Exercise 28A

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Question 1: Differentiate the following functions:
(i) x^{-3} (ii) $x^{1/3}$

Solution:

We know, $d/dx (x^m) = m(x^{m-1})$

(i) $d/dx (x^{-3}) = -3x^{-4}$

(ii) $d/dx (x^{1/3}) = 1/3\ x^{-2/3}$

Question 2: Differentiate the following functions:
(i) 1/x (ii) $1/\sqrt{x}$ (iii) $1/x^{1/3}$

Solution:
We know, $d/dx (x^m) = m(x^{m-1})$

(i) $d/dx (1/x) = x^{-1} = -x^{-2} = -1/x^2$

(ii) $d/dx (1/\sqrt{x}) = x^{-1/2} = -1/2 * x^{-3/2}$

(iii) $d/dx (1/x^{1/3}) = x^{-1/3} = -1/3 * x^{-4/3}$

Question 3: Differentiate the following functions:
(i) $3x^{-5}$ (ii) 1/5x (iii) $6 (x^2)^{3/2}$

Solution:
We know, $d/dx (x^m) = m(x^{m-1})$

(i) $d/dx (3x^{-5}) = 3(-5)x^{-6} = -15\ x^{-6}$

(ii) $d/dx (1/5x) = 1/5 * (d/dx (1/x)) = 1/5 (-1/x^2) = -1/5x^2$

(iii) $d/dx (6 (x^2)^{1/3}) = 6\ d/dx (x^{2/3}) = 6(2/3 * x^{-1/3}) = 4x^{-1/3}$

Question 4: Differentiate the following functions:
(i) $6x^5 + 4x^3 – 3x^2 + 2x – 7$
(ii) $5x^{-3/2} + 4/\sqrt{x} + \sqrt{x} – 7/x$
(iii) $ax^3 + bx^2 + cx + d$, where a, b, c, d are constants

Solution:
We know, d/dx (x^m) = m(x^{m-1})

(i) d/dx($6x^5 + 4x^3 – 3x^2 + 2x – 7$) = $30x^4 + 12x^2 – 6x + 2$

(ii) d/dx($5x^{-3/2} + 4/\sqrt{x} + \sqrt{x} – 7/x$) .....(1)

d/dx($5x^{-3/2}$) = 5 * -3/2 * $x^{-5/2}$ = -15/2 * $x^{-5/2}$

d/dx($4/\sqrt{x}$) = 4 d/dx($1/\sqrt{x}$) = 4 * -1/2 * $x^{-3/2}$ = -2 $x^{-3/2}$

d/dx($\sqrt{x}$) = 1/2 * $x^{-1/2}$

d/dx(7/x) = 7 d/dx(1/x) = 7 * -1/x^2

(1)=>

$$d/dx(5x^{-3/2} + 4/\sqrt{x} + \sqrt{x} - 7/x) = -\frac{15}{2}x^{-\frac{5}{2}} - 2x^{-\frac{3}{2}} + \frac{1}{2}x^{\frac{-1}{2}} + 7x^{-2}$$

(iii) $ax^3 + bx^2 + cx + d$, where a, b, c, d are constants

d/dx($ax^3 + bx^2 + cx + d$) = d/dx(ax^3) + d/dx(bx^2) + d/dx(cx) + d/dx(d)

= $3ax^2 + 2bx + c + 0$

We know derivative of a constant is zero.

d/dx($ax^3 + bx^2 + cx + d$) = $3ax^2 + 2bx + c$

Question 5: Differentiate the following functions:

(i) $4x^3 + 3.2^x + 6.\sqrt[8]{x^{-4}} + 5\cot x$

(ii) $\frac{x}{3} - \frac{3}{x} + \sqrt{x} - \frac{1}{\sqrt{x}} + x^2 - 2^x + 6x^{-2/3} - \frac{2}{3}x^6$

Solution:
We know,
d/dx (x^m) = m(x^{m-1})
d/dx cot x = - $cosec^2$ x

$d/dx\ a^x = \log_n(a)a^x$

(i)

$$\frac{d}{dx}(4x^3 + 3.2^x + 6x^{-\frac{1}{2}} + 5\cot x)$$

$= 12x^2 + 3\log_n(2)2^x + 6 * -1/2 * (x^{-3/2}) + 5(-\text{cosec}^2 x)$

$= 12x^2 + (3\log_n 2)\,2^x - 3x^{-3/2} - 5\,\text{cosec}^2 x$

$$\text{(ii)}\ \frac{d}{dx}\left(\frac{x}{3} - 3x^{-1} + x^{\frac{1}{2}} - x^{-\frac{1}{2}} + x^2 - 2^x + 6x^{-\frac{2}{3}} - \frac{2}{3}x^6\right)$$

$$= \frac{1}{3} - (-1) \times 3x^{-1-1} + \frac{1}{2}x^{\frac{1}{2}-1} - \left(-\frac{1}{2}\right)x^{-\frac{1}{2}-1} + 2x^{2-1} - \log(2).2^x + 6\left(-\frac{2}{3}\right)x^{-\frac{2}{3}-1} - \frac{2}{3} \times 6x^{6-1}$$

$$= \frac{1}{3} + 3x^{-2} + \frac{1}{2}x^{-\frac{1}{2}} + \frac{1}{2}x^{-\frac{3}{2}} + 2x^1 - \log(2).2^x - 4x^{-\frac{5}{3}} - 4x^5$$

Question 6: Differentiate the following functions:

$$\text{(i)}\ 4\cot x - \frac{1}{2}\cos x + \frac{2}{\cos x} - \frac{3}{\sin x} + \frac{6\cot x}{\text{cosec}\,x} + 9$$

(ii) $-5\tan x + 4\tan x\cos x - 3\cot x\sec x + 2\sec x - 13$

Solution:
We know,
$d/dx \sin x = \cos x$

$d/dx \cos x = -\sin x$

$d/dx \tan x = \sec 2x$

$d/dx\ \text{cosec}\ x = -\text{cosec}\ x \cot x$

$d/dx \sec x = \sec x \tan x$

$d/dx \cot x = -\text{cosec}2\ x$

$d/dx\ k = 0$

(k is any constant)

(i)

$$\frac{d}{dx}(4\cot x - \frac{1}{2}\cos x + 2\sec x - 3\operatorname{cosec} x + 6\cos x + 9)$$

$$= 4(-\operatorname{cosec}^2 x) - \tfrac{1}{2}(-\sin x) + 2\sec x \tan x - 3(-\operatorname{cosec} x \cot x) + 6(-\sin x) + 0$$

$$= -4\operatorname{cosec}^2 x + \tfrac{1}{2}\sin x + 2\sec x \tan x + 3\operatorname{cosec} x \cot x - 6\sin x$$

(ii) -5 tan x + 4 tan x cos x – 3 cot x sec x + 2sec x – 13

= -5 tan x + 4 sinx/cosx * cosx – 3 cosx/sinx * 1/cosx + 2sec x – 13

= -5 tan x + 4 sinx – 3 cosec x + 2sec x – 13

Now,
d/dx (-5 tan x + 4 sinx – 3 cosecx + 2sec x – 13)

= -5 $\sec^2$x + 4cosx -3(- cosecx cotx) + 2 secx tanx – 0

= -5 $\sec^2$x + 4cosx + 3 cosecx cotx + 2 secx tanx