Exercise 26A

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Question 1: If a point lies on the z-axis, then find its x-coordinate and y-coordinate.

Solution:
If the point lies on the z axis then its x-coordinate and y-coordinate will be zero.

Question 2: If a point lies on yz-plane then what is its x-coordinate?
Solution:
If the point lies on yz-plane then x-coordinate will be zero.

Question 3: In which plane does the point (4, -3, 0) lie?

Solution:
Given: x, y, z coordinates of the point are 4, -3, 0.

As the distance of point along the z-axis is 0, the plane in which the point lies is the xy-plane.

Question 4: In which octant does each of the given points lie?
(i) (-4, -1, -6)
(ii) (2, 3, -4)
(iii) (-6, 5, -1)
(iv) (4, -3, -2)
(v) (-1, -6, 5)
(vi) (4, 6, 8)

Solution:
The position of a point in a octant is signified by the signs of the x, y, z coordinates.
Signs of x, y, z coordinates in all the octants are as follow:

Number	Sign of X	Sign of Y	Sign of Z
I	+	+	+
II	-	+	+
III	-	-	+
IV	+	-	+
V	+	+	-
VI	-	+	-
VII	-	-	-
VIII	+	-	-

Compare signs with given table and allocate octants to the points

(i) (-4, -1, -6) lies in octant VII

(ii) (2, 3, -4) lies in octant V

(iii) (-6, 5, -1) lies in octant VI

(iv) (4, -3, -2) lies in octant VIII

(v) (-1, -6, 5) lies in octant III

(vi) (4, 6, 8) lies in octant I

Exercise 26B

Question 1: Find the distance between the points :

(i) A(5, 1, 2) and B(4, 6, -1)
(ii) P(1, -1, 3) and Q(2, 3, -5)
(iii) R(1, -3, 4) and S(4, -2, -3)
(iv) C(9, -12, -8) and the origin

Solution:

The distance between two points (x_1, y_1, z_1) and (x_2, y_2, z_2) is given by

$$\text{Distance} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

(i) A(5, 1, 2) and B(4, 6, -1)

Here $(x_1, y_1, z_1) = (5, 1, 2)$ and $(x_2, y_2, z_2) = (4, 6, -1)$

Now,

$$\text{Distance} = \sqrt{(4-5)^2 + (6-1)^2 + (-1-2)^2}$$

$$= \sqrt{(-1)^2 + (5)^2 + (-3)^2}$$

$$= \sqrt{1 + 25 + 9}$$

$$= \sqrt{35}$$

Distance between A and B is $\sqrt{35}$ units

(ii) P(1, -1, 3) and Q(2, 3, -5)

Here $(x_1, y_1, z_1) = (1, -1, 3)$ and $(x_2, y_2, z_2) = (2, 3, -5)$

Distance = $\sqrt{(2-1)^2+(3-(-1))^2+(-5-3)^2}$

$= \sqrt{(1)^2+(4)^2+(-8)^2}$

$= \sqrt{1+16+64}$

$= \sqrt{81}$

$= 9$

Distance between P and Q is 9 units

(iii) R(1, -3, 4) and S(4, -2, -3)

Here $(x_1, y_1, z_1) = (1, -3, 4)$ and $(x_2, y_2, z_2) = (4, -2, -3)$

Distance = $\sqrt{(4-1)^2+(-2-(-3))^2+(-3-4)^2}$

$= \sqrt{(3)^2+(1)^2+(-7)^2}$

$= \sqrt{9+1+49}$

$= \sqrt{59}$

Distance between R and S is √59 units

(iv) C(9, -12, -8) and the origin

Here $(x_1, y_1, z_1) = (9, -12, -8)$ and $(x_2, y_2, z_2) = (0, 0, 0)$

Distance = $\sqrt{(0-9)^2+(0-(-12))^2+(0-(-8))^2}$

$= \sqrt{(-9)^2+(12)^2+(8)^2}$

$= \sqrt{81+144+64}$

$= \sqrt{289}$

$= 17$

Distance between C and Origin is 17 units.

Question 2: Show that the points A(1, -1, -5), B(3, 1,3) and C(9, 1, -3) are the vertices of an equilateral triangle.

Solution:

Given points are the vertices of an equilateral triangle if measure of all the sides are equal.

Apply distance formula:

The distance between two points (x_1, y_1, z_1) and (x_2, y_2, z_2) is given by

Distance = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$

Length AB:

$= \sqrt{(3-1)^2 + (1-(-1))^2 + (3-(-5))^2}$

$= \sqrt{(2)^2 + (2)^2 + (8)^2}$

$= \sqrt{4+4+64}$

$= \sqrt{72}$

$= 6\sqrt{2}$

Length BC:

$= \sqrt{(9-3)^2 + (1-1)^2 + (-3-3)^2}$

$= \sqrt{(6)^2 + (0)^2 + (-6)^2}$

$= \sqrt{36+0+36}$

$= \sqrt{72}$

$= 6\sqrt{2}$

Length AC:

$= \sqrt{(9-1)^2 + (1-(-1))^2 + (-3-(-5))^2}$

$= \sqrt{(8)^2 + (2)^2 + (2)^2}$

$= \sqrt{64+4+4}$

$= \sqrt{72}$

$= 6\sqrt{2}$

=> AB = BC = AC

Therefore, Points A, B, C are vertices of an equilateral triangle.

Question 3: Show that the points A(4, 6, -5), B(0, 2, 3) and C(-4, -4, -1) from the vertices of an isosceles triangle.

Solution:

Given points are the vertices of an isosceles triangle if measure of any two sides are equal.

Apply distance formula:

The distance between two points (x_1, y_1, z_1) and (x_2, y_2, z_2) is given by

Distance = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$

Length AB:

$= \sqrt{(0-4)^2 + (2-6)^2 + (3-(-3))^2}$

$= \sqrt{(-4)^2 + (-4)^2 + (6)^2}$

$= \sqrt{16 + 16 + 36}$

$= \sqrt{68}$

$= 2\sqrt{17}$

Length BC:

$= \sqrt{(-4-0)^2 + (-4-2)^2 + (-1-3)^2}$

$= \sqrt{(-4)^2 + (-6)^2 + (-4)^2}$

$= \sqrt{16 + 36 + 16}$

$= \sqrt{68}$

$= 2\sqrt{17}$

Length AC:

$= \sqrt{(-4-4)^2 + (-4-6)^2 + (-1-(-5))^2}$

$= \sqrt{(-8)^2 + (-10)^2 + (2)^2}$

$= \sqrt{64 + 100 + 4}$

$= \sqrt{168}$

From above result, we have AB = BC

Therefore, vertices A, B, C forms an isosceles triangle.

Question 4: Show that the points A(0, 1, 2), B(2, -1, 3) and C(1, -3, 1) are the vertices of an isosceles right-angled triangle.

Solution:
Given points are the vertices of an isosceles right-angled triangle if sum of squares of two sides is equal to square of third side.

Apply distance formula:
The distance between two points (x_1, y_1, z_1) and (x_2, y_2, z_2) is given by

Distance = $\sqrt{(x_2-x_1)^2 + (y_2-y_1)^2 + (z_2-z_1)^2}$

Length AB:

$= \sqrt{(2-0)^2 + (-1-1)^2 + (3-2)^2}$

$= \sqrt{(2)^2 + (-2)^2 + (1)^2}$

$= \sqrt{9}$

Length BC:

$= \sqrt{(1-2)^2 + (-3+1)^2 + (1-3)^2}$

$= \sqrt{(-1)^2 + (-2)^2 + (-2)^2}$

$= \sqrt{9}$

Length AC:

$= \sqrt{(1-0)^2 + (-3-1)^2 + (1-2)^2}$

$= \sqrt{1 + 16 + 1}$

$= \sqrt{18}$

Now, AB^2 + BC^2 = 9 + 9

= 18

= AC^2

Therefore, Therefore, vertices A, B, C are the vertices of an isosceles right-angled triangle.

Question 5: Show that the points A(1, 1, 1), B(-2, 4, 1), C(1, -5, 5) and D(2, 2, 5) are the vertices of a square.

Solution:

Points A(1, 1, 1), B(-2, 4, 1), C(1, -5, 5) and D(2, 2, 5) are the vertices of a square if all sides are of equal measure.

Apply distance formula, and find the distance between all the points:

AB = $\sqrt{(-2-1)^2+(4-1)^2+(1-1)^2}$

= $\sqrt{9+9+0}$

= $\sqrt{18}$

BC = $\sqrt{(-1+2)^2+(5-4)^2+(5-1)^2}$

= $\sqrt{1+1+16}$

= $\sqrt{18}$

AC = $\sqrt{(-1-1)^2+(5-1)^2+(5-1)^2}$

= $\sqrt{4+16+16}$

= $\sqrt{36}$

CD = $\sqrt{(2+1)^2+(2-5)^2+(5-5)^2}$

= $\sqrt{9+9+0}$

= $\sqrt{18}$

AD = $\sqrt{(2-1)^2+(2-1)^2+(5-1)^2}$

= $\sqrt{1+1+16}$

= $\sqrt{18}$

BD = $\sqrt{(2+2)^2+(2-4)^2+(5-1)^2}$

= $\sqrt{16+4+16}$

= $\sqrt{36}$

From above results, AB = BC = CD = AD, and AC = BD

Hence vertices A, B, C, D form a square.

Question 6: Show that the points A(1, 2, 3), B(-1, -2, -1), C(2, 3, 2) and D(4, 7, 6) are the vertices of a parallelogram. Show that ABCD is not a rectangle.

Solution:

Apply distance formula, and find the distance between all the points:

AB = $\sqrt{(-1-1)^2+(-2-2)^2+(-1-3)^2}$

= $\sqrt{4+16+16}$

= $\sqrt{36}$

BC = $\sqrt{(2+1)^2+(3+2)^2+(2+1)^2}$

= $\sqrt{9+25+9}$

= $\sqrt{43}$

CD = $\sqrt{(4-2)^2+(7-3)^2+(6-2)^2}$

= $\sqrt{4+16+16}$

= $\sqrt{36}$

$AD = \sqrt{(4-1)^2 + (7-2)^2 + (6-3)^2}$

$= \sqrt{9 + 25 + 9}$

$= \sqrt{43}$

$BD = \sqrt{(4+1)^2 + (7+2)^2 + (6+1)^2}$

$= \sqrt{25 + 81 + 49}$

$= \sqrt{155}$

$AC = \sqrt{(2-1)^2 + (3-2)^2 + (2-3)^2}$

$= \sqrt{1 + 1 + 1}$

$= \sqrt{3}$

From above result, we have
AB = CD [opposite sides]
BC = AD [opposite sides]
Diagonal AC ≠ BD, Diagonals are not equal

Hence, Points are vertices of parallelogram and ABCD is not a rectangle.

Question 7: Show that the points P(2, 3, 5), Q(-4, 7, -7), R(-2, 1, -10) and S(4, -3, 2) are the vertices of a rectangle.

Solution:
PQRS is a rectangle if opposite sides are equal and diagonals are also of equal length.

PQ = $\sqrt{(-4-2)^2+(7-3)^2+(-7-5)^2}$

= $\sqrt{36+16+144}$

= $\sqrt{196}$

QR= $\sqrt{(-2+4)^2+(1-7)^2+(-10+7)^2}$

= $\sqrt{4+36+9}$

= $\sqrt{49}$

RS = $\sqrt{(4+2)^2+(-3-1)^2+(2+10)^2}$

= $\sqrt{36+16+144}$

= $\sqrt{196}$

PS = $\sqrt{(4-2)^2+(-3-3)^2+(2-5)^2}$

= $\sqrt{4+36+9}$

= $\sqrt{49}$

QS = $\sqrt{(4+4)^2+(-3-7)^2+(2+7)^2}$

= $\sqrt{64+100+81}$

= $\sqrt{245}$

PR = $\sqrt{(-2-2)^2+(1-3)^2+(-10-5)^2}$

= $\sqrt{16+4+225}$

= $\sqrt{245}$

Here, PQ = RS and QR = PS and PR = QS

Hence, points are vertices of a rectangle.

Question 8: Show that the points P(1, 3, 4), Q(-1, 6, 10), R(-7, 4, 7) and S(-5, 1, 1) are the vertices of a rhombus.

Solution:
Apply distance formula and find the length of each side.

PQ = $\sqrt{(-1-1)^2+(6-3)^2+(10-4)^2}$

= $\sqrt{4+9+36}$

= $\sqrt{49}$

QR = $\sqrt{(-7+1)^2+(4-6)^2+(7-10)^2}$

= $\sqrt{36+34+9}$

= $\sqrt{49}$

RS = $\sqrt{(-5+7)^2+(1-4)^2+(1-7)^2}$

= $\sqrt{4+9+36}$

= $\sqrt{49}$

PS = $\sqrt{(-5-1)^2+(1-3)^2+(1-4)^2}$

= $\sqrt{36+4+9}$

= $\sqrt{49}$

QS = $\sqrt{(-5+1)^2+(1-6)^2+(1-10)^2}$

= $\sqrt{16+25+81}$

= $\sqrt{122}$

PR = $\sqrt{(-7-1)^2+(4-3)^2+(7-4)^2}$

= $\sqrt{64+1+9}$

= $\sqrt{74}$

=> PQ = RS =QR = PS

And, diagonals: PR ≠ QS

Hence, points are vertices of a rhombus.

Exercise 26C

Question 1: Find the coordinates of the point which divides the join of A(3, 2, 5) and B(-4, 2, -2) in the ratio 4 : 3.

Solution:

Section formula: The coordinates of point R that divides the line segment joining points A(x_1, y_1, z_1) and B(x_2, y_2, z_2) in the ratio m: n are

$$\left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}, \frac{mz_2 + nz_1}{m+n}\right)$$

Here, Point A(3, 2, 5) and B(-4, 2, -2),
m = 4 and n = 3

Using the above formula, we get,

$$= \left(\frac{4 \times -4 + 3 \times 3}{4+3}, \frac{4 \times 2 + 3 \times 2}{4+3}, \frac{4 \times -2 + 3 \times 5}{4+3}\right)$$

$$= \left(\frac{-7}{7}, \frac{14}{7}, \frac{7}{7}\right)$$

= (-1, 2, 1)

(-1, 2, 1) is the coordinates of the point which divides AB in the ratio 4 : 3.

Question 2: Let A(2, 1, -3) and B(5, -8, 3) be two given points. Find the coordinates of the point of trisection of the segment AB.

Solution:

Given: Point A(2, 1, -3) and B(5, -8, 3)
m = 2 and n = 1

Using section formula:

$$= \left(\frac{2 \times 5 + 1 \times 2}{2+1}, \frac{2 \times -8 + 1 \times 1}{2+1}, \frac{2 \times 3 + 1 \times -3}{2+1}\right)$$

$$= \left(\frac{12}{3}, \frac{-15}{3}, \frac{3}{3}\right)$$

= (4, -5, 1)

Therefore, (4,-5, 1), is the point of trisection of the segment AB.

Question 3: Find the coordinates of the point that divides the join of A(-2, 4, 7) and B(3, -5, 8) extremally in the ratio 2 : 1.

Solution:
Given: Point A(-2, 4, 7) and B(3, -5, 8)
m = 2 and n = 1

Using section formula:

$$\left(\frac{2 \times 3 - 1 \times -2}{2-1}, \frac{2 \times -5 - 1 \times 4}{2-1}, \frac{2 \times 8 - 1 \times 7}{2-1}\right)$$

= (8, -14, 9),

Therefore, (8, -14, 9), is the point that divides AB externally in the ratio 2:1.

Question 4: Find the ratio in which the point R(5, 4, -6) divides the join of P(3, 2, -4) and Q(9, 8, -10).

Solution:
Given: Point P(3, 2, -4) and Q(9, 8, -10) divided by a point R(5, 4, -6)

Let the ratio be k:1 in which point R divides point P and point Q.

We know below are the coordinates of a point which divides a line segment in ratio, k:1.

Using section formula, we have

$$(5,4,-6) = \frac{k \times 9 + 1 \times 3}{k+1}, \frac{k \times 8 + 1 \times 2}{k+1}, \frac{k \times -10 + 1 \times -4}{k+1})$$

Find the value of k:

Choosing first co-ordinate,

5 = (9k+3)/(k+1)

5k + 5 = 9k + 3

4k = 2

Therefore, the required ratio be 1:2.

Question 5: Find the ratio in which the point C(5, 9, -14) divides the join of A(2, -3, 4) and B(3, 1, -2).

Solution:

Given: Point A(2, -3, 4) and B(3, 1, -2) divided by a point C(5, 9, -14)

Let the ratio be k:1 in which point C divides point A and point B.

We know below are the coordinates of a point which divides a line segment in ratio, k:1.

Using section formula, We have

$$(5, 9, -14) = \left(\frac{k \times 3 + 1 \times 2}{k+1}, \frac{k \times 1 + 1 \times -3}{k+1}, \frac{k \times -2 + 1 \times 4}{k+1}\right)$$

Find the value of k:

Choosing first co-ordinate,

5 = (3k+2)/(k+1)

5k + 5 = 3k + 2

2k = -3

Or k = -3/2

Since, the ratio is -3:2, division is external. Therefore, external division ratio is 3:2.

Question 6: Find the ratio in which the line segment having the end points A(-1, -3, 4) and B(4, 2, -1) is divided by the xz-plane. Also, find the coordinates of the point of division.

Solution: Let the plane XZ divides the points A(-1, -3, 4) and B(4, 2, -1) in ratio k:1.

Using section formula, we have

$$(x, 0, z) = \left(\frac{k \times 4 + 1 \times -1}{k+1}, \frac{k \times 2 + 1 \times -3}{k+1}, \frac{k \times -1 + 1 \times 4}{k+1}\right)$$

Since we are given with xz-plane, so y-coordinate is zero here.

Choosing y-coordinate, we have

0 = (2k-3)/(k+1)

or k = 3/2

The ratio is 3:2 in XZ plane which divides AB.