

# CBSE Class 12 Maths Question Paper Solution 2019

## QUESTION PAPER CODE 65/5/1 EXPECTED ANSWER/VALUE POINTS

### SECTION A

$$1. \sin \theta = \frac{(\hat{i} + \hat{j} + \hat{k}) \cdot (3\hat{i} - \hat{j} + 2\hat{k})}{\sqrt{1+1+1} \sqrt{9+1+4}} \quad \frac{1}{2}$$

$$\Rightarrow \theta = \sin^{-1} \left( \frac{4}{\sqrt{42}} \right) \quad \frac{1}{2}$$

OR

$$\text{Any point on } \frac{x+2}{1} = \frac{y-5}{3} = \frac{z+1}{5} = \lambda \dots(1) \quad \frac{1}{2}$$

is  $(\lambda - 2, 3\lambda + 5, 5\lambda - 1)$

Line (1) cuts yz plane at  $\lambda - 2 = 0$  i.e.,  $\lambda = 2$

hence required point is  $(0, 11, 9)$   $\frac{1}{2}$

$$2. y = 5e^{7x} + 6e^{-7x} \Rightarrow \frac{dy}{dx} = 35e^{7x} - 42e^{-7x} \quad \frac{1}{2}$$

$$\frac{d^2y}{dx^2} = 245e^{7x} + 294e^{-7x} \Rightarrow \frac{d^2y}{dx^2} = 49y \quad \frac{1}{2}$$

$$3. |2AA'| = 4|A||A'| \quad \frac{1}{2}$$

$$= 4 \times 4 \times 4 = 64 \quad \frac{1}{2}$$

$$4. \text{Order} = 2 \quad 1$$

### SECTION B

$$5. \begin{bmatrix} 2x-3 & 6 \\ 15 & 2y-4 \end{bmatrix} = \begin{bmatrix} 7 & 6 \\ 15 & 14 \end{bmatrix} \quad 1$$

$$\Rightarrow 2x - 3 = 7 \text{ and } 2y - 4 = 14 \Rightarrow x = 5, y = 9 \Rightarrow x - y = -4 \quad 1$$

$$6. (y + 3x^2) \frac{dx}{dy} = x \Rightarrow \frac{dy}{dx} - \frac{1}{x}y = 3x \quad \frac{1}{2}$$

$$\text{I.F.} = e^{\int \frac{-1}{x} dx} = e^{-\log(x)} = \frac{1}{x} \quad \frac{1}{2}$$

$$\text{Solution is } y \cdot \frac{1}{x} = \int 3x \cdot \frac{1}{x} dx \Rightarrow y = 3x^2 + cx \quad 1$$

$$\begin{aligned} 7. \int e^x (\cos x - \sin x) \operatorname{cosec}^2 x \, dx &= -\int e^x [\operatorname{cosec} x - \operatorname{cosec} x \cot x] \, dx & 1 \\ &= -e^x \operatorname{cosec} x + c & 1 \end{aligned}$$

8. Let P(2, -1, 3), Q(3, -5, 1) and R(-1, 11, 9) be three point.

$$\overline{PQ} = \hat{i} - 4\hat{j} - 2\hat{k} \quad \frac{1}{2}$$

$$\overline{PR} = -3\hat{i} + 12\hat{j} + 6\hat{k} = -3(\hat{i} - 4\hat{j} - 2\hat{k}) \quad \frac{1}{2}$$

$\therefore \overline{PR} = -3\overline{PQ}$ , since P is common.

Therefore the points P, Q and R are collinear. 1

OR

$$(\vec{a} \times \vec{b})^2 = \vec{a}^2 \vec{b}^2 - (\vec{a} \cdot \vec{b})^2$$

$$\text{LHS} = (\vec{a} \times \vec{b})^2$$

$$= (|\vec{a}| |\vec{b}| \sin \theta \hat{n})^2 = |\vec{a}|^2 |\vec{b}|^2 \sin^2 \theta \quad 1$$

$$= |\vec{a}|^2 |\vec{b}|^2 (1 - \cos^2 \theta)$$

$$= |\vec{a}|^2 |\vec{b}|^2 - (|\vec{a}| |\vec{b}| \cos \theta)^2$$

$$= \vec{a}^2 \vec{b}^2 - (\vec{a} \cdot \vec{b})^2 \quad 1$$

$$9. \int \frac{x-1}{(x-2)(x-3)} dx = \int \left( \frac{-1}{x-2} + \frac{2}{x-3} \right) dx \quad 1$$

$$= -\log|x-2| + 2\log|x-3| + C \quad 1$$

OR

$$\int \frac{e^x}{\sqrt{5-4e^x-e^{2x}}} dx \quad \text{Put } e^x = t \text{ so that } e^x dx = dt \quad \frac{1}{2}$$

$$\int \frac{dt}{\sqrt{5-4t-t^2}} = \int \frac{dt}{\sqrt{3^2-(t+2)^2}} \quad 1$$

$$= \sin^{-1}\left(\frac{t+2}{3}\right) + C = \sin^{-1}\left(\frac{e^x+2}{3}\right) + C \quad \frac{1}{2}$$

10.  $P(B|A) = 0.4 \Rightarrow \frac{P(B \cap A)}{P(A)} = 0.4 \Rightarrow P(B \cap A) = 0.24 \quad \frac{1}{2}$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.6 + 0.5 - 0.24 = 0.86 \quad 1$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{0.24}{0.5} = 0.48 \quad \frac{1}{2}$$

11. (i)  $\forall a, b \in \mathbb{Z}, a * b = 2a^2 + b \in \mathbb{Z} \therefore *$  is binary 1

(ii)  $1, 2 \in \mathbb{Z}, 1 * 2 = 2 \times 1^2 + 2 = 4, 2 * 1 = 2 \times (2)^2 + 1 = 9$

$$1 * 2 \neq 2 * 1$$

$\therefore *$  is not commutative. 1

12.  $n = 4, p = \frac{1}{4}, q = \frac{3}{4} \quad \frac{1}{2}$

$$P(\text{at least 3 are diamonds}) = P(X = 3) + P(X = 4)$$

$$= {}^4C_3 \left(\frac{1}{4}\right)^3 \left(\frac{3}{4}\right) + {}^4C_4 \left(\frac{1}{4}\right)^4 \quad 1$$

$$= \left(\frac{1}{4}\right)^4 [12 + 1] = \frac{13}{256} \quad \frac{1}{2}$$

OR

Let  $E_1$  : A coming on time.

$E_2$  : B coming on time.

$$P(\bar{E}_1) = \frac{5}{7}, P(\bar{E}_2) = \frac{3}{7} \quad \frac{1}{2}$$

P(only one on time)

$$= P(E_1) P(\bar{E}_2) + P(\bar{E}_1) P(E_2)$$

$$= \frac{2}{7} \times \frac{3}{7} + \frac{5}{7} \times \frac{4}{7}$$

$$= \frac{26}{49}$$

1

 $\frac{1}{2}$ 

13.  $x^p y^q = (x+y)^{p+q} \Rightarrow p \log x + q \log y = (p+q) \log(x+y)$

 $\frac{1}{2}$ 

Differentiating w.r.t x,  $\frac{p}{x} + \frac{q}{y} \frac{dy}{dx} = \frac{p+q}{x+y} \left(1 + \frac{dy}{dx}\right)$

1

$$\Rightarrow \left(\frac{q}{y} - \frac{p+q}{x+y}\right) \frac{dy}{dx} = \frac{p+q}{x+y} - \frac{p}{x}$$

$$\Rightarrow \left[\frac{qx - py}{y(x+y)}\right] \frac{dy}{dx} = \frac{qx - py}{x(x+y)} \Rightarrow \frac{dy}{dx} = \frac{y}{x}$$

1

Differentiating again w.r.t x

$$\frac{d^2 y}{dx^2} = \frac{x \frac{dy}{dx} - y}{x^2}$$

1

$$\Rightarrow \frac{d^2 y}{dx^2} = \frac{x \left(\frac{y}{x}\right) - y}{x^2} \Rightarrow \frac{d^2 y}{dx^2} = 0$$

 $\frac{1}{2}$ 

14.  $\int (\sin x \sin 2x \sin 3x) dx = \frac{1}{2} \int (2 \sin x \sin 2x) \sin 3x dx$

 $\frac{1}{2}$ 

$$= \frac{1}{2} \int [\cos(x) - \cos(3x)] \sin 3x dx$$

1

$$= \frac{1}{4} \int 2 \cos x \sin 3x dx - \frac{1}{4} \int 2 \cos 3x \sin 3x dx$$

$$= \frac{1}{4} \int (\sin 4x + \sin 2x) dx - \frac{1}{4} \int \sin 6x dx$$

1

$$= -\frac{1}{4} \left( \frac{\cos 4x}{4} + \frac{\cos 2x}{2} \right) + \frac{1}{24} \cos 6x + C \quad 1 \frac{1}{2}$$

15. Let  $y = \tan^{-1} \left( \frac{3x - x^3}{1 - 3x^2} \right)$ , Put  $x = \tan \theta$  1  $\frac{1}{2}$

$$y = \tan^{-1} \left( \frac{3 \tan \theta - \tan^3 \theta}{1 - 3 \tan^2 \theta} \right) \Rightarrow y = \tan^{-1} (\tan 3\theta) = 3\theta$$

$$y = 3 \tan^{-1} x \Rightarrow \frac{dy}{dx} = \frac{3}{1+x^2} \quad \dots(i) \quad 1$$

Let  $z = \tan^{-1} \left( \frac{x}{\sqrt{1-x^2}} \right)$ , put  $x = \sin \phi$  1  $\frac{1}{2}$

$$z = \tan^{-1} \left( \frac{\sin \phi}{\sqrt{1-\sin^2 \phi}} \right) \Rightarrow z = \tan^{-1} (\tan \phi) = \phi$$

$$z = \phi = \sin^{-1} x \Rightarrow \frac{dz}{dx} = \frac{1}{\sqrt{1-x^2}} \quad \dots(ii) \quad 1$$

Using (i) & (ii),  $\frac{dy}{dz} = \frac{dy/dx}{dz/dx} = \frac{3\sqrt{1-x^2}}{1+x^2}$  1

OR

$$\sqrt{1-x^2} + \sqrt{1-y^2} = a(x-y), \text{ put } x = \sin \theta, y = \sin \phi \quad 1$$

$$\sqrt{1-\sin^2 \theta} + \sqrt{1-\sin^2 \phi} = a(\sin \theta - \sin \phi)$$

$$\Rightarrow \cos \theta + \cos \phi = a(\sin \theta - \sin \phi)$$

$$\Rightarrow 2 \cos \left( \frac{\theta+\phi}{2} \right) \cos \left( \frac{\theta-\phi}{2} \right) = 2a \cos \left( \frac{\theta+\phi}{2} \right) \sin \left( \frac{\theta-\phi}{2} \right) \quad 1$$

$$\Rightarrow \tan \left( \frac{\theta-\phi}{2} \right) = \frac{1}{a}$$

$$\Rightarrow \frac{\theta-\phi}{2} = \tan^{-1} \left( \frac{1}{a} \right) \Rightarrow \sin^{-1} x - \sin^{-1} y = 2 \tan^{-1} \left( \frac{1}{a} \right) \quad 1$$

Differentiating both sides w.r.t x

$$\frac{1}{\sqrt{1-x^2}} - \frac{1}{\sqrt{1-y^2}} \frac{dy}{dx} = 0 \quad \frac{1}{2}$$

$$\frac{1}{\sqrt{1-y^2}} \frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}} \text{ or } \frac{dy}{dx} = \frac{\sqrt{1-y^2}}{\sqrt{1-x^2}} \quad \frac{1}{2}$$

16.  $(1+e^{2x})dy + (1+y^2)e^x dx = 0$

$$\Rightarrow \int \frac{dy}{1+y^2} = -\int \frac{e^x}{1+e^{2x}} dx \quad 1$$

$$\Rightarrow \tan^{-1} y = -\int \frac{e^x}{1+e^{2x}} dx \quad \frac{1}{2}$$

Put  $e^x = t$ , so that  $e^x dx = dt$

$$\tan^{-1} y = -\int \frac{dt}{1+t^2} \Rightarrow \tan^{-1} y = -\tan^{-1}(e^x) + C \quad \dots(i) \quad 1$$

Substituting  $y = 1$ , when  $x = 0$  in equation (i)

$$\tan^{-1}(1) = -\tan^{-1}(1) + C \Rightarrow C = \frac{\pi}{2} \quad \frac{1}{2}$$

$$\text{Substituting } C = \frac{\pi}{2} \text{ in equation (i)} \Rightarrow \tan^{-1} y + \tan^{-1}(e^x) = \frac{\pi}{2} \quad \frac{1}{2}$$

OR

$$x \frac{dy}{dx} \sin\left(\frac{y}{x}\right) + x - y \sin\left(\frac{y}{x}\right) = 0$$

$$\Rightarrow \frac{dy}{dx} = \frac{y}{x} - \frac{1}{\sin\left(\frac{y}{x}\right)} \quad \dots(i) \quad \frac{1}{2}$$

Put  $\frac{y}{x} = v$  i.e.,  $y = vx$  in (i) so that  $\frac{dy}{dx} = v + x \frac{dv}{dx}$  1

$$v + x \frac{dv}{dx} = v - \frac{1}{\sin v}$$

$$\int \sin v \, dv = \int -\frac{1}{x} \, dx \quad \frac{1}{2}$$

$$\Rightarrow -\cos v = -\log |x| + C \Rightarrow \cos\left(\frac{y}{x}\right) = \log |x| + C \quad \dots(\text{ii}) \quad 1$$

Substituting  $y = \frac{\pi}{2}$  when  $x = 1$  in (ii)

$$\cos\left(\frac{\pi}{2}\right) = \log 1 + C \Rightarrow C = 0 \quad \frac{1}{2}$$

$$\text{Required solution is } \cos\left(\frac{y}{x}\right) = \log |x| \quad \frac{1}{2}$$

17. (i) **Reflexive:**  $\forall a \in A, |a - a| = 0$  which is even

$$\Rightarrow (a, a) \in R, \text{ hence } R \text{ is reflexive} \quad 1$$

(ii) **Symmetric:** Let  $(a, b) \in R \Rightarrow |a - b|$  is even

$$\Rightarrow |-(b - a)| \text{ is even} \Rightarrow |b - a| \text{ is even}$$

$$\text{so, } (b, a) \in R$$

hence  $R$  is symmetric. 1

(iii) **Transitive:** Let  $(a, b), (b, c) \in R$

so,  $|a - b|$  is even and  $|b - c|$  is even

$$\Rightarrow a - b = 2\lambda, b - c = 2\mu \text{ where } \lambda, \mu \in \mathbb{Z}$$

$$\text{Now, } a - c = (a - b) + (b - c) = 2(\lambda + \mu)$$

$$\Rightarrow |a - c| \text{ is even, so } (a, c) \in R$$

hence  $R$  is transitive. 1  $\frac{1}{2}$

Since  $R$  is reflexive, symmetric and transitive therefore its an equivalence relation 1  $\frac{1}{2}$

OR

Let for  $x_1, x_2 \in A, f(x_1) = f(x_2)$  1  $\frac{1}{2}$

$$\frac{4x_1+3}{6x_1-4} = \frac{4x_2+3}{6x_2-4}$$

$$\Rightarrow (4x_1+3)(6x_2-4) = (6x_1-4)(4x_2+3)$$

$$\Rightarrow 34x_1 = 34x_2 \Rightarrow x_1 = x_2, \text{ hence } f \text{ is one-one.}$$

1

For any  $y \in A$  such that  $y = \frac{4x+3}{6x-4}$  there exists  $x$  such that

$$6xy - 4y = 4x + 3 \Rightarrow (6y - 4)x = 4y + 3$$

$$\Rightarrow x = \frac{4y+3}{6y-4}, y \in A, x = \frac{4y+3}{6y-4} \in A$$

1

$\Rightarrow f$  is onto.

1

Since  $f$  is one-one and onto, therefore  $f^{-1}$  exists in  $A$

$$\text{and } f^{-1}(y) = \frac{4y+3}{6y-4} \text{ or } f^{-1}(x) = \frac{4x+3}{6x-4}$$

 $\frac{1}{2}$ 

18.  $f(x) = \cos\left(2x + \frac{\pi}{4}\right) \Rightarrow f'(x) = -2\sin\left(2x + \frac{\pi}{4}\right)$

 $1\frac{1}{2}$ 

$$\text{As } \frac{3\pi}{8} < x < \frac{5\pi}{8} \Rightarrow \frac{3\pi}{4} < 2x < \frac{5\pi}{4}$$

$$\Rightarrow \pi < 2x + \frac{\pi}{4} < \frac{3\pi}{2}$$

 $1\frac{1}{2}$ 

$$\Rightarrow \sin\left(2x + \frac{\pi}{4}\right) < 0 \Rightarrow f'(x) > 0$$

$$\therefore f(x) \text{ is increasing in } \left(\frac{3\pi}{8}, \frac{5\pi}{8}\right)$$

1

19. Equation of the required plane is

$$a(x+1) + b(y-3) + c(z-2) = 0 \quad \dots(i)$$

1

Since plane (i) is perpendicular to planes  $x+2y+3z=5$  and  $3x+3y+z=0$ , therefore

$$\text{and } \left. \begin{array}{l} a+2b+3c=0 \quad \dots(ii) \\ 3a+3b+c=0 \quad \dots(iii) \end{array} \right\}$$

1



Solving (ii) and (iii) we get

$$\frac{a}{7} = \frac{b}{-8} = \frac{c}{3} \quad 1$$

Equation of plane is

$$7 \cdot (x + 1) - 8(y - 3) + 3(z - 2) = 0$$

$$\Rightarrow 7x - 8y + 3z + 25 = 0 \quad 1$$

20. LHS =  $\sin^{-1} \frac{4}{5} + \tan^{-1} \frac{5}{12} + \cos^{-1} \frac{63}{65}$

$$= \tan^{-1} \frac{4}{3} + \tan^{-1} \frac{5}{12} + \cos^{-1} \frac{63}{65} \quad 1$$

$$= \tan^{-1} \left( \frac{\frac{4}{3} + \frac{5}{12}}{1 - \frac{4}{3} \times \frac{5}{12}} \right) + \cos^{-1} \frac{63}{65} \quad 1$$

$$= \tan^{-1} \left( \frac{63}{16} \right) + \cot^{-1} \left( \frac{63}{16} \right) \quad 1$$

$$= \frac{\pi}{2} = \text{RHS} \quad 1$$

21.  $\int_1^5 (|x-1| + |x-2| + |x-4|) dx$

$$= \int_1^2 (5-x) dx + \int_2^4 (x+1) dx + \int_4^5 (3x-7) dx \quad 1 \frac{1}{2}$$

$$= \left[ 5x - \frac{x^2}{2} \right]_1^2 + \left[ \frac{x^2}{2} + x \right]_2^4 + \left[ \frac{3x^2}{2} - 7x \right]_4^5 \quad 1 \frac{1}{2}$$

$$= \frac{7}{2} + 8 + \frac{13}{2} = 18 \quad 1$$

22. A(x, 5, -1), B(3, 2, 1), C(4, 5, 5), D(4, 2, -2)

$$\left. \begin{aligned} \overline{BA} &= (x-3)\hat{i} + 3\hat{j} - 2\hat{k} \\ \overline{BC} &= \hat{i} + 3\hat{j} + 4\hat{k} \\ \overline{BD} &= \hat{i} + 0\hat{j} - 3\hat{k} \end{aligned} \right\} \quad 1 \frac{1}{2}$$

$$\begin{vmatrix} x-3 & 3 & -2 \\ 1 & 3 & 4 \\ 1 & 0 & -3 \end{vmatrix} = 0$$

1

$$\text{i.e., } (x-3)(-9) - 3(-7) - 2(-3) = 0$$

1

$$\Rightarrow x = 6$$

 $\frac{1}{2}$ 23.  $\Delta = 0$ 

$$\Rightarrow \begin{vmatrix} x & x^2 & x^3 \\ y & y^2 & y^3 \\ z & z^2 & z^3 \end{vmatrix} - \begin{vmatrix} x & x^2 & 1 \\ y & y^2 & 1 \\ z & z^2 & 1 \end{vmatrix} = 0$$

1

$$\Rightarrow xyz \begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix} - (-1)^2 \begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix} = 0$$

1

$$\Rightarrow \begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix} (xyz - 1) = 0$$

 $\frac{1}{2}$ 

$$\Rightarrow \begin{vmatrix} 0 & x-z & x^2-z^2 \\ 0 & y-z & y^2-z^2 \\ 1 & z & z^2 \end{vmatrix} (xyz-1) = 0 \quad \begin{array}{l} R_1 \rightarrow R_1 - R_3 \\ R_2 \rightarrow R_2 - R_3 \end{array}$$

1

$$\Rightarrow (x-y)(y-z)(z-x)(xyz-1) = 0$$

$$\therefore x \neq y \neq z \text{ therefore } xyz = 1$$

 $\frac{1}{2}$ 

## SECTION D

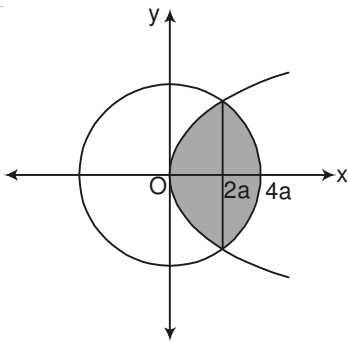
24.

Correct figure

1

$$\text{Solving } x^2 + y^2 = 16a^2 \quad \dots(1)$$

$$\text{and } y^2 = 6ax \quad \dots(2) \text{ we get}$$



$$x = 2a \text{ (as } -8a \text{ is not possible)}$$

1

$$\text{Required Area} = 2 \left[ \int_0^{2a} \sqrt{6ax} \, dx + \int_{2a}^{4a} \sqrt{(4a)^2 - x^2} \, dx \right]$$

2

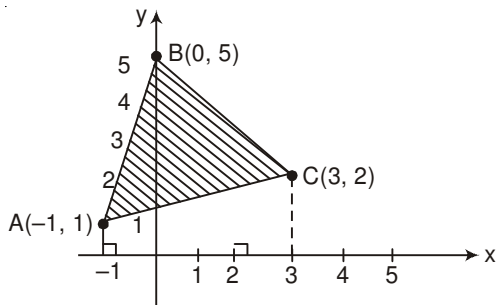
$$= 2 \left[ \sqrt{6a} \left( \frac{2}{3} x^{3/2} \right) \right]_0^{2a} + 2 \left[ \frac{x}{2} \sqrt{(4a)^2 - x^2} + 8a^2 \sin^{-1} \left( \frac{x}{4a} \right) \right]_{2a}^{4a}$$

1

$$= \frac{8}{3} \sqrt{12} a^2 + \frac{16}{3} \pi a^2 - 4\sqrt{3} a^2$$

$$= \frac{4}{3} a^2 (4\pi + \sqrt{3})$$

1



OR

Correct figure

1

$$4x - y + 5 = 0$$

...(1)

$$x + y - 5 = 0$$

...(2)

$$x - 4y + 5 = 0$$

...(3)

Coordinates of A(-1, 1), B(0, 5) and C(3, 2)

1  $\frac{1}{2}$ 

$$\text{Required Area} = \int_{-1}^0 (4x + 5) dx + \int_0^3 (5 - x) dx - \frac{1}{4} \int_{-1}^3 (x + 5) dx$$

2

$$= \left[ 2x^2 + 5x \right]_{-1}^0 + \left[ 5x - \frac{x^2}{2} \right]_0^3 - \frac{1}{4} \left[ \frac{x^2}{2} + 5x \right]_{-1}^3$$

1

$$= \frac{15}{2}$$

1  $\frac{1}{2}$ 

25. Required equation of the line is

$$\vec{r} = 2\hat{i} + \hat{j} - \hat{k} + \mu(2\hat{i} - \hat{j} + \hat{k})$$

2

$$\text{Let } \vec{a}_1 = 2\hat{i} + \hat{j} - \hat{k}, \vec{a}_2 = \hat{i} + \hat{j}, \vec{b} = 2\hat{i} - \hat{j} + \hat{k}$$

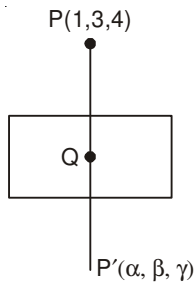
$$\text{The required distance} = \left| \frac{(\vec{a}_2 - \vec{a}_1) \times \vec{b}}{|\vec{b}|} \right|$$

$$= \frac{|(\hat{i} - \hat{k}) \times (2\hat{i} - \hat{j} + \hat{k})|}{|2\hat{i} - \hat{j} + \hat{k}|} \quad 1$$

$$(\vec{a}_2 - \vec{a}_1) \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 0 & -1 \\ 2 & -1 & 1 \end{vmatrix} = -\hat{i} - 3\hat{j} - \hat{k} \quad 2$$

$$\text{Required distance} = \frac{\sqrt{1+9+1}}{\sqrt{4+1+1}} = \frac{\sqrt{11}}{\sqrt{6}} \text{ or } \frac{\sqrt{66}}{6} \quad 1$$

OR



Correct figure

$$\text{Equation of line PQ is } \frac{x-1}{2} = \frac{y-3}{-1} = \frac{z-4}{1} = \lambda \quad 1$$

$$\text{The coordinates of Q are } (2\lambda + 1, -\lambda + 3, \lambda + 4) \quad \frac{1}{2}$$

$$\because \text{Q lies on plane } 2x - y + z + 3 = 0$$

$$\therefore 2(2\lambda + 1) - (-\lambda + 3) + (\lambda + 4) + 3 = 0 \quad 1$$

$$\Rightarrow 6\lambda + 6 = 0 \text{ i.e., } \lambda = -1 \quad \frac{1}{2}$$

$$\text{The coordinates of Q are } (-1, 4, 3) \quad \frac{1}{2}$$

$$PQ = \sqrt{(-1-1)^2 + (4-3)^2 + (3-4)^2} = \sqrt{6} \quad 1$$

Let  $P'(\alpha, \beta, \gamma)$  be the image of P.

$$\text{then } \frac{\alpha+1}{2} = -1, \frac{\beta+3}{2} = 4, \frac{\gamma+4}{2} = 3 \quad \frac{1}{2}$$

$$\Rightarrow \alpha = -3, \beta = 5, \gamma = 2$$

$$\therefore \text{the image } P' \text{ is } (-3, 5, 2) \quad \frac{1}{2}$$

26. Let numbers of souvenirs of type A be  $x$  and number of souvenirs of type B be  $y$

$\therefore$  L.P.P is

Maximize  $P = 100x + 120y$

Subject to constraints

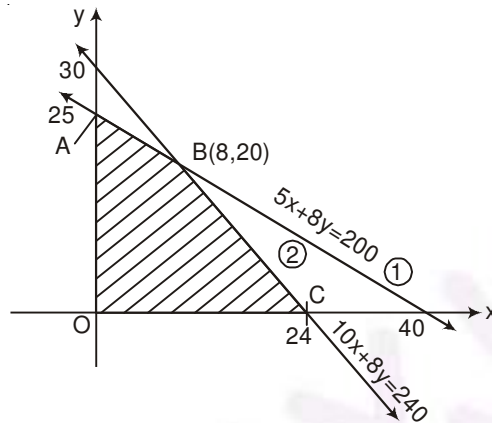
$5x + 8y \leq 200$  ... (1)

$10x + 8y \leq 240$  ... (2)

$x, y \geq 0$

$\frac{1}{2}$

$2\frac{1}{2}$



2

Values at corner points

Points	P
A(0, 25)	3000
B(8, 20)	3200 (Max)
C(24, 0)	2400

1

So, 8 type A Souvenirs and 20 type B Souvenirs should be made to maximize profit.

27. Let  $E_1 =$  Student guesses the answer  
 $E_2 =$  Student copies the answer  
 $E_3 =$  Student knows the answer  
 $A =$  Student answers the question correctly.

1

$P(E_1) = \frac{1}{4}, P(E_2) = \frac{1}{4}, P(E_3) = 1 - \left(\frac{1}{4} + \frac{1}{4}\right) = \frac{1}{2}$

$1\frac{1}{2}$

$P(A | E_1) = \frac{1}{4}, P(A | E_2) = \frac{3}{4}, P(A | E_3) = 1$

$1\frac{1}{2}$

The required probability

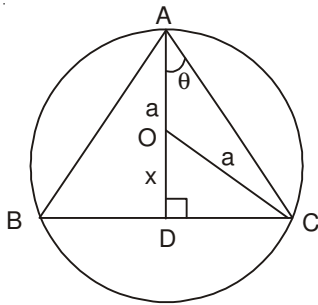
$$= P(E_3 | A) = \frac{P(E_3) \times P(A | E_3)}{\sum_{i=1}^3 P(E_i) \times P(A | E_i)}$$

$$= \frac{\frac{1}{2} \times 1}{\frac{1}{4} \times \frac{1}{4} + \frac{1}{4} \times \frac{3}{4} + \frac{1}{2} \times 1}$$

$$= \frac{1}{\frac{1}{8} + \frac{3}{8} + 1} = \frac{8}{12} = \frac{2}{3}$$

 $\frac{1}{2}$  $\frac{1}{2}$ 

28.



Correct figure

1

$$CD = \sqrt{a^2 - x^2}$$

 $\frac{1}{2}$ 

$$\text{Area, } A = \frac{1}{2} \times 2\sqrt{a^2 - x^2} (a + x)$$

 $\frac{1}{2}$ 

$$Z = A^2 = (a - x)(a + x)^3$$

 $\frac{1}{2}$ 

$$\frac{dZ}{dx} = 2(a + x)^2 (a - 2x)$$

 $\frac{1}{2}$ 

$$\frac{dZ}{dx} = 0 \Rightarrow x = \frac{a}{2}$$

 $\frac{1}{2}$ 

$$\frac{d^2Z}{dx^2} = -12(a + x)x$$

 $\frac{1}{2}$ 

$$\left( \frac{d^2Z}{dx^2} \right)_{x=\frac{a}{2}} = -9a^2 < 0$$

 $\frac{1}{2}$ 

$\therefore Z$  is maximum when  $x = \frac{a}{2}$

i.e., Area is maximum when  $x = \frac{a}{2}$

 $\frac{1}{2}$

For maximum area

$$\tan \theta = \frac{CD}{AD} = \frac{\sqrt{a^2 - \frac{a^2}{4}}}{a + \frac{a}{2}} = \frac{1}{\sqrt{3}}$$

$$\theta = \frac{\pi}{6}$$

1

29. Let  $A = \begin{bmatrix} 3 & 0 & -1 \\ 2 & 3 & 0 \\ 0 & 4 & 1 \end{bmatrix}$

Then  $A = IA$

$$\Rightarrow \begin{bmatrix} 3 & 0 & -1 \\ 2 & 3 & 0 \\ 0 & 4 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

1

$$\Rightarrow \begin{bmatrix} 1 & -3 & -1 \\ 2 & 3 & 0 \\ 0 & 4 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A \quad (R_1 \rightarrow R_1 - R_2)$$

$$\Rightarrow \begin{bmatrix} 1 & -3 & -1 \\ 0 & 9 & 2 \\ 0 & 4 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 0 \\ -2 & 3 & 0 \\ 0 & 0 & 1 \end{bmatrix} A \quad (R_2 \rightarrow R_2 - 2R_1)$$

$$\Rightarrow \begin{bmatrix} 1 & -3 & -1 \\ 0 & 1 & 0 \\ 0 & 4 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 0 \\ -2 & 3 & -2 \\ 0 & 0 & 1 \end{bmatrix} A \quad (R_2 \rightarrow R_2 - 2R_3)$$

$$\Rightarrow \begin{bmatrix} 1 & -3 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 0 \\ -2 & 3 & -2 \\ 8 & -12 & 9 \end{bmatrix} A \quad (R_3 \rightarrow R_3 - 4R_2)$$

$$\Rightarrow \begin{bmatrix} 1 & -3 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 9 & -13 & 9 \\ -2 & 3 & -2 \\ 8 & -12 & 9 \end{bmatrix} A \quad (R_1 \rightarrow R_1 - R_3)$$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & -4 & 3 \\ -2 & 3 & -2 \\ 8 & -12 & 9 \end{bmatrix} A \quad (R_1 \rightarrow R_1 - 3R_2)$$

4

$$\therefore A^{-1} = \begin{bmatrix} 3 & -4 & 3 \\ -2 & 3 & -2 \\ 8 & -12 & 9 \end{bmatrix} \quad 1$$

OR

The given system of equations is

$$AX = B,$$

$$\text{where } A = \begin{bmatrix} 2 & 3 & 10 \\ 4 & -6 & 5 \\ 6 & 9 & -20 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, B = \begin{bmatrix} 4 \\ 1 \\ 2 \end{bmatrix}$$

$$|A| = 1200 \neq 0 \quad 1$$

$$\Rightarrow A^{-1} \text{ exists.}$$

$$X = A^{-1}B \quad 1$$

$$\text{adj } A = \begin{bmatrix} 75 & 150 & 75 \\ 110 & -100 & 30 \\ 72 & 0 & -24 \end{bmatrix} \quad 2$$

$$A^{-1} = \frac{1}{|A|} \text{adj } A = \frac{1}{1200} \begin{bmatrix} 75 & 150 & 75 \\ 110 & -100 & 30 \\ 72 & 0 & -24 \end{bmatrix} \quad \frac{1}{2}$$

$$X = A^{-1}B = \frac{1}{1200} \begin{bmatrix} 75 & 150 & 75 \\ 110 & -100 & 30 \\ 72 & 0 & -24 \end{bmatrix} \begin{bmatrix} 4 \\ 1 \\ 2 \end{bmatrix}$$

$$= \frac{1}{1200} \begin{bmatrix} 600 \\ 400 \\ 240 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} \\ \frac{1}{3} \\ \frac{1}{5} \end{bmatrix} \quad 1$$

$$\therefore x = \frac{1}{2}, y = \frac{1}{3}, z = \frac{1}{5} \quad \frac{1}{2}$$