MARKING SCHEME

SECTION -A

1. $\frac{7}{11}$ [1]
2. 136 [1]
3. 60° [1]
4. 34° [1]

SECTION-B

5. Here $a=4$, $d=4$ and $a_n=96$ [1/2]
   So, $a_n = a + (n-1)d$
   $96 = 4 + (n-1)4$
   $n=24$ [1/2]
   Now, $S_n = \frac{(a+a_n)}{2}$
   $= 1200$ [1/2]

6. Let $A(-1,3)$, $B(2,p)$ and $C(5,-1)$ be 3 collinear points. [1/2]

   Then Area $\Delta ABC = 0$ [1/2]

   Then, $\frac{1}{2} | -1(p+1)+2(-1-3)+5(3-p) | = 0$ [1]

   i.e. $-p-1-8+15-5p=0$
   i.e. $6=6p$
   i.e. $p=1$ [1/2]

7. For equal roots, $b^2-4ac=0$ [1/2]

   Here, $a=k$, $b=-k$ and $c=1$

   $\therefore k^2 - 4(k)(1)=0$ [1/2]

   i.e. $k(k-4)=0$

   i.e. $k=0$ or $k=4$ [1/2]

   rejecting $k=0$, we get $k=4$. [1/2]

8. Perimeter of $\Delta ABC = AB+BC+CA$

   $= AB+[BP+CP]+CA$ [1/2]

   $= AB+BQ+CR+CA$ (Tangents from an external point are equal) [1/2]

   $= AQ+AR$ [1/2]

   $= AR+AR$ (Tangents from an external point are equal)

   $= 2AR$ [1/2]

Co-ordinates of point R are given by
\[
x = \frac{(2 \times 3 + 3 \times 2)}{5} = 0 \\
y = \frac{(2 \times 4 + 3 \times -1)}{5} = 1
\]

So, the required point R is (0,1)

10. Tangents drawn to a circle from same external point are equal in length. So,

\[ AE = CE \quad \text{----------------- (1)} \]

And \[ EB = ED \quad \text{----------------- (2)} \]

Adding (1) and (2) , we get,

\[ AB = CD. \]

11. \[ x^2 + 12x - 45 = 0 \]

Using the method of completing the square,
\[
x^2 + 12x - 45 + 36 = 36 \\
i.e. x^2 + 12x + 36 = 36 + 45 \\
i.e. (x+6)^2 = 81
\]

i.e. \( x = 3 \) or -15

12. \[ \frac{a+9d}{a+29d} = \frac{1}{3} \]

i.e. 3a + 27d = a + 29d
i.e. $a=d$ \hspace{1cm} (1) \hspace{1/2}

Also, $S_6 = 42$

i.e. $\frac{6}{2} (2a+5d) = 42$ \hspace{1cm} (1)

i.e. $3(2a+5a) = 42$ Using... (1)

i.e. $3(7a) = 42$

i.e. $a = 2$ \hspace{1cm} (1/2)

13. Let $AB$ represent the lighthouse.

$\angle ACB = 45^\circ$ and $\angle ADB = 30^\circ$

![Fig 1]

In $\triangle ABC$,

$\tan 45^\circ = \frac{AB}{BC}$

$1 = \frac{75}{BC}$

i.e. $BC = 75m$ \hspace{1cm} (1)

Now, in $\triangle ABD$,

$\tan 30^\circ = \frac{AB}{BD}$

i.e. $\frac{1}{\sqrt{3}} = \frac{75}{(BC+CD)}$

i.e. $\frac{1}{\sqrt{3}} = \frac{75}{(75 + CD)}$

i.e. $75 + CD = 75\sqrt{3}$

i.e. $CD = 75(\sqrt{3} - 1) m$ \hspace{1cm} (1)

14. Let $A(-1,3)$, $B(1,-1)$ and $C(5,1)$ be the vertices of $\triangle ABC$.

Median through $C$ would be the line joining $C$ and midpoint of side $AB$. Let it be point $D$

$D = \left(\frac{-1+1}{2}, \frac{3-1}{2}\right)$ \hspace{1cm} (1)

Coordinates of $D$ are $(0,1)$ \hspace{1cm} (1/2)

Length of median $CD = \sqrt{(5-0)^2 + (1-1)^2}$

$= 5$ units. \hspace{1cm} (1/2)

15. No. of cards left $= 52 - 3 = 49$

$P($face card$) = \frac{9}{49}$ \hspace{1cm} (1)

$P($red card$) = \frac{23}{49}$ \hspace{1cm} (1)

$P($a king$) = \frac{3}{49}$ \hspace{1cm} (1)
16. \( \frac{\theta}{360} \times 2\pi r = 44 \)  
Putting \( r=42\text{cm} \), we get \( \theta = 60^\circ \)  

Now, Area of minor segment = Area of minor sector - Area of \( \Delta \)  
Since \( \theta = 60^\circ \), so the triangle formed will be an equilateral \( \Delta \)  
\[ \therefore \text{Area of minor segment} = \text{Area of minor sector} - \text{Area of equilateral } \Delta \]

i.e. \[ \text{Area of minor segment} = \frac{\theta}{360} \times \pi r^2 - \frac{\sqrt{3}}{4} a^2 \]  
\[ = 924 - 441 \sqrt{3} \text{ cm}^2 \]

17. Time required to fill the conical vessel = Volume of cone / volume of water coming out of cylindrical pipe per unit time  
\[ \frac{\frac{1}{3} \pi r_1^2 h_1}{\frac{1}{3} \pi r_2^2 h_2} = \frac{[\frac{1}{3} \pi (40)^2 \times 72]/[\pi (2)^2 \times 20 \times 100]}{} \]
\[ = 4.8 \text{ minutes} \]

18. Area of shaded region = Area of semicircle with diameter PS – Area of semicircle with diameter QS + Area of semicircle with diameter PQ.  
So, required area = \( \frac{1}{2} \pi (6)^2 - \frac{1}{2} \pi (4)^2 + \frac{1}{2} \pi (2)^2 \)  
\[ = \frac{1}{2} \pi [36-16+4] \text{ cm}^2 \]
\[ = 37.71 \text{ cm}^2 \]

19. No. of lead shots = \( \frac{\text{Volume of cuboid}}{\text{Volume of sphere}} \)  
\[ = \frac{l_1 b_1 h_1}{\frac{1}{3} \pi r_2^3} \]
\[ = \frac{24 \times 22 \times 12 \times 3}{\pi \times 3 \times 3 \times 3 \times 4} \]
\[ = 56 \]

20. Required surface area = \( 2 \pi rh + 2 \times [2 \pi r^2] \)  
\[ = 2 \times \pi x 3.5 \times 10 + 4 \pi (3.5)^2 \]
\[ = 374 \text{ cm}^2 \]
Cost of polishing = Rs.374 x 10 = Rs.3740

21. Correct Construction of \( \Delta ABC \)  
Correct construction of similar triangle

22. Let the speed of the train be \( x \) km/hr.
According to question,
\[
\frac{90}{x} = \frac{90}{x+15} \Rightarrow \frac{1}{x} = \frac{2}{x+15} [1]
\]
i.e. \(x^2 + 15x - 2700 = 0\) [1]
Solving for \(x\) we get,
\(x = -60\) or \(45\) [1]
Rejecting \(x = -60\), we get, \(x = 45\) [1]
So, \(x = 45\) km/hr [1/2]

Time = Distance / Speed
\[
\frac{90}{45} = 2\text{ hours} [1/2]
\]

23. (i) Cards marked with numbers which are multiples of 3 are 3, 9, 15, 21, 27, 33, 39 and 45.
So, \(P(\text{getting a number divisible by 3}) = \frac{8}{25}\) [1]
(ii) \(P(\text{composite number}) = \frac{10}{25}\) [1]
(iii) \(P(\text{not a perfect square}) = 1 - P(\text{perfect square}) = 1 - \frac{4}{25} = \frac{21}{25}\) [1]
(iv) \(P(\text{multiple of 3 and 5}) = \frac{2}{25}\) [1]

24.

Construction: Join OR, OC and OS. [1/2]

In \(\triangle ORA\) and \(\triangle OCA\)

\(\text{OR} = \text{OC} \) (radii)
\(\text{AO} = \text{AO} \) (common)
\(\text{AR} = \text{AC} \) (tangents from an external point)

\(\triangle ORA \cong \triangle OCA \) (By SSS rule) [1]
\(\therefore \angle RAO = \angle CAO \) (CPCT) .................. (1) [1/2]
Similarly \(\triangle OSB \cong \triangle OCB \) (By SSS rule)
\(\therefore \angle SBO = \angle CBO \) (CPCT) .................. (2) [1/2]
\(\angle RAB + \angle SBA = 180^\circ \) (Co-interior angles)
\(2\angle OAB + 2\angle OBA = 180^\circ \) (From (1) & (2))
\(\angle OAB + \angle OBA = 90^\circ \) .................. (3) [1]
In \(\triangle AOB\),
\(\angle OAB + \angle OBA + \angle AOB = 180^\circ \) (Angle sum property)
\(90^\circ + \angle AOB = 180^\circ \) (From 3)
\(\angle AOB = 90^\circ \) [1/2]
25. Quadratic formula, \( x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \) \[1\]
\( a = p^2, b = (p^2 - q^2), c = -q^2 \) \[1/2\]
x= \([- (p^2 - q^2) \pm \sqrt{(p^2 - q^2)^2 - 4p^2(-q^2)}]/2p^2 \) \[1\]
x= \([- p^2 + q^2 \pm (p^2 + q^2)]/2p^2 \) \[1/2\]
x= \( q^2/p^2 \) or -1 \[1\]

26. Diagonals of a parallelogram bisect each other, So, midpoint of AC = midpoint of BD \[1/2\]
i.e. \( (\frac{1+k}{2}, \frac{2-2}{2}) = (\frac{2-4}{2}, \frac{3-3}{2}) \) \[1/2\]
i.e. \( (\frac{1+k}{2}) = -1 \)
i.e. \( k = -3 \) \[1/2\]
Now ar ABCD= 2 Area of ΔABD
=\(2 \times \frac{1}{2} \times [1(6) + 2(-1) - 4(-5)]\)
= 24 sq units. \[1\]
\( AB = \sqrt{(1-2)^2 + (-2-3)^2} \)
= \(\sqrt{26}\) units \[1/2\]

Ar (ABCD) = base x height
= AB x h
So, \( 24 = \sqrt{26} \times h \)
So, \( h = \frac{24}{\sqrt{26}} \) units \[1\]

27.

Let FC be the lake and D be a point 100m above the lake.
Let A be the helicopter at height \( h \) metre above the lake and let E be its reflection \( \therefore CE = h \) metre
\( \angle BDE = 60^\circ, \angle ADB = 30^\circ \) and DB = \( x \) metre \[1 \text{ mark for correct figure and description}\]

\[\tan 30^\circ = \frac{h-100}{x}\]
\[
\frac{1}{\sqrt{3}} = \frac{h-100}{x}
\]
\[ h = x\sqrt{3} + 100 \] ........................................... (1) [1]

\[ \tan 60^\circ = \frac{h+100}{x} \]

\[ \sqrt{3} x = h+100 \]

\[ h = \sqrt{3}x - 100 \] ........................................... (2) [1]

From equation 1 & 2

\[ x\sqrt{3} + 100 = \sqrt{3}x - 100 \]

\[ x = 100\sqrt{3}m \]

and so \( h=200m \) [1]

i.e. height of the helicopter is 200m.

28. (i) Volume of each container = \[ \frac{1}{3} \pi h(r_1^2 + r_2^2 + r_1 r_2) \] \[ = \frac{1}{3} \times \frac{22}{7} \times 30(20^2 + 40^2 + 20\times40) \]

\[ = 88000 cm^3 = 88 l \] [1]

Total milk = 880 l

a) Milk in 1 container = 88 l

So number of containers = \[ \frac{880}{88} = 10 \] [1]

b) Cost = 880\times35 = Rs.30800 [1/2]

c) Any relevant Value inculcated [1]

29.

\[ \angle APB = 90^\circ \text{ (angle in a semicircle)} \]

\[ \angle ODB = 90^\circ \text{ (tangent is perpendicular to the radius)} \]

\[ \triangle APB \text{ and } \triangle ODB \]

\[ \angle APB = \angle ODB = 90^\circ \] [1/2]

\[ \angle ABP = \angle OBD \text{ (common)} \] [1/2]

\[ \triangle APB \sim \triangle ODB \text{ (AA)} \] [1/2]
30. (i) \( a_3 = 600 \) \( \therefore a + 2d = 600 \) \( \tag{1} \)

\( a_7 = 700 \) \( \therefore a + 6d = 700 \) \( \tag{2} \)

From (1) & (2)

\( d = 25 \), \( a = 550 \)

(i) \( a_1 = 550 \)

(ii) \( a_{10} = a + 9d = 550 + 9 \times 25 = 775 \)

(iii) \( S_7 = \frac{7}{2}(2 \times 550 + 6 \times 25) \)

\( = 4375 \)

31. \( r = 7\text{cm}, h = 50 \times 0.5 = 25\text{cm} \)

Total Surface Area = \( 2\pi r (r + h) \)

\( = 2 \times \frac{22}{7} \times 7 \times (7 + 25) \)

\( = 1408 \text{ cm}^2 \)

Volume of the box = \( 25 \times 25 \times 25 = 15625 \text{ cm}^3 \)

Volume of the solid formed = \( \pi r^2 h \)

\( = \frac{22}{7} \times 7 \times 7 \times 25 = 3850 \text{ cm}^3 \)

Space left = \( 15625 - 3850 = 11775 \text{ cm}^3 \)

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