QUESTION PAPER CODE 65/4/1
EXPECTED ANSWER/VALUE POINTS

SECTION A

1. $xy = A + 5x \Rightarrow x\dfrac{dy}{dx} + y = 5$ $\quad \frac{1}{2}+\frac{1}{2}$

2. $|2\text{ adj }A| = 2^3\ |A|^{3-1} = 8 \times 81 = 648$ $\quad \frac{1}{2}+\frac{1}{2}$

3. $\cos\theta = \dfrac{3+12-4}{3\times 7} \Rightarrow \theta = \cos^{-1}\left(\dfrac{11}{21}\right)$ $\quad \frac{1}{2}+\frac{1}{2}$

OR

$\dfrac{x}{5/2}+\dfrac{y}{5}+\dfrac{z}{-5}=1$, length of $x-$intercept $=\dfrac{5}{2}$ $\quad 1$

4. $\dfrac{dy}{dx} = \dfrac{-\sin e^x}{\cos e^x}\cdot e^x$ or $-e^x \cdot \tan e^x$ $\quad 1$

SECTION B

5. $\displaystyle\int_{-\pi/4}^{0} \tan\left(\frac{\pi}{4}+x\right)dx = \log\left|\sec\left(\frac{\pi}{4}+x\right)\right|_{-\pi/4}^{0}$ $\quad 1+\frac{1}{2}$

$= \log\sqrt{2}$ or $\dfrac{1}{2}\log 2$ $\quad \frac{1}{2}$

6. $a * b = 2a + b \in R \quad \forall\ a, b \in R \qquad \therefore *$ is binary $\quad 1$

$a * (b * c) = a * (2b + c) = 2a + 2b + c$

$(a * b) * c = (2a + b) * c = 4a + 2b + c$

In genral $a * (b * c) \neq (a * b) * c \qquad \therefore *$ is not associative $\quad 1$

7. Position vector of $z = \dfrac{2(\vec{a}-3\vec{b})-1(3\vec{a}+\vec{b})}{2-1}$ $\quad 1$

$= -\vec{a}-7\vec{b}$ $\quad 1$

OR

$\vec{a}+\vec{b} = 4\hat{i}+\hat{j}-\hat{k}$ $\quad \frac{1}{2}$

$\vec{a}-\vec{b} = -2\hat{i}+3\hat{j}-5\hat{k}$ $\quad \frac{1}{2}$

$(\vec{a}+\vec{b})\cdot(\vec{a}-\vec{b}) = -8+3+5 = 0$ $\frac{1}{2}$

so $(\vec{a}+\vec{b}) \perp (\vec{a}-\vec{b})$ $\frac{1}{2}$

8. Given: A′ = A, B′ = B $\frac{1}{2}$

(AB – BA)′ = (AB)′ – (BA)′ $\frac{1}{2}$

= B′A′ – A′B′ $\frac{1}{2}$

= BA – AB

= –(AB – BA), Hence AB – BA is shew symmetric $\frac{1}{2}$

9. A: card bears odd number
B: Number on the card is greater than 5 } $\frac{1}{2}$

$A \cap B = \{7, 9, 11\}$ $\frac{1}{2}$

$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{3/12}{7/2} = \frac{3}{7}$ $\frac{1}{2}+\frac{1}{2}$

10. Required probability $= \frac{{}^3C_2 \times {}^5C_2}{{}^8C_4}$ 1

$= \frac{3}{7}$ 1

OR

$n = 5 \quad p = \frac{1}{3} \quad q = \frac{2}{3}$ $\frac{1}{2}$

$P(X \geq 4) = P(X = 4) + P(X = 5)$

$= 5C_4\left(\frac{1}{3}\right)^4\left(\frac{2}{3}\right)^1 + {}^5C_5\left(\frac{1}{3}\right)^5$ 1

$= \frac{11}{243}$ $\frac{1}{2}$

11. I.F. = e^x $\frac{1}{2}$

Solution is $y.e^x = \int (\cos x - \sin x) e^x \, dx + C$ $\frac{1}{2}$

$y.e^x = e^x \cos x + C$ or $y = \cos x + Ce^{-x}$ 1

12. $I = \tan^{-1} x \cdot \frac{x^2}{2} - \int \frac{x^2}{2(1+x^2)} dx$ 1

$= \frac{x^2}{2} \tan^{-1} x - \frac{1}{2}[x - \tan^{-1} x] + C$ 1

OR

$I = \frac{1}{\sqrt{2}} \int \frac{dx}{\sqrt{\frac{5}{2} - 2x - x^2}}$ $\frac{1}{2}$

$= \frac{1}{\sqrt{2}} \int \frac{dx}{\sqrt{\left(\frac{\sqrt{7}}{\sqrt{2}}\right)^2 - (x+1)^2}}$ 1

$= \frac{1}{\sqrt{2}} \sin^{-1}\left[\frac{(x+1)\sqrt{2}}{\sqrt{7}}\right] + C$ $\frac{1}{2}$

SECTION C

13. $R_1 \to R_1 + R_2 + R_3$ & taking 12 + x common

$$(12+x)\begin{vmatrix} 1 & 1 & 1 \\ 4+x & 4-x & 4+x \\ 4+x & 4+x & 4-x \end{vmatrix} = 0$$ $1+\frac{1}{2}$

$C_2 \to C_2 - C_1 \quad C_3 \to C_3 - C_1$

$$(12+x)\begin{vmatrix} 1 & 0 & 0 \\ 4+x & -2x & 0 \\ 4+x & 0 & -2x \end{vmatrix} = 0$$ $1\frac{1}{2}$

$4x^2\ (12 + x) = 0$ $\frac{1}{2}$

$x = 0$ or $x = -12$ $\frac{1}{2}$

14. Given differential equation can be written as

$$\frac{dy}{dx} = (1 + x^2)\,(1 + y^2)$$ 1

$$\int \frac{dy}{1 + y^2} = \int (x^2 + 1)\,dx$$ 1

$$\tan^{-1} y = \frac{x^3}{3} + x + C$$ 1

$$x = 0,\ y = 1 \Rightarrow C = \frac{\pi}{4}$$ $\frac{1}{2}$

So particular solution is $\tan^{-1} y = \frac{x^3}{3} + x + \frac{\pi}{4}$ $\frac{1}{2}$

OR

Clearly given differential equation can be written as $\frac{dy}{dx} = \frac{y/x}{1 + (y/x)^2}$

Let $y = vx \Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$ 1

Given equation becomes

$$v + x\frac{dv}{dx} = \frac{v}{1 + v^2}$$

$$x\frac{dv}{dx} = \frac{-v^3}{1 + v^2}$$

$$\int \frac{(1 + v^2)}{v^3}\,dV = -\int \frac{dx}{x}$$ 1

$$-\frac{1}{2v^2} + \log|v| = -\log|x| + C$$ 1

$$-\frac{x^2}{2y^2} + \log|y| = C$$

$x = 0,\ y = 1 \Rightarrow C = 0$ $\frac{1}{2}$

So, particular solution is $\log|y| = \frac{x^2}{2y^2}$ $\frac{1}{2}$

15. Let $x_1, x_2 \in R - \{2\}$ such that $f(x_1) = f(x_2)$

$\Rightarrow \frac{x_1 - 1}{x_1 - 2} = \frac{x_2 - 1}{x_2 - 2}$ $\frac{1}{2}$

$\Rightarrow x_1x_2 - 2x_1 - x_2 + 2 = x_1x_2 - 2x_2 - x_1 + 2$

$\Rightarrow x_1 = x_2$ 1

So f is one – one

For range let f(x) = y

$\frac{x - 1}{x - 2} = y$ $\frac{1}{2}$

$x = \frac{2y - 1}{y - 1}$ 1

Range of $f = R - \{1\}$ = co domain B $\frac{1}{2}$

So f is onto.

$f^{-1}(y) = \frac{2y - 1}{y - 1}$ or $f^{-1}(x) = \frac{2x - 1}{x - 1}$ $\frac{1}{2}$

OR

For reflexive 1

For symmetric 1

For transitive

Let $(a, b) \in S$ & $(b, c) \in S$

$|a - b| = 3m, |b - c| = 3n$

$a - b = \pm 3m \qquad b - c = \pm 3n$

$a - c = 3(\pm m \pm n) \Rightarrow a - c$ is divisible by 3 $\quad 1\frac{1}{2}$

$\Rightarrow |a - c|$ is divisible by 3

$\Rightarrow (a, c) \in S$

S is transitive

As S is reflexive, symmetric & transitive

$\therefore$ S is an equivalence reation. $\quad \frac{1}{2}$

16. $I = \int \frac{\cos(x+b+a-b)}{\sin(x+b)} dx \quad 1$

$= \int [\cot(x+b)\cos(a-b) - \sin(a-b)] dx \quad 1$

$= \cos(a - b) \log |\sin(x + b)| - x \sin(a - b) + C \quad 2$

17. $\frac{dx}{dt} = \cos t, \quad \frac{dy}{dt} = p\cos pt \Rightarrow \frac{dy}{dx} = \frac{p\cos pt}{\cos t} \quad 1$

$\frac{dy}{dx} = \frac{p\sqrt{1-y^2}}{\sqrt{1-x^2}} \quad 1$

$\Rightarrow (1-x^2)\left(\frac{dy}{dx}\right)^2 = p^2(1-y^2)$ differentiating both sides w.r.t x $\quad \frac{1}{2}$

$\Rightarrow 2(1-x^2)\frac{dy}{dx}\frac{d^2y}{dx^2} - 2x\left(\frac{dy}{dx}\right)^2 = -2p^2y\frac{dy}{dx} \quad 1$

$\Rightarrow (1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} + p^2y = 0 \quad \frac{1}{2}$

OR

Let $\theta = \cos^{-1} x^2 \Rightarrow x^2 = \cos\theta \quad 1$

$\Rightarrow y = \tan^{-1}\left(\frac{\sqrt{1+\cos\theta} - \sqrt{1-\cos\theta}}{\sqrt{1+\cos\theta} + \sqrt{1-\cos\theta}}\right)$

$$= \tan^{-1}\left(\frac{\cos\frac{\theta}{2}-\sin\frac{\theta}{2}}{\cos\frac{\theta}{2}+\sin\frac{\theta}{2}}\right)$$ 1

$$= \frac{\pi}{4}-\frac{1}{2}\theta$$ 1

$$\therefore \quad \frac{dy}{d\theta} = -\frac{1}{2}$$ 1

18. LHS becomes $\tan^{-1}\frac{5}{12}+\tan^{-1}\frac{3}{4}$ 1

$$= \tan^{-1}\left(\frac{\frac{5}{12}+\frac{3}{4}}{1-\frac{5}{12}\cdot\frac{3}{4}}\right) = \tan^{-1}\frac{56}{33}$$ 1+1

$$= \sin^{-1}\frac{56}{65} = \text{RHS}$$ 1

19. Let $u = x^{\cos x}$, $v = (\cos x)^{\sin x}$

$y = u + v$

$$\Rightarrow \frac{dy}{dx} = \frac{du}{dx}+\frac{dv}{dx} \quad \text{...(1)}$$ 1

$$\log u = \cos x \log x \Rightarrow \frac{1}{u}\frac{du}{dx} = \frac{\cos x}{x} - \sin x \log x$$

$$\frac{du}{dx} = x^{\cos x}\left(\frac{\cos x}{x} - \sin x \log x\right)$$ 1

$$\log v = \sin x \log \cos x \Rightarrow \frac{1}{v}\frac{dv}{dx} = -\sin x \tan x + \cos x \log \cos x$$

$$\Rightarrow \frac{dv}{dx} = (\cos x)^{\sin x}(-\sin x \tan x + \cos x \log \cos x)$$ 1

So, $$\frac{dy}{dx} = x^{\cos x}\left(\frac{\cos x}{x} - \sin x \log x\right) + (\cos x)^{\sin x}(-\sin x \tan x + \cos x \log \cos x)$$ 1

20. Let $a - x = t \Rightarrow -dx = dt$ $\frac{1}{2}$

RHS $= -\int_a^0 f(t)\,dt = \int_0^a f(t)\,dt = \int_0^a f(x)\,dx$ $\frac{1}{2}$

$$I = \int_0^1 x^2 (1-x)^n\,dx$$

$$= \int_0^1 (1-x)^2 x^n\,dx \qquad 1$$

$$= \int_0^1 [x^n + x^{n+2} - 2x^{n+1}]\,dx \qquad \frac{1}{2}$$

$$= \left.\frac{x^{n+1}}{n+1} + \frac{x^{n+3}}{n+3} - \frac{2x^{n+2}}{n+2}\right|_0^1 \qquad 1$$

$$= \frac{1}{n+1} + \frac{1}{n+3} - \frac{2}{n+2} \qquad \frac{1}{2}$$

21.

$$\left.\begin{aligned} \overrightarrow{BA} &= (x-4)\hat{i} - 6\hat{j} - 2\hat{k} \\ \overrightarrow{BC} &= -\hat{i} + 4\hat{j} + 3\hat{k} \\ \overrightarrow{BD} &= -8\hat{i} - \hat{j} + 3\hat{k} \end{aligned}\right\} \qquad 1\frac{1}{2}$$

As points are coplanar

$$\therefore \begin{vmatrix} x-4 & -6 & -2 \\ -1 & 4 & 3 \\ -8 & -1 & 3 \end{vmatrix} = 0 \qquad 1$$

$15(x - 4) + 6 \times 21 - 2 \times 33 = 0$ 1

$15x = 0$

$x = 0$ $\frac{1}{2}$

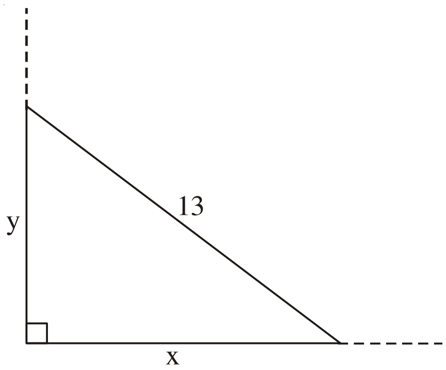
22. 

Figure $\frac{1}{2}$

$$\frac{dx}{dt} = 2\text{ cm/sec}$$ $\frac{1}{2}$

$$y = \sqrt{169 - x^2}$$ 1

$$\frac{dy}{dt} = -\frac{x}{\sqrt{169 - x^2}}\frac{dx}{dt}$$ 1

$$\left(\frac{dy}{dt}\right)_{x=5} = \frac{-5}{6}\text{ cm/sec}$$ 1

Hence height is decreasing at the rate $\frac{5}{6}$ cm / sec

23. Equation of required plane is

$$\begin{vmatrix} x-3 & y+1 & z-2 \\ 2 & 3 & 2 \\ -4 & 0 & 4 \end{vmatrix} = 0$$ 2

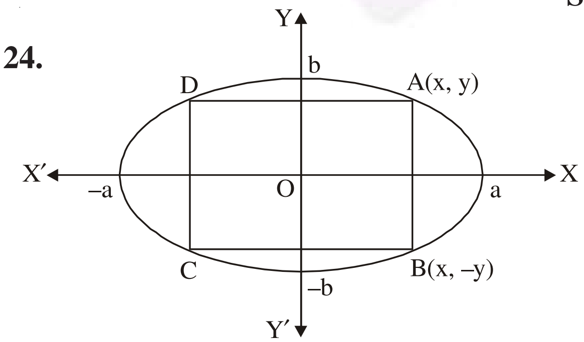
$$12(x - 3) - 16(y + 1) + 12(z - 2) = 0$$

$$3x - 4y + 3z = 19$$

$$\vec{r} \cdot (3\hat{i} - 4\hat{j} + 3\hat{k}) = 19$$ 1

Distance from origin $= \frac{19}{\sqrt{34}}$ or $\frac{19\sqrt{34}}{34}$ 1

SECTION D

24. 

Correct Figure 1

Let ABCD is a rectangle

$A = 4xy$

$$A^2 = 16x^2(a^2 - x^2)\frac{b^2}{a^2} = f(x)$$ 1

$$f'(x) = \frac{16b^2}{a^2}(2a^2x - 4x^3)$$ 1

$$f'(x) = 0 \Rightarrow x = \frac{a}{\sqrt{2}} \Rightarrow y = \frac{b}{\sqrt{2}}$$ 1

$$f''(x) = \frac{16b^2}{a^2}(2a^2 - 12x^2) = \frac{16b^2}{a^2}(-4a^2) < 0 \text{ at } x = \frac{a}{\sqrt{2}}$$

Area is maximum at $x = \frac{a}{\sqrt{2}}, y = \frac{b}{\sqrt{2}}$ 1

Required Area $= 4\int_0^{a/\sqrt{2}} \frac{b}{\sqrt{2}} dx$ $\frac{1}{2}$

$= 4\frac{b}{\sqrt{2}}\frac{a}{\sqrt{2}} = 2ab$ $\frac{1}{2}$

Note: Since finding maximum/minimum area using integration is not discussed in prescribed books, so a student who tried to attempt but could not complete may be given full marks.

25. E_1: Selected person is cyclist
E_2: Selected person is scooter driver
E_3: Selected person is car driver
A: insured person met with an accident
} 1

$P(E_1) = \frac{3}{18}, \ P(E_2) = \frac{6}{18}, \ P(E_3) = \frac{9}{18}$
$P(A|E_1) = 0.3, P(A|E_2) = 0.05, P(A|E_3) = 0.02$
} 2

$$P(E_1/A) = \frac{P(E_1)\,P(A|E_1)}{P(E_1)\,P(A|E_1) + P(E_2)\,P(A|E_2) + P(E_3)\,P(A|E_3)}$$

$$= \frac{\frac{3}{18}\times\frac{30}{100}}{\frac{3}{18}\times\frac{30}{100} + \frac{6}{18}\times\frac{5}{100} + \frac{9}{18}\times\frac{2}{100}}$$ 2

$$= \frac{90}{138} \text{ or } \frac{15}{23}$$ 1

26. $A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$ we know that A = IA i.e., $\begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$ 1

$R_1 \to R_1 - R_3$

$$\begin{bmatrix} 1 & -4 & 7 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} A$$

$R_2 \to R_2 - 3R_1, R_3 \to R_3 - R_1$

$$\begin{bmatrix} 1 & -4 & 7 \\ 0 & 14 & -25 \\ 0 & 5 & -9 \end{bmatrix} = \begin{bmatrix} 1 & 0 & -1 \\ -3 & 1 & 3 \\ -1 & 0 & 2 \end{bmatrix} A$$

$R_2 \to -(R_2 - 3R_3)$

$$\begin{bmatrix} 1 & -4 & 7 \\ 0 & 1 & -2 \\ 0 & 5 & -9 \end{bmatrix} = \begin{bmatrix} 1 & 0 & -1 \\ 0 & -1 & 3 \\ -1 & 0 & 2 \end{bmatrix} A$$

$R_1 \to R_1 + 4R_2$

$R_3 \to R_3 - 5R_3$

$$\begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -4 & 11 \\ 0 & -1 & 3 \\ -1 & 5 & -13 \end{bmatrix} A$$ 4

$R_1 \to R_1 + R_3$

$R_2 \to R_2 + 2R_3$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & -4 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix} A$$

$\Rightarrow A^{-1} = \begin{bmatrix} 0 & 1 & -4 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix}$ 1

OR

$|A| = 67 \neq 0 \qquad \therefore X = A^{-1}B$ $1+\frac{1}{2}$

adj A = $\begin{bmatrix} -6 & 17 & 13 \\ 14 & 5 & -8 \\ -15 & 9 & -1 \end{bmatrix}$ 2

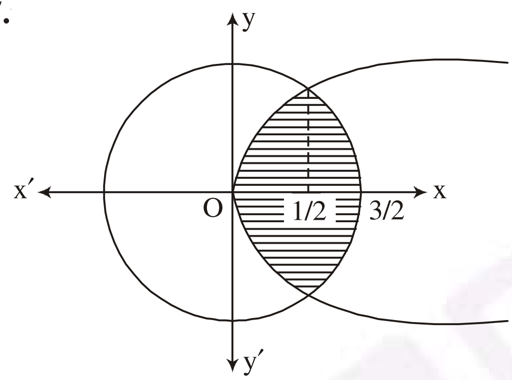
$$A^{-1} = \frac{1}{67}\begin{bmatrix} -6 & 17 & 13 \\ 14 & 5 & -8 \\ -15 & 9 & -1 \end{bmatrix}$$ $\frac{1}{2}$

$$\text{So } X = \frac{1}{67}\begin{bmatrix} -6 & 17 & 13 \\ 14 & 5 & -8 \\ -15 & 9 & -1 \end{bmatrix}\begin{bmatrix} -4 \\ 2 \\ 11 \end{bmatrix}$$

$$= \frac{1}{67}\begin{bmatrix} 201 \\ -134 \\ 67 \end{bmatrix} = \begin{bmatrix} 3 \\ -2 \\ 1 \end{bmatrix}$$ $1\frac{1}{2}$

$x = 3, y = -2, z = 1$ $\frac{1}{2}$

27.



Correct Figure 1

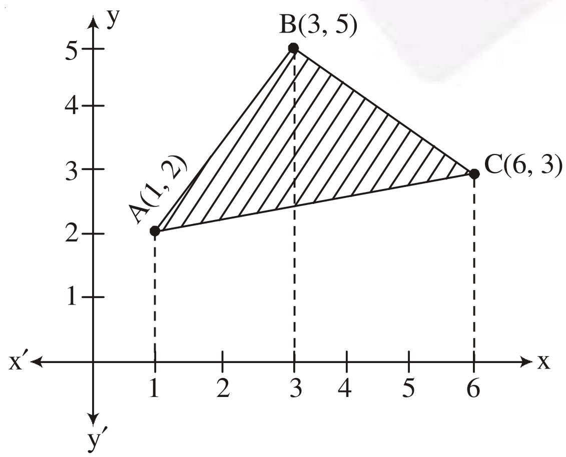
x coordinate of Point of intersection $= \frac{1}{2}$ 1

$$\text{Required Area} = 2\left[\int_0^{\frac{1}{2}} 2\sqrt{x}\,dx + \int_{\frac{1}{2}}^{3/2} \sqrt{\left(\frac{3}{2}\right)^2 - x^2}\,dx\right]$$ 2

$$= 2\left[\frac{4}{3}x^{3/2}\Big|_0^{\frac{1}{2}} + \frac{x}{2}\sqrt{\frac{9}{4} - x^2} + \frac{9}{8}\sin^{-1}\frac{2x}{3}\Big|_{\frac{1}{2}}^{3/2}\right]$$ 1

$$= \frac{\sqrt{2}}{6} + \frac{9\pi}{8} - \frac{9}{4}\sin^{-1}\frac{1}{3}$$ 1

OR



Correct figure 1

Correct points of intersection $1\frac{1}{2}$

$$\text{Required Area} = \int_1^3 \frac{3x+1}{2}dx + \int_3^6 \frac{21-2x}{3}dx - \int_1^6 \frac{x+9}{5}dx$$ 2

$$= \left(\frac{3x^2}{4}+\frac{x}{2}\right)\Bigg|_1^3 + \left(7x-\frac{x^2}{3}\right)\Bigg|_3^6 - \left(\frac{x^2}{10}+\frac{9x}{5}\right)\Bigg|_1^6$$ 1

$$= 7+12-\frac{25}{2}$$

$$= \frac{13}{2}$$ $\frac{1}{2}$

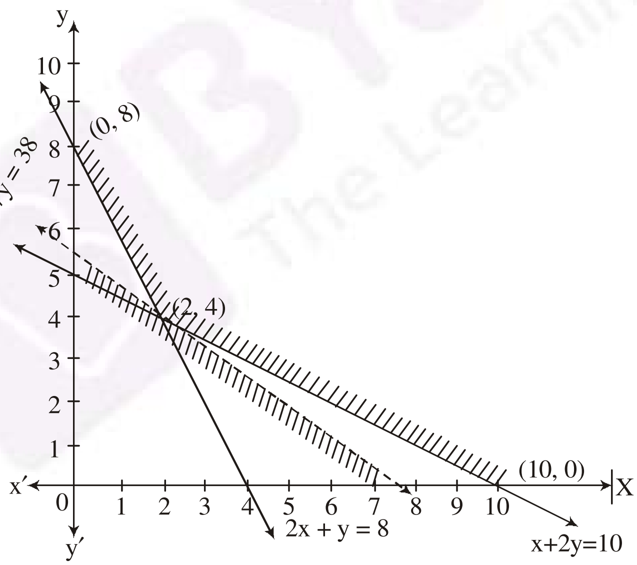
28. Let Quantity of Food I = x kg

Quantity of Food II = y kg

Out L.P.P. is

Minimize Z = 50x + 70y subject to $\frac{1}{2}$

$$\left.\begin{aligned} 2x + y &\geq 8 \\ x + 2y &\geq 10 \\ x \geq 0,\ y &\geq 0 \end{aligned}\right\}$$ $2\frac{1}{2}$



Correct graph 2

Corner points	Value of Z	
(10, 0)	500	
(2, 4)	380	→ minimum
(0, 8)	560	

$\frac{1}{2}$

Consider 50x + 70y < 380

which has no point common with feasible region

So minimum value of Z = ₹ 380 $\frac{1}{2}$

at x = 2 kg, y = 4 kg

29. Required equation of line is

$$\vec{r} = 2\hat{i} + 3\hat{j} + 2\hat{k} + \mu(2\hat{i} - 3\hat{j} + 6\hat{k})$$ 2

$$\vec{a}_1 = -2\hat{i} + 3\hat{j}, \vec{a}_2 = 2\hat{i} + 3\hat{j} + 2\hat{k}, \quad \vec{b} = 2\hat{i} - 3\hat{j} + 6\hat{k}$$

$$\vec{a}_2 - \vec{a}_1 = 4\hat{i} + 2\hat{k}$$ 1

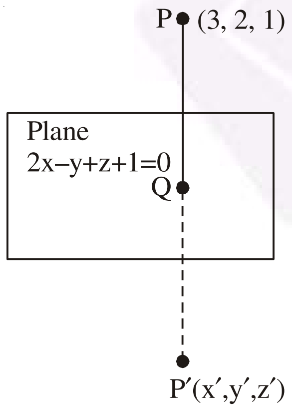
$$d = \frac{|(\vec{a}_2 - \vec{a}_1) \times \vec{b}|}{|\vec{b}|}$$

$$(\vec{a}_2 - \vec{a}_1) \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & 0 & 2 \\ 2 & -3 & 6 \end{vmatrix}$$

$$= 6\hat{i} - 20\hat{j} - 12\hat{k}$$ 2

$$d = \frac{\sqrt{580}}{7}$$ 1

OR



Correct figure 1

Equation of PQ

$$\frac{x-3}{2} = \frac{y-2}{-1} = \frac{z-1}{1} = \lambda$$ $\frac{1}{2}$

Coodinates of Q $(2\lambda + 3, -\lambda + 2, \lambda + 1)$ $\frac{1}{2}$

As Q lies on plane

$\therefore 4\lambda + 6 + \lambda - 2 + \lambda + 1 = -1$

gives, $\lambda = -1$ 1

Coordinates of Q (1, 3, 0) $\frac{1}{2}$

$PQ = \sqrt{6}$ 1

For unique $P'(x', y', z')$

$\frac{x'+3}{2} = 1, \ \frac{y'+2}{2} = 3, \ \frac{z'+1}{2} = 0$ 1

$x' = -1 \ \ y' = 4 \ \ z' = -1$

image is (–1, 4, –1) $\frac{1}{2}$