SAMPLE QUESTION PAPER

Summative Assessment – II

Class – X (2015–16)

Mathematics - Marking Scheme

Section A

Ans 1. K = 3 [1]

Ans 2. $\sqrt{200}$ [1]

Ans 3. — [1]

Ans 4. a= - 9 [1]

Section B

Ans 5.

2 is the root of $x^2+kx+12=0$

$\Rightarrow (2)^2 + 2k+12=0$

$\Rightarrow 2k+16=0$

$k=-8$ [1/2]

Put k=-8 in $x^2 +kx+q=0$

$\Rightarrow x^2 -8x+q=0$ [1/2]

For equal roots

$(-8)^2 – 4 (1)q=0$ [1/2]

$64 -4q =0$

$4q = 64$

$q = 16$ [1/2]

Ans 6.

Two digit numbers which are divisible by 7 are

14,21,28,…….98. [1/2]

It forms an A.P.

$a=14$, $d=7$, $a_n=98$ [1/2]

$a_n=a+(n-1)\,d$

$98=14+(n-1)7$ [1/2]

$98-14=7n-7$

$84+7=7n$

$7n=91$

$n=13$ [1/2]

Ans 7

Let P(x,y) is equidistant from A(-5,3) and B(7,2)

AP=BP [1/2]

$\Rightarrow\sqrt{((x+5)^2+(y-3)^2)} = \sqrt{((x-7)^2+(y-2)^2)}$

$\Rightarrow x^2+10x+25+y^2-6y+9 = x^2-14x+49+y^2-4y+4$ [1/2]

$10x-6y+34 = -14x-4y+53$

$10x+14x-6y+4y = 53-34$

$24x-2y = 19$

$24x – 2y – 19 = 0$ is the required relation [1]

Ans 8

Perimeter of the shaded region

= AD+ BC + lengths of semi circles APB&CPD [1]

= 21+21+2(2x—x —) [1/2]

= 42+2(66)

= 42+ 132

=174 cm [1/2]

Ans 9

Let the water level raised in cylindrical vessel be h cm

Volume of Sphere = Volume of water displaced in cylinder [1/2]

$-\pi (3)^3 = \pi (6)^2 h$ [1]

–x27 = 36 h

36 = 36h

h = 1cm [1/2]

Ans 10

Volume of Coin = π r^2h

= $\frac{22}{}x(0.75)^2$ x0.2 cm^3 [1/2]

Volume of Cylinder = $\frac{22}{}x(2.25)^2$ x10 cm^3 [1/2]

No. of Coins = Volume of Cylinder / Volume of Coin [1/2]

=($\frac{22}{}x(2.25)^2$ x10) / ($\frac{22}{}x(0.75)^2$ x0.2)

=450 [1/2]

Section C

Ans 11

——— = – + – + –

$\Rightarrow$——— – = – + – [1/2]

$\Rightarrow$————— = —— [1/2]

$\Rightarrow$————— = ——

$\Rightarrow$————— = —— [1/2]

$\Rightarrow$ x(a + b + x) = - ab

$\Rightarrow$ $x^2 + (a+b)x - ab = 0$

$\Rightarrow$ (x+a)(x+b) = 0 [1]

$\Rightarrow$ x = -a or x = -b [1/2]

Ans 12

= ———

$a_n = S_n - S_{n-1}$ [1]

$\Rightarrow a_{25} = S_{25} - S_{24}$

=————— - ————— [1]

$= \frac{1}{-}\{3(25^2-24^2) + 13(25-24)\}$

=– (3x49 + 13) = 80 [1]

Ans 13

Let the first term of A.P be a and common difference be d.

$a_9 = 7a_2$

$\Rightarrow a+8d = 7(a+d)$ …………….. (1) [1/2]

$a_{12} = 5a_3+2$

$\Rightarrow a + 11d = 5(a + 2d) + 2$ …………….. (2) [1]

From (1), $a + 8d = 7a + 7d$

$-6a + d = 0$ …………….. (3)

From (2), $a + 11d = 5a + 10d + 2$

$-4a + d = 2$ …………….. (4)

Subtracting (4) from (3)

$-2a = -2$

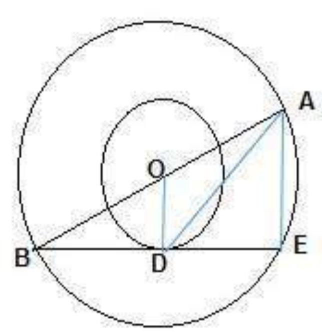
$\Rightarrow a = 1$ [1]

From (3),

$-6 + d = 0$

$d = 6$ [1/2]

Ans 14



Join OD and AE [1/2]

$\angle ODB = 90^o$ ………………. (radius is perpendicular to tangent at point of contact)

$\angle AEB = 90^o$ ………………. (angle in a semicircle)

OD | | AE ……………………. (Corresponding Angles) [1/2]

AE = 2 x OD

= 2 x 8 = 16 cm [1/2]

In right Δ ODB, $BD^2 = 13^2 - 8^2$ [1/2]

= 169-64 = 105

BD = $\sqrt{105}$ cm

DE = $\sqrt{105}$ cm [1/2]

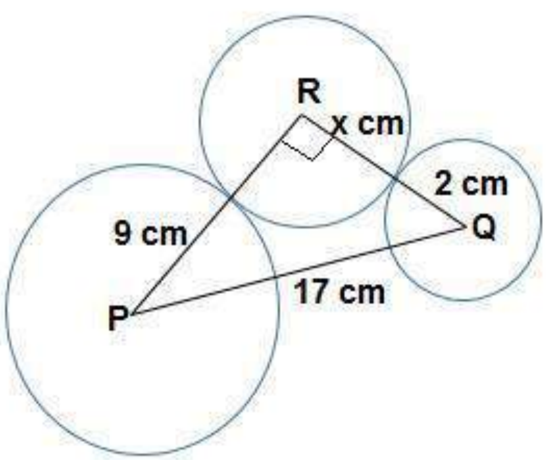
In right Δ AED, $AD^2 = AE^2 + DE^2$

$=16^2 + (\sqrt{105})^2$

=256+105 = 361

AD = 19 cm [1/2]

Ans15



[1/2]

In right ΔPQR, by Pythagoras theorem

$PQ^2 = PR^2 + PQ^2$

$\Rightarrow$ $17^2 = (x+9)^2 + (x+2)^2$ [1]

$\Rightarrow x^2 + 11x - 102 = 0$ [1/2]

$\Rightarrow x^2 + 17x - 6x - 102 = 0$

$\Rightarrow x(x+17) - 6(x+17) = 0$

$\Rightarrow$ $(x-6)(x+17) = 0$

$\Rightarrow$ $x = 6$ or $x = -17$ [1/2]

$\Rightarrow$ $x = 6$ cm (x can't be negative) [1/2]

Ans 16

For correct construction [3]

Ans 17

Total number of cards = 52

Number of non face cards = 52 – 12

= 40

P(non-face cards) = $\frac{40}{52} = \frac{10}{13}$ [1]

Number of black kings = 2

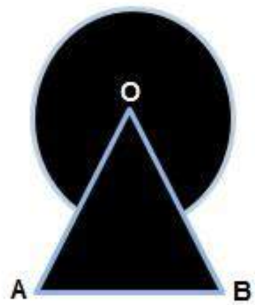
Number of red queens = 2

P(a black King or a red queen) = $\frac{4}{52}$ [1]

Number of spade cards = 13

P (Spade cards) = $\frac{13}{52}$ [1]

Ans 18



$\angle AOB = 60^{O}$ [1/2]

Area of shaded region

= Area of ΔAOB + Area of major sector of circle [1]

$= \frac{\sqrt{3}}{4}\ (12)^2 + \frac{300^0}{360^0} \times \frac{22}{7} \times (6)^2 \ \ cm^2$ [1]

$= 36\sqrt{3} + \frac{660}{7}\ cm^2$ [1/2]

Ans 19

Length of water that flows out in 30 minutes

= (0.7 X 100 X 60 X 30) cm

= 126000 cm [1]

Volume of water that flows out in 30 minutes

$= \pi\ (1)^2 \times 126000\ cm^3$

$= 126000\ \pi\ cm^3$ [1/2]

Let the depth of water in the tank be x cm

Volume of water in tank

$= \pi\ (40)^2 \ X\ x\ cm^3$ [1/2]

According to the question

$\pi\ (40)^2\ X\ x = 126000\pi$ [1/2]

$\Rightarrow x = 78.75$ cm [1/2]

Ans 20

Let R and r be the radii of the circular ends of the frustum. (R> r)

$2\pi R = 207.24$

$R = 207.24/ (2 \times 3.14)$

$R = 33$ cm [1]

$2\pi r = 169.56$ cm

$r = 169.56 / (2 \times 3.14)$

$r = 27$ cm [1/2]

$l^2 = h^2 + (R-r)^2$

$= 8^2 + (33-27)^2$ [1/2]

$l = 10$ cm [1/2]

Whole surface area of the frustum

$= \pi (R^2 + r^2 + (R+r)l)$

$= 3.14 ((33)^2 + (27)^2 + (33+27)10)$ [1/2]

$= 3.14 (1089 +729 + 600)$

$= 3.14 \times 2418$ cm^2

$= 7592.52$ cm^2[1]

Section D

Ans 21

Let the total number of students be x

$\frac{3}{8}x = 16 + \sqrt{x}$ [1]

$\Rightarrow \frac{3}{8}x - 16 = \sqrt{x}$

$\Rightarrow 3x - 128 = 8\sqrt{x}$

$\Rightarrow$ $3x - 8\sqrt{x} - 128 = 0$ [1/2]

Let $\sqrt{x} = y$

$3y^2 - 8y - 128 = 0$ [1/2]

$\Rightarrow$ $3y^2 - 24y + 16y - 128 = 0$

$\Rightarrow$ $3y(y-8) + 16(y-8) = 0$

$\Rightarrow$ $(y-8)(3y+16) = 0$

$y = 8$ or $y = -16/3$

$y = 8 \Rightarrow x = 64$

$y = -16/3 \Rightarrow x = 256/9$

number of students = 64 [1]

Values inculcated [1]

Ans 22

$a = 8$, $d = 1/3$ years, $S_n = 168$ [1/2]

$S_n = \frac{n}{2}[2a + (n-1)d]$

$\Rightarrow$ $168 = \frac{n}{2}[2(8) + (n-1)\frac{1}{3}]$ [1/2]

$n^2 + 47n - 1008 = 0$ [1]

$\Rightarrow$ $n^2 + 63n - 16n - 1008 = 0$

$\Rightarrow$ $(n-16)(n+63) = 0$

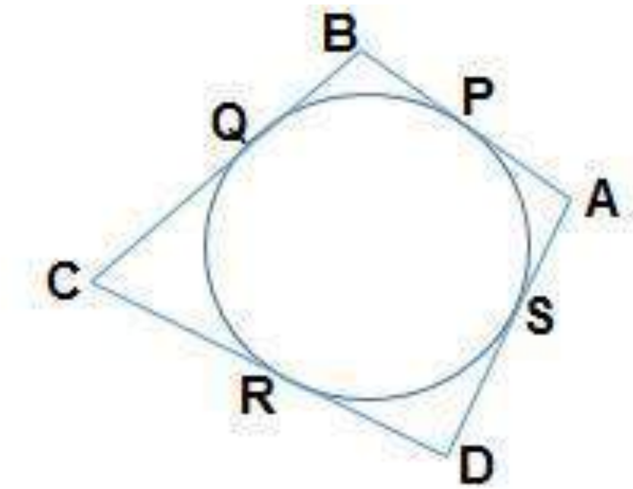
$\Rightarrow$ $n = 16$ or $n = -63$

$n = 16$ (n cannot be negative) [1]

Age of the eldest participant = $a + 15d = 13$ years [1]

Ans 23

Correct Proof of the theorem [2]



In the given figure,

Using the above theorem

AP = AS ………………..(1)

BP = BQ …………………(2)

DR = DS ………………….(3)

CR = CQ…………………..(4) [1]

Adding (1), (2),(3) and (4), we get

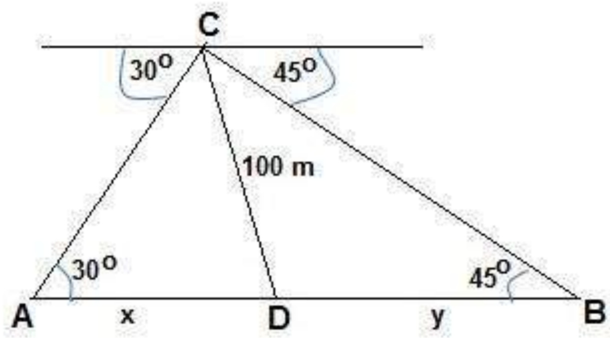
(AB+BP) + (DR+CR) = (AS+DS) + (BQ+CQ)

$\Rightarrow$ AB +CD = AD + BC [1]

Ans 24

For correct constructions [4]

Ans 25



Correct diagram [1]

In right ΔADC

$\tan 30^o = \frac{CD}{AD}$ [1/2]

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{100}{x}$

$\Rightarrow x = 100\sqrt{3}$ ……………………. (1) [1/2]

In right ΔBDC

$\tan 45^o = \frac{CD}{DB}$ [1/2]

$\Rightarrow 1 = \frac{100}{y}$ [1/2]

$\Rightarrow y = 100$ m

Distance between two cars

= AB = AD + DB [1/2]

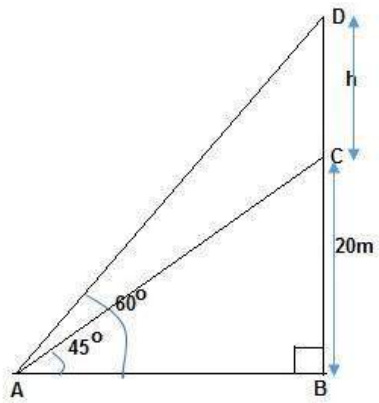
= $(100\sqrt{3} + 100)$m

= (100 X 1.73 + 100)m

= (173 + 100) m

= 273 m [1/2]

Ans 26



Let BC be building of height 20m and CD be the tower of height h m.
Let A be point on the ground at a distance of x m from the foot of the building. [1]
In right Δ ABC,

tan 45° = $\frac{BC}{AB}$

⇒ 1 = $\frac{20}{X}$

⇒ x = 20m ………………(1) [1]

In right Δ ABD,

tan 60° = $\frac{BD}{AB}$

⇒√3 = $\frac{h+20}{X}$

⇒√3 = $\frac{h+20}{20}$ [1]

⇒ h = 20 √3 – 20

= 20 (√3 – 1)

= 20 x 0.732

= 14.64 m

Height of tower = 14.64 m [1]

Ans 27

Total number of cards = 48

Probability of an event = $\frac{Total\ number\ of\ favourable\ outcomes}{Total\ number\ of\ outcomes}$ [1]

Number of cards divisible by 7 = 7

P(cards divisible by 7) = $\frac{7}{48}$ [1]

Number of cards having a perfect square = 6

P(cards having a perfect square)= $\frac{6}{48} = \frac{1}{8}$ [1]

Number of multiples of 6 from 3 to 50 = 8

P (multiple of 6 from 3 to 50) = $\frac{8}{48} = \frac{1}{6}$ [1]

Ans 28

By Section formula

$9a-2 = \frac{3(8a)+1(3a+1)}{3+1}$ ……………..(1) [1]

$-b = \frac{3(5)+1(-3)}{3+1}$ ………………(2) [1]

From (2)

$-b = \frac{15-3}{4} = 3$

$b = -3$ [1]

From (1)

$9a - 2 = \frac{24\ a+3a+1}{4}$

$4 (9a - 2) = 27a + 1$

$36a - 8 = 27a + 1$

$9a = 9$

$a = 1$ [1]

Ans 29

Let P(x_1, y_1) and Q(x_2, y_2) are two points which divide AB in three equal parts.

By Section formula

P(x_1, y_1) = $\left(\frac{1X\ (-4)+2X\ (2))}{1+2}, \frac{1X\ (-6)+2X\ (-3))}{1+2} \right)$ [1]

$= \left(\frac{-4\ +4}{3}, \frac{-6\ +(-6)}{3}\right)$

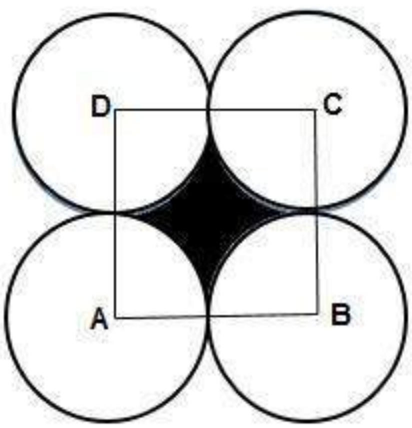
$= (0, -4)$ [1]

$Q(x_2, y_2) = \left(\frac{2X(-4)+1X(2))}{2+1}, \frac{2X(-6)+1X(-3))}{2+1} \right)$ [1]

$= \left(\frac{-8+2}{3}, \frac{-12+(-3)}{3} \right)$

$= (-2, -5)$ [1]

Ans 30



Let r cm be the radius of each circle.

Area of square – Area of 4 sectors = $\frac{24}{7}$ cm^2 [1/2]

$\Rightarrow (2r)^2 - 4\left(\frac{90^0}{360^0} \times \pi r^2\right) = \frac{24}{7}$ [1]

$\Rightarrow 4r^2 - \frac{22}{7}r^2 = \frac{24}{7}$ [1/2]

$\Rightarrow \frac{28r^2 - 22r^2}{7} = \frac{24}{7}$

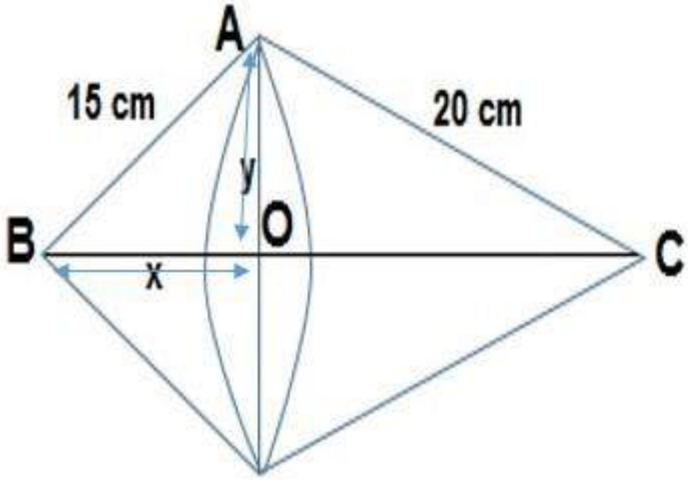
$\Rightarrow 6r^2 = 24$

$\Rightarrow r^2 = 4$ [1]

$\Rightarrow r = \pm 2$

$\Rightarrow$ radius of each circle is 2 cm (r cannot be negative) [1]

Ans 31



In rightΔ BAC, by Pythagoras theorem

$BC^2 = AB^2 + AC^2$

$= 15^2 + 20^2$

$= 225 + 400$

$= 625$

BC = 25cm [1/2]

Let OA = y cm and OB = x cm

$x^2+y^2 = 15^2$ [1/2]

$(25-x)^2 + y^2 = 20^2$ [1/2]

Solving we get x=9 and y=12 [1/2]

$\therefore$ OA= 12 cm and OB = 9 cm

Volume of double cone = $\frac{1}{3}\pi\ (OA)^2 \times OC + \frac{1}{3}\pi\ (OA)^2 \times OB$

$= \frac{1}{3} \times 3.14 \times (12)^2 \times (OC + OB)$ [1/2]

$= \frac{1}{3} \times 3.14 \times 144 \times 25$

$= 3768\ cm^3$ [1/2]

Surface area of double cone = $\pi \times OA \times AC + \pi \times OA \times AB$

$= \pi \times 12 \times 20 + \pi \times 12 \times 15$ [1/2]

$= 420\ \pi\ cm^2$

$= 1318.8\ cm^2$ [1/2]