

Exercise 29C

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Question 1: Rewrite the following statement in five different ways conveying the same meaning.

If a given number is a multiple of 6, then it is a multiple of 3.

Solution:

- (i) A given number is a multiple of 6, it implies that it is a multiple of 3 as well.
- (ii) For a given number to be a multiple of 6, it is necessary that it is a multiple of 3.
- (iii) A given number is a multiple of 6 only if it is a multiple of 3.
- (iv) If the given number is not a multiple of 3, then it is not a multiple of 6.
- (v) For a given number to be a multiple of 3, it is sufficient that the number is multiple of 6.

Question 2: Write each of the following statements in the form 'if then' :

- (i) A rhombus is a square only if each of its angles measures 90° .
- (ii) When a number is a multiple of 9, it is necessarily a multiple of 3.
- (iii) You get a job implies that your credentials are good.
- (iv) Atmospheric humidity increase only if it rains.
- (v) If a number is not a multiple of 3, then it is not a multiple of 6.

Solution:

- (i) If each of the angles of a rhombus measures 90° , then the rhombus is a square.
- (ii) If a number is a multiple of 9, then the number is multiple of 3.
- (iii) If you get a job, then your credentials are good.
- (iv) If it rains, then the atmospheric humidity increases.
- (v) If a number is a multiple of 6, then it is a multiple of 3.

Question 3: Write the converse and contrapositive of each of the following :

- (i) If x is a prime number, then x is odd.
- (ii) If a positive integer n is divisible by 9, then the sum of its digits is divisible by 9.
- (iii) If the two lines are parallel, then they do not intersect in the same plane.
- (iv) If the diagonal of a quadrilateral bisect each other, then it is a parallelogram.
- (v) If A and B are subsets of X such that $A \subseteq B$, then $(X - B) \subseteq (X - A)$
- (vi) If $f(2) = 0$, then $f(x)$ is divisible by $(x - 2)$.
- (vii) If you were born in India, then you are a citizen of India.
- (viii) If it rains, then I stay at home.

Solution:

(i) Converse: If x is an odd number, then it is a prime.

Contrapositive: If x is not an odd number, then it is not a prime.

(ii) Converse: If x is an odd number, then it is a prime.

Contrapositive: If the sum of the digits of a positive integer n is not divisible by 9, then n is not divisible by 9.

(iii) Converse: If the two lines do not intersect in the same plane, then they are not parallel.

Contrapositive: If two lines intersect in the same plane, then they are not parallel.

(iv) Converse: If a quadrilateral is a parallelogram, then its diagonals bisect each other.

Contrapositive: If the quadrilateral is not a parallelogram, then its diagonals do not bisect each other.

(v) Converse: If A and B are subsets of X such that $(X - B) \subseteq (X - A)$, then $A \subseteq B$.

Contrapositive: If A and B are subsets of X such that $(X - B)$ is not a subset of $(X - A)$, then A is not a subset of B .

(vi) Converse: If $f(x)$ is divisible by $(x-2)$, then $f(2) = 0$.

Contrapositive: If $f(x)$ is not divisible by $(x-2)$, then $f(2) \neq 0$.

(vii) Converse: If you are a citizen of India, then you were born in India.

Contrapositive: If you are not a citizen of India, then you were not born in India.

(viii) Converse: If I stay at home, then it rains.

Contrapositive: If I do not stay at home, then it does not rain.

Question 4: Given below are some pairs of statements. Combine each pair using if and only if:

(i) p : If a quadrilateral is equiangular, then it is a rectangle.

q : If a quadrilateral is a rectangle, then it is equiangular.

(ii) p : If the sum of the digits of a number is divisible by 3, then the number is divisible by 3.

q : If a number is divisible by 3, then the sum of its digits is divisible by 3.

(iii) p : A quadrilateral is a parallelogram if its diagonals bisect each other.

q : If the diagonals of a quadrilateral bisect each other, then it is a parallelogram.

(iv) p : If $f(a) = 0$, then $(x - a)$ is a factor of polynomial $f(x)$.

q : If $(x - a)$ is a factor of polynomial $f(x)$, then $f(a) = 0$.

(v) p : If a square matrix A is invertible, then $|A|$ is nonzero.

q : If A is a square matrix such that $|A|$ is nonzero, then A is invertible.

Solution:

- (i) A quadrilateral is a rectangle if and only if it is equiangular.
- (ii) A number is divisible by 3 if and only if the sum of the digits of the number is divisible by 3.
- (iii) A quadrilateral is a parallelogram if and only if its diagonals bisect each other.
- (iv) $(x-a)$ is a factor of polynomial $f(x)$ if and only if $f(a) = 0$.
- (v) Square matrix A is invertible if and only if $|A|$ is nonzero.

Question 5 : write each of the following using 'if and only if' :

- (i) In order to get A grade, it is necessary and sufficient that you do all the homework regularly.
- (ii) If you watch television, then your mind is free, and if your mind is free, then you watch television.

Solution:

- (i) You get an A grade if and only if you do all your homework regularly.
- (ii) You watch television if and only if your mind is free.