Exercise 31A

Question 1: A coin is tossed once. Find the probability of getting a tail.

Solution:
We know,
Probability of occurrence of an event = (Total number of favorable outcomes) / (Total numbers of outcomes)

In this case:

Total numbers of outcomes = {H, T} = 2

Number of outcomes in which tail comes = 1

Now, Probability (getting a tail) = 1/2

Question 2: A die is thrown. Find the probability of

(i) getting a 5
(ii) getting a 2 or a 3
(iii) getting an odd number
(iv) getting a prime number
(v) getting a multiple of 3
(vi) getting a number between 3 and 6

Solution:
We know,
Probability of occurrence of an event = (Total number of favorable outcomes) / (Total numbers of outcomes)

Total outcomes are 1, 2, 3, 4, 5, 6.

Total numbers of outcomes = 6

(i) getting a 5

Total number of desired outcomes i.e. getting 5 = 1

Probability (getting a 5) = 1/6

(ii) getting a 2 or a 3

Total number of desired outcomes i.e. getting 2 or 3 = 2

Probability (getting 2 or 3) = 2/6 = 1/3

(iii) getting an odd number

Total number of desired outcomes i.e. an odd number = 1, 3, 5 = 3

Probability (getting an odd number) = 3/6 = 1/2

(iv) getting a prime number

Total number of desired outcomes i.e. prime number = 2, 3, 5 = 3

Probability (getting a prime number) = 3/6 = 1/2

(v) getting a multiple of 3

Multiple of 3 = 3, 6

Total number of desired outcomes = 2

Probability (getting a multiple of 3) = 2/6 = 1/3

(vi) getting a number between 3 and 6

Number between 3 and 6 = 4, 5

Total number of desired outcomes = 2

Probability (getting a number between 3 and 6) = 2/6 = 1/3

Question 3: In a single throw of two dice, find the probability of

(i) getting a sum less than 6
(ii) getting a doublet of odd numbers
(iii) getting the sum as a prime number

Solution:
Probability of occurrence of an event = (Total number of favorable outcomes) / (Total numbers of outcomes)

Possible outcomes are as follow:
(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6),
(2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6),
(3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6),
(4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6) ,
(5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6) ,
(6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)

Total number of outcomes = 36

(i) getting a sum less than 6

Pick entries having sum less than 6:
(1, 1), (1, 2), (1, 3), (1, 4), (2, 1), (2, 2), (2, 3), (3, 1), (3, 2), (4, 1)

Total number of favorable outcomes = 10

Probability (getting a sum less than 6) = 10/36 or 5/18

(ii) getting a doublet of odd numbers

Pick entries having doublet of odd numbers:
(1, 1), (3, 3), (5, 5)

Total number of favorable outcomes = 3

Probability (getting a doublet of odd numbers) = 3/36 or 1/12

(iii) getting the sum as a prime number

Pick entries having sum as a prime number:
(1, 1), (1, 2), (1, 4), (1, 6), (2, 1), (2, 3), (2, 5), (3, 2), (3, 4), (4, 1), (4, 3), (5, 2), (5, 6), (6, 1), (6, 5)

Total number of favorable outcomes = 15

Probability (getting the sum as a prime number) = 15/36 or 5/12

Question 4: In a single throw of two dice, find
(i) P (an odd number on the first die and a 6 on the second)
(ii) P (a number greater than 3 on each die)
(iii) P (a total of 10)
(iv) P (a total greater than 8)
(v) P (a total of 9 or 11)

Solution:
Probability of occurrence of an event = (Total number of favorable outcomes) / (Total numbers of outcomes)
Possible outcomes are as follow:
(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6),
(2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6),
(3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6),
(4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6),
(5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6),
(6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)

Total number of outcomes = 36

(i) Pick favorable entries: (1, 6), (3, 6), (5, 6)

Total number of favorable outcomes = 3

P (an odd number on the first die and a 6 on the second) = 3/36 = 1/12

(ii) Pick favorable entries: (4, 4), (4, 5), (4, 6), (5, 4), (5, 5), (5, 6), (6, 4), (6, 5), (6, 6)

Total number of favorable outcomes = 9

P (a number greater than 3 on each die) = 9/36 = 1/4

(iii) Pick favorable entries: (4, 6), (5, 5), (6, 4)

Total number of favorable outcomes = 3

P (a total of 10) = 3/36 = 1/12

(iv) Pick favorable entries: (3, 6), (4, 5), (4, 6), (5, 4), (5, 5), (5, 6), (6, 3), (6, 4), (6, 5), (6, 6)

Total number of favorable outcomes = 10

P (a total greater than 8) = 10/36 = 5/18

(v) Pick favorable entries: (3, 6), (4, 5), (5, 4), (6, 3), (6, 5) , (5, 6)

Total number of favorable outcomes = 6

P (a total of 9 or 11) = 6/36 = 1/6

Question 5: A bag contains 4 white and 5 black balls. A ball is drawn at random from the bag. Find the probability that the ball is drawn is white.

Solution:
Given: A bag contains 4 white and 5 black balls

Total number of balls = 4 + 5 = 9

Total number of white balls = 4

We know,
Probability of occurrence of an event = (Total number of favorable outcomes) / (Total numbers of outcomes)

So, P(getting white ball) = 4/9

Question 6: An urn contains 9 red, 7 white, and 4 black balls. A ball is drawn at random. Find the probability that the ball is drawn is

(i) red (ii) white (iii) red or white (iv) white or black (v) not white

Solution:
Total number of Red balls = 9
Total number of white balls = 7
Total number of black balls = 4

Total number of balls = 9 + 7 + 4 = 20

We know,
Probability of occurrence of an event = (Total number of favorable outcomes) / (Total numbers of outcomes)

(i) P(getting a red ball) = 9/20

(ii) P(getting a white ball) = 7/20

(iii) P(getting a red or white) = (9+7)/20 = 16/20 = 4/5

(iv) P(getting a white or black) = (7+4)/20 = 11/20

(v) P(getting not white ball) = 1 - P(getting a white ball) = 1 – 7/20 = 13/20

Question 7: In a lottery, there are 10 prizes and 25 blanks. Find the probability of getting a prize.

Solution:
Total number of outcomes = 10 + 25 = 35
Total number of favorable outcomes (i.e. getting a prize) = 10

We know,
Probability of occurrence of an event = (Total number of favorable outcomes) / (Total numbers of outcomes)

Therefore, probability of getting a prize = 10/35 = 2/7

Question 8: If there are two children in a family, find the probability that there is at least one boy in the family.

Solution:
Let G for Girl and B for Boy, then total possible outcomes: BB, GB, BG, GG

Total numbers of outcomes = 4

Since, we have to find the probability that there is at least one boy in the family.
So, favorable outcomes are: BB, BG, GB

Total number of favorable outcomes = 3

We know,
Probability of occurrence of an event = (Total number of favorable outcomes) / (Total numbers of outcomes)

Therefore, the probability of at least one boy in the family = 3/4

Question 9: Three unbiased coins are tossed once. Find the probability of getting
(i) exactly 2 tails (ii) exactly one tail (iii) at most 2 tails

(iv) at least 2 tails (v) at most 2 tails or at least 2 heads

Solution:
When 3 unbiased coins are tossed once, then possible outcomes are:

TTT, TTH, THT, HTT, THH, HTH, HHT, HHH

Total number of outcomes = 8
We know,
Probability of occurrence of an event = (Total number of favorable outcomes) / (Total numbers of outcomes)

(i) exactly 2 tails

Possible outcomes: TTH, THT, HTT

Total numbers of outcomes = 3

Therefore, the probability of getting exactly 2 tails = 3/8

(ii) exactly one tail

Possible outcomes: THH, HTH, HHT

Total numbers of outcomes = 3

Therefore, the probability of getting exactly one tail = 3/8

(iii) at most 2 tails

Possible outcomes: THH, HTH, HHT, TTH, THT, HTT, HHH

Total numbers of outcomes = 7

Therefore, the probability of getting at most 2 tails = 7/8

(iv) at least 2 tails
Possible outcomes: TTH, THT, HTT, TTT

Total numbers of outcomes = 4

Therefore, the probability of getting at least 2 tails = 4/8 = 1/2

(v) at most 2 tails or at least 2 heads
Possible outcomes: TTH, THT, HTT, THH, HTH, HHT, HHH

Total numbers of outcomes = 7

Therefore, the probability of getting at most 2 tails or at least 2 heads = 7/8

Question 10: In a single throw of two dice, determine the probability of not getting the same number on the two dice.

Solution:
In a single throw of two dice.

Possible outcomes are as follow:
(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6),
(2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6),
(3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6),
(4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6) ,
(5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6) ,
(6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)

Total number of outcomes = 36

Favorable outcomes (i.e. not getting the same number**)** = All outcomes except (1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6)

We know,
Probability of occurrence of an event = (Total number of favorable outcomes) / (Total numbers of outcomes)

Therefore, the probability of at least one boy in the family = 30/36 = 5/6

Question 11: If a letter is chosen at random from the English alphabet, find the probability that the letter is chosen is
(i) a vowel (ii) a consonant

Solution:
Total number of possible outcomes = Total number of alphabets = 26

We know,
Probability of occurrence of an event = (Total number of favorable outcomes) / (Total numbers of outcomes)

(i) a vowel

Favorable outcomes are a, e, i, o, u

Total number of favorable outcomes = 5

Therefore, the probability that the letter is chosen is a vowel = 5/26

(ii) a consonant

Total number of consonant = 26 - 5 = 21

Therefore, the probability that the letter is chosen is a consonant = 21/26

Question 12: A card is drawn at random from a well-shuffled pack of 52 cards. What is the probability that the card bears a number greater than 3 and less than 10?
Solution:
Total number of cards = 52
i.e. Total numbers of outcomes = 52

There will be 4 sets of each card naming A, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, J, Q, K.
So, there will be a total of 24 cards between 3 and 10

Total number of favorable outcomes = 24

We know,
Probability of occurrence of an event = (Total number of favorable outcomes) / (Total numbers of outcomes)

Therefore, the probability of picking card between 3 and 10 = 24/52 = 6/13

Exercise 31B Page No: 976

Question 1: If A and B are two events associated with a random experiment for which P(A) = 0.60, P(A or B) = 0.85 and P(A and B) = 0.42, find P(B).

Solution:
Given: P(A) = 0.60, P(A or B) = 0.85 and P(A and B) = 0.42

We know that, P(A or B) = P(A) + P(B) - P(A and B)

By substituting values in the above formula, we get

0.85 = 0.60 + P(B) – 0.42

0.85 = 0.18 + P(B)

0.85 – 0.18 = P(B)

0.67 = P(B)

or P(B) = 0.67

Question 2: Let A and B be two events associated with a random experiment for which P(A) = 0.4, P(B) = 0.5 and P(A or B) = 0.6. Find P(A and B).

Solution:
Given: P(A) = 0.4, P(A or B) = 0.6 and P(B) = 0.5

We know that, P(A or B) = P(A) + P(B) - P(A and B)

By substituting values in the above formula, we get

0.6 = 0.4 + 0.5 – P(A and B)

0.6 = 0.9 - P(A and B)

P(A and B) = 0.9 – 0.6

P(A and B) = 0.3

P(A and B) = 0.3

Question 3: In a random experiment, let A and B be events such that P(A or B) = 0.7, P(A and B) = 0.3 and $P(\bar{A}) = 0.4$. Find P(B).

Solution:
Given: P(A or B) = 0.7, P(A and B) = 0.3 and

$P(\bar{A}) = 0.4$

We know, P(A) = 1 – $P(\bar{A}) = 0.4$

= 1 – 0.4 = 0.6

So, P(A) = 0.6

Again, P(A or B) = P(A) + P(B) - P(A and B)

By substituting values in the above formula, we get

0.7 = 0.6 + P(B) – 0.3

0.7 = 0.3 + P(B)

0.7 – 0.3 = P(B)

Or P(B) = 0.4

Question 4: If A and B are two events associated with a random experiment such that P(A) = 0.25, P(B) = 0.4 and P(A or B) = 0.5, find the values of

(i) P(A and B)
(ii)
$P(A \text{ and } \bar{B})$

Solution:
Given : P(A) = 0.25 P(B) = 0.4 and P(A or B) = 0.5
(i)
We know, P(A or B) = P(A) + P(B) - P(A and B)

Substituting values in the above formula, we get

0.5 = 0.25 + 0.4 – P(A and B)

0.5 = 0.65 - P(A and B)

P(A and B) = 0.65 – 0.5

P(A and B) = 0.15

(ii)

We know, P(A and $\bar{B}$) = P(A) - P(A and B)

Substituting values in the above formula, we get

P(A and $\bar{B}$) = 0.25 – 0.15

P(A and $\bar{B}$) = 0.10

P(A and $\bar{B}$) = 0.10

Question 5: If A and B be two events associated with a random experiment such that P(A) = 0.3, P(B) = 0.2 and P(A ∩ B) = 0.1, find

(i) $P(\overline{A} \cap B)$

(ii) $P(A \cap \overline{B})$

Solution:

Given: P(A) = 0.3, P(B) = 0.2 and P(A ∩ B) = 0.1

(i)

$P(\overline{A} \cap B)$ = P(B) - P(A ∩ B)

$P(\overline{A} \cap B) = 0.2 - 0.1$

$P(\overline{A} \cap B) = 0.1$

(ii)

$P(A \cap \overline{B})$ = P(A) - P(A ∩ B)

$P(\overline{A} \cap B) = 0.3 - 0.1$

$P(\overline{A} \cap B) = 0.2$

Question 6: If A and B are two mutually exclusive events such that P(A) = (1/2) and P(B) = (1/3), find P(A or B).

Solution:
P(A) = 1/2 , P(B) = 1/3

We know, P(A or B) = P(A) + P(B) - P(A and B)

For mutually exclusive events A and B, P(A and B) = 0

P(A or B) = 1/2 + 1/3 – 0

P(A or B) = 5/6

Question 7: Let A and B be two mutually exclusive events of a random experiment such that P(not A) = 0.65 and P(A or B) = 0.65, find P(B).

Solution: P(A) = 1 – P(not A) = 1 - 0.65 = 0.35

=>P(A) = 0.35

Again, P(A or B) = P(A) + P(B) - P(A and B)

Here P(A and B) = 0
(Events are mutually exclusive)

0.65 = 0.35 + P(B)

P(B) = 0.65 – 0.35

P(B) = 0.30

Question 8: A, B, C are three mutually exclusive and exhaustive events associated with a random experiment.

If P(B) = (3/2) P(A) and P(C) = (1/2) P(B), find P(A).

Solution: Here, P(A) + P(B) + P(C) = 1 ...(1)

For mutually exclusive events A, B, and C,
P(A and B) = P(B and C) = P(A and C)= 0

Given: P(B) = (3/2) P(A) and P(C) = (1/2) P(B)

(1) => P(A) + (3/2) P(A) + (1/2) P(B) = 1

=> P(A) + (3/2) P(A) + (1/2){(3/2) P(A)} = 1

=> P(A) + (3/2) P(A) + (3/4) P(A) = 1

=> 13/4 P(A) = 1

or P(A) = 4/13

Question 9: The probability that a company executive will travel by plane is (2/5) and that he will travel by train is (1/3). Find the probability of his travelling by plane or train.

Solution:
Let P(A) is the probability that a company executive will travel by plane and P(B) is the probability that he will travel by train.
Then,
P(A) = 2/5 and P(B) = 1/3

As he cannot be travel by plane and train at the same time, so P(A and B) = 0

Using formula, P(A or B) = P(A) + P(B) - P(A and B)

P(A or B) = 2/5 + 1/3 - 0

= 11/15

Therefore, probability of a company executive will be travelling by plane or train= P(A or B) = 11/15.

Question 10: From a well-shuffled pack of 52 cards, a card is drawn at random. Find the probability of its being a king or a queen.

Solution:
In a pack of 52 cards, there are 4 king cards and 4 queen cards

Let A denote the event that the card drawn is queen and B denote the event that card drawn is king.
Then,
P(A) = 4/52 and P(B) = 4/52

As a card cannot be both king and queen in the same time, so P(A and B)= 0

Using formula, P(A or B) = P(A) + P(B) - P(A and B)

P(A or B) = 4/52 + 4/52 - 0

= 2/13

Probability of a card drawn is king or queen = P(A or B) = 2/13

Question 11: From a well-shuffled pack of cards, a card is drawn at random. Find the probability of its being either a queen or a heart.

Solution:
In a pack of 52 cards, there are 4 queen cards and 13 heart cards.

Let A denote the event that the card drawn is queen and B denote the event that card drawn is heart. Then,
P(A) = 4/52 and P(B) = 13/52

As there is one card which is both queen and heart (queen of hearts), so P(A and B)= 1/52

Using formula, P(A or B) = P(A) + P(B) - P(A and B)

P(A or B) = 4/52 + 13/52 – 1/52

= 16/52

= 4/13

Probability of a card drawn is either a queen or heart = P(A or B) = 4/13