Exercise 22

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Question 1: Find the coordinates of the focus and the vertex, the equations of the directrix and the axis, and length of the latus rectum of the parabola:

(i) $y^2 = 12x$

(ii) $y^2 = 10x$

(iii) $3y^2 = 8x$

Solution:
The general form of a parabola: $y^2 = 4ax$ ....(1)
Focus : F(a,0)
Vertex : A(0,0) (at any point A)
Equation of the directrix : x + a = 0
Axis: y = 0
Length of latus rectum : 4a

(i) $y^2 = 12x$

On comparing given equation with (1), we have

4a = 12 => a = 3

Now,

Focus : F(a,0) = F(3,0)

Vertex : A(0,0)

Equation of the directrix : x + a = 0

=> x + 3 = 0

or x = -3

Axis: y = 0

Length of latus rectum : 4a = 4 x 3 = 12 units

(ii) $y^2 = 10x$

On comparing given equation with (1), we have

$4a = 10 \Rightarrow a = 2.5$

Now,

Focus : F(a,0) = F(2.5,0)

Vertex : A(0,0)

Equation of the directrix : x + a = 0

$\Rightarrow x + 2.5 = 0$
or $x = -2.5$

Axis: y = 0

Length of latus rectum : 4a = 4 x (2.5) = 10 units

(iii) $3y^2 = 8x$

or $y^2 = 8/3\ x$

On comparing given equation with (1), we have

$4a = 8/3 \Rightarrow a = 2/3$

Now,
Focus : F(a,0) = F(2/3,0)

Vertex : A(0,0)

Equation of the directrix : x + a = 0

$\Rightarrow x + 2/3 = 0$
or $x = -2/3$

Axis: y = 0

Length of latus rectum : 4a = 4 x 2/3 = 8/3 units

Question 2: Find the coordinates of the focus and the vertex, the equations of the directrix and the axis, and length of the latus rectum of the parabola:

(i) $y^2 = -8x$
(ii) $y^2 = -6x$
(iii) $5y^2 = -16x$

Solution:
The general form of a parabola: $y^2 = -4ax$ ....(1)
Focus : F(-a,0)
Vertex : A(0,0) (at any point A)
Equation of the directrix : x - a = 0
Axis: y = 0
Length of latus rectum : 4a

(i) $y^2 = -8x$

On comparing given equation with (1), we have

4a = 8 => a = 2

Now,

Focus : F(-2,0)

Vertex : A(0,0) (at any point A)

Equation of the directrix : x - 2 = 0 or x = 2

Axis: y = 0

Length of latus rectum : 4a = 4 x 2 = 8 units

(ii) $y^2 = -6x$
On comparing given equation with (1), we have

4a = 6 => a = 3/2

Now,

Focus : F(-3/2,0)

Vertex : A(0,0) (at any point A)

Equation of the directrix : x - 3/2 = 0 or x = 3/2 or 2x - 3 = 0

Axis: y = 0

Length of latus rectum : 4a = 4 x 3/2 = 6 units

(iii) $5y^2 = -16x$
or $y^2 = -16/5\ x$

On comparing given equation with (1), we have

4a = 16/5 => a = 4/5

Now,

Focus : F(-4/5,0)

Vertex : A(0,0) (at any point A)

Equation of the directrix : x - 4/5 = 0 or 5x - 4 = 0

Axis: y = 0

Length of latus rectum : 4a = 4 x 4/5 = 16/5 units

Question 3: Find the coordinates of the focus and the vertex, the equations of the directrix and the axis, and length of the latus rectum of the parabola :

(i) $x^2 = 16y$
(ii) $x^2 = 10y$
(iii) $3x^2 = 8y$

Solution:
The general form of a parabola: $x^2 = 4ay$ ....(1)
Focus : F(0,a)
Vertex : A(0,0) (at any point A)
Equation of the directrix : y + a = 0
Axis: x = 0
Length of latus rectum : 4a

(i) $x^2 = 16y$

On comparing given equation with (1), we have

$4a = 16 \Rightarrow a = 4$

Now,

Focus : F(0, 4)

Vertex : A(0, 0)

Equation of the directrix : y + 4 = 0

Axis: x = 0

Length of latus rectum : 4a = 4 x 4 = 16 units

(ii) $x^2 = 10y$
On comparing given equation with (1), we have

$4a = 10 \Rightarrow a = 2.5$

Now,

Focus : F(0, 2.5)

Vertex : A(0, 0)

Equation of the directrix : y + 2.5 = 0

Axis: x = 0

Length of latus rectum : 4a = 4 x 2.5 = 10 units

(iii) $3x^2 = 8y$
or $x^2 = 8/3\ y$

On comparing given equation with (1), we have

$4a = 8/3 \Rightarrow a = 2/3$

Now,

Focus : F(0, 2/3)

Vertex : A(0, 0)

Equation of the directrix : y + 2/3 = 0 or 3y + 2 = 0

Axis: x = 0

Length of latus rectum : 4a = 4 x 2/3 = 8/3 units

Question 4: Find the coordinates of the focus and the vertex, the equations of the directrix and the axis, and length of the latus rectum of the parabola :

(i) $x^2 = -8y$
(ii) $x^2 = -18y$
(iii) $3x^2 = -16y$

Solution:
The general form of a parabola: $x^2 = -4ay$ ....(1)
Focus : F(0, -a)
Vertex : A(0,0) (at any point A)
Equation of the directrix : y - a = 0
Axis: x = 0
Length of latus rectum : 4a

(i) $x^2 = -8y$
On comparing given equation with (1), we have

4a = 8 => a = 2

Now,

Focus : F(0, -2)

Vertex : A(0, 0)

Equation of the directrix : y - 2 = 0

Axis: x = 0

Length of latus rectum : 4a = 4 x 2 = 8 units

(ii) $x^2 = -18y$

On comparing given equation with (1), we have

4a = 18 => a = 9/2

Now,

Focus : F(0, -9/2)

Vertex : A(0, 0)

Equation of the directrix : y - 9/2 = 0 or 2y - 9 = 0

Axis: x = 0

Length of latus rectum : 4a = 4 x 9/2 = 18 units

(iii) $3x^2 = -16y$

Or $x^2 = -16/3\ y$

On comparing given equation with (1), we have

4a = 16/3 => a = 4/3

Now,

Focus : F(0, -4/3)

Vertex : A(0, 0)

Equation of the directrix : y - 4/3 = 0 or 3y - 4 = 0

Axis: x = 0

Length of latus rectum : 4a = 4 x 4/3 = 16/3 units

Question 5: Find the equation of the parabola with vertex at the origin and focus at F(-2, 0).

Solution:
Give: Vertex : A (0,0) and focus, F(-2,0)

We know, For Vertex A(0,0) and Focus F(-a,0), equation of parabola is: $y^2 = - 4ax$

Here, a = 2

Therefore, equation of parabola: $y^2 = - 8x$

Question 6: Find the equation of the parabola with focus F(4, 0) and directrix x = -4.

Solution:
We are given with focus F(4, 0) and directrix x = -4 or x + 4 = 0

We know, For directrix with equation x + a = 0 and focus (a, 0), equation of the parabola is, $y^2 = 4ax$

Here, a = 4

Therefore, equation of parabola: $y^2 = 16x$

Question 7: Find the equation of the parabola with focus F(0, -3) and directrix y = 3.

Solution:
Given, focus F(0, -3) and directrix y = 3 or y - 3 = 0.

We know, For directrix with equation y - a = 0 and focus (0, -a), equation of the parabola is: $x^2 = - 4ay$

Here, a = 3

Therefore, equation of parabola: $x^2 = - 12y$

Question 8: Find the equation of the parabola with vertex at the origin and focus F(0, 5).
Solution:
We have to find equation of the parabola with origin and focus F(0, 5).

We know, For vertex A(0, 0) (origin at point A) and focus, F(0, a), equation of the parabola is: $x^2 = 4ay$

Here, a = 5

Therefore, equation of parabola: $x^2 = 20y$

Question 9: Find the equation of the parabola with vertex at the origin, passing through the point P(5, 2) and symmetric with respect to the y-axis.

Solution:

The equation of a parabola with vertex at the origin and symmetric about the y-axis: $x^2 = 4ay$

As we are given, parabola is passing through the point P(5,2).

Putting x = 5 and y = 2 in $x^2 = 4ay$

=> 25 = 4a(2) = 8a

=> a = 25/8

Therefore, equation of parabola:

$x^2 = 4(25/8)y = 25/2\ y$

or $2x^2 = 25y$

Question 10: Find the equation of the parabola, which is symmetric about the y-axis and passes through the point P(2, -3).

Solution: The equation of a parabola with vertex at the origin and symmetric about the y-axis: $x^2 = 4ay$

As we are given, parabola is passing through the point P(2, -3).

Putting x = 2 and y = -3 in $x^2 = 4ay$

=> 4 = -12a

=> a = -1/3

Therefore, equation of parabola:

$x^2 = 4(-1/3)y = -4/3\ y$

or $3x^2 = -4y$