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Question 1: Find the (i) lengths of major and minor axes, (ii) coordinates of the vertices, (iii) coordinates of the foci, (iv) eccentricity, and (v) length of the latus rectum of each of the following ellipses. (Question 1 to question 12)

Question 1: $x^2/25 + y^2/9 = 1$
Solution:

For general form of ellipse:

$x^2/a^2 + y^2/b^2 = 1$ ….(1)

Major axis	Minor Axis	vertices	foci	eccentricity	latus rectum
2a	2b	(±a, 0)	(±c, 0)	e = c/a	$2b^2/a$

On comparing given equation with (1), we get

a = 5 and b = 3

Then,

$c^2 = a^2 – b^2$

$c^2 = 25 – 9 = 16$

or c = 4

Now,
(i) Lengths of major and minor axes:

Major axis: 2a = 10 units

Minor axis : 2b = 6 units

(ii) Coordinates of the vertices: (±a, 0) = (±5, 0)

(iii) Coordinates of the foci: (±c, 0) = (±4, 0)

(iv) Eccentricity: e = c/a = 4/5

(v) Length of the latus rectum: $2b^2/a$ = 18/5 units

Question 2: x2/49 + y2/36 = 1
Solution:

For general form of ellipse:

$x^2/a^2 + y^2/b^2 = 1$ ....(1)

Major axis	Minor Axis	vertices	foci	eccentricity	latus rectum
2a	2b	(±a, 0)	(±c, 0)	e = c/a	$2b^2/a$

On comparing given equation with (1), we get

a = 7 and b = 6

Then,

$c^2 = a^2 - b^2$

$c^2 = 49 - 36 = 13$

or $c = \sqrt{13}$

Now,
(i) Lengths of major and minor axes:

Major axis: 2a = 14 units

Minor axis : 2b = 12 units

(ii) Coordinates of the vertices: (±a, 0) = (±7, 0)

(iii) Coordinates of the foci: (±c, 0) = $(\pm\sqrt{13}, 0)$

(iv) Eccentricity: $e = c/a = \sqrt{13}/7$

(v) Length of the latus rectum: $2b^2/a = 2(6)^2/7 = 72/7$ units

Question 3: $16x^2 + 25y^2 = 400$

Solution:
Divide both sides by 400
$16/400\ x^2 + 25/400\ y^2 = 1$

or $x^2/25 + y^2/16 = 1$

For general form of ellipse:

$x^2/a^2 + y^2/b^2 = 1$ ....(1)

Major axis	Minor Axis	vertices	foci	eccentricity	latus rectum
2a	2b	(±a, 0)	(±c, 0)	e = c/a	$2b^2/a$

On comparing given equation with (1), we get

a = 5 and b = 4

Then,

$c^2 = a^2 – b^2$

$c^2 = 25 – 16 = 9$

or c = 3

Now,
(i) Lengths of major and minor axes:

Major axis: 2a = 10 units

Minor axis : 2b = 8 units

(ii) Coordinates of the vertices: (±a, 0) = (±5, 0)

(iii) Coordinates of the foci: (±c, 0) = (±3, 0)

(iv) Eccentricity: e = c/a = 3/5

(v) Length of the latus rectum: $2b^2/a = 2(4)^2/5 =$ 32/5 units

Question 4: $x^2 + 4y^2 = 100$

Solution:
Divide both sides by 100

$x^2/100 + y^2/25 = 1$

For general form of ellipse:

$x^2/a^2 + y^2/b^2 = 1$ ….(1)

Major axis	Minor Axis	vertices	foci	eccentricity	latus rectum
2a	2b	(±a, 0)	(±c, 0)	e = c/a	$2b^2/a$

On comparing given equation with (1), we get

a = 10 and b = 5

Then,

$c^2 = a^2 – b^2$

$c^2 = 100 – 25 = 75$

or $c = 5\sqrt{3}$

Now,
(i) Lengths of major and minor axes:

Major axis: 2a = 20 units

Minor axis : 2b = 10 units

(ii) Coordinates of the vertices: (±a, 0) = (±10, 0)

(iii) Coordinates of the foci: (±c, 0) = $(\pm 5\sqrt{3}, 0)$

(iv) Eccentricity: $e = c/a = 5\sqrt{3}/10 = \sqrt{3}/2$

(v) Length of the latus rectum: $2b^2/a = 2(5)^2/10 = 5$ units

Question 5: $9x^2 + 16y^2 = 144$

Solution:

Divide both sides by 144

$x^2/16 + y^2/9 = 1$

For general form of ellipse:

$x^2/a^2 + y^2/b^2 = 1$ ….(1)

Major axis	Minor Axis	vertices	foci	eccentricity	latus rectum
2a	2b	(±a, 0)	(±c, 0)	e = c/a	$2b^2/a$

On comparing given equation with (1), we get

a = 4 and b = 3

Then,

$c^2 = a^2 – b^2$

$c^2 = 16 – 9 = 7$

or $c = \sqrt{7}$

Now,

(i) Lengths of major and minor axes:

Major axis: 2a = 8 units

Minor axis : 2b = 6 units

(ii) Coordinates of the vertices: (±a, 0) = (±4, 0)

(iii) Coordinates of the foci: (±c, 0) = $(\pm\sqrt{7}, 0)$

(iv) Eccentricity: $e = c/a = \sqrt{7}/4 = \sqrt{3}/2$

(v) Length of the latus rectum: $2b^2/a = 2(3)^2/4 = 9/2$ units

Question 6: $4x^2 + 9y^2 = 1$

Solution:

$4x^2 + 9y^2 = 1$ can be written as $x^2/(1/4) + 9y^2/(1/9) = 1$

For general form of ellipse:

$x^2/a^2 + y^2/b^2 = 1$ ….(1)

Major axis	vertices	foci	eccentricity	latus rectum
2a	(±a, 0)	(±c, 0)	e = c/a	$2b^2/a$

On comparing given equation with (1), we get

a = 1/2 and b = 1/3

Then,

$c^2 = a^2 – b^2$

$c^2 = 1/4 – 1/9 = 5/36$

or $c = \sqrt{5}/6$

Now,
(i) Lengths of major and minor axes:

Major axis: 2a = 1 units

Minor axis : 2b = 2/3 units

(ii) Coordinates of the vertices: (±a, 0) = (±1/2, 0)

(iii) Coordinates of the foci: (±c, 0) = ($\pm\sqrt{5}/6$, 0)

(iv) Eccentricity: e = c/a = $\sqrt{5}/3$

(v) Length of the latus rectum: $2b^2/a$ = 4/9 units

Question 7: $x^2/4 + y^2/25 = 1$

Solution:

In this case, coefficient of y^2 > coefficient of x^2

Which is of the form $x^2/b^2 + y^2/a^2 = 1$

For general form of ellipse:

$x^2/b^2 + y^2/a^2 = 1$ ....(1)

Major axis	Minor Axis	vertices	foci	eccentricity	latus rectum
2a	2b	(0, ±a)	(0, ±c)	e = c/a	$2b^2/a$

On comparing given equation with (1), we get

a = 5 and b = 2

Then,

$c^2 = a^2 - b^2$

$c^2 = 25 - 4 = 21$

or $c = \sqrt{21}$

Now,
(i) Lengths of major and minor axes:

Major axis: 2a = 10 units

Minor axis : 2b = 4 units

(ii) Coordinates of the vertices: (0, ±a) = (0, ±5)

(iii) Coordinates of the foci: (0, ±c) = $(0, \pm\sqrt{21})$

(iv) Eccentricity: $e = c/a = \sqrt{21}/5$

(v) Length of the latus rectum: $2b^2/a = 8/5$ units

Question 8: $x^2/9 + y^2/16 = 1$

Solution:
In this case, coefficient of y^2 > coefficient of x^2

Which is of the form $x^2/b^2 + y^2/a^2 = 1$

For general form of ellipse:

$x^2/b^2 + y^2/a^2 = 1$ ….(1)

Major axis	Minor Axis	vertices	foci	eccentricity	latus rectum
2a	2b	(0, ±a)	(0, ±c)	e = c/a	$2b^2/a$

On comparing given equation with (1), we get

a = 4 and b = 3

Then,

$c^2 = a^2 – b^2$

$c^2 = 16 – 9 = 7$

or c = √7

Now,
(i) Lengths of major and minor axes:

Major axis: 2a = 8 units

Minor axis : 2b = 6 units

(ii) Coordinates of the vertices: (0, ±a) = (0, ±4)

(iii) Coordinates of the foci: (0, ±c) = (0, ±√7)

(iv) Eccentricity: e = c/a = √7/5

(v) Length of the latus rectum: $2b^2/a$ = 9/2 units

Question 9: $3x^2 + 2y^2 = 18$

Solution:
Divide each side by 18
$x^2/6 + y^2/9 = 1$

In this case, coefficient of y^2 > coefficient of x^2

Which is of the form $x^2/b^2 + y^2/a^2 = 1$

For general form of ellipse:
$x^2/b^2 + y^2/a^2 = 1$ ….(1)

Major axis	Minor Axis	vertices	foci	eccentricity	latus rectum
2a	2b	(0, ±a)	(0, ±c)	e = c/a	$2b^2/a$

On comparing given equation with (1), we get

a = 3 and b = $\sqrt{6}$

Then,

$c^2 = a^2 - b^2$

$c^2 = 9 - 6 = 3$

or c = $\sqrt{3}$

(i) Lengths of major and minor axes:

Major axis: 2a = 6 units

Minor axis : 2b = $2\sqrt{6}$ units

(ii) Coordinates of the vertices: (0, ±a) = (0, ±3)

(iii) Coordinates of the foci: (0, ±c) = (0, $\pm\sqrt{3}$)

(iv) Eccentricity: e = c/a = $1/\sqrt{3}$

(v) Length of the latus rectum: $2b^2/a$ = 4 units

Question 10: $9x^2 + y^2 = 36$

Solution:
Divide each side by 36, we get
$x^2/4 + y^2/36 = 1$

Which is of the form $x^2/b^2 + y^2/a^2 = 1$

For general form of ellipse:

$x^2/b^2 + y^2/a^2 = 1$ ....(1)

Major axis	Minor Axis	vertices	foci	eccentricity	latus rectum
2a	2b	(0, ±a)	(0, ±c)	e = c/a	$2b^2/a$

On comparing given equation with (1), we get

a = 6 and b = 2

Then,

$c^2 = a^2 – b^2$

$c^2 = 36 – 4 = 32$

or $c = 4\sqrt{2}$

Now,
(i) Lengths of major and minor axes:

Major axis: 2a = 12 units

Minor axis : 2b = 4 units

(ii) Coordinates of the vertices: (0, ±a) = (0, ±6)

(iii) Coordinates of the foci: (0, ±c) = $(0, \pm 4\sqrt{2})$

(iv) Eccentricity: e = c/a = $2\sqrt{2}/3$

(v) Length of the latus rectum: $2b^2/a$ = 4/3 units

Question 11: $16x^2 + y^2 = 16$

Solution:

Divide each side by 16, we get

$x^2/1 + y^2/16 = 1$

Which is of the form $x^2/b^2 + y^2/a^2 = 1$

For general form of ellipse:

$x^2/b^2 + y^2/a^2 = 1$ ….(1)

Major axis	Minor Axis	vertices	foci	eccentricity	latus rectum
2a	2b	(0, ±a)	(0, ±c)	e = c/a	$2b^2/a$

On comparing given equation with (1), we get

a = 4 and b = 1

Then,

$c^2 = a^2 – b^2$

$c^2 = 16 – 1 = 15$

or $c = \sqrt{15}$

Now,

(i) Lengths of major and minor axes:

Major axis: 2a = 8 units

Minor axis : 2b = 2 units

(ii) Coordinates of the vertices: (0, ±a) = (0, ±4)

(iii) Coordinates of the foci: (0, ±c) = (0, $\pm\sqrt{15}$)

(iv) Eccentricity: $e = c/a = \sqrt{15}/4$

(v) Length of the latus rectum: $2b^2/a$ = 1/2 units

Question 12: $25x^2 + 4y^2 = 100$

Solution:
Divide each side by 100, we get
$x^2/4 + y^2/25 = 1$

Which is of the form $x^2/b^2 + y^2/a^2 = 1$

For general form of ellipse:

$x^2/b^2 + y^2/a^2 = 1$ ....(1)

Major axis	Minor Axis	vertices	foci	eccentricity	latus rectum
2a	2b	(0, ±a)	(0, ±c)	e = c/a	$2b^2/a$

On comparing given equation with (1), we get

a = 5 and b = 2

Then,

$c^2 = a^2 - b^2$

$c^2 = 25 - 4 = 21$

or $c = \sqrt{21}$

Now,
(i) Lengths of major and minor axes:

Major axis: 2a = 10 units

Minor axis : 2b = 4 units

(ii) Coordinates of the vertices: (0, ±a) = (0, ±5)

(iii) Coordinates of the foci: (0, ±c) = $(0, \pm\sqrt{21})$

(iv) Eccentricity: $e = c/a = \sqrt{21}/5$

(v) Length of the latus rectum: $2b^2/a = 8/5$ units

Question 13: Find the equation of ellipse whose vertices are at (±6, 0) and foci at (±4, 0).

Solution:

Vertices are of the form (±a, 0), hence major axis is along x-axis.

So, the equation is of the form: $x^2/a^2 + y^2/b^2 = 1$ .....(1)

Given: vertices of ellipse (±6, 0)

Here a = 6

Again,

Foci is of the form (±c, 0)

Given: foci of ellipse at (±4, 0)

=> c = 4

Find b:

We know, $c^2 = a^2 – b^2$

$16 = 36 – b^2$

or $b^2 = 20$

Equation (1)=>

$x^2/36 + y^2/20 = 1$

Which is required equation.

Question 14: Find the equation of ellipse whose vertices are at (0, ±4) and foci at (0, ±√7).

Solution:

Vertices are of the form (0, ±a), hence major axis is along x-axis.

So, the equation is of the form: $x^2/b^2 + y^2/a^2 = 1$ .....(1)

Given: vertices of ellipse (0, ±4)

Here a = 4

Again,

Foci is of the form (0, ±c)

Given: foci of ellipse at (0, ±√7)

=> c = √7

Find b:
We know, $c^2 = a^2 – b^2$

$7 = 16 – b^2$

or $b^2 = 9$

Equation (1)=>

$x^2/9 + y^2/16 = 1$

Which is required equation.

Question 15: Find the equation of ellipse the ends whose major and minor axes are (±4, 0) and (0, ±3) respectively.

Solution: Major axis of the given ellipse lie on x-axis.

Equation of the ellipse will be of the form: $x^2/a^2 + y^2/b^2 = 1$
Whose vertices (±a, 0) and foci (±c, 0)

Here, a = 4

Ends of minor axis: (0, ±3), let A(0, -3) and B(0, 3)
=> AB = 6 units = length of minor axis.
=> 2b = 6 or b = 3

Therefore, required equation is : $x^2/16 + y^2/9 = 1$

Question 16: The length of the major axis of an ellipse is 20 units and its foci are (±5√3, 0) Find the equation of the ellipse.

Solution: Foci of the equation is in the form (±c, 0)

Equation of the ellipse will be of the form: $x^2/a^2 + y^2/b^2 = 1$
Whose vertices (±a, 0) and foci (±c, 0)

Here, $c = 5\sqrt{3}$

Length of the major axis of an ellipse is 20 units (given)
So, length of the semi-major axis = 20/2 = 10 units, which is the value of a.

=> a = 10

Find b:

We know, $c^2 = a^2 - b^2$

$(5\sqrt{3})^2 = 100 - b^2$

or $b^2 = 100 - 75 = 25$

Equation (1)=>

$x^2/100 + y^2/25 = 1$

Which is required equation.