Exercise 27A

Question 1: Evaluate

$$\lim_{x \to 2}(5 - x)$$

Solution:

We know, $\lim_{x \to a} f(x) = f(a)$

As $x \to 2$

$$\lim_{x \to 2}(5 - x) = 5 - 2 = 3$$

The value of $\lim_{x \to 2}(5 - x)$ is 3.

Question 2: Evaluate

$$\lim_{x \to 1}(6x^2 - 4x + 3)$$

Solution:

We know, $\lim_{x \to a} f(x) = f(a)$

As $x \to 1$

$$\lim_{x \to 1}(6x^2 - 4x + 3) = 6\,(1^2) - 4(1) + 3 = 5$$

Question 3: Evaluate

$$\lim_{x \to 3}\left(\frac{x^2 + 9}{x + 3}\right)$$

Solution:

We know, $\lim_{x \to a} f(x) = f(a)$

As $x \to 3$

$$\lim_{x \to 3}\frac{x^2 + 9}{x + 3} = \frac{3^2 + 9}{3 + 3} = \frac{18}{6} = 3$$

Question 4: Evaluate

$$\lim_{x\to 3}\left(\frac{x^2-4x}{x-2}\right)$$

Solution:

We know, $\lim_{x\to a} f(x) = f(a)$

As $x \to 3$

$$\lim_{x\to 3}\frac{x^2-4x}{x-2} = \frac{3^2-4(3)}{3-2}$$

$$= \frac{3^2-4(3)}{3-2}$$

$$= -3$$

Question 5: Evaluate

$$\lim_{x\to 5}\left(\frac{x^2-25}{x-5}\right)$$

Solution:

We know, $\lim_{x\to a} f(x) = f(a)$

As $x \to 5$

$$\lim_{x\to 5}\frac{x^2-25}{x-5} = \lim_{x\to 5}\frac{(x+5)(x-5)}{x-5}$$

$$= \lim_{x\to 5}(x+5)$$

$$= 5+5$$

$$= 10$$

Question 6: Evaluate

$$\lim_{x \to 1}\left(\frac{x^3 - 1}{x - 1}\right)$$

Solution:

We know, $\lim_{x \to a} f(x) = f(a)$

As $x \to 1$

$$\lim_{x \to 1}\frac{x^3 - 1}{x - 1} = \lim_{x \to 1}\frac{(x - 1)(x^2 + x + 1)}{x - 1}$$

$$= \lim_{x \to 1}(x^2 + x + 1)$$

$$= 1 + 1 + 1$$

$$= 3$$

Question 7: Evaluate

$$\lim_{x \to -2}\left(\frac{x^3 + 8}{x + 2}\right)$$

Solution:

We know, $\lim_{x \to a} f(x) = f(a)$

As $x \to -2$

$$\lim_{x \to -2}\frac{x^3 + 8}{x + 2} = \lim_{x \to -2}\frac{(x + 2)(x^2 - 2x + 4)}{x + 2}$$

$$= \lim_{x \to -2}(x^2 - 2x + 4)$$

$$= 4 + 4 + 4$$

$$= 12$$

Question 8: Evaluate

$$\lim_{x \to 3}\left(\frac{x^4 - 81}{x - 3}\right)$$

Solution:

We know, $\lim_{x \to a} f(x) = f(a)$

As $x \to 3$

$$\lim_{x \to 3}\frac{x^4 - 81}{x - 3} = \lim_{x \to 3}\frac{(x^2+9)(x^2-9)}{x-3}$$

$$= \lim_{x \to 3}\frac{(x^2+9)(x+3)(x-3)}{x-3}$$

$$= \lim_{x \to 3}(x^2+9)(x+3)$$

$$= (9+9)(3+3)$$

$$= 108$$

Question 9: Evaluate

$$\lim_{x \to 3}\left(\frac{x^2 - 4x + 3}{x^2 - 2x - 3}\right)$$

Solution:

We know, $\lim_{x\to a} f(x) = f(a)$

As $x \to 3$

$$\lim_{x\to 3}\frac{x^2-4x+3}{x^2-2x-3} = \lim_{x\to 3}\frac{(x-3)(x-1)}{(x-3)(x+1)}$$

$$= \lim_{x\to 3}\frac{(x-1)}{(x+1)}$$

$$= \frac{2}{4} \text{ or } \frac{1}{2}$$

Question 10: Evaluate

$$\lim_{x\to \frac{1}{2}}\left(\frac{4x^2-1}{2x-1}\right)$$

Solution:

We know, $\lim_{x\to a} f(x) = f(a)$

As $x \to \frac{1}{2}$

$$\lim_{x\to \frac{1}{2}}\frac{4x^2-1}{2x-1} = \lim_{x\to \frac{1}{2}}\frac{(2x+1)(2x-1)}{2x-1}$$

$$= \lim_{x\to \frac{1}{2}}(2x+1)$$

$$= 2$$

Question 11: Evaluate

$$\lim_{x\to 4}\left(\frac{x^3-64}{x^2-16}\right)$$

Solution:

$$\lim_{x\to 4}\frac{x^3-64}{x^2-16}=\lim_{x\to 4}\frac{(x-4)(x^2+4x+16)}{(x+4)(x-4)}$$

$$=\lim_{x\to 4}\frac{(x^2+4x+16)}{(x+4)}$$

$$=\frac{48}{8}$$

$$=6$$

Question 12: Evaluate

$$\lim_{x\to 2}\left(\frac{x^5-32}{x^3-8}\right)$$

Solution:

$$\lim_{x\to 2}\frac{x^5-32}{x^3-8}=\lim_{x\to 2}\frac{x^5-2^5}{x^3-2^3}$$

We know,

$$\lim_{x\to a}\frac{x^m-a^m}{x-a}=m\,a^{m-1}$$

so,

$$\lim_{x\to 2}\frac{x^5-32}{x^3-8}=\lim_{x\to 2}\frac{\frac{x^5-2^5}{x-2}}{\frac{x^3-2^3}{x-2}}$$

$$=\lim_{x\to 2}\frac{5(2)^4}{3(2)^2}$$

$$=\frac{20}{3}$$

Question 13: Evaluate

$$\lim_{x \to a}\left(\frac{x^{5/2} - a^{5/2}}{x - a}\right)$$

Solution:

We know,

$$\lim_{x \to a} \frac{x^m - a^m}{x - a} = m\,a^{m-1}$$

$$\lim_{x \to a} \frac{x^{\frac{5}{2}} - a^{\frac{5}{2}}}{x - a} = \frac{5}{2} a^{\frac{5}{2}-1} = \frac{5}{2} a^{\frac{3}{2}}$$

Question 14: Evaluate

$$\lim_{x \to a}\left\{\frac{(x+2)^{5/3} - (a+2)^{5/3}}{x - a}\right\}$$

Solution:

$$\lim_{x \to a}\left\{\frac{(x+2)^{\frac{5}{3}} - (a+2)^{\frac{5}{3}}}{x - a}\right\} = \lim_{x \to a}\left\{\frac{(x+2)^{\frac{5}{3}} - (a+2)^{\frac{5}{3}}}{(x+2) - (a+2)}\right\}$$

We know,

$$\lim_{x \to a} \frac{x^m - a^m}{x - a} = m\,a^{m-1}$$

$$\lim_{x \to a}\left\{\frac{(x+2)^{\frac{5}{3}} - (a+2)^{\frac{5}{3}}}{x - a}\right\} = \frac{5}{3}(a+2)^{\frac{5}{3}-1} = \frac{5}{3}(a+2)^{\frac{2}{3}}$$

Question 15: Evaluate

$$\lim_{x \to 1}\left(\frac{x^n - 1}{x - 1}\right)$$

Solution:

$$\lim_{x \to a}\frac{x^n - 1}{x - 1} = \lim_{x \to a}\frac{x^n - 1^n}{x - 1}$$

We know,

$$\lim_{x \to a}\frac{x^m - a^m}{x - a} = m\,a^{m-1}$$

$$\lim_{x \to a}\frac{x^n - 1}{x - 1} = n$$

Question 16: Evaluate

$$\lim_{x \to a}\left(\frac{\sqrt{x} - \sqrt{a}}{x - a}\right)$$

Solution:

$$\lim_{x \to a}\left(\frac{\sqrt{x} - \sqrt{a}}{x - a}\right) = \lim_{x \to a}\frac{x^{\frac{1}{2}} - a^{\frac{1}{2}}}{x - a}$$

We know,

$$\lim_{x \to a}\frac{x^m - a^m}{x - a} = m\,a^{m-1}$$

$$\lim_{x \to a}\frac{x^{\frac{1}{2}} - a^{\frac{1}{2}}}{x - a} = \frac{1}{2}a^{\frac{1}{2}-1} = \frac{1}{2\sqrt{a}}$$

Question 17: Evaluate

$$\lim_{h\to 0}\left(\frac{\sqrt{x+h}-\sqrt{x}}{h}\right)$$

Solution:

$$\lim_{h\to 0}\left(\frac{\sqrt{x+h}-\sqrt{x}}{h}\right) = \lim_{h\to 0}\frac{\sqrt{x+h}-\sqrt{x}}{h} \times \frac{\sqrt{x+h}+\sqrt{x}}{\sqrt{x+h}+\sqrt{x}}$$

$$= \lim_{h\to 0}\frac{x+h-x}{h\left(\sqrt{x+h}+\sqrt{x}\right)}$$

$$= \lim_{h\to 0}\frac{1}{\sqrt{x+h}+\sqrt{x}}$$

$$= \frac{1}{2\sqrt{x}}$$

Question 18: Evaluate

$$\lim_{h\to 0}\frac{1}{h}\left\{\frac{1}{\sqrt{x+h}}-\frac{1}{\sqrt{x}}\right\}$$

Solution:

$$\lim_{h\to 0}\frac{1}{h}\left\{\frac{1}{\sqrt{x+h}}-\frac{1}{\sqrt{x}}\right\} = \lim_{h\to 0}\frac{\frac{1}{\sqrt{x+h}}-\frac{1}{\sqrt{x}}}{h}$$

$$= \lim_{h\to 0}\frac{1}{h\sqrt{x+h}}-\frac{1}{h\sqrt{x}}$$

$$= \lim_{h\to 0}\frac{\sqrt{x}}{h\sqrt{x+h}\sqrt{x}}-\frac{\sqrt{x+h}}{h\sqrt{x+h}\sqrt{x}}$$

$$= \lim_{h\to 0}\frac{\sqrt{x}-\sqrt{x+h}}{h\sqrt{x(x+h)}} \times \frac{\sqrt{x}+\sqrt{x+h}}{\sqrt{x}+\sqrt{x+h}}$$

$$= \lim_{h\to 0}\frac{x-(x+h)}{\left(h\sqrt{x(x+h)}\right)\left(\sqrt{x}+\sqrt{x+h}\right)}$$

$$= \frac{-1}{2x\sqrt{x}}$$

Question 19: Evaluate

$$\lim_{x\to 0}\frac{\sqrt{1+x}-1}{x}$$

Solution:

$$\lim_{x\to 0}\frac{\sqrt{1+x}-1}{x}=\lim_{x\to 0}\frac{\sqrt{1+x}-1}{x}\text{ x }\frac{\sqrt{1+x}+1}{\sqrt{1+x}+1}$$

$$=\lim_{x\to 0}\frac{1+x-1}{x\left(\sqrt{1+x}+1\right)}$$

$$=\frac{1}{2}$$

Question 20: Evaluate

$$\lim_{x\to 0}\frac{\sqrt{2-x}-\sqrt{2+x}}{x}$$

Solution:

$$\lim_{x\to 0}\frac{\sqrt{2-x}-\sqrt{2+x}}{x}=\lim_{x\to 0}\frac{\sqrt{2-x}-\sqrt{2+x}}{x}\text{ x }\frac{\sqrt{2-x}+\sqrt{2+x}}{\sqrt{2-x}+\sqrt{2+x}}$$

$$=\lim_{x\to 0}\frac{2-x-2-x}{x\left(\sqrt{2-x}+\sqrt{2+x}\right)}$$

$$=\frac{-1}{\sqrt{2}}$$

Exercise 27B

Question 1: Evaluate

$$\lim_{x\to 0}\frac{\sin 4x}{6x}$$

Solution:

$$\lim_{x\to 0}\frac{\sin 4x}{6x} = \lim_{x\to 0}\left(\frac{\sin 4x}{4x}\right)\times\frac{4}{6}$$

We know, $\lim_{x\to 0}\frac{\sin x}{x} = 1$

$$\lim_{x\to 0}\frac{\sin 4x}{6x} = \frac{4}{6} = \frac{2}{3}$$

Question 2: Evaluate

$$\lim_{x\to 0}\frac{\sin 5x}{\sin 8x}$$

Solution:

$$\lim_{x\to 0}\frac{\sin 5x}{\sin 8x} = \lim_{x\to 0}\left(\frac{\sin 5x}{5x}\right)\times\lim_{x\to 0}\frac{8x}{\sin 8x}\times\lim_{x\to 0}\frac{5x}{8x}$$

We know, $\lim_{x\to 0}\frac{\sin x}{x} = 1$

$$\lim_{x\to 0}\frac{\sin 5x}{\sin 8x} = \frac{5}{8}$$

Question 3: Evaluate

$$\lim_{x\to 0}\frac{\tan 3x}{\tan 5x}$$

Solution:

$$\lim_{x\to 0}\frac{\tan 3x}{\tan 5x} = \lim_{x\to 0}\left(\frac{\tan 3x}{3x}\right)\times \lim_{x\to 0}\frac{5x}{\tan 5x}\times \lim_{x\to 0}\frac{3x}{5x}$$

We know, $\lim_{x\to 0}\frac{\tan x}{x} = 1$

$$\lim_{x\to 0}\frac{\tan 3x}{\tan 5x} = \frac{3}{5}$$

Question 4: Evaluate

$$\lim_{x\to 0}\frac{\tan \alpha x}{\tan \beta x}$$

Solution:

$$\lim_{x\to 0}\frac{\tan \alpha x}{\tan \beta x} = \lim_{x\to 0}\left(\frac{\tan \alpha x}{\alpha x}\right)\times \lim_{x\to 0}\frac{\beta x}{\tan \beta x}\times \lim_{x\to 0}\frac{\alpha x}{\beta x}$$

We know, $\lim_{x\to 0}\frac{\tan x}{x} = 1$

$$\lim_{x\to 0}\frac{\tan \alpha x}{\tan \beta x} = \frac{\alpha}{\beta}$$

Question 5: Evaluate

$$\lim_{x\to 0}\frac{\sin 4x}{\tan 7x}$$

Solution:

$$\lim_{x\to 0}\frac{\sin 4x}{\tan 7x} = \lim_{x\to 0}\left(\frac{\sin 4x}{4x}\right)\times \lim_{x\to 0}\frac{7x}{\sin 7x}\times \lim_{x\to 0}\frac{4x}{7x}$$

We know, $\lim_{x\to 0}\frac{\tan x}{x} = 1$ and $\lim_{x\to 0}\frac{\sin x}{x} = 1$

$$\lim_{x\to 0}\frac{\sin 4x}{\tan 7x} = \frac{4}{7}$$

Question 6: Evaluate

$$\lim_{x\to 0}\frac{\tan 3x}{\sin 4x}$$

Solution:

$$\lim_{x\to 0}\frac{\tan 3x}{\sin 4x} = \lim_{x\to 0}\left(\frac{\tan 3x}{3x}\right)\times \lim_{x\to 0}\frac{4x}{\sin 4x}\times \lim_{x\to 0}\frac{3x}{4x}$$

We know, $\lim_{x\to 0}\frac{\tan x}{x} = 1$ and $\lim_{x\to 0}\frac{\sin x}{x} = 1$

$$\lim_{x\to 0}\frac{\tan 3x}{\sin 4x} = \frac{3}{4}$$

Question 7: Evaluate

$$\lim_{x\to 0}\frac{\sin mx}{\tan nx}$$

Solution:

$$\lim_{x\to 0}\frac{\sin mx}{\tan nx} = \lim_{x\to 0}\left(\frac{\sin mx}{mx}\right)\times \lim_{x\to 0}\frac{nx}{\tan nx}\times \lim_{x\to 0}\frac{mx}{nx}$$

We know, $\lim_{x\to 0}\frac{\tan x}{x} = 1$ and $\lim_{x\to 0}\frac{\sin x}{x} = 1$

$$\lim_{x\to 0}\frac{\sin mx}{\tan nx} = \frac{m}{n}$$

Question 8: Evaluate

$$\lim_{x\to 0}\frac{\sin x - 2\sin 3x + \sin 5x}{x}$$

Solution:

$$\lim_{x\to 0}\frac{\sin x - 2\sin 3x + \sin 5x}{x} = \lim_{x\to 0}\left(\frac{\sin x}{x} - \frac{2\sin 3x}{x} + \frac{\sin 5x}{x}\right)$$

$$= \lim_{x\to 0}\left(\frac{\sin x}{x} - \frac{6\sin 3x}{3x} + \frac{5\sin 5x}{5x}\right)$$

We know, $\lim_{x\to 0}\frac{\sin x}{x} = 1$

$$\lim_{x\to 0}\frac{\sin x - 2\sin 3x + \sin 5x}{x} = 1 - 6 + 5 = 0$$

Question 9: Evaluate

$$\lim_{x\to \pi/6}\frac{(2\sin^2 x + \sin x - 1)}{(2\sin^2 x - 3\sin x + 1)}$$

Solution:

Let sin x = y

Numerator: 2y^2 + y – 1 = (2y – 1)(y + 1)

Denominator: 2y^2 – 3y + 1 = (2y – 1)(y - 1)

Eliminate common term from numerator and denominator which is (2y-1).

$$\lim_{x\to \frac{\pi}{6}}\frac{(2\sin^2 x + \sin x - 1)}{(2\sin^2 x - 3\sin x + 1)} = \lim_{x\to \frac{\pi}{6}}\frac{(\sin x + 1)}{(\sin x - 1)}$$

We know, $\lim_{x\to a} f(x) = f(a)$

$$\lim_{x\to \frac{\pi}{6}}\frac{(2\sin^2 x + \sin x - 1)}{(2\sin^2 x - 3\sin x + 1)} = \frac{(\sin\frac{\pi}{6} + 1)}{(\sin\frac{\pi}{6} - 1)}$$

We know, $\sin\frac{\pi}{6} = \frac{1}{2}$

$$\lim_{x\to \frac{\pi}{6}}\frac{(2\sin^2 x + \sin x - 1)}{(2\sin^2 x - 3\sin x + 1)} = -3$$

Question 10: Evaluate

$$\lim_{x\to 0}\frac{(\sin 2x + 3x)}{(2x + \sin 3x)}$$

Solution:

$$\lim_{x\to 0}\frac{(\sin 2x + 3x)}{(2x + \sin 3x)} = \lim_{x\to 0}\frac{\frac{2x\sin(2x)}{(2x)} + 3x}{2x + \frac{(3x\sin 3x)}{3x}}$$

We know, $\lim_{x\to 0}\frac{\sin x}{x} = 1$

$$\lim_{x\to 0}\frac{(\sin 2x + 3x)}{(2x + \sin 3x)} = \frac{2x + 3x}{2x + 3x} = 1$$

Question 11: Evaluate

$$\lim_{x\to 0}\frac{(\tan 2x - x)}{(3x - \tan x)}$$

Solution:

$$\lim_{x\to 0}\frac{(\tan 2x - x)}{(3x - \tan x)} = \lim_{x\to 0}\frac{\frac{2x\tan 2x}{(2x)} - x}{3x - \frac{x\tan x}{x}}$$

We know, $\lim_{x\to 0}\frac{\tan x}{x} = 1$

$$\lim_{x\to 0}\frac{(\tan 2x - x)}{(3x - \tan x)} = \frac{2x - x}{3x - x} = \frac{1}{2}$$

Question 12: Evaluate

$$\lim_{x\to 0}\frac{(x^2-\tan 2x)}{\tan x}$$

Solution:

$$\lim_{x\to 0}\frac{(x^2-\tan 2x)}{\tan x}=\lim_{x\to 0}\frac{x^2-\frac{2x\tan 2x}{(2x)}}{\frac{x\tan x}{x}}$$

We know, $\lim_{x\to 0}\frac{\tan x}{x}=1$

$$\lim_{x\to 0}\frac{(x^2-\tan 2x)}{\tan x}=\lim_{x\to 0}\frac{x^2-2x}{x}=-2$$

Question 13: Evaluate

$$\lim_{x\to 0}\frac{x\cos x+\sin x}{x^2+\tan x}$$

Solution:

$$\lim_{x\to 0}\frac{x\cos x+\sin x}{x^2+\tan x}=\lim_{x\to 0}\frac{x(\cos x+\frac{\sin x}{x})}{x(x+\frac{\tan x}{x})}$$

We know, $\lim_{x\to 0}\frac{\tan x}{x}=1$ and $\lim_{x\to 0}\frac{\sin x}{x}=1$

$$\lim_{x\to 0}\frac{x\cos x+\sin x}{x^2+\tan x}=\lim_{x\to 0}\frac{\cos x+1}{x+1}=2$$

Question 14: Evaluate

$$\lim_{x\to 0} \frac{\tan x - \sin x}{\sin^3 x}$$

Solution:

$$\frac{\tan x - \sin x}{\sin^3 x} = \lim_{x\to 0} \frac{\frac{\sin x}{\cos x} - \sin x}{\sin^3 x}$$

$$= \lim_{x\to 0} \frac{\sin x - \sin x \cos x}{\cos x \ \sin^3 x}$$

$$= \lim_{x\to 0} \frac{1-\cos x}{\sin^2 x \cos x}$$

We know, sin x = 1 - cos x = (1 - cos x)(1 + cos x)

$$= \lim_{x\to 0} \frac{1-\cos x}{\cos x(1 - \cos x)(1 + \cos x)}$$

$$= \lim_{x\to 0} \frac{1}{(\cos x)(1 + \cos x)}$$

$$= \frac{1}{2}$$

Question 15: Evaluate

$$\lim_{x\to 0} x \operatorname{cosec} x$$

Solution:

$$\lim_{x\to 0} x \operatorname{cosec} x = \lim_{x\to 0} \frac{x}{\sin x}$$

$$= \frac{1}{\lim_{x\to 0} \frac{\sin x}{x}}$$

We know, $\lim_{x\to 0} \frac{\sin x}{x} = 1$

$$\lim_{x\to 0} x \operatorname{cosec} x = 1$$

Question 16: Evaluate

$$\lim_{x\to 0}(x\cot 2x)$$

Solution:

$$\lim_{x\to 0}(x\cot 2x) = \lim_{x\to 0} x \frac{\cos 2x}{\sin 2x}$$

$$= \lim_{x\to 0} x \frac{\cos 2x}{\sin 2x} \frac{2x}{2x}$$

$$= \lim_{x\to 0} x \frac{\cos 2x}{2x} \frac{2x}{\sin 2x}$$

We know, $\lim_{x\to 0}\frac{\sin x}{x} = 1$

$$= \lim_{x\to 0} \frac{\cos 2x}{2}$$

$$\lim_{x\to 0}(x\cot 2x) = \frac{1}{2}$$

Question 17: Evaluate

$$\lim_{x\to 0}\frac{\sin x \cos x}{3x}$$

Solution:

$$\lim_{x\to 0}\frac{\sin x \cos x}{3x} = \lim_{x\to 0} \frac{\sin x}{x} \frac{\cos x}{3}$$

We know, $\lim_{x\to 0}\frac{\sin x}{x} = 1$

$$= \lim_{x\to 0} \frac{\cos x}{3}$$

$$= \frac{1}{3}$$

Question 18: Evaluate

$$\lim_{x\to 0}\frac{\sin(x/4)}{x}$$

Solution:

$$\lim_{x\to 0}\frac{\sin(x/4)}{x} = \lim_{x\to 0}\frac{\sin(x/4)}{4\,x/4}$$

We know, $\lim_{x\to 0}\frac{\sin x}{x} = 1$

$$= \frac{1}{4}$$

Exercise 27C

Question 1: If f(x) = |x - 3|, find lim(x->3) f(x)

Solution:

$$\lim_{x\to3^+} f(x) = \lim_{x\to3^+} |x-3| = \lim_{x\to3^+}(x-3) = 3 - 3 = 0$$

$$\lim_{x\to3^-} f(x) = \lim_{x\to3^-} |x-3| = \lim_{x\to3^-} -(x-3) = 3 - 3 = 0$$

So, $\lim_{x\to3} f(x) = 0$

Question 2:

Let f(x) = $\begin{cases} \frac{x}{|x|}, & x \neq 0 \\ 0, & x = 0 \end{cases}$

Show that $\lim_{x\to0} f(x)$ does not exist.

Solution:

$$\lim_{x\to0^-} f(x) = \lim_{x\to0^-} \frac{x}{|x|} = \lim_{x\to0^-} \frac{x}{(-x)} = -1$$

$$\lim_{x\to0^+} f(x) = \lim_{x\to0^+} \frac{x}{|x|} = \lim_{x\to0^+} \frac{x}{(+x)} = 1$$

=> $\lim_{x\to0^-} f(x) \neq \lim_{x\to0^+} f(x)$

So, $\lim_{x\to0} f(x)$ does not exist.

Question 3:

Let f(x) = $\begin{cases} \frac{|x-3|}{(x-3)}, & x \neq 3 \\ 0, & x = 3 \end{cases}$

Show that $\lim_{x\to3} f(x)$ does not exist

Solution:

$$\lim_{x\to 3^+} f(x) = \lim_{x\to 3^+} \frac{|x-3|}{x-3} = \lim_{x\to 3^+} \frac{(x-3)}{x-3} = 1$$

$$\lim_{x\to 3^-} f(x) = \lim_{x\to 3^-} \frac{|x-3|}{x-3} = \lim_{x\to 3^-} \frac{-(x-3)}{x-3} = -1$$

$$\Rightarrow \lim_{x\to 3^+} f(x) \neq \lim_{x\to 3^-} f(x)$$

So, $\lim_{x\to 3} f(x)$ does not exist.

Question 4:

Let f(x) = $\begin{cases} 1+x^2, 0 \le x \le 1 \\ 2-x, \ x > 1 \end{cases}$

Show that $\lim_{x\to 1} f(x)$ does not exist

Solution:

$$\lim_{x\to 1^+} f(x) = \lim_{x\to 1^+} 2-x = 2 - 1 = 1$$

$$\lim_{x\to 1^-} f(x) = \lim_{x\to 1^-} 1+x^2 = 1 + 1 = 2$$

$$\lim_{x\to 1^-} f(x) \neq \lim_{x\to 1^+} f(x)$$

So, $\lim_{x\to 1} f(x)$ does not exist

Question 5:

Let f(x) = $\begin{cases} \frac{x-|x|}{x}, x \neq 0 \\ 2, \quad x = 0 \end{cases}$

Show that $\lim_{x\to 0} f(x)$ does not exist

Solution:

$$\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} \frac{x-|x|}{x} = \lim_{x\to 0^+} \frac{x-(x)}{x} = 0$$

$$\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} \frac{x-|x|}{x} = \lim_{x\to 0^-} \frac{x-(-x)}{x} = 2$$

$$\lim_{x\to 0^-} f(x) \neq \lim_{x\to 0^+} f(x)$$

$\lim_{x\to 0} f(x)$ does not exist

Question 6:

Let f(x) = $\begin{cases} 5x-4, & 0 < x \le 1 \\ 4x^3 - 3x, & 1 < x < 2 \end{cases}$

Find $\lim_{x\to 1} f(x)$

Solution:

$$\lim_{x\to 1^+} f(x) = \lim_{x\to 0^+} 4x^3 - 3x = 4 - 3 = 1$$

$$\lim_{x\to 1^-} f(x) = \lim_{x\to 1^-} 5x - 4 = 5 - 4 = 1$$

$$\lim_{x\to 0^-} f(x) = \lim_{x\to 0^+} f(x)$$

Therefore, $\lim_{x\to 1} f(x) = 1$

Question 7:

Let f(x) = $\begin{cases} 4x-5, x \le 2 \\ x-a, x > 2 \end{cases}$

If $\lim_{x\to 2} f(x)$ exists then find the value of a.

Solution:

$$\lim_{x\to 2^+} f(x) = \lim_{x\to 2^+} x - a = 2 - a$$

$$\lim_{x\to 2^-} f(x) = \lim_{x\to 2^-} 4x - 5 = 8 - 5 = 3$$

As, $\lim_{x\to 2} f(x)$ exists (given)

Then, 2 - a = 3

or a = -1

Question 8:

Let f(x) = $\begin{cases} \dfrac{3x}{|x| + 2x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$

Show that $\lim_{x\to 0} f(x)$ does not exist

Solution:

$$\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} \frac{3x}{|x| + 2x} = \lim_{x\to 0^+} \frac{3x}{(x) + 2x} = 1$$

$$\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} \frac{3x}{|x| + 2x} = \lim_{x\to 0^-} \frac{3x}{(-x) + 2x} = 3$$

$$\lim_{x\to 0^-} f(x) \neq \lim_{x\to 0^+} f(x)$$

So, $\lim_{x\to 0} f(x)$ does not exists.