Total No. of Printed Pages—7

HS/XII/A. Sc. Com/M/19

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MATHEMATICS

Full Marks : 100

Time : 3 hours

General Instructions :

(i) Write all the answers in the Answer Script.

(ii) The question paper consists of three Sections—A, B and C.

(iii) Section—A consists of 15 questions, carrying *2* marks each.

(iv) Section—B consists of 10 questions, carrying *4* marks each, out of which 2 questions have internal choices.

(v) Section—C has 5 questions, carrying *6* marks each, out of which 2 questions have internal choices.

SECTION—A

1. Show that the function $f : \mathbb{R} \to \mathbb{R}$ defined by $f(x) = x^3$ is one-one and onto.

2. Evaluate :

$$\sin\left(\frac{\pi}{3} - \sin^{-1}\left(-\frac{1}{2}\right)\right)$$

3. If A $\begin{matrix} 1 & 3 \\ 2 & 5 \end{matrix}$ and B $\begin{matrix} 2 & 5 \\ 3 & 4 \end{matrix}$, then find A $4B$.

4. Express the matrix A $\begin{matrix} 2 & 3 \\ 1 & 4 \end{matrix}$ as a sum of a symmetric and a skew-symmetric matrix.

5. For what value of k the function

$$f(x) \quad \frac{k\cos x}{2x}, \text{ when } x \quad \frac{}{2}$$
$$3 \qquad , \text{ when } x \quad \frac{}{2}$$

is continuous at x $\frac{}{2}$?

6. Show that

$$\frac{d}{dx}(\sin 2x \sin 4x) \quad 3\sin 6x \quad \sin 2x$$

7. The side of a square sheet of metal is increasing at 3 cm per minute. At what rate the area is increasing when the side is 10 cm long?

8. Evaluate :

$$^{/2}_{\;/2} |\sin x| dx$$

9. Solve the equation

$$\log \frac{dy}{dx} \quad ax \quad by$$

10. Evaluate :

$$e^{x} \quad \frac{1 \quad x \log x}{x} \; dx$$

11. Find the value of the integral

$$_{0}^{/2} \; \frac{\sin x}{1 \quad \cos^2 x} \, dx$$

12. Find the unit vector perpendicular to both $\vec{a}$ and $\vec{b}$ where

$$\vec{a} \quad 3\hat{i} \quad \hat{j} \quad 2\hat{k} \text{ and } \vec{b} \quad 2\hat{i} \quad 3\hat{j} \quad \hat{k}$$

13. Find the value of k so that the lines

$$\frac{1 \quad x}{3} \quad \frac{7y \quad 14}{2k} \quad \frac{z \quad 3}{2} \text{ and } \frac{7 \quad 7x}{3k} \quad \frac{5 \quad y}{1} \quad \frac{6 \quad z}{5}$$

are at right angles.

14. If $\vec{a}$, $\vec{b}$, $\vec{c}$ are unit vectors such that $\vec{a} \quad \vec{b} \quad \vec{c} \quad 0$, find the value of

$$\vec{a} \;\; \vec{b} \quad \vec{b} \;\; \vec{c} \quad \vec{c} \;\; \vec{a}$$

15. The probability that a student selected at random from a class will pass in Hindi is $\frac{4}{5}$ and the probability that he passes in Hindi and English is $\frac{1}{2}$. What is the probability that he will pass in English if it is known that the student has passed in Hindi?

SECTION—B

16. Let $\mathbb{N}$ be the set of natural numbers and let R be a relation on $\mathbb{N} \quad \mathbb{N}$ defined by

$$(a,\, b) R (c,\, d) \qquad ad \quad bc$$

Prove that R is an equivalence relation.

17. Using the properties of determinants, show that

$$\begin{vmatrix} 1 & a & a^2 & bc \\ 1 & b & b^2 & ca \\ 1 & c & c^2 & ab \end{vmatrix} \quad 0$$

18. If $x \quad \dfrac{1 \quad \log t}{t^2}$ and $y \quad \dfrac{3 \quad 2\log t}{t}$, then show that $\dfrac{dy}{dx} \quad t$.

19. Evaluate :

$$(\sin\ ^{1} x)^2\, dx$$

20. Using differential, find the approximate value of $\sqrt[3]{127}$.

21. Find the interval in which the function $f(x) \quad 5 \quad 36x \quad 3x^2 \quad 2x^3$ is *(a)* strictly increasing and *(b)* strictly decreasing.

Or

Find the equation of the tangent to the curve $x^2 \quad 3y \quad 3$ which is parallel to the line $y \quad 4x \quad 5 \quad 0$.

22. Using the properties of definite integral, show that

$$_{/6}^{/3} \frac{1}{1 \quad \sqrt{\tan x}} dx \quad \frac{}{12}$$

23. Evaluate the following integral as the limit of a sum :

$$_{1}^{4} (3x^2 \quad 2x)\, dx$$

24. Find the equation of the plane passing through the point $(1,\ 0,\ \ 2)$ and perpendicular to each of the planes $2x \quad y \quad z \quad 2 \quad 0$ and $x \quad y \quad z \quad 3 \quad 0$.

Or

Find the length and foot of the perpendicular from the point (7, 14, 5) to the plane $2x \quad 4y \quad z \quad 2$.

25. Find the shortest distance between the lines

$$\vec{r} \quad (6\hat{i} \quad 3\hat{k}) \qquad (2\hat{i} \quad \hat{j} \quad 4\hat{k}) \text{ and}$$

$$\vec{r} \quad (\ 9\hat{i} \quad \hat{j} \quad 10\hat{k}) \qquad (4\hat{i} \quad \hat{j} \quad 6\hat{k})$$

SECTION—C

26. Using integration, find the area of the region in the first quadrant enclosed by the X-axis, the line $y \quad x$ and the circle $x^2 \quad y^2 \quad 32$.

27. Using matrices, solve the following system of equations :

$$\frac{2}{x} \quad \frac{3}{y} \quad \frac{3}{z} \quad 10$$
$$\frac{1}{x} \quad \frac{1}{y} \quad \frac{1}{z} \quad 10$$
$$\frac{3}{x} \quad \frac{1}{y} \quad \frac{2}{z} \quad 13$$

28. Show that the maximum volume of the cylinder which can be inscribed in a sphere of radius $5\sqrt{3}$ cm is $(500 \quad)$ cm^3.

Or

A window is in the form of a rectangle, surmounted by a semicircular opening. The total perimeter of the window is 10 metres. Find the dimensions of the window to admit maximum light through it.

29. A box contains 16 bulbs, out of which 4 bulbs are defective. 3 bulbs are drawn one by one from the box without replacement. Let X be the number of defective bulbs drawn. Find the mean and variance of X.

30. A manufacturer produces two types of steel trunks. He has two machines *A* and *B*. The first type of trunk requires 3 hours on machine *A* and 3 hours on machine *B*. The second type requires 3 hours on machine *A* and 2 hours on machine *B*. Machines *A* and *B* can work at most 18 hours and 15 hours per day respectively. He earns a profit of ₹ 30 and ₹ 25 per trunk of first and second type respectively. How many trunks of each type must he make each day to make maximum profit?

Or

Two tailors *A* and *B*, earn ₹ 300 and ₹ 400 per day respectively. *A* can stitch 6 shirts and 4 pairs of trousers per day while *B* can stitch 10 shirts and 4 pairs of trousers per day. How many days should each of them work if it is desired to produce at least 60 shirts and 32 pairs of trousers at a minimum labour cost?

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