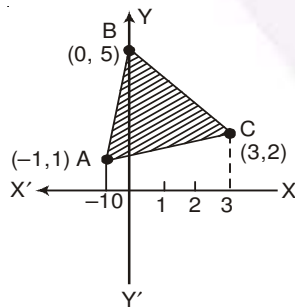
1

\therefore For volume to be maximum, height = radius

26.

Let the points be A (-1, 1), B (0, 5) and C (3, 2)

Correct Figure 1



$$\text{Equation of AB : } y = 4x + 5$$

$$\text{BC : } y = 5 - x$$

$$\text{AC : } y = \frac{1}{4}(x + 5)$$

 $\frac{1}{2}$

$$\text{Req. Area} = \int_{-1}^0 (4x + 5) dx + \int_0^3 (5 - x) dx - \int_{-1}^3 \frac{1}{4}(x + 5) dx$$

 $\frac{1}{2}$

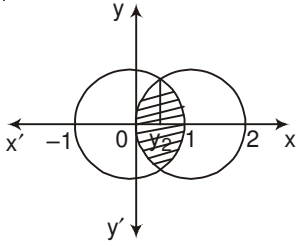
$$\therefore A = \left[\frac{(4x+5)^2}{8} \right]_{-1}^0 + \left[\frac{(5-x)^2}{-2} \right]_0^3 - \frac{1}{4} \left[\frac{(x+5)^2}{2} \right]_{-1}^3$$

1

$$= 3 + \frac{21}{2} - 6 = \frac{15}{2}$$

1

OR



Correct Figure

1

$$(x-1)^2 + y^2 = 1$$

$$\text{and } x^2 + y^2 = 1 \Rightarrow (x-1)^2 = x^2$$

$$\Rightarrow x = \frac{1}{2}$$

1

$$\therefore \text{Required area} = 2 \left[\int_0^{\frac{1}{2}} \sqrt{1-(x-1)^2} dx + \int_{\frac{1}{2}}^1 \sqrt{1-x^2} dx \right]$$

2

$$= 2 \left[\frac{x-1}{2} \sqrt{1-(x-1)^2} + \frac{1}{2} \sin^{-1}(x-1) \right]_0^{\frac{1}{2}} + 2 \left[\frac{x}{2} \sqrt{1-x^2} + \frac{1}{2} \sin^{-1} x \right]_{\frac{1}{2}}^1$$

1

$$= 2 \left[\frac{-\sqrt{3}}{8} + \frac{\pi}{6} \right] + 2 \left[\frac{-\sqrt{3}}{8} + \frac{\pi}{6} \right] = \left(\frac{2\pi}{3} - \frac{\sqrt{3}}{2} \right)$$

1

27. Equation of plane passing through (2, 5, -3), (-2, -3, 5) and (5, 3, -3) is

$$\begin{vmatrix} x-2 & y-5 & z+3 \\ -4 & -8 & 8 \\ 3 & -2 & 0 \end{vmatrix} = 0$$

1

$$\Rightarrow 16(x-2) + 24(y-5) + 32(z+3) = 0$$

$$\text{i.e. } 2x + 3y + 4z - 7 = 0$$

...(i)

1

which in vector form can be written as $\vec{r} \cdot (2\hat{i} + 3\hat{j} + 4\hat{k}) = 7$

1

Equation of line passing through (3, 1, 5) and (-1, -3, -1) is

$$\frac{x-3}{4} = \frac{y-1}{4} = \frac{z-5}{6} \text{ or } \frac{x-3}{2} = \frac{y-1}{2} = \frac{z-5}{3}$$

...(ii)

1

Any point on (ii) is $(2\lambda + 3, 2\lambda + 1, 3\lambda + 5)$

 $\frac{1}{2}$

If this is point of intersection with plane (i), then

$$2(2\lambda + 3) + 3(2\lambda + 1) + 4(3\lambda + 5) - 7 = 0$$

$$22\lambda + 22 = 0 \Rightarrow \lambda = -1$$

\(\therefore\) Point of intersection is (1, -1, 2)

OR

Equation of plane through the intersection of planes

$$\vec{r} \cdot (\hat{i} + \hat{j} + \hat{k}) - 1 = 0 \text{ and } \vec{r} \cdot (2\hat{i} + 3\hat{j} - \hat{k}) + 4 = 0, \text{ is}$$

$$[\vec{r} \cdot (\hat{i} + \hat{j} + \hat{k}) - 1] + \lambda[\vec{r} \cdot (2\hat{i} + 3\hat{j} - \hat{k}) + 4] = 0$$

$$\Rightarrow \vec{r} \cdot [(1 + 2\lambda)\hat{i} + (1 + 3\lambda)\hat{j} + (1 - \lambda)\hat{k}] - 1 + 4\lambda = 0 \dots(i)$$

$$\text{Plane (i) is } \parallel \text{ to x-axis} \Rightarrow 1 + 2\lambda = 0 \Rightarrow \lambda = \frac{-1}{2}$$

$$\therefore \text{Equation of plane is } \vec{r} \cdot \left(-\frac{1}{2}\hat{j} + \frac{3}{2}\hat{k}\right) - 3 = 0$$

$$\text{or } \vec{r} \cdot (-\hat{j} + 3\hat{k}) - 6 = 0$$

Distance of this plane from x-axis

$$= \frac{|-6|}{\sqrt{(-1)^2 + (3)^2}} = \frac{6}{\sqrt{10}} \text{ units}$$

28. Let the events be

$$\left. \begin{array}{l} E_1 : \text{bag I is selected} \\ E_2 : \text{bag II is selected} \\ A : \text{getting a red ball} \end{array} \right\}$$

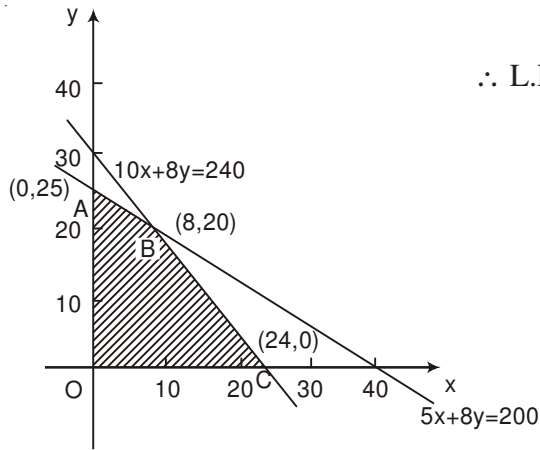
$$P(E_1) = P(E_2) = \frac{1}{2}$$

$$P(A/E_1) = \frac{3}{9} = \frac{1}{3}; \quad P(A/E_2) = \frac{5}{5+n}$$

$$P(E_2/A) = \frac{3}{5} = \frac{\frac{1}{2} \cdot \frac{5}{5+n}}{\frac{1}{2} \cdot \frac{1}{3} + \frac{1}{2} \cdot \frac{5}{5+n}} \quad 2$$

$$\Rightarrow \frac{3}{5} = \frac{15}{5+n+15} \Rightarrow n = 5. \quad 1$$

29.



Let number of Souvenirs of type A be x , and that of type B be y .

\therefore L.P.P is maximise $P = 50x + 60y$

such that $5x + 8y \leq 200$

$10x + 8y \leq 240$

$x, y \geq 0$

 $\frac{1}{2}$
 $2\frac{1}{2}$

Correct Graph 2

$P(\text{at A}) = ₹1500$

$P(\text{at B}) = ₹(400 + 1200) = ₹1600$

$P(\text{at C}) = ₹(1200)$

\therefore Max Profit = ₹ 1600, when number of Souvenirs of type A = 8 and number of Souvenirs of type B = 20.

1