Exercise 29A

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Question 1: Which of the following sentences are statements? In case of a statement mention whether it is true or false.

(i) The sun is a star.
(ii) √7 is an irrational number.
(iii) The sum of 5 and 6 is less than 10.
(iv) Go to your class.
(v) Ice is always cold.
(vi) Have you ever seen the Red Fort?
(vii) Every relation is a function.
(viii) The sum of any two sides of a triangle is always greater than the third side.
(ix) May God bless you!

Solution:
(i) The sun is a star is a statement.

It is a scientifically proven fact, therefore this sentence is always true.

(ii) An irrational number is any number which cannot be expressed as a fraction of two integers.
Here, √7 cannot be expressed as a fraction of two integers, so √7 is an irrational number.

Therefore, "√7 is an irrational number" is a statement, and it is true.

(iii) Sum of 5 and 6 = 5 + 6 = 11 > 10

Sum of 5 and 6 is 11, which is greater than 10.

Therefore, "The sum of 5 and 6 is less than 10" is a statement, but not true.

(iv) The sentence 'Go to your class' is an order.
This is an Imperative sentence. Hence it is not a statement.

(v) Ice is always cold is a statement.
It is scientifically proven the fact, therefore the sentence is always true.

(vi) The sentence 'Have you ever seen the Red Fort?
This is an interrogative sentence. Hence not a statement.

(vii) 'Every relation is a function' is a statement.
There are relations which are not functions.
Therefore, the sentence is false.

(viii) 'The sum of any two sides of a triangle is always greater than the third side'
It is a statement and mathematically proven result.
Hence the statement is true.

(ix) 'May God bless you!' is an exclamation sentence. Hence it is not a statement.

Question 2: Which of the following sentences are statements? In case of a statement, mention whether it is true or false.

(i) Paris is in France.
(ii) Each prime number has exactly two factors.
(iii) The equation $x^2 + 5|x| + 6 = 0$ has no real roots.
(iv) $(2 + \sqrt{3})$ is a complex number.
(v) Is 6 a positive integer?
(vi) The product of -3 and -2 is -6.
(vii) The angles opposite the equal sides of an isosceles triangle are equal.
(viii) Oh! it is too hot.
(ix) Monika is a beautiful girl.
(x) Every quadratic equation has at least one real root.

Solution:
(i) Paris is in France, is a statement.
Paris is located in France, so the sentence is true.
So, the statement is true.

(ii) Each prime number has exactly two factors, is a statement.
This is a mathematically proven fact.
So, the statement is true.

(iii) The equation $x^2 + 5|x| + 6 = 0$ has no real roots.

Find the roots of $x^2 + 5|x| + 6 = 0$:

Case 1: $x \geq 0$
$x^2 + 5x + 6 = (x+2)(x+3) = 0 \Rightarrow x = -2, -3$ but we already assumed $x \geq 0$, which is a contradiction.

Case 2: $x < 0$
$x^2 - 5x + 6 = (x-2)(x-3) = 0 \Rightarrow x = (2,3)$ but we already assumed $x < 0$, which is a contradiction.

So, equation $x^2 + 5|x| + 6 = 0$ has no real roots.

Therefore, the given sentence is true, and it is a statement.

(iv) $(2 + \sqrt{3})$ is a complex number, is a statement.

Complex numbers are in the form 'a+ib'.
$(2 + \sqrt{3})$ cannot be expressed in 'a+ib' form,.
$2 + \sqrt{3}$ is not a complex number.
The given sentence is a statement, and it is false.

(v) Is 6 a positive integer?
This is an interrogative sentence, so it is not a statement.

(vi) The product of -3 and -2 is -6, is a statement.

Product of -3 and -2 = -3 x -2 = 6 ≠ -6

This statement is false.

(vii) The angles opposite the equal sides of an isosceles triangle are equal, is a statement.

It is a mathematically proven result.

So the given sentence is true.

(viii) Oh! it is too hot.
This is an exclamatory sentence, so it is not a statement.

(ix) Monika is a beautiful girl, is not a statement.
The given sentence is an opinion, can be true for some cases, false for some other case.

(x) Every quadratic equation has at least one real root, is a statement.
Because not every quadratic equation will have a real root.
So the given sentence is false.

Question 3: Which of the following statements are true and which are false? In each case give a valid reason for your answer.
(i) p: √11 is an irrational number.
(ii) q: Circle is a particular case of an ellipse.
(iii) r: Each radius of a circle is a chord of the circle.
(iv) S: The center of a circle bisects each chord of the circle.
(v) t: If a and b are integers such that a < b, then –a > -b.
(vi) y: The quadratic equation $x^2 + x + 1 = 0$ has no real roots.

Solution:
(i) p: √11 is an irrational number.

True statement.

Reason:
An irrational number is any number which cannot be expressed as a fraction of two integers.
√11 cannot be expressed as a fraction of two integers, so √11 is an irrational number.

(ii) q: Circle is a particular case of an ellipse.

True statement.

Reason:
The equation of an ellipse is $x^2/a^2 + y^2/b^2 = 1$
Special case: When a = b
Then $x^2 + y^2 = 1$, which is an equation of circle.
So, we can say that, a circle is a particular case of an ellipse with the same radius in all points.

(iii) r: Each radius of a circle is a chord of the circle.

False statement.

Reason:
A chord intersects the circle at two points, but radius intersects the circle only at one point.
So the radius is not a chord of the circle.

(iv) S: The center of a circle bisects each chord of the circle.

False statement.

Reason:
The only diameter of a circle is bisected by the center of the circle. Except for diameter, no other chords are passes through the center of a circle.

(v) t: If a and b are integers such that a < b, then –a > -b.

True statement.

Reason:
a < b, then –a > -b [By rule of inequality]

(vi) y: The quadratic equation $x^2 + x + 1 = 0$ has no real roots.

True statement.

Reason:
General form of a quadratic equation, $ax^2 + bx + c = 0$, has no real roots if discriminant, D < 0.
Where $D= b^2 – 4ac < 0$.

Given equation; $x^2 + x + 1 = 0$
Here, a= 1, b = 1 and c = 1
Now, $b^2 – 4ac = 1 – 4 \times 1 \times 1 = -3 < 0$
So, there is no real root.