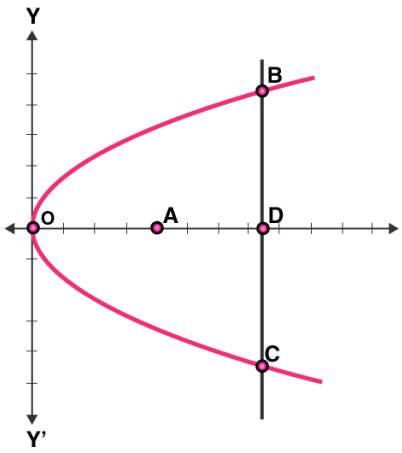
Exercise 25 Page No: 781

Question 1: The focus of a parabolic mirror is at a distance of 6 cm from its vertex. If the mirror is 20 cm deep, find its diameter.

Solution:



Here O is the vertex and A is the focus of a parabolic mirror.
So,
OA = a = 6 cm

Required equation of the parabola is, $y^2 = 4ax$

or $y^2 = 24x$

Let OD is the deep of the mirror, OD = 20 cm

BC is the diameter of the mirror and x-coordinate of the points B, C and D is 20

From figure, points, B and C satisfies the equation of the parabola.
Therefore, $y^2 = 24 \times 20 = 480$

or $y = \pm 21.9$

Coordinate of B is (20, 21.9) and the coordinate of C is (20, -21.9).

Therefore, diameter of the mirror is = (21.9 + 21.9) cm = 43.8 cm

Question 2: A parabolic reflector is 5 cm deep and its diameter is 20 cm. How far is its focus from the vertex?

Solution: Reflector depth = OD = 5 cm
Diameter of the mirror = BC = 20 cm

Consider the equation of the parabola is $y^2 = 4ax$

To find : Length of a, which is distance of the focus from the vertex.

Since parabolic reflector is 5 cm deep, which shows x-coordinate of the points B and C is 5.

Again, BD = CD = 10 cm
[D is the middle point of BC]

Therefore, coordinate of B is (5, 10)

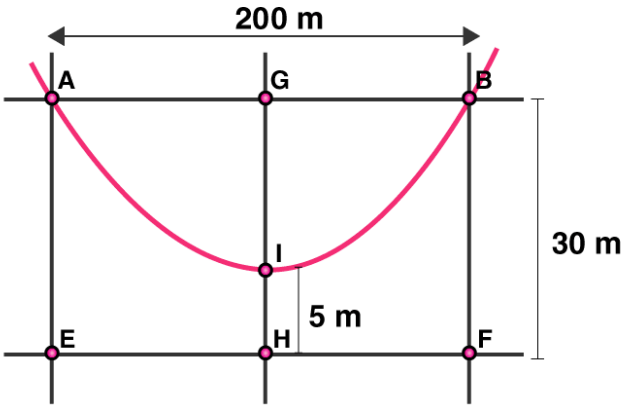
Point B lies on the parabolic reflector, which means it satisfy the equation $y^2 = 4ax$

100 = 20a

or a = 5

Question 3: The towers of bridge, hung in the form of a parabola, have their tops 30 m above the roadway, and are 200 m apart. If the cable is 5 m above the roadway at the center of the bridge, find the length of the vertical supporting cable, 30 m from the center.

Solution:



A and B are the top of the towers. AE and BF are the height of the towers. H is the center of the bridge. HI is the 5 m above from the roadway.

Here, the equation of the parabola will be: $x^2 = 4ay$

From figure, x-cordinate of top of tower is 100 m and y-coordinate is 25 m.

(1)=> 100^2 = 4a(25)

or a = 100

Again, length of the vertical supporting cable, 30 m from the center is y + 5.

Find y co-ordinate

(1)=> 900 = 4(100)y
[x-coordinate of the point is 30]

or y = 9/4

Vertical length = y + 5 = 9/4 + 5 = 29/4

Therefore, the vertical length of cable is 29/4 m

Question 4: A rod of length 15 cm moves with its ends always touching the coordinate axes. Find the equation of the locus of a point P on the rod, which is at a distance of 3 cm from the end in contact with the x-axis.

Solution:

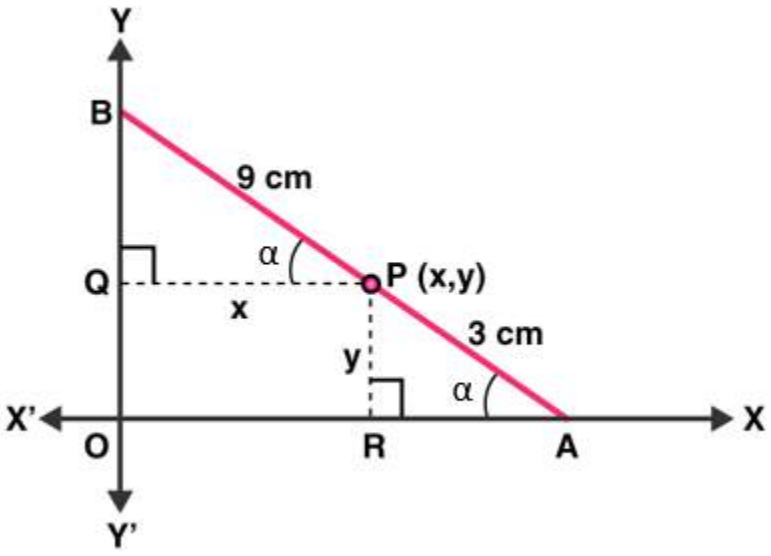
Let AB is the rod making an angle with the x-axis.

Here AP = 3, then

PB = AB – AP = (12 – 3)cm = 9 cm

Let (x, y) be the coordinates of point P.

Draw PQ and PR perpendiculars on x-axis and on y-axis respectively.



From figure,
In △PAR

Sin α = y/AP = y/3

and in △BPQ

Cos α = x/BP = x/9

We know, $\sin^2\alpha + \cos^2\alpha = 1$

$(y/3)^2 + (x/3)^2 = 1$

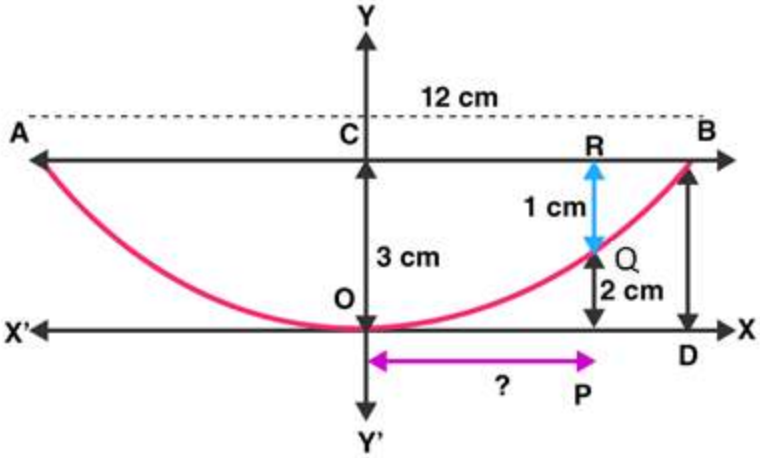
$y^2/9 + x^2/81 = 1$

or $x^2/81 + y^2/9 = 1$

Which is required result.

Question 5: A beam is supported at its ends by supports which are 12 m apart. Since the load is concentrated at its center, there is a deflection of 3 cm at the center, and the deflected beam is in the shape of a parabola. How far from the center is the deflection 1 cm?

Solution:



Beam is always facing upwards with the axis vertical.

Here, the equation of the parabola is: $x^2 = 4ay$ ...(1)

Find the coordinate of B:
Width of beam = AB = 12 m

BC = 12/2 = 6 m
[Parabola is symmetric about its axis]

Again, there is deflection of 3 cm at the center. (given)

BD = OC = 3 cm or 3/100 m

Point B is B(6, 3/100)

Since B lies on $x^2 = 4ay$

Here x = 6 and y = 3/100

36 = 12/100 a

or a = 300 m

Now, find how far from the center is the deflection 1 cm

From figure, RQ = 1 cm
QP = (3-1)cm = 2cm or 2/100 m

Let OP = x

Coordinates of Q is Q(x, 2/100)

Since Q lies on $x^2 = 4ay$

Here y = 2/100

$x^2 = 4a(2/100)$
[Using value of a]

$x^2 = 4 \times 300 \times (2/100)$

$x^2 = 24$

or $x = 2\sqrt{6}$

Therefore, the required distance is $2\sqrt{6}$ m.