EXERCISE 1.1

Write the correct answer in each of the following:

1. Every rational number is

(A) a natural number
(B) an integer
(C) a real number
(D) a whole number

Solution:

(C) a real number

Explanation:

We know that rational and irrational numbers taken together are known as real numbers. Therefore, every real number is either a rational number or an irrational number. Hence, every rational number is a real number.

Hence, (C) is the correct option.

2. Between two rational numbers

(A) there is no rational number
(B) there is exactly one rational number
(C) there are infinitely many rational numbers
(D) there are only rational numbers and no irrational numbers

Solution:

(C) there are infinitely many rational numbers

Explanation:

Between two rational numbers there are infinitely many rational number.

Hence, (C) is the correct option.

3. Decimal representation of a rational number cannot be

(A) terminating
(B) non-terminating
(C) non-terminating repeating
(D) non-terminating non-repeating

Solution:

(D) non-terminating non-repeating

Explanation:

The decimal representation of a rational number cannot be non-terminating and non- repeating.

Hence, (D) is the correct option

4. The product of any two irrational numbers is

(A) always an irrational number
(B) always a rational number
(C) always an integer
(D) sometimes rational, sometimes irrational

Solution:

(D) sometimes rational, sometimes irrational

Explanation:
The product of any two irrational numbers is sometimes rational and sometimes irrational.
Hence, (D) is the correct option

5. The decimal expansion of the number √2 is
(A) a finite decimal
(B) 1.41421
(C) non-terminating recurring
(D) non-terminating non-recurring

Solution:
(D) non-terminating non-recurring
Explanation:
The decimal expansion of the number √2 = 1.41421356237…
Hence, (D) is the correct option

6. Which of the following is irrational?
(A) √4/√9
(B) √12/√3
(C) √7
(D) √81

Solution:
(C) √7
Explanation:
(A) √4/√9 = 2/3
(B) √12/√3 = 2√3/√3 = 2
(C) √7 = 2.64575131106
(D) √81 = 9
Here, (C) √7 = 2.64575131106, is a non terminating decimal expansion.
Hence, (C) is the correct option

7. Which of the following is irrational?
(A) 0.14
(B) $0.14\overline{16}$
(C) $0.\overline{1416}$
(D) 0.4014001400014...

Solution:
(D) 0.4014001400014…
Explanation:
A number is irrational if and only of its decimal representation is non-terminating and non-recurring.
(A) is a terminating decimal and therefore cannot be an irrational number.
(B) is a non-terminating and recurring decimal and therefore cannot be irrational.
(C) is a non-terminating and recurring decimal and therefore cannot be irrational.
(D) is a non-terminating and non-recurring decimal and therefore is an irrational number.
Hence, (D) is the correct option.

8. A rational number between 2 and 3 is

(A) (√2+√3)/2
(B) (√2. √3)/2
(C) 1.5
(D) 1.8

Solution:

(C) 1.5

Explanation:

√2 =1.4142135.... and √3 =1.732050807....

(A) (√2+√3)/2 = 1.57313218497… is a non-terminating and non-recurring decimal and therefore is an irrational number.

(B) (√2. √3)/2 = 1.22474487139… is a non-terminating and non-recurring decimal and therefore is an irrational number.

(C) 1.5 is a terminating decimal and therefore is a rational number.

(D) 1.8 is a terminating decimal and therefore is a rational number.

Here both 1.5 and 1.8 are rational numbers. But, 1.8 does not lie in between √2 =1.4142135.... and √3 =1.732050807.... Whereas 1.5 lies in between √2 =1.4142135.... and √3 =1.732050807....

Hence, (C) is the correct option.

9. The value of 1.999... in the form *p/q*, where *p* and *q* are integers and *q* ≠ 0 , is

(A) 19/10
(B) 1999/1000
(C) 2
(D) 1/9

Solution:

(C) 2

Explanation:

(A) 19/10 = 1.9
(B) 1999/1000= 1.999
(C) 2
(D) 1/9= 0.111….

Let x = 1.9999..... --- (1)
Multiply equation (1) with 10
10x = 19.9999..... --- (2)
Subtract equation (1) from equation(2) ,
We get,
9x = 18
x = 18 / 9
x = 2
Therefore,
x = 1.9999... = 2
Hence, (C) is the correct option.

10. 2√3 + √3 is equal to

(A) 2√6
(B) 6

(C) $3\sqrt{3}$
(D) $4\sqrt{6}$

Solution:

(C) $3\sqrt{3}$

Explanation:

$2\sqrt{3} + \sqrt{3}$

Taking $\sqrt{3}$ common,

We get,

$\sqrt{3}(2+1) = \sqrt{3}(3) = 3\sqrt{3}$

Hence, (C) is the correct option.

EXERCISE 1.2 PAGE NO: 6

1. Let x and y be rational and irrational numbers, respectively. Is x + y necessarily an irrational number? Give an example in support of your answer.

Solution:

Yes, if x and y are rational and irrational numbers, respectively, then x+ y is an irrational number.

For example,

Let x = 5 and y = $\sqrt{2}$.

Then, x+y = 5 + $\sqrt{2}$ = 5 + 1.414… = 6.414...

Here, 6.414 is a non-terminating and non-recurring decimal and therefore is an irrational number.

Hence, x + y is an irrational number.

2. Let x be rational and y be irrational. Is xy necessarily irrational? Justify your answer by an example.

Solution:

No, if x is rational number and y is irrational number, then, xy is not necessarily an irrational number. It can be rational if x = 0, which is a rational number.

For Example:

Let y = $\sqrt{2}$, which is irrational.

Consider x = 2, which is rational.

Then, x × y = 2 × $\sqrt{2}$ = 2$\sqrt{2}$, which is irrational.

Consider x = 0, which is rational.

Then xy = 0 × $\sqrt{2}$ = 0, which is rational.

∴, we can conclude that, the product of a rational and an irrational number is always irrational, only if the rational number is not zero.

EXERCISE 1.3 PAGE NO: 9

1. Find which of the variables x, y, z and u represent rational numbers and which irrational numbers:

(i) $x^2 = 5$
(ii) $y^2 = 9$
(iii) $z^2 = .04$
(iv) $u^2 = 17/4$

Solution:

(i) $x^2 = 5$
On solving, we get
$\Rightarrow x = \pm \sqrt{5}$
Hence, x is an irrational number.

(ii) $y^2 = 9$
On solving, we get
$\Rightarrow y = \pm 3$
Hence, y is a rational number.

(iii) $z^2 = .04$
On solving, we get
$\Rightarrow z = \pm 0.2$
Hence, z is a rational number.

(iv) $u^2 = 17/4$
On solving, we get
$\Rightarrow u = \pm \sqrt{17}/2$
$\sqrt{17}$ is irrational.
Hence, u is an irrational number

2. Find three rational numbers between

(i) –1 and –2
(ii) 0.1 and 0.11
(iii) 5/7 and 6/7
(iv) 1/4 and 1/5

Solution:

(i) –1 and –2
Three rational numbers between –1 and –2 are –1.1, –1.2 and –1.3.

(ii) 0.1 and 0.11
Three rational numbers between 0.1 and 0.11 are 0.101, 0.102 and 0.103.

(iii)5/7 and 6/7
5/7 can be written as $(5 \times 10)/(7 \times 10) = 50/70$
Similarly,

6/7 can be written as (6 × 10)/(7 × 10) = 60/70
Three rational numbers between 5/7 and 6/7 = three rational numbers between 16/80 and 20/80.
Three rational numbers between 5/7 and 6/7 are 51/70, 52/70, 53/70.

(iv)1/4 and 1/5
Here, according to the question,
LCM of 4 and 5 is 20.
Let us make the denominators common, 80.
(4 × 20) = 80 and (5 × 16) = 80
Hence,
1/4 can be written as (1 × 20)/(4 × 20) = 20/80
Similarly,
1/5 can be written as (1 × 16)/(5 × 16) = 16/80
Three rational numbers between 1/4 and 1/5 = three rational numbers between 16/80 and 20/80.
Therefore, the three rational numbers are 17/80, 18/80 and 19/80.

3. Insert a rational number and an irrational number between the following:
(i) 2 and 3
(ii) 0 and 0.1
(iii) 1/3 and 1/2
(iv) – 2/5 and 1/2
(v) 0.15 and 0.16
(vi) √2 and √3
(vii) 2.357 and 3.121
(viii) .0001 and .001
(ix) 3.623623 and 0.484848
(x) 6.375289 and 6.375738.

Solution:

(i) 2 and 3
So, rational number between 2 and 3 = 2.5
And, irrational number between 2 and 3 = 2.040040004...

(ii) 0 and 0.1
So, rational number between 0 and 0.1 = 0.05
And, irrational number between 0 and 0.1 = 0.007000700007…

(iii) 1/3 and 1/2
LCM of 3 and 2 is 6.
1/3 = 0.33
1/3 can be written as (1 × 20)/(3 × 20) = 20/60
½ = 0.5
1/2 can be written as (1 × 30)/(2 × 30) = 30/60
So, rational number between 1/3 and 1/2 = 25/60
And, irrational number between 1/3 and 1/2 = irrational number between 0.33 and 0.5 = 0.414114111...

(iv) – 2/5 and 1/2
LCM of 5 and 2 is 10.
-2/5 = -0.4
-2/5 can be written as (-2 × 2)/(5 × 2) = -4/10
1/2 = 0.5
1/2 can be written as (1 × 5)/(2 × 5) = 5/10
So, rational number between -2/5 and 1/2 = rational number between -4/10 and 5/10 = 1/10
And, irrational number between -2/5 and 1/2 = irrational number between -0.4 and 0.5 = 0.414114111...

(v) 0.15 and 0.16
Rational number between 0.15 and 0.16 = 0.151
Irrational number between 0.15 and 0.16 = 0.151551555...

(vi) √2 = 1.41 and √3 = 1.732
Rational number between √2 and √3 = rational number between 1.41 and 1.732 = 1.5
Irrational number between √2 and √3 = irrational number between 1.41 and 1.732 = 1.585585558...

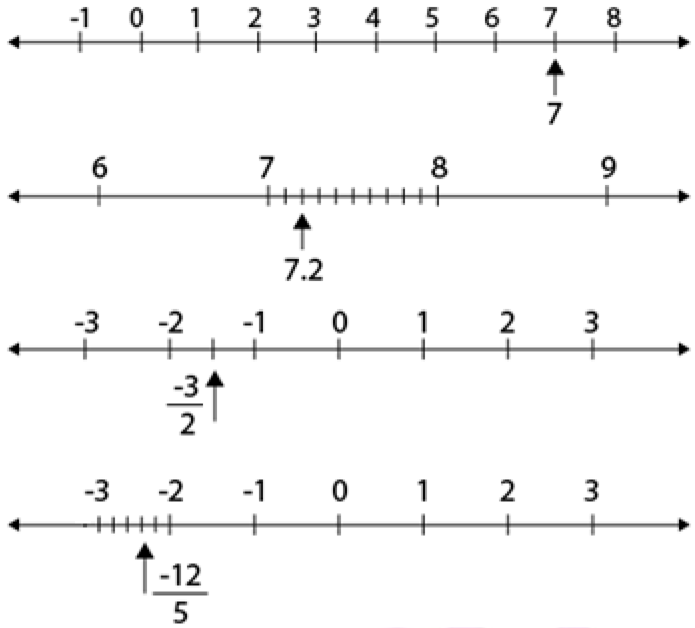
(vii) 2.357 and 3.121
Rational number between 2.357 and 3.121 = 3
Irrational number between 2.357 and 3.121 = 3.101101110...

(viii) .0001 and .001
Rational number between .0001 and .001 = 0.00011
Irrational number between .0001 and .001 = 0.0001131331333...

(ix) 3.623623 and 0.484848
Rational number between 3.623623 and 0.484848 = 1
Irrational number between 3.623623 and 0.484848 = 1.909009000...

(x) 6.375289 and 6.375738.
Rational number between 6.375289 and 6.375738 = 6.3753
Irrational number between 6.375289 and 6.375738 = 6.375414114111...

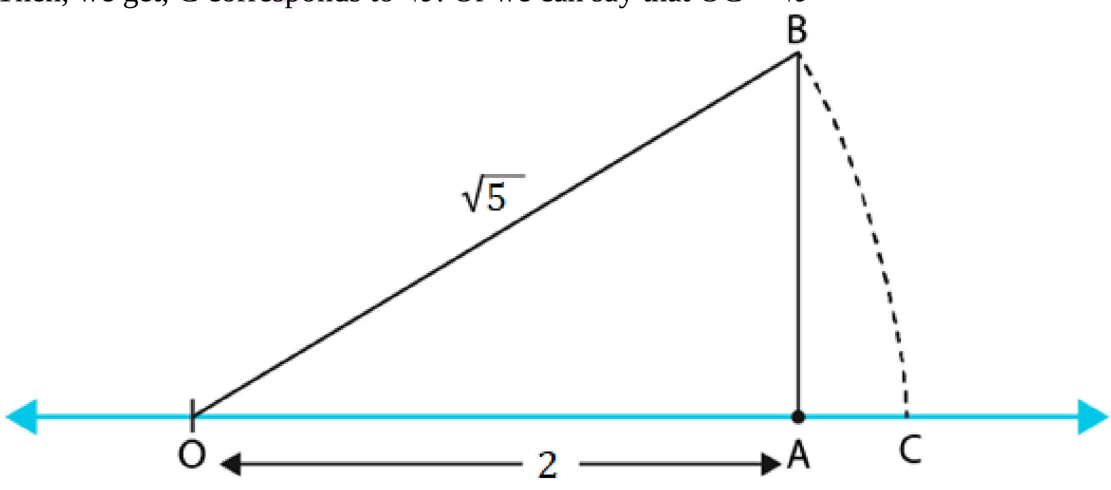
4. Represent the following numbers on the number line:
7, 7.2, −3/2 , −12/5
Solution:



5. Locate √5, √10 and √17 on the number line.
Solution:

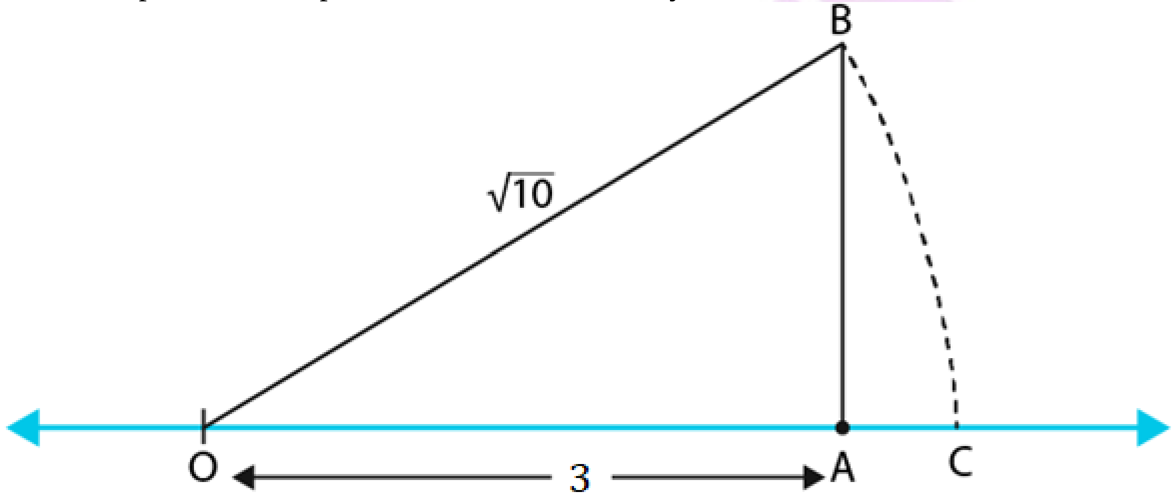
<u>√5 on the number line:</u>

5 can be written as the sum of the square of two natural numbers:
i.e., $5 = 1 + 4 = 1^2 + 2^2$
On the number line,
Take OA = 2 units.
Perpendicular to OA, draw BA = 1 unit.
Join OB.
Using Pythagoras theorem,
We have, OB= √5
Draw an arc with centre O and radius OB using a compass such that it intersects the number line at the point C.
Then, we get, C corresponds to √5. Or we can say that OC = √5



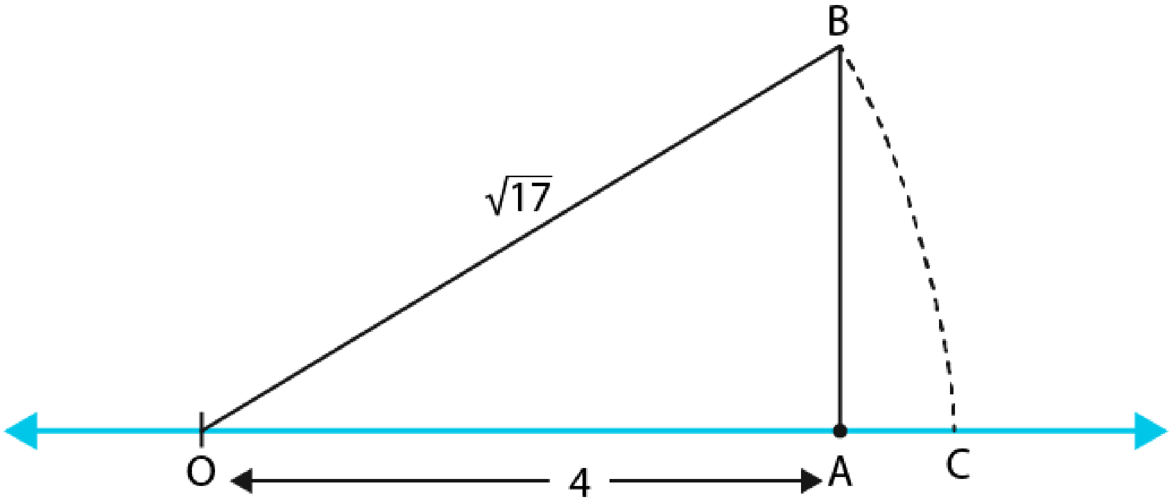
√10 on the number line:

10 can be written as the sum of the square of two natural numbers:
i.e., $10 = 1 + 9 = 1^2 + 3^2$
On the number line,
Take OA = 3 units.
Perpendicular to OA, draw BA = 1 unit.
Join OB.
Using Pythagoras theorem,
We have, OB= √5
Draw an arc with centre O and radius OB using a compass such that it intersects the number line at the point C.
Then, the point C corresponds to √10. Or we can say that OC = √10



√17 on the number line:

17 can be written as the sum of the square of two natural numbers:
i.e., $17 = 1 + 16 = 1^2 + 4^2$
On the number line,
Take OA = 4 units.
Perpendicular to OA, draw BA = 1 unit.
Join OB.
Using Pythagoras theorem,
We have, OB= √17
Draw an arc with centre O and radius OB using a compass such that it intersects the number line at the point C.
Then, the point C corresponds to √17. Or, we can say that OC = √17



6. Represent geometrically the following numbers on the number line:

(i) √4.5

(ii) √5.6

(iii) √8.1

(iv) √2.3

Solution:

(i) √4.5

Draw a line segment such that AB = 4.5 units.

Mark C at a distance of 1 unit from B.

Mark O, the mid-point of AC.

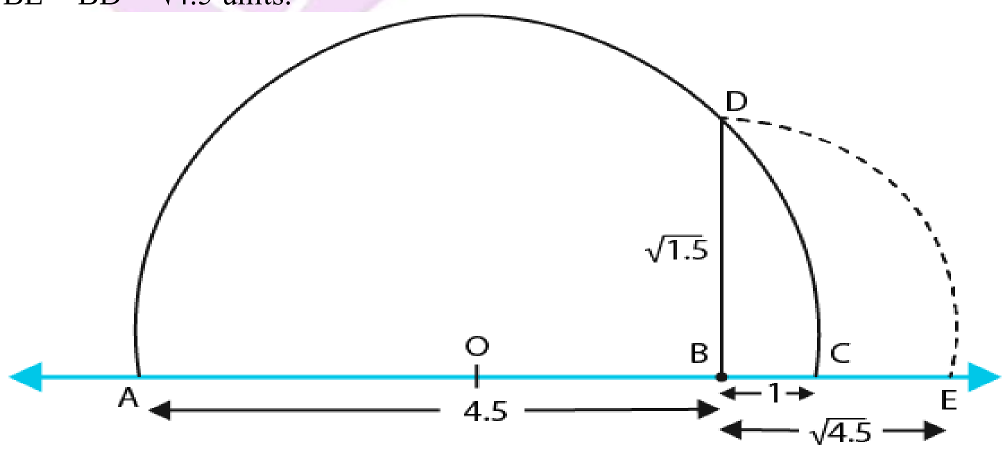
Draw a semicircle with centre O and radius OC.

Draw a line perpendicular to AC, passing through B and intersecting the semicircle at D.

Now, BD = √4.5.

Draw an arc with centre B and radius BD, meeting AC produced at E.

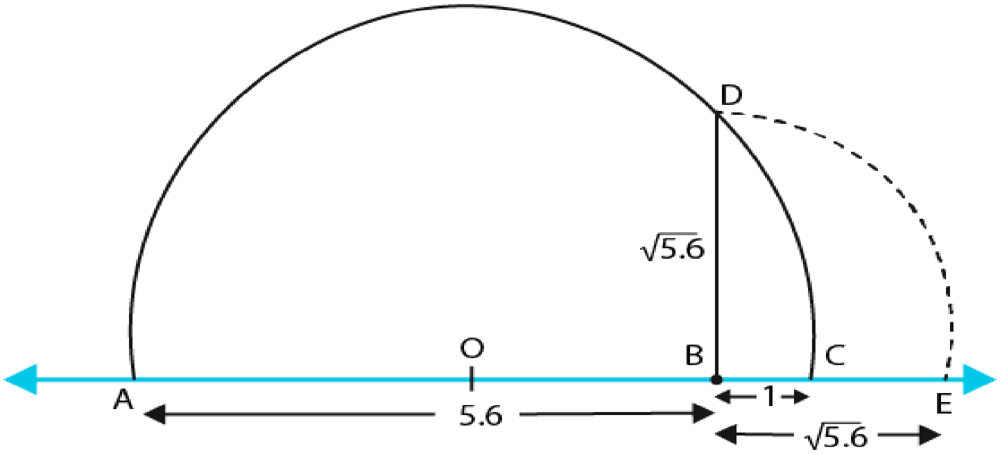
Then BE = BD = √4.5 units.



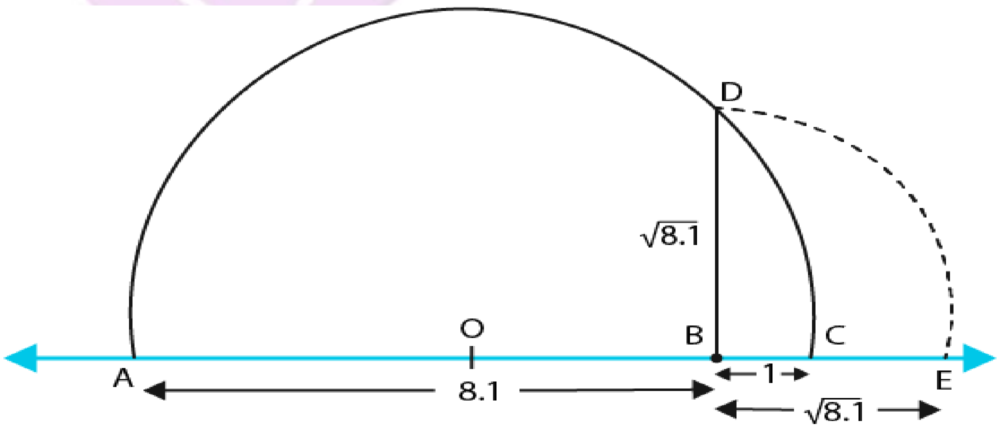
(ii) √5.6

Draw a line segment such that AB = 5.6 units.

Mark C at a distance of 1 unit from B.
Mark O, the mid-point of AC.
Draw a semicircle with centre O and radius OC.
Draw a line perpendicular to AC, passing through B and intersecting the semicircle at D.
Now, BD = $\sqrt{5.6}$
Draw an arc with centre B and radius BD, meeting AC produced at E.
Then BE = BD = $\sqrt{5.6}$ units.

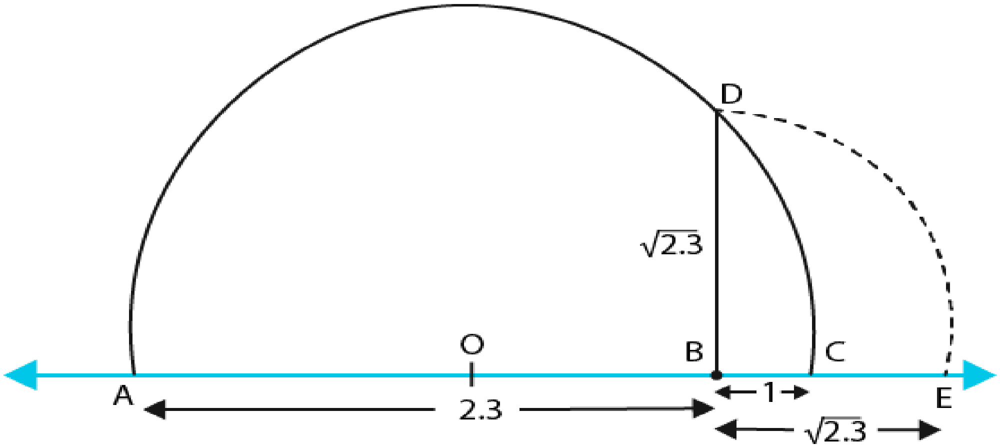


(iii) $\sqrt{8.1}$
Draw a line segment such that AB = 8.1 units.
Mark C at a distance of 1 unit from B.
Mark O, the mid-point of AC.
Draw a semicircle with centre O and radius OC.
Draw a line perpendicular to AC, passing through B and intersecting the semicircle at D.
Now, BD = $\sqrt{8.1}$.
Draw an arc with centre B and radius BD, meeting AC produced at E.
Then BE = BD = $\sqrt{8.1}$ units.



(iv) $\sqrt{2.3}$
Draw a line segment such that AB = 2.3 units.
Mark C at a distance of 1 unit from B.
Mark O, the mid-point of AC.
Draw a semicircle with centre O and radius OC.

Draw a line perpendicular to AC, passing through B and intersecting the semicircle at D.
Now, BD = √2.3.
Draw an arc with centre B and radius BD, meeting AC produced at E.
Then BE = BD = √2.3 units.



7. Express the following in the form p/q, where p and q are integers and q ≠ 0 :
(i) 0.2
(ii) 0.888...
(iii) $5.\overline{2}$
(iv) $0.\overline{001}$
(v) 0.2555...
(vi) $0.1\overline{34}$
(vii) .00323232...
(viii) .404040...
Solution:

(i) 0.2

We know that,
0/2 can be written as,
0.2 = 2/10 = 1/5

(ii) 0.888...

Assume that $x = 0.888\ \ldots$
$\Rightarrow x = 0.8$ ……………. Eq.(1)
Multiply L.H.S and R.H.S by 10,
We get
$10\ x = 8.8$ ……………. Eq.(2)
Subtracting equation (1) from (2),
We get
$10\ x - x = 8.8 - 0.8$
$\Rightarrow 9x = 8$
$\Rightarrow x = 8/9$

(iii) $5.\overline{2}$

Assume that $x = 5.\overline{2}$ ……………. Eq.(1)
Multiply L.H.S and R.H.S by 10,
We get
$10\,x = 52.\overline{2}$ …………… Eq. (2)
Subtracting equation (1) from (2),
We get
$10\,x - x = 52.\overline{2} - 5.\overline{2}$
$\Rightarrow 9x = 47$
$\Rightarrow x = 47/9$

(iv) $0.\overline{001}$

Assume that $x = 0.\overline{001}$ ……………. Eq. (1)
Multiply L.H.S and R.H.S by 1000,
We get
$1000\,x = 1.\overline{001}$ …………… Eq. (2)
Subtracting equation (1) from (2),
We get
$1000x - x = 1.\overline{001} - 0.\overline{001}$
$\Rightarrow 999x = 1$
$\Rightarrow x = 1/999$

(v) 0.2555...

Assume that $x = 0.2555$ …
$\Rightarrow$ x = $0.2\overline{5}$ ……………. Eq. (1)
Multiply L.H.S and R.H.S by 10,
We get
10 x = $2.\overline{5}$ ……………. Eq. (2)
Multiply L.H.S and R.H.S by 100,
We get
100 x = $25.\overline{5}$ ………… Eq. (3)
Subtracting equation (2) from (3),
We get
100 x-10x = $25.\overline{5}$ - $2.\overline{5}$
$\Rightarrow 90x = 23$
$\Rightarrow x = 23/90$

(vi) $0.1\overline{34}$

Let $x = 0.1\overline{34}$ …………….. Eq. (1)
Multiply L.H.S and R.H.S by 10,
We get
$10\,x = 1.\overline{34}$ ………………. Eq. (2)
Multiply L.H.S and R.H.S by 1000,
We get
$1000\,x = 134.\overline{34}$ ………… Eq. (3)
Subtracting equation (2) from (3),

We get
$1000\,x - 10x = 134.34 - 1.34$
$\Rightarrow 990x = 133$
$\Rightarrow x = 133/990$

(vii) .00323232...
Let $x = 0.00323232\ \ldots$
⇒ x = 0.0032 ……………… Eq. (1)
Multiply L.H.S and R.H.S by 100,
We get,
100x = 0.32 ……………. Eq. (2)
Multiply L.H.S and R.H.S by 10000,
We get
10000 x = 32.32 …………. Eq. (3)
Subtracting equation (2) from (3),
We get
10000 x-100x = 32.32 – 0.32
$\Rightarrow 9900x = 32$
$\Rightarrow x = 32/9900 = 8/2475$

(viii) .404040...
Let $x = 0.404040\ \ldots$
$\Rightarrow x = 0.\ 40$ ……………… (1)
Multiply L.H.S and R.H.S by 100,
We get
$100\ x = 40.40$ …………. (2)
Subtracting equation (1) from (2),
We get
$100\ x - x = 40.40 - 0.40$
$\Rightarrow 99x = 40$
$\Rightarrow x = 40/99$

EXERCISE 1.4 PAGE NO: 12

1. Express $0.6+0.\overline{7}+0.4\overline{7}$ in the form p/q, where *p* and *q* are integers and $q \neq 0$.

Solution:

Let x = 0.6
Multiply by 10 on L.H.S and R.H.S,
10x = 6
x = 6/10
x = 3/5
So, the p/q form of 0.6 = 3/5
Let y = 0.77777…
Multiply by 10 on L.H.S and R.H.S,
10y = 7.7777…
10y – y = 7.7777777……. – 0.7777777…………..
9y = 7
y = 7/9
So the p/q form of 0.7777… = 7/9
Let z = 0.47777…
Multiply by 10 on L.H.S and R.H.S,
10z = 4.7777…
10z – z = 4.7777777… – 0.47777777…
9z = 4.2999
z ≈ 4.3/9
z = 43/90
So the p/q form of 0.4777… = 43/90
Therefore, p/q form of $0.6+0.\overline{7}+0.4\overline{7}$ is,
x+y+z = 3/5 + 7/9 + 43/90
= (54 + 70 + 43)/90
= 167/90

2. Simplify:

$$\frac{7\sqrt{3}}{\sqrt{10}+\sqrt{3}} - \frac{2\sqrt{5}}{\sqrt{6}+5} - \frac{3\sqrt{2}}{\sqrt{15}+3\sqrt{2}}$$

Solution:

$$\frac{7\sqrt{3}}{\sqrt{10}+\sqrt{3}} - \frac{2\sqrt{5}}{\sqrt{6}+5} - \frac{3\sqrt{2}}{\sqrt{15}+3\sqrt{2}}$$

Let us first make the denominators same,

To make the denominators same, Cross multiply the first and second terms of the equation.

$$\Rightarrow \frac{7\sqrt{3}\times(\sqrt{6}+\sqrt{5})-(\sqrt{10}+\sqrt{3})\times 2\sqrt{5}}{(\sqrt{10}+\sqrt{3})\times(\sqrt{6}+\sqrt{5})}-\frac{3\sqrt{2}}{\sqrt{15}+3\sqrt{2}}$$

$$\Rightarrow \frac{7\sqrt{3}\times\sqrt{6}+7\sqrt{3}\times\sqrt{5}-(2\sqrt{5}\times\sqrt{10}+2\sqrt{5}\times\sqrt{3})}{(\sqrt{10}+\sqrt{3})\times(\sqrt{6}+\sqrt{5})}-\frac{3\sqrt{2}}{\sqrt{15}+3\sqrt{2}}$$

$$\Rightarrow \frac{21\sqrt{2}+7\sqrt{15}-10\sqrt{2}-2\sqrt{15}}{2\sqrt{15}+5\sqrt{2}+3\sqrt{2}+\sqrt{15}}-\frac{3\sqrt{2}}{\sqrt{15}+3\sqrt{2}}$$

$$\Rightarrow \frac{11\sqrt{2}-5\sqrt{15}}{3\sqrt{15}+8\sqrt{2}}-\frac{3\sqrt{2}}{\sqrt{15}+3\sqrt{2}}$$

Now, again make the denominators same by cross-multiplying the obtained term and the third term of the given equation in the question.

$$\Rightarrow \frac{(11\sqrt{2}-5\sqrt{15})\times(\sqrt{15}+3\sqrt{2})-(3\sqrt{15}+8\sqrt{2})\times(3\sqrt{2})}{(3\sqrt{15}+8\sqrt{2})\times\sqrt{15}+3\sqrt{2}}$$

$$\Rightarrow \frac{11\sqrt{30}+66-75-15\sqrt{30}-9\sqrt{30}-48}{45+9\sqrt{30}+8\sqrt{30}+48}$$

$$\Rightarrow \frac{-13\sqrt{30}-57}{17\sqrt{30}+93}$$

3. If √2 =1.414, √3 =1.732, then find the value of

$$\frac{4}{3\sqrt{3}-2\sqrt{2}}+\frac{3}{3\sqrt{3}+2\sqrt{2}}$$

Solution:

$$\frac{4}{3\sqrt{3}-2\sqrt{2}}+\frac{3}{3\sqrt{3}+2\sqrt{2}}$$

Let us first make the denominators same by cross multiplication method

$$\Rightarrow \frac{4\times(3\sqrt{3}+2\sqrt{2})+3\times(3\sqrt{3}-2\sqrt{2})}{(3\sqrt{3}-2\sqrt{2})\times(3\sqrt{3}+2\sqrt{2})}$$

Observing the denominator, we can say that,

Denominators is of the form,

$(a + b) \times (a - b) = (a^2 - b^2)$

Here $a = 3\sqrt{3}$

$b = 2\sqrt{2}$

$a^2 = (3\sqrt{3})^2 = 27$
$b^2 = (2\sqrt{2})^2 = 8$

$$\Rightarrow \frac{12\sqrt{3} + 8\sqrt{2} + 9\sqrt{3} - 6\sqrt{2}}{27 - 8}$$

$$\Rightarrow \frac{21\sqrt{3} + 2\sqrt{2}}{19}$$