

2017

**MATHEMATICS**

Full Marks : 100

Time : 3 hours

*General Instructions :*

- (i) Write all the answers in the Answer Script.
- (ii) The question paper consists of three Sections — A, B and C.
- (iii) Section — A consists of 15 questions, carrying 2 marks each.
- (iv) Section — B consists of 10 questions, carrying 4 marks each, out of which 2 questions have internal choices.
- (v) Section — C has 5 questions, carrying 6 marks each, out of which 2 questions have internal choices.

SECTION — A

( Answer all the Questions )

- 1. Find  $(A \cup B)'$ , if  $U = \{1, 2, 3, 4, 5, 6\}$ ,  
 $A = \{1, 3, 5\}$  and  $B = \{3, 4, 5\}$  2
- 2. If  $P = \{a, b, c\}$  and  $Q = \{q, r\}$ , Find  
 $P \times Q$  and  $Q \times P$  2

- 3. If  $A = \{a, b, c, d, e\}$ ,  $B = \{a, c, e, g\}$   
and  $C = \{b, e, f, g\}$ , Find  $A \cap (B - C)$  2
- 4. Find the domain and range of the following relation 2  
 $R = \left\{ \left( x, \frac{1}{x} \right) : x \text{ is an integer and } 0 < x < 6 \right\}$ .
- 5. Solve:  $\log(x + 1) - \log(x - 1) = 1$  2
- 6. Show that  $(1 + i^{14} + i^{18} + i^{22})$  is a real number. 2
- 7. Find the 17<sup>th</sup> term of the G. P. 2  
 $2, 2\sqrt{2}, 4, 4\sqrt{2}, \dots$
- 8. If  $(n+1)! = 12 \times (n-1)!$ , find the value of  $n$ . 2
- 9. If  ${}^{18}C_r = {}^{18}C_{r+2}$ , find  ${}^rC_5$  2
- 10. Evaluate:  $\lim_{x \rightarrow 3} \frac{x^4 - 81}{x - 3}$  2
- 11. Write the truth value of the following statement 1 + 1 = 2  
(a)  $\sqrt{3}$  is an irrational number.  
(b)  $2 + \sqrt{3}$  is a complex number.

( 3 )

12. Find the value of  $\tan (-120^\circ)$ . 2
13. Prove that,  
 $\cos(n+2)x \cos(n+1)x + \sin(n+2)x \sin(n+1)x = \cos x$  2
14. Find the equation of the line passing through the point  $(2, 3)$  and perpendicular to the line  $4x + 3y = 10$  2
15. A die is tossed once. What is the probability of getting a number greater than 4. 2

SECTION — B

( Answer all the Questions )

16. In a class of certain school 50 Students offered mathematics, 42 offered biology and 24 offered both the subjects. Find the number of students. 2 + 2 = 4  
(i) offering mathematics only.  
(ii) offering biology only.
17. Let  $A = \{2, 3, 5, 7\}$  and  $B = \{3, 5, 9, 13, 15\}$ .  
Let  $f = \{(x, y) : x \in A, y \in B \text{ and } y = 2x - 1\}$ .  
Write  $f$  in roster form. Show that  $f$  is a function from  $A$  to  $B$ . Find domain and range of  $f$ . 4

( 4 )

18. In a circle of diameter 30 cm, the length of a chord is 15 cm. Find the length of the minor arc. 4

Or

Prove that  $\frac{\cos 9^\circ + \sin 9^\circ}{\cos 9^\circ - \sin 9^\circ} = \tan 54^\circ$  4

19. If the sum of first  $p$ -terms of an AP is the same as the sum of its first  $q$ -terms, show that the sum of the first  $(p + q)$  terms is zero. 4

20. If  $(x + iy) = \frac{a + ib}{a - ib}$ , prove that  $x^2 + y^2 = 1$  4

21. Find the derivative of the function  $f(x) = \sqrt{ax + b}$  from the first principle. 4

22. Find the general solution of the equation  
 $\sqrt{3} \cos x + \sin x = 1$  4

Or

In any  $\triangle ABC$ , Prove that,

$\sin\left(\frac{B-C}{2}\right) = \left(\frac{b-c}{a}\right) \cos \frac{A}{2}$  4

( 5 )

23. If  $E_1$  and  $E_2$  are two events such that  $P(E_1)=0.5$ ,  $P(E_2)=0.3$  and  $P(E_1 \text{ and } E_2)=0.1$ . Find 4

(i)  $P(E_1 \text{ or } E_2)$  and

(ii)  $P(\text{neither } E_1 \text{ nor } E_2)$ .

24. Find the equation of the circle whose centre is  $(2, -3)$  and which passes through the point of intersection of the lines  $3x + 2y = 11$  and  $2x + 3y = 4$ . 4

25. Solve the following simultaneous inequalities graphically

$$x + 2y \leq 10, x + y \leq 6, x \geq 0, y \geq 0 \quad 4$$

SECTION — C

( Answer all the Questions )

26. Using the principle of mathematical induction prove that 6

$$\frac{1}{1.3} + \frac{1}{3.5} + \frac{1}{5.7} + \dots + \frac{1}{(2n-1)(2n+1)} = \frac{n}{(2n+1)}$$

( 6 )

27. (a) For the post of 5 teachers, there are 23 applicants. 2 post are reserved for SC candidate and there are 7 SC candidate among the applicants. In how many ways can the selection be made? 3

(b) Find the term independent of  $x$  in the expansion of  $\left(x^2 - \frac{2}{x^3}\right)^{15}$  3

28. (a) If  ${}^{15}P_{r-1} : {}^{16}P_{r-2} = 3 : 4$ , find  $r$  3

(b) Solve :  $x^2 + 3ix + 10 = 0$  3

29. (a) Evaluate :  $\lim_{h \rightarrow 0} \frac{\sqrt{x+h} - \sqrt{x}}{h}$  3

(b) Find  $\frac{dy}{dx}$ , if  $y = \frac{2-3\cos x}{\sin x}$  3

Or

(a) Differentiate :  $\frac{x^2 + 3x - 1}{x + 2}$  3

(b) If  $y = \frac{\cos x + \sin x}{\cos x - \sin x}$ , Prove that  $\frac{dy}{dx} = \sec^2\left(x + \frac{\pi}{4}\right)$  3

( 7 )

30. Find mean, variance and standard deviation from the following data.

6

Class	0-10	10-20	20-30	30-40	40-50
Frequency	5	8	15	16	6

Or

Find the mean deviation about the median for the following data:

6

$x_i$	10	15	20	25	30	35	40	45
$f_i$	7	3	8	5	6	8	4	9

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