Exercise 30A

Page No: 926

Find the mean deviation about the mean for the following data: (Question 1 to Question 3)
Formula used:

Mean Deviation about the mean

$$M.D.(\bar{x}) = \frac{\sum_{i=1}^{n} |x_i - \bar{x}|}{n}$$

Where $\bar{x}$ = mean

Question 1: Find the mean deviation about the mean for 7, 8, 4, 13, 9, 5, 16, 18

Solution:

Step 1: Find the mean

$$\bar{x} = \frac{7+8+4+13+9+5+16+18}{8} = \frac{80}{8} = 10$$

Step 2: Mean deviation using formula

$$M.D.(\bar{x}) = \frac{\sum_{i=1}^{8} |x_i - \bar{x}|}{8}$$

$$= \frac{3+2+6+3+1+5+6+8}{8} = \frac{34}{8} = 4.25$$

Question 2: Find the mean deviation about the mean for 39, 72, 48, 41, 43, 55, 60, 45, 54, 43.

Solution:

Step 1: Find the mean

$$\bar{x} = \frac{39+72+48+41+43+55+60+45+54+43}{10} = \frac{500}{10} = 50$$

Step 2: Mean deviation using formula

$$M.D.(\bar{x}) = \frac{\sum_{i=1}^{10}|x_i - \bar{x}|}{10}$$

$$= \frac{11 + 22 + 2 + 9 + 7 + 5 + 10 + 5 + 4 + 7}{10} = \frac{82}{10} = 8.2$$

Question 3: Find the mean deviation about the mean for 17, 20, 12, 13, 15, 16, 12, 18, 15, 19, 12, 11.

Solution:

Step 1: Find the mean

$$\bar{x} = \frac{17 + 20 + 12 + 13 + 15 + 16 + 12 + 18 + 15 + 19 + 12 + 11}{12}$$

Step 2: Mean deviation using formula

$$M.D.(\bar{x}) = \frac{\sum_{i=1}^{12}|x_i - \bar{x}|}{12}$$

$$= \frac{2 + 5 + 3 + 2 + 0 + 1 + 3 + 3 + 0 + 4 + 3 + 4}{12} = \frac{30}{12} = 2.5$$

Find the mean deviation about the median for the following data: (Question 4 to Question 7)
Formula used:

Mean Deviation about the median

$$M.D.(M) = \frac{\sum_{i=1}^{n}|x_i - M|}{n}$$

Where M= median

Question 4: Find the mean deviation about the median for 12, 5, 14, 6, 11, 13, 17, 8, 10.

Solution:

Step 1: Find the median
Arranging the data into ascending order:

5, 6, 8, 10, 11, 12, 13, 14, 17

Total number of observations = 9, which is odd.

$$\text{Median(M)} = \left(\frac{9+1}{2}\right)^{\text{th}}$$

$$\text{or } 5^{\text{th}} \text{ observation} = 11$$

Step 2: Mean deviation using formula

$$\text{M.D.(M)} = \frac{\sum_{i=1}^{9} |x_i - M|}{9}$$

$$= \frac{6+5+3+1+0+1+2+3+6}{9} = \frac{27}{9} = 3$$

Question 5: Find the mean deviation about the median for 4, 15, 9, 7, 19, 13, 6, 21, 8, 25, 11.

Solution:

Step 1: Find the median
Arranging the data into ascending order:

4, 6, 7, 8, 9, 11, 13, 15, 19, 21, 25

Total number of observations = 11, which is odd.

$$\text{Median(M)} = \left(\frac{11+1}{2}\right)^{\text{th}}$$

$$\text{or } 6^{\text{th}} \text{ observation} = 11$$

Step 2: Mean deviation using formula

$$\text{M.D.(M)} = \frac{\sum_{i=1}^{11} |x_i - M|}{11}$$

$$= \frac{7+5+4+3+2+0+2+4+8+10+14}{11} = \frac{59}{11} = 5.3$$

Question 6: Find the mean deviation about the median for 34, 23, 46, 37, 40, 28, 32, 50, 35, 44.

Solution:

Step 1: Find the median
Arranging the data into ascending order:

23, 28, 32, 34, 35, 37, 40, 44, 46, 50

Total number of observations = 10, which is Even.

$$\text{Median(M)} = \left(\frac{5^{\text{th}} \text{ observation} + 6^{\text{th}} \text{ observation}}{2}\right) = \frac{35+37}{2} = 36$$

Step 2: Mean deviation using formula

$$\text{M.D.(M)} = \frac{\sum_{i=1}^{10} |x_i - M|}{10}$$

$$= \frac{13+8+4+2+1+1+4+8+10+14}{10} = \frac{65}{10} = 6.5$$

Question 7: Find the mean deviation about the median for 70, 34, 42, 78, 65, 45, 54, 48, 67, 50, 56, 63.

Solution:

Step 1: Find the median
Arranging the data into ascending order:

34, 42, 45, 48, 50, 54, 56, 63, 65, 67, 70, 78

Total number of observations = 12, which is Even.

$$\text{Median(M)} = \left(\frac{6^{\text{th}} \text{ observation} + 7^{\text{th}} \text{ observation}}{2}\right) = \frac{54+56}{2} = 55$$

Step 2: Mean deviation using formula

$$\text{M.D.}(M) = \frac{\sum_{i=1}^{12} |x_i - M|}{12}$$

$$= \frac{21 + 13 + 10 + 7 + 5 + 1 + 1 + 8 + 10 + 12 + 15 + 23}{12} = \frac{126}{12} = 10.5$$

Find the mean deviation about the mean for the following data: (Question 8 to Question 10)

Question 8: Find the mean deviation about the mean for below data:

x_i	6	12	18	24	30	36
f_i	5	4	11	6	4	6

Solution:

Mean = $\bar{x} = \frac{\sum_{i=1}^{6} f_i x_i}{\sum_{i=1}^{6} f_i} = \frac{756}{36} = 21$

x_i	f_i	$f_i x_i$	$\lvert x_i - \bar{x}\rvert$	$f_i\lvert x_i - \bar{x}\rvert$
6	5	30	15	75
12	4	48	9	36
18	11	198	3	33
24	6	144	3	18
30	4	120	9	36
36	6	216	15	90
	36	756		288

Mean Deviation about the mean

$$\bar{x} = \frac{\sum_{i=1}^{6} f_i\lvert x_i - \bar{x}\rvert}{\sum_{i=1}^{6} f_i} = \frac{288}{36} = 8$$

Question 9: Find the mean deviation about the mean for below data:

x_i	2	5	6	8	10	12
f_i	2	8	10	7	8	5

Solution:

$$\text{Mean} = \bar{x} = \frac{\sum_{i=1}^{6} f_i x_i}{\sum_{i=1}^{6} f_i} = \frac{300}{40} = 7.5$$

x_i	f_i	$f_i x_i$	$\lvert x_i - \bar{x}\rvert$	$f_i \lvert x_i - \bar{x}\rvert$
2	2	4	5.5	11
5	8	40	2.5	20
6	10	60	1.5	15
8	7	56	0.5	3.5
10	8	80	2.5	20
12	5	60	4.5	22.5
	40	300		92

Mean Deviation about the mean

$$\bar{x} = \frac{\sum_{i=1}^{6} f_i \lvert x_i - \bar{x}\rvert}{\sum_{i=1}^{6} f_i} = \frac{92}{40} = 2.3$$

Question 10: Find the mean deviation about the mean for below data:

x_i	3	5	7	9	11	13
f_i	6	8	15	25	8	4

Solution:

Mean = $\bar{x} = \frac{\sum_{i=1}^{6} f_i x_i}{\sum_{i=1}^{6} f_i} = \frac{528}{66} = 8$

x_i	f_i	$f_i x_i$	$\|x_i - \bar{x}\|$	$f_i\|x_i - \bar{x}\|$
3	6	18	5	30
5	8	40	3	24
7	15	105	1	15
9	25	225	1	25
11	8	88	3	24
13	4	52	5	20
	66	528		138

Mean Deviation about the mean

$$\bar{x} = \frac{\sum_{i=1}^{6} f_i|x_i - \bar{x}|}{\sum_{i=1}^{6} f_i} = \frac{138}{66} = 2.09$$

Exercise 30B

Question 1: Find the mean, variance and standard deviation for the numbers 4, 6, 10, 12, 7, 8, 13, 12.

Solution:
Given data: 4, 6, 10, 12, 7, 8, 13, 12

Sum of observations = 4 + 6 + 10 + 12 + 7 + 8 + 13 + 12 = 72

Total number of observation = 8

Find Mean:
Mean = (Sum of observations) / (Total number of observation)

= 72/8 = 9

Mean = 9

Find Variance:

x_i	$x_i - \bar{x}$	$(x_i - \bar{x})^2$
4	4 – 9 = -5	25
6	6 – 9 = -3	9
10	10 – 9 = 1	1
12	12 – 9 = 3	9
7	7 – 9 = -2	4
8	8 – 9 = -1	1
13	13 – 9 = 4	16
12	12 – 9 = 3	9
		Sum = 74

$$\text{Variance} = \sigma^2 = \frac{\sum(x_i - \bar{x})^2}{n} = \frac{74}{8}$$

Find Standard Deviation:

$$\text{Standard Deviation } (\sigma) = \sqrt{\text{Variance}}$$

$$= \sqrt{\frac{74}{8}}$$

$$= \sqrt{9.25}$$

$$= 3.04$$

Question 2: Find the mean, variance and standard deviation for first six odd natural numbers.

Solution:

Given data: First six odd natural numbers = 1, 3, 5, 7, 9, 11

Sum of observations = 1 + 3 + 5 + 7 + 9 + 11 = 36

Total number of observation = 6

Find Mean:

Mean = (Sum of observations) / (Total number of observation)

= 36/6 = 6

Mean = 6

Find Variance:

x_i	$x_i - \bar{x}$	$(x_i - \bar{x})^2$
1	1 - 6 = -5	25
3	3 - 6 = -3	9
5	5 - 6 = -1	1
7	7 - 6 = 1	1
9	9 - 6 = 3	9
11	11 - 6 = 5	25
		Sum = 70

$$\text{Variance} = \sigma^2 = \frac{\sum (x_i - \bar{x})^2}{n} = \frac{70}{6} = 11.67$$

Find Standard Deviation:

$$\text{Standard Deviation } (\sigma) = \sqrt{\text{Variance}}$$
$$= \sqrt{11.67}$$
$$= 3.41$$

Question 3: Using short cut method, find the mean, variance and standard deviation for the data :

x_i	4	8	11	17	20	24	32
f_i	3	5	9	5	4	3	1

Solution:

x_i	f_i	$x_i f_i$	$x_i - \bar{x}$ $(\bar{x} = 14)$	$(x_i - \bar{x})^2$	$f_i(x_i - \bar{x})^2$
4	3	12	-10	100	300
8	5	40	-6	36	180
11	9	99	-3	9	81
17	5	85	3	9	45
20	4	80	6	36	144
24	3	72	10	100	300
32	1	32	18	324	324
	$\sum f_i = 30$	$\sum f_i x_i = 420$			$\sum f_i(x_i - \bar{x})^2 = 1374$

Mean:

$$\text{Mean } (\bar{x}) = \frac{\sum f_i x_i}{\sum f_i}$$

$$= \frac{420}{30}$$

$$= 14$$

Variance:

$$\sigma^2 = \frac{\sum f_i (x_i - \bar{x})^2}{N}$$

$$= \frac{1374}{30}$$

$$= 45.8$$

Standard deviation

$$\sigma = \sqrt{\text{Variance}}$$

$$= \sqrt{45.8}$$

$$= 6.77$$

Question 4: Using short cut method, find the mean, variance and standard deviation for the data :

x_i	6	10	14	18	24	28	30
f_i	2	4	7	12	8	4	3

Solution:

x_i	f_i	$x_i f_i$	$x_i - \bar{x}$ ($\bar{x}$ = 19)	$(x_i - \bar{x})^2$	$f_i(x_i - \bar{x})^2$
6	2	12	-13	169	338
10	4	40	-9	81	324
14	7	98	-5	25	175
18	12	216	-1	1	12
24	8	192	5	25	200
28	4	112	9	81	324
30	3	90	11	121	363
	$\sum f_i = 40$	$\sum f_i x_i = 760$			$\sum f_i(x_i - \bar{x})^2 = 1736$

Mean:

$$\text{Mean } (\bar{x}) = \frac{\sum f_i x_i}{\sum f_i}$$

$$= \frac{760}{40}$$

$$= 19$$

Variance:

$$\sigma^2 = \frac{\sum f_i (x_i - \bar{x})^2}{N}$$

$$= \frac{1736}{40}$$

$$= 43.4$$

Standard deviation:

$$\sigma = \sqrt{\text{Variance}}$$
$$= \sqrt{43.4}$$
$$= 6.59$$

Question 5: Using short cut method, find the mean, variance and standard deviation for the data :

x_i	10	15	18	20	25
f_i	3	2	5	8	2

Solution:

x_i	f_i	$x_i f_i$	$x_i - \bar{x}$ ($\bar{x}$=19.5)	$(x_i - \bar{x})^2$	$f_i(x_i - \bar{x})^2$
10	3	30	-9.5	90.25	270.75
15	2	30	-4.5	20.25	40.5
18	5	90	-1.5	2.25	11.25
20	8	160	0.5	0.25	2
25	2	50	5.5	30.25	60.5
	$\sum f_i = 20$	$\sum f_i x_i = 390$			$\sum f_i(x_i - \bar{x})^2 = 385$

Mean:

$$\text{Mean } (\bar{x}) = \frac{\sum f_i x_i}{\sum f_i}$$
$$= \frac{390}{20}$$
$$= 19.5$$

Variance:

$$\sigma^2 = \frac{\sum f_i(x_i - \bar{x})^2}{N}$$

$$= \frac{385}{20}$$

$$= 19.25$$

Standard deviation:

$$\sigma = \sqrt{\text{Variance}}$$

$$= \sqrt{19.25}$$

$$= 4.39$$

Question 6: Using short cut method, find the mean, variance and standard deviation for the data :

x_i	92	93	97	98	102	104	109
f_i	3	2	3	2	6	3	3

Solution:

x_i	f_i	$x_i f_i$	$x_i - \bar{x}$ $(\bar{x} = 100)$	$(x_i - \bar{x})^2$	$f_i(x_i - \bar{x})^2$
92	3	276	-8	64	192
93	2	186	-7	49	98
97	3	291	-3	9	27
98	2	196	-2	4	8
102	6	612	2	4	24
104	3	312	4	16	48
109	3	327	9	81	243
	$\sum f_i = 22$	$\sum f_i x_i = 2200$			$\sum f_i(x_i - \bar{x})^2 = 640$

Mean:

$$\text{Mean }(\overline{x}) = \frac{\sum f_i x_i}{\sum f_i}$$

$$= \frac{2200}{22}$$

$$= 100$$

Variance:

$$\sigma^2 = \frac{\sum f_i (x_i - \overline{x})^2}{N}$$

$$= \frac{640}{22}$$

$$= 29.09$$

Standard deviation:

$$\sigma = \sqrt{\text{Variance}}$$

$$= \sqrt{29.09}$$

$$= 5.39$$

Exercise 30C

Question 1: If the standard deviation of the numbers 2, 3, 2x, 11 is 3.5, calculate the possible values of x.

Solution:

Standard Deviation (σ) = 3.5

Sum of observations: 2 + 3 + 2x + 11 = 16 + 2x

Total number of observations = 4

Mean = (16+2x)/4 = (8+x)/2

x_i	$x_i - \bar{x}$	$(x_i - \bar{x})^2$
2	$2 - \frac{8+x}{2} = \frac{-4-x}{2}$	$\frac{16+8x+x^2}{4}$
3	$3 - \frac{8+x}{2} = \frac{-2-x}{2}$	$\frac{4+4x+x^2}{4}$
2x	$2x - \frac{8+x}{2} = \frac{3x-8}{2}$	$\frac{64-48x+9x^2}{4}$
11	$11 - \frac{8+x}{2} = \frac{14-x}{2}$	$\frac{196-28x+x^2}{4}$

$$\text{Variance}, \sigma^2 = \frac{1}{n}\sum (x_i - \bar{x})^2$$

$$(3.5)^2 = \frac{1}{4}\left[\frac{16+8x+x^2}{4} + \frac{4+4x+x^2}{4} + \frac{64-48x+9x^2}{4} + \frac{9-6x+x^2}{4}\right]$$

$12.25 \times 16 = 280 - 64x + 12x^2$

$196 = 280 - 64x + 12x^2$

$12x^2 - 64x + 84 = 0$

or $3x^2 - 16x + 21 = 0$

or $(3x - 7)(x - 3) = 0$

=> Either $3x - 7 = 0$ or $x - 3 = 0$

=>$x = 7/3$ or $x = 3$

Therefore, possible values of x are 3 and 7/3.

Question 2: The variance of 15 observations is 6. If each observation is increased by 8, find the variance of the resulting observations.

Solution: Let $x_1, x_2, x_3, x_4, \ldots, x_{15}$ are any random 15 observations.

Variance = 6 and n = 15 (Given)

We know that,

$$\text{Variance}, \sigma^2 = \frac{1}{n}\sum(x_i - \bar{x})^2$$

$$6 = \frac{1}{15}\sum(x_i - \bar{x})^2$$

$$90 = \sum(x_i - \bar{x})^2 \quad \ldots(1)$$

If each observation is increased by 8, then let new observations be $y_1, y_2, y_3, y_4, \ldots, y_{15}$; where $y_i = x_i + 8$ …….(2)

Now, find the variance for new observations:

$$\text{New Variance} = \frac{1}{n}\sum(y_i - \bar{y})^2$$

Mean of new observations,

$$\bar{y} = \frac{\sum_{i=1}^{n} y_i}{n}$$

$$\bar{y} = \frac{\sum_{i=1}^{15} x_i + 8}{15}$$

[Using equation (2)]

$$\bar{y} = \frac{1}{15}\left[\sum_{i=1}^{15} x_i + 8\sum_{i=1}^{15} 1\right]$$

$$\bar{y} = \frac{1}{15}\sum_{i=1}^{15} x_i + 8 \times \frac{15}{15}$$

$$\bar{y} = \bar{x} + 8 \quad .....(3)$$

Using equation (2) and (3) in equation (1), we get

$$\sum (x_i - \bar{x})^2 = 90$$

$$\sum (y_i - 8 - (\bar{y} - 8))^2 = 90$$

$$\sum (y_i - 8 - \bar{y} + 8)^2 = 90$$

$$\sum (y_i - \bar{y})^2 = 90$$

$$\text{New Variance } = \frac{1}{n}\sum (y_i - \bar{y})^2$$

$$= \frac{1}{15} \times 90$$

$$= 6$$

Question 3: The variance of 20 observations is 5. If each observation is multiplied by 2. Find the variance of the resulting observations.

Solution:

Given: Variance = 5 and n = 20

Let $x_1, x_2, x_3, x_4, \ldots, x_{20}$ are any random 20 observations.

$$\text{Variance}, \sigma^2 = \frac{1}{n}\sum(x_i - \bar{x})^2$$

$$5 = \frac{1}{20}\sum(x_i - \bar{x})^2$$

$$100 = \sum(x_i - \bar{x})^2 \ .....(1)$$

If each observation is multiplied by 2, then let new observations be $y_1, y_2, y_3, y_4, \ldots\ldots\ldots, y_{20}$.
where $y_i = 2x_i$ ....(2)

Now, find the variance for new observations:

$$\text{New Variance} = \frac{1}{n}\sum(y_i - \bar{y})^2$$

Mean of new observations,

$$\bar{y} = \frac{\sum y_i}{n}$$

$$\bar{y} = \frac{\sum(2x_i)}{20}$$

[Using equation (2)]

$$\bar{y} = 2\left(\frac{\sum x_i}{20}\right)$$

$$\bar{y} = 2\bar{x} \ ...(3)$$

Using equation (2) and (3) in equation (1), we get

$$\sum(x_i - \bar{x})^2 = 100$$

$$\sum\left(\frac{1}{2}y_i - \frac{1}{2}\bar{y}\right)^2 = 100$$

$$\left(\frac{1}{2}\right)^2\sum(y_i - \bar{y})^2 = 100$$

$$\sum(y_i - \bar{y})^2 = 400$$

$$\text{New Variance} = \frac{1}{n}\sum(y_i - \bar{y})^2$$

$$= \frac{1}{20} \times 400$$

$$= 20$$

Question 4: The mean and variance of five observations are 6 and 4 respectively. If three of these are 5, 7 and 9, find the other two observations.

Solution:

Mean of five observations = 6 and

Variance of five observations = 4

Let the other two observations be x and y, then new set of observations be 5, 7, 9, x and y

Total number of observations = 5

Sum of all the observations = 5 + 7 + 9 + x + y = 21 + x + y

We know, Mean = (Sum of all the observations) / (Total number of observations)

=> 6 = (21 + x + y)/5

=> 9 = x + y ……(1)

Also,

x_i	$x_i - \bar{x}$	$(x_i - \bar{x})^2$
5	5 - 6 = -1	1
7	7 - 6 = 1	1
9	9 - 6 = 3	9
x	x - 6	$(x - 6)^2$
y	y - 6	$(y - 6)^2$

$$\sum (x_i - \bar{x})^2 = 11 + (x-6)^2 + (y-6)^2$$

$$\text{Variance}, \sigma^2 = \frac{\sum (x_i - \bar{x})^2}{n}$$

$$4 = \frac{11 + (x-6)^2 + (y-6)^2}{5}$$

$20 = 11 + (x^2 + 36 – 12x) + (y^2 + 36 – 12y)$

$9 = x^2 + y^2 + 72 – 12(x + y)$

$x^2 + y^2 + 72 – 12(9) – 9 = 0$
(using equation (1))

$x^2 + y^2 + 63 – 108 = 0$

$x^2 + y^2 – 45 = 0$

or $x^2 + y^2 = 45$ ....(2)

Form (1); $x + y = 9$

Squaring both sides,
$(x + y)^2 = (9)^2$

$(x^2 + y^2) + 2xy = 81$

$45 + 2xy = 81$ (using equation (2))

$2xy = 81 – 45$

or $xy = 18$

or $x = 18/y$

(1)=> $18/y + y = 9$

$y^2 – 9y +18 = 0$

$(y – 3)(y – 6) = 0$

Either $(y – 3) = 0$ or $(y – 6) = 0$
=> $y = 3, 6$
For $y = 3$
$x = 18/3 = 6$
and for $y = 6$
$x = 18/6 = 3$

Thus, remaining two observations are 3 and 6.

Question 5: The mean and variance of five observations are 4.4 and 8.24 respectively. If three of these are 1, 2 and 6, find the other two observations.

Solution:

Mean of five observations = 4.4 and
Variance of five observations = 8.24
Let the other two observations be x and y, then new set of observations be 1, 2, 6, x and y.

Total number of observations = 5
Sum of all the observations = 1 + 2 + 6 + x + y = 9 + x + y

We know, Mean = (Sum of all the observations) / (Total number of observations)

=> 4.4 = (9 + x + y)/5

=> 13 = x + y ……(1)

Also,

x_i	$x_i - 4.4$	$(x_i - \bar{x})^2$
1	-3.4	11.56
2	-2.4	5.76
6	1.6	2.56
x	x - 4.4	$(x - 4.4)^2$
y	y - 4.4	$(y - 4.4)^2$

$$\sum (x_i - \bar{x})^2 = 19.88 + (x - 4.4)^2 + (y - 4.4)^2$$

$$\text{Variance}, \sigma^2 = \frac{\sum (x_i - \bar{x})^2}{n}$$

$$8.24 = \frac{19.88 + (x - 4.4)^2 + (y - 4.4)^2}{5}$$

$41.2 = 19.88 + (x^2 + 19.36 – 8.8x) + (y^2 + 19.36 – 8.8y)$

$21.32 = x^2 + y^2 + 38.72 – 8.8(x + y)$
$x^2 + y^2 + 38.72 – 8.8(13) – 21.32 = 0$
(using equation (1))

$x^2 + y^2 – 97 = 0$ ...(2)

Squaring equation (1) both the sides, we get

$(x + y)^2 = (13)2$

$x^2 + y^2 + 2xy = 169$

$97 + 2xy = 169$
(using equation (2))

$xy = 36$

or $x = 36/y$

(1)=> $36/y + y = 13$

$y^2 + 36 = 13y$

$y^2 – 13y + 36 = 0$

$(y – 4)(y – 9) = 0$

Either $(y – 4) = 0$ or $(y – 9) = 0$

=> $y = 4$ or $y = 9$

For $y = 4$

$x = 36/y = 36/4 = 12$

For $y = 9$

$x = 36/9 = 4$

Thus, remaining two observations are 4 and 9.

Exercise 30D

Page No: 944

Question 1: The following results show the number of workers and the wages paid to them in two factories F_1 and F_2.

Factory	A	B
Number of workers	3600	3200
Mean wages	Rs 5300	Rs 5300
Variance of distribution of wage	100	81

Which factory has more variation in wages?

Solution:

Mean wages of both the factories = Rs. 5300

Find the coefficient of variation (CV) to compare the variation.

We know, CV = SD/Mean x 100, where SD is the standard deviation.

The variance of factory A is 100 and the variance of factory B is 81.

Now, SD of factory A = √100 = 10

And, SD of factory B = √81 = 9

Therefore,

The CV of factory A = 10/5300 x 100 = 0.189

The CV of factory B = 9/5300 x 100 = 0.169

Here, the CV of factory A is greater than the CV of factory B.

Hence, factory A has more variation in wages.

Question 2: Coefficient of variation of the two distributions are 60% and 80% respectively, and their standard deviations are 21 and 16 respectively. Find their arithmetic means.

Solution:

Step 1: Coefficient of variation (CV) is 60%, and the standard deviation (SD) is 21.

We know, CV = SD/Mean x 100, where SD is the standard deviation.

or Mean = SD/CV x 100

= 21/60 x 100

= 35

Step 2: Coefficient of variation (CV) is 80%, and the standard deviation (SD) is 16.

Now, Mean = SD/CV x 100

= 16/80 x 100

= 20

Therefore, the arithmetic mean of both the distribution are 35 and 20.

Question 3: The mean and variance of the heights and weights of the students of a class are given below:

	Heights	Weights
Mean	**63.2 inches**	**63.2 kg**
SD	**11.5 inches**	**5.6 kg**

Which shows more variability, heights or weights?

Solution:
Step 1: In case of heights

Mean = 63.2 inches and SD = 11.5 inches.

Coefficient of variation:

We know, CV = SD/Mean x 100, where SD is the standard deviation.

CV = 11.5/63.2 x 100 = 18.196

Step 2: In case of weights

Mean = 63.2 inches and SD = 5.6 inches.

Coefficient of variation:

CV = 5.6/63.2 x 100 = 8.86

From both the steps, we found

CV of heights > CV of weights

So, heights show more variability.

Question 4: The mean and variance of the heights and weights of the students of a class are given below:

Firm	**A**	**B**
Number of workers	**560**	**650**
Mean monthly wages	**Rs 5460**	**Rs 5460**
Variance of distribution of wage	**100**	**121**

(i) Which firm pays a larger amount as monthly wages?
(ii) Which firm shows greater variability in individual wages?

Solution:
(i)
Both the factories pay the same mean monthly wages i.e. Rs 5460

Number of workers for factory A = 560
Number of workers for factory B = 650

Factory A totally pays as monthly wage = Rs.(5460 x 560) = Rs. 3057600

Factory B totally pays as monthly wage = Rs.(5460 x 650) = Rs. 3549000

That means, factory B pays a larger amount as monthly wages.

(ii)

Find the coefficient of variation (CV) to compare the variation.

We know, CV = SD/Mean x 100, where SD is the standard deviation.

The variance of factory A is 100 and the variance of factory B is 121.

Now,

SD of factory A = √100 = 10

SD of factory B = √121 = 11

Therefore,

The CV of factory A = 10/5460 x 100 = 0.183

The CV of factory B = 11/5460 x 100 = 0.201

Here, CV of factory B is greater than the CV of factory A.

Hence, factory B shows greater variability.

Question 5: The sum and the sum of squares of length x (in cm) and weight y (in g) of 50 plant products are given below:

$$\sum_{i=1}^{50} x_i = 212, \sum_{i=1}^{50} x_i^2 = 902.8, \sum_{i=1}^{50} y_i = 261 \text{ and } \sum_{i=1}^{50} y_i^2 = 1457.6$$

Which is more variable, the length or weight?

Solution:

Compare the coefficients of variation (CV) to get required result.

Here the number of products are n = 50 for length and weight both.

Step 1: For length

$$\text{Mean} = \frac{\sum x_i}{n} = \frac{212}{50} = 4.24$$

$$\text{Variance} = \frac{1}{n^2}[n \sum x_i^2 - (\sum x_i)^2]$$

$$= \frac{1}{50^2}[(50 \times 902.8) - (212)^2]$$

$$= \frac{196}{2500}$$

$$= 0.0784$$

$$\text{SD} = \sqrt{\text{Variance}} = \sqrt{0.0784} = 0.28$$

<u>Coefficient of variation of length:</u>

$$\text{CV} = \frac{0.28}{4.24} \times 100 = 6.603$$

Step 2: For weight

$$\text{Mean} = \frac{\sum y_i}{n} = \frac{261}{50} = 5.22$$

$$\text{Variance} = \frac{1}{n^2}[n \sum y_i^2 - (\sum y_i)^2]$$

$$= \frac{1}{50^2}[(50 \times 1457.6) - (261)^2]$$

$$= \frac{4759}{2500}$$

$$= 1.9036$$

$$\text{SD} = \sqrt{\text{Variance}} = \sqrt{1.9036} = 1.37$$

<u>Coefficient of variation of length:</u>

$$\text{CV} = \frac{1.37}{5.22} \times 100 = 26.245$$

From above results, we can say CV of weight) > CV of length

Therefore, the weight is more variable than height.