Exercise 29D

Question 1: Let p : If x is an integer and x^2 is even, then x is even,

Using the method of contrapositive, prove that p is true.

Solution:
Let p: x is an integer and x^2 is even.
q: x is even

For contrapositive,
~p = x is an integer and x^2 is not even.
~q = x is not even.

Now, the statement is: If x is an integer and x^2 is not even, then x is not even.

Proof:
Let x be an odd integer and x = 2n + 1

=>$x^2 = (2n+1)^2 = 4n^2 + 4n + 1$ (odd integer)

Thus, if x is an integer and x^2 is not even, then x is not even.

Question 2: Consider the statement:
q : For any real numbers a and b, $a^2 = b^2 \Rightarrow a = b$
By giving a counter-example, prove that q is false.

Solution:
Let us take the numbers a= +7 and b= -7.

$a^2 = (+7)^2 = 49$

$b^2 = (-7)^2 = 49$

=> $a^2 = b^2$

But, +7 ≠ -7

=> a ≠ b.

Thus q is false.

Question 3: By giving a counter-example, show that the statement is false :

p : If n is an odd positive integer, then n is prime.

Solution:

Prime number definition, a number must only have itself and 1 as its factors.

Let us take an odd positive integer, n = 15

Since 15 is an odd positive integer but not prime number.

Thus, statement p is false.

Question 4: Use contradiction method to prove that :

"p: √3 is irrational" is a true statement.

Solution:
Contradiction statement: √3 is a rational number.

Proof:
If √3 is a rational number, then √3 = p/q where (p, q) co-prime.

or q = p/√3

or q^2 = p^2/3 ....(1)

Thus, p^2 must be divisible by 3. Hence p will also be divisible by 3.

We can write p = 3k, where k is a constant.

=> p^2 = 9c^2

(1)=>

q^2 = 9c^2/3 = 3c^2

or c^2 = q^2/3

Thus, q^2 must be divisible by 3, which implies that q will also be divisible by 3.

Thus, both p and q are divisible by 3.

Which is a contradiction, as we assume that p and q are co-prime.

Thus, $\sqrt{3}$ is irrational.

Hence, the statement p is true.

Question 5: By giving a counter-example, show that the following statement is false:
p: If all the sides of a triangle are equal, then the triangle is obtuse angled.

Solution:
We know, Obtuse angles lie between 90° and 180°.

By the properties of triangles, if all sides of the triangle are equal, then all its angles are also equal.

Let each angle of the triangle be x°, then

$x° + x° + x° = 180°$

$3x° = 180°$

$x° = 60°$

[The sum of all angles of a triangle is 180°]

Thus, all angles of the triangle measure 60° which is an acute angle.

Thus, the statement p is false.