EXERCISE 2.1

Write the correct answer in each of the following:
1. Which one of the following is a polynomial?

(A) $\frac{x^2}{2}-\frac{2}{x^2}$ (B) $\sqrt{2x}-1$

(C) $x^2+\frac{3x^{\frac{3}{2}}}{\sqrt{x}}$ (D) $\frac{x-1}{x+1}$

Solution:

(C)

$$x^2+\frac{3x^{\frac{2}{3}}}{\sqrt{x}}=x^2+3x$$

Explanation:

(A)

$$\frac{x^2}{2}-\frac{2}{x^2}=\frac{x^2}{2}-2x^{-2}$$

The equation contains the term x^2 and $-2x^{-2}$.
Here, the exponent of x in second term = – 2, which is not a whole number.
Hence, the given algebraic expression is not a polynomial.

(B)

$$\sqrt{2x}-1=\sqrt{2}x^{\frac{1}{2}}-1$$

The equation contains the term $\sqrt{2}x^{½}$.
Here, the exponent of x in first term = ½ , which is not a whole number.
Hence, the given algebraic expression is not a polynomial.

(C)

$$x^2+\frac{3x^{\frac{2}{3}}}{\sqrt{x}}=x^2+3x$$

The equation contains the term x^2 and 3x .
Here, the exponent of x in first term and second term= 2and 1 respectively, which is a whole number.
Hence, the given algebraic expression is a polynomial.

(D)

$$\frac{x-1}{x+1}$$

The equation is a rational function.
Here, the given equation is not in the standard form of polynomial.
Hence, the given algebraic expression is not a polynomial.

Hence, option C is the correct answer

2. √2 is a polynomial of degree

(A) 2
(B) 0
(C) 1
(D) ½

Solution:

(B) 0

Explanation:

√2 can be written as $\sqrt{2}x^0$

i.e., $\sqrt{2} = \sqrt{2}x^0$

Therefore, the degree of the polynomial = 0

Hence, option B is the correct answer

3. Degree of the polynomial $4x^4 + 0x^3 + 0x^5 + 5x + 7$ is

(A) 4
(B) 5
(C) 3
(D) 7

Solution:

(A) 4

Explanation:

Degree of a polynomial = Highest power of the variable in a polynomial.

The highest power of variable x in the polynomial $4x^4 + 0x^3 + 0x^5 + 5x + 7$ is 4.

Therefore, degree of the polynomial of $4x^4 + 0x^3 + 0x^5 + 5x + 7 = 4$

Hence, option A is the correct answer

4. Degree of the zero polynomial is

(A) 0
(B) 1
(C) Any natural number
(D) Not defined

Solution:

(D) Not defined

Explanation:

Degree of a zero polynomial is not defined.

Hence, option D is the correct answer

5. If $p(x)= x^2 – 2\sqrt{2}x + 1$, then $p(2\sqrt{2})$ is equal to

(A) 0
(B) 1
(C) $4\sqrt{2}$
(D) $8\sqrt{2} +1$

Solution:

(B) 1

Explanation:

According to the question,

$p(x) = x^2 - 2\sqrt{2}x + 1$

To get $p(2\sqrt{2})$,

We substitute $x = 2\sqrt{2}$,

$p(2\sqrt{2}) = (2\sqrt{2})^2 - (2\sqrt{2} \times (2\sqrt{2})) + 1$

$= (4 \times 2) - (4 \times 2) + 1$

$= 8 - 8 + 1$

$= 1$

Hence, option B is the correct answer

6. The value of the polynomial $5x - 4x^2 + 3$, when $x = -1$ is

(A) – 6

(B) 6

(C) 2

(D) – 2

Solution:

(A) – 6

Explanation:

According to the question,

$p(x) = 5x - 4x^2 + 3$

To get $p(-1)$,

We substitute $x = -1$,

$p(-1) = 5(-1) - 4(-1)^2 + 3$

$= 5(-1) - 4(1) + 3$

$= -5 - 4 + 3$

$= -9 + 3$

$= -6$

Hence, option A is the correct answer

7. If $p(x) = x + 3$, then $p(x) + p(-x)$ is equal to

(A) 3

(B) 2x

(C) 0

(D) 6

Solution:

(D) 6

Explanation:

$p(x) = x + 3$

$p(-x) = -x + 3$

Therefore,

$p(x) + p(-x) = (x + 3) + (-x + 3)$

$= x + 3 - x + 3$

$= 6$

Hence, option D is the correct answer

8. Zero of the zero polynomial is

(A) 0
(B) 1
(C) Any real number
(D) Not defined

Solution:

(C) Any real number

Explanation:

Zero polynomial is a constant polynomial whose coefficients are all equal to 0.
Zero of a polynomial is the value of the variable that makes the polynomial equal to zero.
Therefore, zero of the zero polynomial is any real number.

Hence, option C is the correct answer

9. Zero of the polynomial p(x) = 2x + 5 is

(A) – 2/5
(B) – 5/2
(C) 2/5
(D) 5/2

Solution:

(B) – 5/2

Explanation:

Zero of the polynomial ⇒ p(x) = 0
p(x) = 0
2x + 5 = 0
2x = – 5
x = – 5/2

Hence, option B is the correct answer

10. One of the zeroes of the polynomial $2x^2 + 7x - 4$ is

(A) 2
(B) ½
(C) – ½
(D) –2

Solution:

(B) ½

Explanation:

Zero of the polynomial ⇒ p(x) = 0
p(x) = 0
$2x^2 + 7x - 4 = 0$
$2x^2 - 1x + 8x - 4 = 0$
x(2x – 1) + 4(2x – 1) = 0
(x + 4)(2x – 1) = 0
Consider, x + 4
x + 4 = 0
x = – 4
Consider, 2x – 1

$2x - 1 = 0$

$2x = 1$

$x = ½$

Hence, option B is the correct answer

EXERCISE 2.2

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1. Which of the following expressions are polynomials? Justify your answer:

(i) 8 (ii) $\sqrt{3}x^2 - 2x$ (iii) $1 - \sqrt{5x}$

(iv) $\frac{1}{5x^{-2}} + 5x + 7$ (v) $\frac{(x-2)(x-4)}{x}$ (vi) $\frac{1}{x+1}$

(vii) $\frac{1}{7}a^3 - \frac{2}{\sqrt{3}}a^2 + 4a - 7$ (viii) $\frac{1}{2x}$

Solution:

(i) 8

8 can be written as $8x^0$.

i.e., $8 = 8x^0$,

Here, the power of x = 0, which is a whole number.

Hence, 8 is a polynomial.

(ii) $\sqrt{3}x^2 - 2x$

$\sqrt{3}x^2 - 2x$

Here, the power of x are 2 and 1 respectively

2 and 1 both are whole numbers.

Hence, $\sqrt{3}x^2 - 2x$ is a polynomial.

(iii) $1 - \sqrt{(5x)}$

$1 - \sqrt{5}\sqrt{x} = 1 - \sqrt{5}\, x^{½}$

Here, the power of x = ½, which is not a whole number.

Hence, $1 - \sqrt{5x}$ is not a polynomial

(iv)

$$\frac{1}{5x^{-2}} + 5x + 7$$

$1/5x^{-2} + 5x + 7 = 5x^2 + 5x + 7$

Here, the power of x are 2 and 1 respectively

2 and 1 both are whole numbers.

Hence, $1/5x^{-2} + 5x + 7$ is a polynomial.

(v)

$$\frac{(x-2)(x-4)}{x}$$

$((x - 2)(x - 4))/x$ $= (x^2 - 4x - 2x + 8)/x$

$= (x^2 - 6x + 8)/x$

$= x - 6 + (8/x)$

$= x - 6 + 8x^{-1}$

Here, the power of x = – 1, which is not a whole number, but a negative number.
Hence, ((x – 2)(x – 4))/x is not a polynomial

(vi)

$$\frac{1}{x+1}$$

$1/(x+1) = (x+1)^{-1}$
Here, the power of x is not a whole number.
Hence, 1/(x+1) is not a polynomial

(vii)

$$\frac{1}{7}a^3 - \frac{2}{\sqrt{3}}a^2 + 4a - 7$$

$(1/7)a^3 – (2/\sqrt{3})a^2 + 4a – 7$
Here, the power of a are 3, 2 and 1 respectively
3, 2 and 1 are all whole numbers.
Hence, $(1/7)a^3 – (2/\sqrt{3})a^2 + 4a – 7$ is a polynomial.

(viii)

$$\frac{1}{2x}$$

$1/2x = (x^{-1}/2)$
Here, the power of x = – 1, which is not a whole number, but a negative number.
Hence, 1/2x is not a polynomial

EXERCISE 2.3

1. **Classify the following polynomials as polynomials in one variable, two variables etc.**

(i) $x^2 + x + 1$
(ii) $y^3 – 5y$
(iii) $xy + yz + zx$
(iv) $x^2 – 2xy + y^2 + 1$

Solution:

(i) $x^2 + x + 1$
Here, the polynomial contains only one variable, i.e., x.
Hence, the given polynomial is a polynomial in **one** variable.

(ii) $y^3 – 5y$
Here, the polynomial contains only one variable, i.e., y.
Hence, the given polynomial is a polynomial in **one** variable.

(iii) $xy + yz + zx$
Here, the polynomial contains three variables, i.e., x, y and z.
Hence, the given polynomial is a polynomial in **three** variable.

(iv) $x^2 – 2xy + y^2 + 1$
Here, the polynomial contains two variables, i.e., x and y.
Hence, the given polynomial is a polynomial in **two** variable.

2. Determine the degree of each of the following polynomials:

(i) $2x – 1$
(ii) $–10$
(iii) $x^3 – 9x + 3x^5$
(iv) $y^3 (1 – y^4)$

Solution:

Degree of a polynomial in one variable = highest power of the variable in algebraic expression
(i) $2x – 1$
Power of x = 1
Highest power of the variable x in the given expression = 1
Hence, degree of the polynomial $2x – 1 = 1$

(ii) $–10$
There is no variable in the given term.
Let us assume that the variable in the given expression is x.
$– 10 = –10x^0$
Power of x = 0
Highest power of the variable x in the given expression = 0
Hence, degree of the polynomial $– 10 = 0$

(iii) $x^3 – 9x + 3x^5$

Powers of x = 3, 1 and 5 respectively.
Highest power of the variable x in the given expression = 5
Hence, degree of the polynomial $x^3 - 9x + 3x^5 = 5$

(iv) $y^3 (1 - y^4)$
The equation can be written as,
$y^3 (1 - y^4) = y^3 - y^7$
Powers of y = 3 and 7 respectively.
Highest power of the variable y in the given expression = 7
Hence, degree of the polynomial $y^3 (1 - y^4) = 7$

3. For the polynomial

$$\frac{x^3 + 2x + 1}{5} - \frac{7}{2}x^2 - x^6$$

, write

(i) the degree of the polynomial
(ii) the coefficient of x^3
(iii) the coefficient of x^6
(iv) the constant term

Solution:

The given polynomial is

$$\frac{(x^3 + 2x + 1)}{5} - \frac{7}{2}x^2 - x^6$$

(i)Powers of x = 3, 1, 2 and 6 respectively.
Highest power of the variable x in the given expression = 6
Hence, degree of the polynomial = 6

(ii) The given equation can be written as,

$$\frac{(x^3 + 2x + 1)}{5} - \frac{7}{2}x^2 - x^6 = \frac{1}{5}x^3 + \frac{2}{5}x + \frac{1}{5} - \frac{7}{2}x^2 - x^6$$

Hence, the coefficient of x^3 in the given polynomial is 1/5.

(iii) The coefficient of x^6 in the given polynomial is – 1

(iv) Since the given equation can be written as,

$$\frac{(x^3 + 2x + 1)}{5} - \frac{7}{2}x^2 - x^6 = \frac{1}{5}x^3 + \frac{2}{5}x + \frac{1}{5} - \frac{7}{2}x^2 - x^6$$

The constant term in the given polynomial is 1/5 as it has no variable x associated with it.

4. Write the coefficient of x^2 in each of the following:

(i) $(\pi/6)x + x^2 - 1$
(ii) $3x - 5$
(iii) $(x - 1)(3x - 4)$
(iv) $(2x - 5)(2x^2 - 3x + 1)$

Solution:

(i) $(\pi/6)\ x + x^2-1$

$(\pi/6)\ x + x^2-1 = (\pi/6)\ x + (1)\ x^2-1$

The coefficient of x^2 in the polynomial $(\pi/6)\ x + x^2-1 = 1$.

(ii) $3x - 5$

$3x - 5 = 0x^2 + 3x - 5$

The coefficient of x^2 in the polynomial $3x - 5 = 0$, zero.

(iii) $(x - 1)(3x - 4)$

$(x - 1)(3x - 4) = 3x^2 - 4x - 3x + 4$

$= 3x^2 - 7x + 4$

The coefficient of x^2 in the polynomial $3x^2 - 7x + 4 = 3$.

(iv) $(2x - 5)(2x^2 - 3x + 1)$

$(2x - 5)(2x^2 - 3x + 1)$

$= 4x^3 - 6x^2 + 2x - 10x^2 + 15x - 5$

$= 4x^3 - 16x^2 + 17x - 5$

The coefficient of x^2 in the polynomial $(2x - 5)(2x^2 - 3x + 1) = - 16$

5. Classify the following as a constant, linear, quadratic and cubic polynomials:

(i) $2 - x^2 + x^3$

(ii) $3x^3$

(iii) $5t - \sqrt{7}$

(iv) $4 - 5y^2$

(v) 3

(vi) $2 + x$

(vii) $y^3 - y$

(viii) $1 + x + x^2$

(ix) t^2

(x) $\sqrt{2}x - 1$

Solution:

Constant polynomials: The polynomial of the degree zero.

Linear polynomials: The polynomial of degree one.

Quadratic polynomials: The polynomial of degree two.

Cubic polynomials: The polynomial of degree three.

(i) $2 - x^2 + x^3$

Powers of x = 2, and 3 respectively.

Highest power of the variable x in the given expression = 3

Hence, degree of the polynomial = 3

Since it is a polynomial of the degree 3, it is a cubic polynomial.

(ii) $3x^3$

Power of x = 3.

Highest power of the variable x in the given expression = 3

Hence, degree of the polynomial = 3

Since it is a polynomial of the degree 3, it is a cubic polynomial.

(iii) $5t - \sqrt{7}$

Power of t = 1.
Highest power of the variable t in the given expression = 1
Hence, degree of the polynomial = 1
Since it is a polynomial of the degree 1, it is a linear polynomial.

(iv) $4 - 5y^2$

Power of y = 2.
Highest power of the variable y in the given expression = 2
Hence, degree of the polynomial = 2
Since it is a polynomial of the degree 2, it is a quadratic polynomial.

(v) 3

There is no variable in the given expression.
Let us assume that x is the variable in the given expression.
3 can be written as $3x^0$.
i.e., $3 = x^0$
Power of x = 0.
Highest power of the variable x in the given expression = 0
Hence, degree of the polynomial = 0
Since it is a polynomial of the degree 0, it is a constant polynomial.

(vi) 2 + x

Power of x = 1.
Highest power of the variable x in the given expression = 1
Hence, degree of the polynomial = 1
Since it is a polynomial of the degree 1, it is a linear polynomial.

(vii) $y^3 - y$

Powers of y = 3 and 1, respectively.
Highest power of the variable x in the given expression = 3
Hence, degree of the polynomial = 3
Since it is a polynomial of the degree 3, it is a cubic polynomial.

(viii) $1 + x + x^2$

Powers of x = 1 and 2, respectively.
Highest power of the variable x in the given expression = 2
Hence, degree of the polynomial = 2
Since it is a polynomial of the degree 2, it is a quadratic polynomial.

(ix) t^2

Power of t = 2.
Highest power of the variable t in the given expression = 2
Hence, degree of the polynomial = 2

Since it is a polynomial of the degree 2, it is a quadratic polynomial.

(x) $\sqrt{2}x - 1$

Power of x = 1.

Highest power of the variable x in the given expression = 1

Hence, degree of the polynomial = 1

Since it is a polynomial of the degree 1, it is a linear polynomial.

6. Give an example of a polynomial, which is:

(i) monomial of degree 1

(ii) binomial of degree 20

(iii) trinomial of degree 2

Solution:

(i) Monomial = an algebraic expression that contains one term

An example of a polynomial, which is a monomial of degree 1 = 2t

(ii) Binomial = an algebraic expression that contains two terms

An example of a polynomial, which is a binomial of degree 20 = $x^{20} + 5$

(iii) Trinomial = an algebraic expression that contains three terms

An example of a polynomial, which is a trinomial of degree 2 = $y^2 + 3y + 11$

7. Find the value of the polynomial $3x^3 - 4x^2 + 7x - 5$, when x = 3 and also when x = –3.

Solution:

Given that,

$p(x) = 3x^3 - 4x^2 + 7x - 5$

According to the question,

<u>When x = 3,</u>

p(x) = p(3)

$p(x) = 3x^3 - 4x^2 + 7x - 5$

Substituting x = 3,

$p(3) = 3(3)^3 - 4(3)^2 + 7(3) - 5$

$p(3) = 3(3)^3 - 4(3)^2 + 7(3) - 5$

$= 3(27) - 4(9) + 21 - 5$

$= 81 - 36 + 21 - 5$

$= 102 - 41$

$= 61$

<u>When x = – 3,</u>

p(x) = p(– 3)

$p(x) = 3x^3 - 4x^2 + 7x - 5$

Substituting x = – 3,

$p(-3) = 3(-3)^3 - 4(-3)^2 + 7(-3) - 5$

$p(-3) = 3(-3)^3 - 4(-3)^2 + 7(-3) - 5$

$= 3(-27) - 4(9) - 21 - 5$

$= -81 - 36 - 21 - 5$

$= -143$

8. If p(x) =x^2 – 4x + 3, evaluate: p(2)− p(−1) + p(½).

Given that,

p(x) =x^2 – 4x + 3

According to the question,

When x = 2,

p(x) = p(2)

p(x) =x^2 – 4x + 3

Substituting x = 2,

p(2) = $(2)^2$ – 4(2) + 3

= 4 – 8 + 3

= – 4 + 3

= – 1

When x = – 1,

p(x) = p(– 1)

p(x) =x^2 – 4x + 3

Substituting x = – 1,

p(– 1) = $(– 1)^2$ – 4(– 1) + 3

= 1 + 4 + 3

= 8

When x = ½ ,

p(x) = p(½)

p(x) =x^2 – 4x + 3

Substituting x = ½,

p(½) = $(½)^2$ – 4(½) + 3

= ¼ – 2 + 3

= ¼ + 1

= 5/4

Now,

p(2)− p(−1) + p(½) = – 1 – 8 + (5/4)

= – 9 + (5/4)

= (– 36 + 5)/4

= – 31/4

9. Find p(0), p(1),p(−2) for the following polynomials:

(i) (x)=10x−4x^2 –3

(ii) (y)=(y + 2) (y – 2)

Solution:

(i) According to the question,

p(x) = 10x−4x^2 –3

When x = 0,

p(x) = p(0)

Substituting x = 0,

p(0) = 10(0)−4$(0)^2$ –3

= 0 – 0 – 3

$= -3$

<u>When x = 1,</u>

p(x) = p(1)

Substituting x = 1,

$p(1) = 10(1) - 4(1)^2 - 3$

$= 10 - 4 - 3$

$= 6 - 3$

$= 3$

<u>When x = – 2,</u>

p(x) = p(– 2)

Substituting x = – 2,

$p(-2) = 10(-2) - 4(-2)^2 - 3$

$= -20 - 16 - 3$

$= -36 - 3$

$= -39$

(ii) According to the question,

$p(y) = (y + 2)(y - 2)$

<u>When y = 0,</u>

p(y) = p(0)

Substituting y = 0,

$p(0) = (0 + 2)(0 - 2)$

$= (2)(-2)$

$= -4$

<u>When y = 1,</u>

p(y) = p(1)

Substituting y = 1,

$p(1) = (1 + 2)(1 - 2)$

$= (3)(-1)$

$= -3$

<u>When y = – 2,</u>

p(y) = p(– 2)

Substituting y = – 2,

$p(-2) = (-2 + 2)(-2 - 2)$

$= (0)(-4)$

$= 0$

10. Verify whether the following are true or false:

(i) –3 is a zero of x – 3

(ii) – 1/3 is a zero of 3x + 1

(iii) – 4/5 is a zero of 4 –5y

(iv) 0 and 2 are the zeroes of $t^2 – 2t$
(v) –3 is a zero of $y^2 + y – 6$

Solution:

(i) –3 is a zero of x – 3

False

Zero of x – 3 is given by,

$x – 3 = 0$

$\Rightarrow x=3$

(ii) – 1/3 is a zero of 3x + 1

True

Zero of 3x + 1 is given by,

$3x + 1 = 0$

$\Rightarrow 3x = – 1$

$\Rightarrow x = – 1/3$

(iii) – 4/5 is a zero of 4 –5y

False

Zero of 4 – 5y is given by,

$4 – 5y =0$

$\Rightarrow – 5y = – 4$

$\Rightarrow y = 4/5$

(iv) 0 and 2 are the zeroes of $t^2 – 2t$

True

Zeros of $t^2 – 2t$ is given by,

$t^2 – 2t = t(t – 2) = 0$

$\Rightarrow t = 0 \text{ or } 2$

(v) –3 is a zero of $y^2 + y – 6$

True

Zero of $y^2 + y – 6$ is given by,

$y^2 + y – 6 = 0$

$\Rightarrow y^2 + 3x – 2x – 6 = 0$

$\Rightarrow y (y + 3) – 2(x + 3) = 0$

$\Rightarrow (y – 2) (y + 3) =0$

$\Rightarrow y = 2 \text{ or } – 3$

11. Find the zeroes of the polynomial in each of the following:

(i) p(x) = x – 4
(ii) g(x) = 3 – 6x
(iii) q(x) = 2x –7
(iv) h(y) = 2y

Solution:

(i) p(x) = x – 4

Zero of the polynomial p(x) $\Rightarrow$ p(x) = 0

P(x) = 0
⇒ x – 4= 0
⇒ x = 4
Therefore, the zero of the polynomial is 4.

(ii) g(x) = 3 – 6x
Zero of the polynomial g(x) ⇒ g(x) = 0
g(x) = 0
⇒3 – 6x = 0
⇒ x = 3/6 = ½
Therefore, the zero of the polynomial is ½

(iii) q(x) = 2x –7
Zero of the polynomial q(x) ⇒ q(x) = 0
q(x) = 0
⇒2x – 7 = 0
⇒ x = 7/2
Therefore, the zero of the polynomial is 7/2

(iv) h(y) = 2y
Zero of the polynomial h(y) ⇒ h(y) = 0
h(y) = 0
⇒2y =0
⇒ y = 0
Therefore, the zero of the polynomial is 0

12. Find the zeroes of the polynomial:
$p(x) = (x - 2)^2 - (x + 2)^2$

Solution:

$p(x) = (x - 2)^2 - (x + 2)^2$
We know that,
Zero of the polynomial p(x) = 0
Hence, we get,
$\Rightarrow (x-2)^2 - (x + 2)^2 = 0$
Expanding using the identity, $a^2 - b^2 = (a - b)(a + b)$
⇒ (x – 2 + x + 2) (x – 2 –x – 2) = 0
⇒ 2x (– 4) = 0
⇒ – 8 x= 0
Therefore, the zero of the polynomial = 0

13. By actual division, find the quotient and the remainder when the first polynomial is divided by the second polynomial: $x^4 + 1$; $x - 1$

Solution:

Performing the long division method, we get,

```
x-1) x⁴ + 1 (x³ + x² + x + 1
     x⁴ -x³
     -   +
     ------
     x³ + 1
     x³ - x²
     -   +
     ------
       x² + 1
       x² -x
     -    +
          ------
          X + 1
          x - 1
          -  +
          ------
               2
```

Hence, from the above long division method, we get,

Quotient = $x^3 + x^2 + x + 1$

Remainder = 2.

14. By Remainder Theorem find the remainder, when p(x) is divided by g(x), where

(i) $p(x) = x^3 – 2x^2 – 4x – 1$, $g(x) = x + 1$

(ii) $p(x) = x3 – 3x2 + 4x + 50$, $g(x) = x – 3$

(iii) $p(x) = 4x^3 – 12x^2 + 14x – 3$, $g(x) = 2x – 1$

(iv) $p(x) = x^3 – 6x^2 + 2x – 4$, $g(x) = 1 – 3/2\, x$

Solution:

(i) Given $p(x) = x^3 – 2x^2 – 4x – 1$ and $g(x) = x + 1$

Here zero of g(x) = – 1

By using the remainder theorem

P(x) divided by g(x) = p(– 1)

$P(– 1) = (– 1)^3 – 2(– 1)^2 – 4(– 1) – 1 = 0$

Therefore, the remainder = 0

(ii) given $p(x) = x^3 – 3x^2 + 4x + 50$, $g(x) = x – 3$

Here zero of g(x) = 3

By using the remainder theorem p(x) divided by g(x) = p(3)

$p(3) = 3^3 – 3 \times (3)^2 + 4 \times 3 + 50 = 62$

Therefore, the remainder = 62

(iii) $p(x) = 4x^3 – 12x^2 + 14x – 3$, $g(x) = 2x – 1$

Here zero of g(x) = ½

By using the remainder theorem p(x) divided by g(x) = p (½)

$P(½) = 4(½)^3 – 12(½)^2 + 14(½) – 3$

$= 4/8 – 12/4 + 14/2 – 3$

$= ½ + 1$

$= 3/2$

Therefore, the remainder = 3/2

(iv) $p(x) = x^3 – 6x^2 + 2x – 4$, $g(x) = 1 – 3/2\ x$
Here zero of g(x) = 2/3
By using the remainder theorem p(x) divided by g(x) = p(2/3)
$p(2/3) = (2/3)^3 – 6(2/3)^2 + 2(2/3) – 4$
$= – 136/27$
Therefore, the remainder = – 136/27

15. Check whether p(x) is a multiple of g(x) or not:
(i) $p(x) = x^3 – 5x^2 + 4x – 3$, $g(x) = x – 2$
(ii) $p(x)= 2x^3 – 11x^2 – 4x + 5$, $g(x)= 2x + 1$
Solution:
(i)
According to the question,
g(x)=x – 2,
Then, zero of g(x),
g(x) = 0
x – 2 = 0
x = 2
Therefore, zero of g(x) = 2
So, substituting the value of x in p(x), we get,
$p(2) = (2)^3 – 5(2)^2 + 4(2) – 3$
$= 8 – 20 + 8 – 3$
$= – 7 \neq 0$
Hence, p(x) is not the multiple of g(x) since the remainder ≠ 0.

(ii)
According to the question,
g(x)= $2x + 1$
Then, zero of g(x),
g(x) = 0
2x + 1 = 0
2x = – 1
x = – ½
Therefore, zero of g(x) = – ½
So, substituting the value of x in p(x), we get,
$p(–½) = 2 \times (– ½)^3 – 11 \times (– ½)^2 – 4 \times (– 1/2) + 5$
$= – ¼ - 11/4 + 7$
$= 16/4$
$= 4 \neq 0$
Hence, p(x) is not the multiple of g(x) since the remainder ≠ 0.

16. Show that:
(i) $x + 3$ is a factor of $69 + 11x−x^2 + x^3$.
(ii) $2x−3$ is a factor of $x + 2x^3 – 9x^2 + 12$
Solution:
(i)According to the question,

Let $p(x) = 69 + 11x - x^2 + x^3$ and $g(x) = x + 3$
$g(x) = x + 3$
zero of $g(x) \Rightarrow g(x) = 0$
$x + 3 = 0$
$x = -3$
Therefore, zero of $g(x) = -3$
So, substituting the value of x in p(x), we get,
$p(-3) = 69 + 11(-3) - (-3)^2 + (-3)^3$
$= 69 - 69$
$= 0$
Since, the remainder = zero,
We can say that,
$g(x) = x + 3$ is factor of $p(x) = 69 + 11x - x^2 + x^3$

(ii) According to the question,
Let $p(x) = x + 2x^3 - 9x^2 + 12$ and $g(x) = 2x - 3$
$g(x) = 2x - 3$
zero of $g(x) \Rightarrow g(x) = 0$
$2x - 3 = 0$
$x = 3/2$
Therefore, zero of $g(x) = 3/2$
So, substituting the value of x in p(x), we get,
$P(3/2) = 3/2 + 2(3/2)^3 - 9(3/2)^2 + 12$
$= (81 - 81) / 4$
$= 0$
Since, the remainder = zero,
We can say that,
$g(x) = 2x - 3$ is factor of $p(x) = x + 2x^3 - 9x^2 + 12$

17. Determine which of the following polynomials has x – 2 a factor:
(i) $3x^2 + 6x - 24$.
(ii) $4x^2 + x - 2$.

Solution:

(i) According to the question,
Let $p(x) = 3x^2 + 6x - 24$ and $g(x) = x - 2$
$g(x) = x - 2$
zero of $g(x) \Rightarrow g(x) = 0$
$x - 2 = 0$
$x = 2$
Therefore, zero of $g(x) = 2$
So, substituting the value of x in p(x), we get,
$p(2) = 3(2)^2 + 6(2) - 24$
$= 12 + 12 - 24$
$= 0$
Since, the remainder = zero,
We can say that,

g(x) = x – 2 is factor of p(x) = $3x^2 + 6x - 24$

(ii) According to the question,
Let p(x) = $4x^2 + x - 2$ and g(x) = x – 2
g(x) = x – 2
zero of g(x) ⇒ g(x) = 0
x – 2 = 0
x = 2
Therefore, zero of g(x) = 2
So, substituting the value of x in p(x), we get,
p(2) $= 4(2)^2 + 2 - 2$
$= 16 \neq 0$
Since, the remainder = zero,
We can say that,
g(x) = x – 2 is factor of p(x) = $4x^2 + x - 2$

18. Show that p – 1 is a factor of $p^{10} - 1$ and also of $p^{11} - 1$.
Solution:

According to the question,
Let h(p) = $p^{10} - 1$,and g(p) = $p - 1$
zero of g(p) ⇒ g(p) = 0
p – 1 = 0
p = 1
Therefore, zero of g(x) = 1
We know that,
According to factor theorem if g(p) is a factor of h(p) , then h(1) should be zero
So,
$h(1) = (1)^{10} - 1 = 1 - 1 = 0$
⟹ g (p) is a factor of h(p).
Now, we have h(p) = $p^{11} - 1$, g (p) = $p - 1$
Putting g (p) = 0 $\Longrightarrow p - 1 = 0 \Longrightarrow p = 1$
According to factor theorem if g (p) is a factor of h(p) ,
Then h(1) = 0
$\Longrightarrow (1)^{11} - 1 = 0$
Therefore, g(p) = $p - 1$ is the factor of h(p) = $p^{10} - 1$

19. For what value of m is $x^3 - 2mx^2 + 16$ divisible by x + 2?
Solution:

According to the question,
Let p(x) = $x^3 - 2mx^2 + 16$, and g(x) = x + 2
g(x) = 0
⟹ x + 2 = 0
⟹ x = – 2
Therefore, zero of g(x) = – 2
We know that,
According to factor theorem,

if p(x) is divisible by g(x), then the remainder p(−2) should be zero.
So, substituting the value of x in p(x), we get,
$p(-2) = 0$
$\Rightarrow (-2)^3 - 2m(-2)^2 + 16 = 0$
$\Rightarrow 0 - 8 - 8m + 16 = 0$
$\Rightarrow 8m = 8$
$\Rightarrow m = 1$

20. If $x + 2a$ is a factor of $x^5 - 4a^2x^3 + 2x + 2a + 3$, find a.
Solution:

According to the question,
Let $p(x) = x^5 - 4a^2x^3 + 2x + 2a + 3$ and $g(x) = x + 2a$
$g(x) = 0$
$\Rightarrow x + 2a = 0$
$\Rightarrow x = -2a$
Therefore, zero of $g(x) = -2a$
We know that,
According to the factor theorem,
If g(x) is a factor of p(x), then $p(-2a) = 0$
So, substituting the value of x in p(x), we get,
$p(-2a) = (-2a)^5 - 4a^2(-2a)^3 + 2(-2a) + 2a + 3 = 0$
$\Rightarrow -32a^5 + 32a^5 - 2a + 3 = 0$
$\Rightarrow -2a = -3$
$\Rightarrow a = 3/2$

EXERCISE 2.4 PAGE NO: 23

1. If the polynomials $az^3 + 4z^2 + 3z - 4$ and $z^3 - 4z + a$ leave the same remainder when divided by $z - 3$, find the value of a.

Solution:

Zero of the polynomial,
$g_1(z) = 0$
$z-3 = 0$
$z = 3$
Therefore, zero of $g(z) = -2a$
Let $p(z) = az^3+4z^2+3z-4$
So, substituting the value of z = 3 in p(z), we get,
$p(3) = a(3)^3+4(3)^2+3(3)-4$
$\Rightarrow p(3) = 27a+36+9-4$
$\Rightarrow p(3) = 27a+41$
Let $h(z) = z^3-4z+a$
So, substituting the value of z = 3 in h(z), we get,
$h(3) = (3)^3-4(3)+a$
$\Rightarrow h(3) = 27-12+a$
$\Rightarrow h(3) = 15+a$
According to the question,
We know that,
The two polynomials, p(z) and h(z), leaves same remainder when divided by z-3
So, $h(3)=p(3)$
$\Rightarrow 15+a = 27a+41$
$\Rightarrow 15-41 = 27a - a$
$\Rightarrow -26 = 26a$
$\Rightarrow a = -1$

2. The polynomial $p(x) = x^4 - 2x^3 + 3x^2 - ax + 3a - 7$ when divided by $x + 1$ leaves the remainder 19. Find the values of a. Also find the remainder when $p(x)$ is divided by $x + 2$.

Solution:

$p(x) = x^4 - 2x^3 + 3x^2 - ax + 3a - 7$.
Divisor = x + 1
x + 1 = 0
x = -1
So, substituting the value of x = – 1 in p(x), we get,
$p(-1) = (-1)^4 - 2(-1)^3 + 3(-1)^2 - a(-1) + 3a - 7$.
$19 = 1 + 2 + 3 + a + 3a - 7$
$19 = 6 - 7 + 4a$
$4a - 1 = 19$
$4a = 20$
$a = 5$
Since, a = 5.
We get the polynomial,

$p(x) = x^4 - 2x^3 + 3x^2 - (5)x + 3(5) – 7$
$p(x) = x^4 - 2x^3 + 3x^2 - 5x + 15 – 7$
$p(x) = x^4 - 2x^3 + 3x^2 - 5x + 8$
As per the question,
When the polynomial obtained is divided by (x + 2),
We get,
$x + 2 = 0$
$x = - 2$
So, substituting the value of x = – 2 in p(x), we get,
$p(-2) = (-2)^4 - 2(-2)^3 + 3(-2)^2 - 5(-2) + 8$
$\Rightarrow p(-2) = 16 + 16 + 12 + 10 + 8$
$\Rightarrow p(-2) = 62$
Therefore, the remainder = 62.

3. If both $x – 2$ and $x – ½$ are factors of $px^2 + 5x + r$, show that $p = r$.
Solution:

Given, $f(x) = px^2+5x+r$ and factors are x-2, x - ½
$g_1(x) = 0$,
$x – 2 = 0$
$x = 2$
Substituting x = 2 in place of equation, we get
$f(x) = px^2+5x+r$
$f(2) = p(2)^2+5(2)+r=0$
$= 4p + 10 + r = 0$ … eq.(i)
$x – ½ = 0$
$x = ½$
Substituting x = ½ in place of equation, we get,
$f(x) = px^2+5x+r$
$f(½) = p(½)^2 + 5(½) + r =0$
$= p/4 + 5/2 + r = 0$
$= p + 10 + 4r = 0$ … eq(ii)
On solving eq(i) and eq(ii),
We get,
$4p + r = – 10$ and $p + 4r = – 10$
Since the RHS of both the equations are same,
We get,
$4p + r = p + 4r$
$3p=3r$
$p = r$.
Hence Proved.

4. Without actual division, prove that $2x^4 – 5x^3 + 2x^2 – x + 2$ is divisible by $x^2 – 3x + 2$. [Hint: Factorise $x^2 – 3x + 2$]
Solution:

x^2-3x+2
$x^2-2x-1x+2$

x(x-2)-1(x-2)
(x-2)(x-1)
Therefore,(x-2)(x-1)are the factors.

Considering (x-2),
x-2=0
x=2
Then, p(x) becomes,
p(x)=2
$p(x)=2x^4-5x^3+2x^2-x+2$
$p(2)=2(2)^4-5(2)^3+2(2)^2-2+2$
=32-40+8
= -40+40=0
Therefore, (x-2) is a factor.

Considering (x-1),
x-1=0
x=1
Then, p(x) becomes,
p(x)=1
$p(x)=2x^4-5x^3+2x^2-x+2$
$p(1)=2(1)^4-5(1)^3+2(1)^2-1+2$
=2-5+2-1+2
=6-6
=0
Therefore, (x-1) is a factor.