Mathematics
Class VII

Rajasthan State Institute of Educational Research and Training, Udaipur

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Foreword

The Changes in scenario of the Society and the nation entail the changes in the system of education which determines and accelerate the process of development in them. Education, beside other factors, is an important factor, responsible for the development of the society and the nation. To make School education, effective, useful and interesting, the changes in the curriculum from time to time is an essential step. The national curriculum framework 2005 and right to Free and Compulsory Child Education Act, 2009 in the present time make it evident that a child occupies a pivotal place in all the teaching-learning activities, conducted in any educational institution. Keeping this view in mind, our process of causing learning amongst the students should be such that they construct knowledge on their own on the basis of the knowledge acquired through their experiences. A child should be allowed maximum freedom in the process of learning and for that – teacher should act as a guide and helper rather than a preacher to make the curriculum easily accessible to children/students, a text book is an important means. That is why the government of Rajasthan has got the new text book written by making necessary changes in them in the light of the changes made in the curriculum.

While writing a text book it has been kept in view that the text book should be easy and comprehensible, with the help of simple language and interesting and attractive with the inclusion of pictures and varied activities through which the learners may not only imbibe the knowledge and information, contained in them but also associate themselves with the social, neighbourhood and local environment along with the development of and adherence to the knowledge about the historical, cultural glory and democratic values of the country so as to establish themselves as sincere, good and worthy citizens of our country, India.

I very humbly request the teachers that they should not only confine themselves to the completion of the teaching of the text book but also to present it in such a manner that a child gets ample opportunities of learning and accomplishing the objectives of teaching-learning on the basis of the curriculum and his/her experiences.

The state Institute of Educational Research and Training (SIERT), Udaipur acknowledges its thankfulness to all those government and private institutions viz. National Council of Educational Research and Training, New Delhi, State and National Census Departments, Ahad Museum, Udaipur. Directorate of Public Relations, Jaipur, Rajasthan, Rajasthan Text Book Board, Jaipur, Vidya Bharati, All India Educational Institute, Jaipur, Vidya Bhawan Reference Library, Udaipur, different writers, newspapers and magazines, publishers and websites that have

cooperated with us in choosing and making the required material available for writing and developing the text book.

Inspite of best efforts, if the name of any writer, publisher, institution, organization and website has not been included here, we apologize for that and extend our thankfulness to them. In this connection, their names will be incorporated in the next editions of this book in future. It (SIERT) also extends thanks to Mr. Damodar Lal Kabra, Retd. Principal, Chittorgarh for cooperation with us in the translation work of this book.

To enhance the quality of the text books, we have received timely guidance and precious suggestions from Shri Kunji Lal Meena Secretary, Elementary Education, Govt. of Rajasthan, Shri Naresh Pal Gangwar Secretary, Secondary Education Govt. of Rajasthan, and Commissioner National Secondary Education Council, Shri Suwa Lal Meena, Director Secondary Education, Govt. of Rajasthan, Shri Babulal Meena, Director Elementary Education, Govt. of Rajasthan and Shri B.L. Jatawat, Commissioner Elementary Education, Govt. of Rajasthan Jaipur, and as such the institute (SIERT) expresses its heartiest gratefulness to all of them.

This book has been prepared with the financial and the technical support of UNICEF. In this connection we are grateful to Mr. Samuel M, Chief, UNICEF Jaipur, Sulgana Roy, Education Specialist and all the related officers of UNICEF for their timely support and cooperation. Besides them the institute appreciates the efforts of all those officers and other members of the staff who have directly or indirectly cooperated with us in accomplishing the task of book writing and publishing it.

I am highly delighted to submit this book to you all with this belief in mind that it will not only prove beneficial to the teachers and the students but also serve as an effective link in the teaching-learning process and the personality development of the students.

To value thoughts and suggestions is a specific feature of a democracy; therefore the SIERT, Udaipur will always welcome your precious thoughts and suggestions for improving the quality of this book and thus make it better in every respect.

Director
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For the Teachers

The new curriculum and the text book(s) in Mathematics have been prepared with a view to developing the teachers' compentencies in the teaching process and methodology in this subject (Mathematics) in the light of the present changing global scenario.

This curriculum and text book in Mathematics have been prepared with a view to developing the child's understanding regarding the world of education along with the development of his/her latent capacities, enhancement of humane and   moral values in dedicated, sincere citizen as well a patriot.

It is expected of a teacher of Mathematics to imbibe the main guiding principles of teaching. Stated in N.C.F., 2005 and in the light of them make the learners not only understand the subject matter of the book but also to imbibe it for their benefit in future.

The text book contains the following main features in it viz. the students have been made aware of the subject matter of the lessons the help of examples from their neighbourhood. In doing so it has been kept in mind that the teaching-learning material is available to the learners at low cost in their surroundings so that the teachers may use it in their day-today teaching by conducting different activities in the class room with a view to ensuring the learners maximum participation in the teaching-learning process and thereby making his/her teaching effective, useful and purposeful.

Considering the child as the center of the teaching-learning process, the teachers of learning by doing and correcting their mistakes an their own so as to develop insight in them for grasping and imbibing the subject matter of the mathematical lessons properly.

In the light of the provision of the Act of the Right to free and Compulsory Education, 2009, the subject matter has been prepared according to the spirit of the 'Continuous and Comprehensive Evaluation'. Therefore students should be imparted instructions dividing them into groups, according to their standards for inculcating the mathematical competencies in them.

The concepts of Mathematics have been delineated in detail along with pertinent pictures and diagrams for them. Examples and exercise have been combined so that the learners may understand the concepts and thus there by develop the capacity to solve the mathematical problems with maximum participation.

Under the heading 'Learning by Doing' enough activities for the development of the skills of mathematical thinking. Researching of the mathematical facts drawing, lining and measuring have been given for practice. All these activities are to be accomplished by the students with the spirit of cooperation, tolerance and responsibility.

The topics of national concerns viz - Environmental protection. Road safety, Gender Sensitivity, Beti Bachao; Bati Padhao and uprooting of Social evils, etc – have been included at proper places in the text book which the teachers should pay heed to and the same should be conveyed to the learners through the mathematical problems and mathematical solution and other glossary. The learners should be brought home to these national concerns along with the development of the sense of understanding them.

The teacher should judiciously divide the class into groups and sub-groups in order to generate the skill of self learning amongst the learners through various activities given in the text book of Mathematics. At the end of every lesson in the text book the mathematical concepts, definitions and results have been given under the title 'We have Learnt' according to learners capacities and maturity of minds.

At proper places the life history of Indian mathematicians and their contribution to Mathematics have been given in order to make the learners understand and appreciate such great personalities.

The curriculum and the book of Mathematics have been prepared, keeping the child at the center of the teaching-learning process reposing great faith in the teacher who with their great devotion and sincere efforts will work with children to make them understand the mathematical problems, definition, concepts and solutions well with this very belief in mind. The group of writers presents this book of Mathematics to the teachers of Rajasthan.
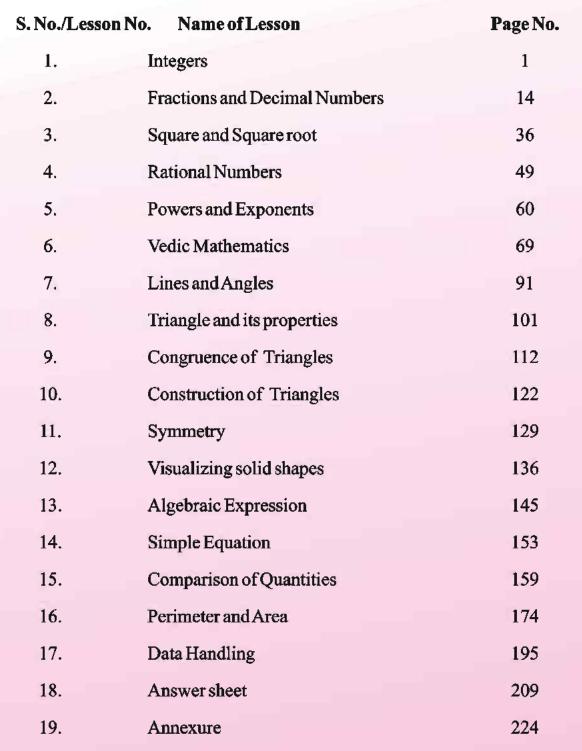
In India Mathematics has had rich tradition(s). Since times immemorial Indian Scholars and mathematicians have done excellent work in this area. In order to use the old knowledge in modern Mathematics and to establish its harmony with a view to enriching it (modern Mathematics), the Indian numerical system (Devnagari) and Vedic Mathematics have been incorporated in this text book - Efforts have been made to make calculations easier through Vedic Mathematics.
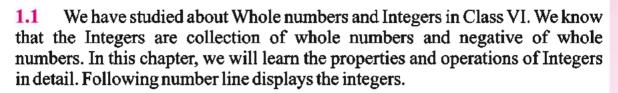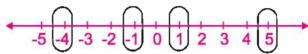
Contents

INTEGERS

1.1 We have studied about Whole numbers and Integers in Class VI. We know that the Integers are collection of whole numbers and negative of whole numbers. In this chapter, we will learn the properties and operations of Integers in detail. Following number line displays the integers.

Write the encircled numbers in increasing order.

We know that the numbers increases from left to right along a number line. Hence

$-4 < -1 < 1 < 5$.

We have also studied in Class VI that on a number line when we

1. Add a positive integer then we move towards right.
2. Add a negative integer then we move towards left.
3. Subtract a positive integer then we move towards left.
4. Subtract a negative integer then we move towards right.

> **Do and learn**
>
> 1. In which direction one should move on the number line to add -5?
> 2. In which direction one will move on the number line to subtract -5 from 3 and will reach on what number?
> $$3 - (-5) = \ldots\ldots\ldots\ldots\ldots$$
> 3. In which direction we will move and on which number will we reach by adding 5 to 3?
> 4. In which direction we will move and on which number will we reach by subtracting + 5 from -3?

Identify true or false statement from the following:

1. Sum of two positive integers is again a positive integer. ( )
2. Sum of two negative integers is again a positive integer. ( )
3. Sum of a positive integer and a negative integer is always a negative integer. ( )
4. Additive inverse of 8 is – 8. ( )
5. $(-7) + 3 = 7 - 3$ ( )
6. $8 + (-7) - (-4) \neq 8 + 7 - 4$ ( )

We check the correctness of above statements as follows:

(1) Statement 1 is true. For example

(i) $7 + 4 = 11$ (ii) $4 + 11 = 15$ (iii) $6 + 7 = 13$ etc.

(2) Statement 2 is false. For example

(i) $(-6) + (-3) = (-9)$

When we add two negative integers, the resulting number is always a negative integer.

(3) Statement 3 is false. For example

$-10 + 15 = 5$, which is not a negative integer.

Therefore the correct statement is that to add a negative and positive integer we take their difference and put the sign of greater integer before it. While choosing the greater integer we neglect the sign.

For example

(i) $(-50) + (70) = 20$ (ii) $12 + (-20) = -8$

(iii) $16 + (-7) = 9$ (iv) $(-14) + (10) = -4$

(4) Statement is true because

$-8 + 8 = 0 = 8 + (-8)$

Addition of additive inverse to a number gives additive identity "0". Give more examples of it.

So, additive inverse of 'a' is '-a' and that of '-a' is 'a'.

(5) Statement is false because

$(-7) + 3 = -4$ and $7 + (-3) = 4$

(6) Statement is true because

$8 + (-7) - (-4) = 5$ and $8 + 7 - 4 = 11$. Hence $8 + (-7) - (-4) \neq 8 + 7 - 4$

1.1 Addition and subtraction properties of Integers
1.2.1 Closure property for addition

We have seen that the addition of two whole numbers is always a whole number, hence we can say that whole numbers are closed for addition. Let us check if integers are also closed for addition.

S. No.	Integer 1	Integer 2	Sum	Is the sum Integer?
1.	+ 2	+ 5	+ 7	Yes
2.	- 3	+ 7		
3.	- 4	+ 4		
4.	+ 3	- 5		

Take various integers and check if this is true only for positive integers or it is true for negative integers also. We observe from the table that **all the integers are closed for addition irrespective of being positive or negative.** Can you tell two such integers whose sum is not an integer? For integers **'a'** and **'b'**, **(a+b)** is always an integer.

1.2.2 Closure property for subtraction

What happens when we subtract an integer from another integer? Is their difference is also an integer? Complete the following table:

	Statement	Observation
1.	$7 - 5 = 2$	Result is an integer.
2.	$4 - 9 = -5$	....
3.	$(-4) - (-5) = $	Result is an integer.
4.	$(-18) - (-18) = $	....
5.	$17 - 0 = $	....

What do you observe? Can we find a pair of integers whose difference is not an integer? Can we say that integers are closed for subtraction? Yes, we can say that **integers are closed for subtraction.**

So, for integers **'a'** and **'b'**, **(a – b)** is always an integer.

Note: It is to be noted that Whole numbers are not closed for subtraction.

1.2.3 Commutative Property

We know that $2 + 4 = 4 + 2 = 6$, i.e., change of order in the addition of whole numbers does not alter the result. Hence, commutative property is followed. Similarly do integers also follow commutative property? Let us check. Are the following same?

$$(-8) + (-4) \text{ and } (-4) + (-8);$$
$$(-2) + 5 \text{ and } 5 + (-2);$$
$$12 + 0 \text{ and } 0 + 12.$$

Add other integers and check if there exist any pair of integers where the result is unchanged by changing the order of addition.

We have seen that if order of addition is changed, the result does not change i.e., **integers follow commutative property under addition operation.** In general, for two integers **'a'** and **'b'** we can say that

$$\textbf{a+b = b + a}$$

We know that the whole numbers do not follow commutative property for subtraction. Does the commutative property applies for subtraction of integers? Consider two integers (-6) and (+4).

Are (-6) – (+4) and (+4) - (-6) same?

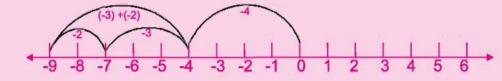No, because

(-6) – (+4) = -10 and (+4) - (-6) = 10

and -10, +10 are not equal.

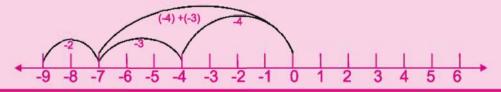Thus, we conclude that **subtraction is not commutative for integers.**

1.2.4 Associative Property

Test for associative property for integers -4, -3 and -2. Calculate
-4 + [(-3) + (-2)] and [(-4) + (-3)]+(-2).

-4 + [(-3) + (-2)] means we first add (-3) and (-2) and then add (-4) to the result.

[(-4) + (-3)] + (-2) means we first add (-4) and (-3) and then add (-2) to the result.

The result is (-9) in both the cases. Give three more such examples. You won't find any example for which the results are different. This shows that **the addition of integers follow the associativity property**, i.e.,

$$a + (b + c) = (a + b) + c.$$

1.2.5 Additive Identity

Observe the following and fill in the blanks:

(i) (-4) + 0 = -4   (ii) 7 + 0 = 7   (iii) 0 + (-14) = …….

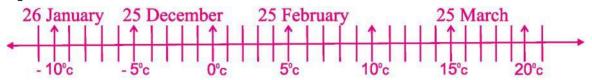(iv) -8 + …... = -8   (v) …... + 0 = 15   (vi) -23 + ……. = -23

It is clear from the above examples that the same integers is obtained when we add 0 to it. Hence, **'0' is the additive identity for integers.** Justify this by taking some more examples.

Exercise 1.1

1.   The temperature in Churu is measured in °C at different time and represented on number line

26 January   25 December   25 February   25 March

$$-10°c \quad -5°c \quad 0°c \quad 5°c \quad 10°c \quad 15°c \quad 20°c$$

(i)   The temperature of Churu on following date from above number line
    (a) 26 January ……………..   (b) 25 December ……………
    (c) 25 February ……………..   (d) 25 March ………………..

(ii)   What is the difference in temperature between the hottest and the coldest day?

(iii)   The temperature of 26 January is how much less than the temperature of 25 February?

(iv)   Can we say that the sum of temperatures on 25 December and 25 February is higher than the temperature on 26 January?

2.   Sheela deposits Rs. 5000 in post office and withdraws Rs. 3700 after one month. If the amount withdrawn is represented in the form of negative number then how will we represent the deposited amount? What is the amount left in the account after withdrawal?

3.   Solve the following:
    (i) $(-4) + (-3)$ (ii) $15 - 8 + (-9)$
    (iii) $400 + (-1000) + (-500)$ (iv) $23 - 41 - 11$
    (v) $-27 + (-3) + 30$

4.   Put the appropriate sign (<, >, =) for the following statements:
    (i) $-14 + 11 + 5$ ( ) $14 - 11 - 5$
    (ii) $30 + (-5) + (-8)$ ( ) $(-5) + (-8) + 30$
    (iii) $7 + 11 + (-5)$ ( ) $(-7) - 11 + 5$
    (iv) $(-14) + 11 + (-12)$ ( ) $14 + 11 + 12$
    (v) $6 + 7 - 13$ ( ) $6 + 7 + (-13)$

5.   Write two such integers whose
    (i) sum is (-7)   (ii) difference is 4   (iii) sum is 0   (iv) difference is -2.

6.   Fill in the blanks.
    (i) $(-3) + 5 = 5 +$ ………..
    (ii) $17 +$ ……… $= 17$
    (iii) ……….. $+ (-5) = 0$
    (iv) $-11 + [(-12) + 4] = [(-11) + (-12)] +$ ……..

7. Examples and some properties of integers are given below. Match the correct property and its example.

	Example		**Property**
(i)	$(a + b) + c = a + (b + c)$	(a)	Identity
(ii)	$3 + 4 = 4 + 3$	(b)	Associativity
(iii)	$(-4) + 0 = (-4)$	(c)	Commutativity
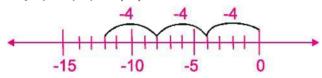
1.3 Multiplication of Integers

1.3.1 Multiplication of Positive Integers with Negative Integers

$3 \times 4 = 4 + 4 + 4 = 12$

$3 \times (-4) = (-4) + (-4) + (-4) = -12$

Similarly, $5 \times (-3) = (-3) + (-3) + (-3) + (-3) + (-3) = -15$

> **Do and learn:** ◆
>
> Solve:
> (i) $4 \times (-8) = \ldots\ldots = \ldots\ldots$ (ii) $3 \times (-3) = \ldots\ldots = \ldots\ldots$
> (iii) $5 \times (-9) = \ldots\ldots = \ldots\ldots$

By using this method we find that the product of a positive integer with a negative integer is a negative integer. But what happens when we multiply a negative integer with a positive integer?

Observe the following pattern:

$3 \times 4 = 12$

$2 \times 4 = 8 = 12 - 4$

$1 \times 4 = 4 = 8 - 4$

$0 \times 4 = 0 = 4 - 4$

$-1 \times 4 = -4 = 0 - 4$

$-2 \times 4 = -8 = -4 - 4$

$-3 \times 4 = -12 = -8 - 4$

We have already obtained that $3 \times (-4) = -12$.

Thus, we find that $(-3) \times 4 = -12 = 3 \times (-4)$.

Similarly, we can also obtain $(-5) \times 3 = -15 = 3 \times (-5)$.

Do and learn : ◆

      Find -

          (i) $15 \times (-5)$ (ii) $27 \times (-10)$

          (iii) -12×12 (iv) -7×4

1.3.2 Product of Two Negative Integers

Observe the following:

$$-3 \times 4 = -12$$
$$-3 \times 3 = -9 = -12 - (-3)$$
$$-3 \times 2 = -6 = -9 - (-3)$$
$$-3 \times 1 = -3 = -6 - (-3)$$
$$-3 \times 0 = 0 = -3 - (-3)$$
$$-3 \times -1 = 3 = 0 - (-3)$$
$$-3 \times -2 = 6 = 3 - (-3)$$

Similarly, complete the following:

    (i) $-3 \times -3 =$ …………….   (ii) $-3 \times -4 =$ ……………

Fill in the blanks in similar manner

$$-5 \times 3 = -15$$
$$-5 \times 2 = -10 = -15 - (-5)$$
$$-5 \times 1 = -5 = -10 - (-5)$$
$$-5 \times 0 = 0 = \text{…………..}$$
$$-5 \times -1 = \text{…………..} = \text{…………}$$
$$-5 \times -2 = \text{…………..} = \text{…………}$$
$$-5 \times -3 = \text{…………..} = \text{…………}$$

Observing this pattern we can say that the product of two negative integers is a positive integer. We multiply two negative integers by considering them whole numbers and put (+) sign before the product value.

For example: $(-10) \times (-14) = 140$, $(-5) \times (-6) = 30$

In general, for two positive integers **'a'** and **'b'**

$$(-a) \times (-b) = a \times b$$

Do and learn : ◆

      Find the following products:

      (i) $(-12) \times (-15)$ (ii) $(-25) \times (-4)$ (iii) $(-17) \times (-11)$

1.3.3 Product of Three or More Negative Integers

We have seen that the product of two negative integers is a positive integer. What will be the product of three or more negative integers? Let us see following examples:

(i) $(-2) \times (-3) = 6$

(ii) $(-2) \times (-3) \times (-4) = [(-2) \times (-3)] \times (-4) = (6) \times (-4) = -24$

(iii) $(-2) \times (-3) \times (-4) \times (-5) = [(-2) \times (-3)] \times [(-4) \times (-5)] = 6 \times 20 = 120$

(iv) $(-2) \times (-3) \times (-4) \times (-5) \times (-6) = [(-2) \times (-3)] \times [(-4) \times (-5)] \times (-6)$
$$= 6 \times 20 \times (-6) \quad = 120 \times (-6) = -720$$

We conclude from the above examples that

(i) The product of two negative integers is a positive integer.

(ii) The product of three negative integers is a negative integer.

(iii) The product of four negative integers is a positive integer.

(iv) What is the product of five negative integers?

(v) In continuation, what is the product of six negative integers?

From above examples, we conclude that if the number of negative integers is even (2,4,6,…) then their product is positive integer and if the number of negative integers is odd (1,3,5,…) then the result is a negative integer.

Do and learn ◆

Find the following products:

(i) $(-1) \times (-1) \times (-1) = $ ………..(ii) $(-1) \times (-1) \times (-1) \times (-1) = $ ………..

1.3.4 Multiplication by Zero

Observe the pattern given below and fill in the blanks:

$-4 \times 3 = -12$ $-4 \times 2 = -8 = -12 - (-4)$

$-4 \times 1 = -4 = -8 - (-4),$ $-4 \times 0 = 0 = -4 - (-4)$

We find that $-4 \times 0 = 0$. Make the pattern in similar manner and check.

Again $3 \times (-5) = -15$

             $2 \times (-5) = -10 = -15 - (-5)$

             $1 \times (-5) = -5 = -10 - (-5)$

             $0 \times (-5) = 0 = -5 - (-5)$

From above pattern, we can say that the product of any integer by zero gives zero.

In general, we can say that for any integer **a**, $\boxed{a \times 0 = 0 = 0 \times a}$

1.3.5 Division of Integers

We know that the division is the inverse of multiplication. For example $4 \times 5 = 20$, $20 \div 4 = 5$ or $20 \div 5 = 4$. Therefore we can say that there exists a division statement for every multiplication statement.

Multiplication Statement	Corresponding Division Statement
$3 \times (-5) = (-15)$	$(-15) \div (3) = -5$, $(-15) \div (-5) = 3$
$(-3) \times 4 = (-12)$	$(-12) \div (-3) = 4$, $(-12) \div 4 = -3$
$(-2) \times (-7) = 14$	$14 \div (-7) = -2$, ...
$(-4) \times 5 = (-20)$	$(-20) \ (-4) = 5$, ...
$5 \times (-9) = -45$	....
$(-6) \times 5 = $	....
$(+5) \times (+2) = $	....

Observe the division statements in the table and accordingly check the following statements and put the sign [$\checkmark$ or $\times$]:

(i)   Negative Integer $\div$ Positive Integer = Negative Integer   (   )

(ii)   Positive Integer $\div$ Negative Integer = Negative Integer   (   )

(iii) Positive Integer $\div$ Positive Integer = Positive Integer   (   )

(iv)   Negative Integer $\div$ Negative Integer = Positive Integer   (   )

Division of integers is carried out in the same manner as in the case of whole numbers. The only point we need to take care whether the result is positive or negative.

In general, **$a \div (-b) = (-a) \div (b)$** (where **b** and **-b** are not zero).

Exercise 1.2

1.   Find the product of following:

   (i) $(-3) \times 4$ (ii) $(-1) \times 24$

   (iii) $(-30) \times (-24)$ (iv) $(-214) \times 0$

   (v) $(-15) \times (-7) \times 6$ (vi) $(-5) \times (-7) \times (-4)$

   (vii) $(-3) \times (-2) \times (-1) \times (-5)$

2.   Start with $(-1) \times 5$ and make pattern to show that $(-1) \times (-1) = +1$.

3.   The rate of decrease in temperature in a fridge is 3°C per minute. A thing whose temperature is 25°C is placed in the fridge. After how much time the temperature of the thing will be -2°C.

4.   In a game if a blue card is chosen then 2 balls are to be given and if a red card is chosen then 3 balls are given. Sheetal has 27 balls. 9 blue cards are chosen in a row in the game. Show that how many balls are with her.

5. Solve the following division problems:
    (i) $(-35) \div 7$ (ii) $15 \div (-3)$
    (iii) $-25 \div (-25)$ (iv) $25 \div (-1)$
    (v) $0 \div (-3)$ (vi) $15 \div [(-2) + 1]$
    (vii) $[(-6) + 3] \div [(-2) + 1]$

6. A shopkeeper earns a profit of Rs.1 by selling a pen and loses 50 Paisa by selling a pencil. Represent the profit and loss in terms of integers.

(i) There is a loss of Rs. 5 in a month. If he had sold 45 pen then find the number of pencils sold in that month.

(ii) There is no profit and no loss in the second month. If he had sold 70 pen the find the number of pencils sold.

7. Fill in the following table by multiplying the integers:

x	2	3	- 4	- 2	1
3					
-2					
-1					
4					
2					

8. If going up a 60 feet high multi-story building by a lift is represented by positive integers then

(i) How will we show the height of flat at 60 feet?

(ii) Represent the parking at 15 feet below in integers.

(iii) If lift goes up at a speed of 5 feet per second then it is represented by +5 and if travels in a opposite direction then what is the integer representing downward direction?

1.4   Properties of Multiplication of Integers

1.4.1 Closure under Multiplication

Integer -1	Integer -2	Product	Product an integer Yes/No
2	-3	-6	Integer
-3	4	-12	Integer
-2	-3		
5	4		
-5	3		

What do you see? Can you find any two integers whose product is not an integer?

No. Hence we can say that the product of two integers is again an integer, i.e., **Integers follow closure property under multiplication.**

1.4.2 Commutativity

We know that the product of whole numbers is commutative. Is the multiplication of integers also commutative?

Observe the following table and complete it:

Integer Pair	Product	Changing the order	Conclusion
5, -4	$5 \times (-4) = -20$	$-4 \times 5 = -20$	$5 \times -4 = -4 \times 5$
-10, 12	$(-10) \times 12 = ...$	$12 \times (-10) = ...$	
-3, -4	$(-3) \times (-4) = ...$		
-5, -7		$(-7) \times (-5) = ...$	
+8, -3	$(+8) \times (-3) = ...$		

What do you see? You will find that product of integers does not depend on their order. Hence, **multiplication of integers is commutative.** In general, for any two integers

$$a \times b = b \times a$$

1.4.3 Multiplicative Identity

We know that the multiplicative identity for whole numbers is 1. Check for the integers.

$$(-3) \times 1 = -3 \qquad\qquad 1 \times 5 = 5$$
$$(-4) \times 1 = \qquad\qquad 1 \times 8 =$$
$$1 \times (-5) = \qquad\qquad 3 \times 1 =$$
$$1 \times (-6) = \qquad\qquad 7 \times 1 =$$

This shows that 1 is multiplicative identity for integers. In general, for any integer

$$a \times 1 = a = 1 \times a$$

> What happens if an integer is multiplied by -1? $-3 \times (-1) = 3$
> $$3 \times -1 = -3$$
> $$-6 \times -1 = 6$$
> $$-1 \times 13 = -13$$
> Is -1 multiplicative identity for integers?

1.4.4 Associative property for Multiplication

Take 3, -4, -2.

$[3 \times (-4)] \times (-2)$
First multiply 3 and -4 and then multiply the product by (-2).
$= (-12) \times (-2) = 24$

Consider $3 \times [(-4) \times (-2)]$
First multiply (-4) and (-2) and then multiply the product by 3.
$= 3 \times (+8) = 24$

Hence, $[3 \times (-4)] \times (-2) = 3 \times [(-4) \times (-2)]$.

Take similar sets of three integers and repeat the above activity. Is the product affected by different sets of integers? In general, for any three integers a, b and c

$$(a \times b) \times c = a \times (b \times c)$$

1.4.5 Distributive Property

We have seen distributive property for whole numbers

$$a \times (b + c) = a \times b + a \times c$$

Let us check if it is true for integers.

(i) $(-7) \times [2 + (-5)]$ $= (-7) \times (-3) = + 21$	$(-7) \times 2 + (-7) \times (-5)$ $= -14 + 35 = + 21$
(ii) $(-4) \times [(-3) + (-7)]$ $= (-4) \times (-10) = 40$	$(-4) \times (-3) + (-4) \times (-7)$ $= 12 + 28 = 40$
(iii) $(-8) \times [(-2) + (-1)]$ $= (-8) \times (-3) = 24$	$(-8) \times (-2) + (-8) \times (-1)$ $= +16 + 8 = 24$

Can we say that the distributive property (distribution of multiplication over addition) is also true for integers? Yes. In general,

$$a \times (b + c) = a \times b + a \times c$$

1.4.6 Division Property of Integers

Complete the following table:

Statement	Conclusion	Statement	Conclusion
$(-8) \div (-2) = 4$	Result is an Integer.	$(-8) \div 4 =$	....
$(-2) \div (-8) = \dfrac{-2}{-8}$	Result is not an Integer.	$3 \div (-8) = \dfrac{3}{-8}$	....

What do you see? We see that the **integers are not closed under division operation.** It is not necessary that division of two integers result in an integer.

Commutativity : We know that the division of whole numbers is not commutative. Let us check for integers. From above table, we can observe.

$$(-8) \div (-2) \neq (-2) \div (-8)$$

Is $[(-6) \div 2]$ and $[2 \div (-6)]$ same?

Hence, we can say that **division is not commutative for integers.**

Exercise 1.3

1. Following are the properties of multiplication of integers and opposite to them are the examples. Match the correct pair.

(i) $(-4) \times 5 = 5 \times (-4)$ (a) Associativity
(ii) $(-4) \times [(-3) + (-2)] = (-4) \times (-3) + (-4) \times (-2)$ (b) Commutativity
(iii) -4 (an integer), $+7$ (another integer), product $(-4) \times (+7) = (-28)$ (an integer) (c) Distributive
(iv) $(-4) \times [(-7) \times (5)] = [(-4) \times (-7)] \times (5)$ (d) Closure

2. Fill in the blanks keeping in view the properties of multiplication of integers:

(i) $26 \times (-48) = (-48) \times$ ....   Commutative
(ii) $(-6) \times [(-2) + (-1)] = (-6) \times (-2) + (-6)$ ....Distributive property
(iii) $100 \times [(-4) \times (-52)] = [100 \times$ ....$] \times (-52)$ Associativity

3.   Find the product by using appropriate property.
(i) $26 \times (-48) + (-48) \times (-56)$ (ii) $8 \times 78 \times (-125)$
(iii) $9 \times (50 - 2)$ (iv) 999×45
4.   Identify True/False. Correct the false statements and write.
(i)   Multiplication of integers is closed.
(ii)   Division of integers is closed.
(iii)   Division of integers is not commutative but multiplication is commutative.
(iv)   Multiplication of integer is distributive over addition.
(v)   Division of integers is distributive over subtraction.

⊨ We Learnt ⊨

1.   Integers can be considered as huge collection of numbers which contains whole numbers and their negatives.
2.   Sum of two positive integers is again a positive integer and sum of two negative integers is again a negative integer.
3.   We have studied the properties satisfied by sum and difference.
(i)   Addition and subtraction of integers is closed, i.e., **a + b** and **a − b** both are integers, where **'a'** and **'b'** are any integers.
(ii)   Addition is commutative for integers, i.e., for all integers **'a'** and **'b'**, **a + b = b + a.**
(iii)   Addition is associative for integers, i.e., for all integers **'a'**, **'b'** and **'c'**, **(a + b) + c = (a + b) + c.**
(iv)   Zero is the additive identity for integers. In addition of two integers of opposite sign we subtract their absolute values. Result will be positive if the absolute value of positive integer is more and result will be negative if the absolute value of negative integer is more.
4.   We have also learnt the product of integers. We have seen that the product of a positive integer and a negative integer is a negative integer and the product of two negative integers is a positive integer.
5.   Following are the properties of integer multiplication:
(I)   Integers are closed under multiplication. If **'a'** and **'b'** are integers then a× b is also an integer.
(ii)   Multiplication of integers is commutative. If **'a'** and **'b'** are integers then $a \times b = b \times a$ hold true.
(iii)   Integer 1 is multiplicative identity, i.e., for any integer **'a'**, $a \times 1 = 1 \times a$
(iv)   Multiplication of integers is associative, i.e., for any three integers $(a \times b) \times c = a \times (b \times c)$ holds true.
6.   Integers exhibit distributive property for addition and multiplication, i.e., for any three integers a, b and c, $[a \times (b + c) = a \times b + a \times c]$ holds true.
7.   Commutativity, associativity and distributive properties under addition and multiplication make our calculations easy.
8.   We have also learnt the division of integers. We found that (a) result obtained by dividing a negative integer by a positive integer or a positive integer by a negative integer is negative. (b) result obtained by dividing a negative integer by a negative integer is positive.
9.   For any integer **'a'**, we find that (i) $a \div 0$ is undefined and (ii) $a \div 1 = a$.

FRACTIONS AND DECIMAL NUMBERS

2.1 You have studied about fractions and decimal numbers in previous classes. Classify the proper and improper fractions from the following.

$$\frac{5}{3}, \frac{6}{11}, \frac{1}{5}, \frac{3}{2}, \frac{3}{7}, \frac{11}{12}, \frac{25}{2}$$

Convert improper fraction so identified into mixed fraction.

You have learnt writing equivalent fraction, addition & subtraction of fractions. Let us refresh them.

Example 1 Write three equivalent fraction of $\frac{2}{5}$.

Solution Equivalent fraction of $\frac{2}{5} = \frac{2 \times 2}{5 \times 2} = \frac{4}{10}$ and $\frac{2 \times 3}{5 \times 3} = \frac{6}{15}, \frac{2 \times 4}{5 \times 4} = \frac{8}{20}$

Answer : $\frac{2}{5} = \frac{4}{10} = \frac{6}{15} = \frac{8}{20}$.

Example 2 Ramesh eats $\frac{4}{5}$ part of the bread and Suresh eats $\frac{5}{7}$ part of the bread. Who eats more bread?

Solution We use equivalent fraction and find the greater fraction in $\frac{4}{5}$ and $\frac{5}{7}$

Equivalent fraction of $\frac{4}{5} = \frac{4 \times 7}{5 \times 7} = \frac{28}{35}$

Equivalent fraction of $\frac{5}{7} = \frac{5 \times 5}{7 \times 5} = \frac{25}{35}$

$\begin{cases} \text{L. C. M. of denominator 5 and 7} \\ = 5 \times 7 = 35 \\ \text{i.e., denominator of equivalent} \\ \text{fraction should be 35} \end{cases}$

Clearly $\frac{28}{35} > \frac{25}{35}$

In simple form

$$\frac{4}{5} > \frac{5}{7}$$, i.e., Ramesh eats more than Suresh.

Do and learn

1. Find five equivalent fractions of $\frac{4}{7}$.

2. Compare and write $(<, >, =)$

(i) $\frac{3}{4} \ \square \ \frac{3}{7}$ (ii) $\frac{2}{5} \ \square \ \frac{3}{8}$ (iii) $\frac{5}{9} \ \square \ \frac{15}{27}$.

You have learnt in previous classes addition and subtraction of fraction. Let us refresh.

Example 3 Raman's home is $\frac{4}{5}$ Km away from school and his aunt's home is $\frac{2}{3}$ Km away from school. Raman wants to go to his aunt's home today. How much distance will he travel in going from home to school and then to aunt's home?

Solution Distance of Raman's home from school $= \frac{4}{5}$ Km

Distance of aunt's home from school $= \frac{2}{3}$ Km

Total distance travelled $= \frac{4}{5} + \frac{2}{3}$

$$= \frac{(4 \times 3) + (2 \times 5)}{15} \qquad \text{(L.C.M. (5,3)=15)}$$

$$= \frac{12 + 10}{15}$$

$$= \frac{22}{15}$$

$$= 1\frac{7}{15} \text{ Km.}$$

Example 4 Dinesh studies for $3\frac{3}{4}$ hours every day after school hours. During this time he studies Science and Mathematics subjects for $1\frac{7}{8}$ Hours. He gives remaining time to other subjects. Find the time of studying other subjects.

Solution Total time of study of Dinesh $= 3\frac{3}{4}$ Hours

Time given to Science and Mathematics $= 1\frac{7}{8}$ Hours

Remaining time $= 3\frac{3}{4} - 1\frac{7}{8}$

$$= \frac{15}{4} - \frac{15}{8}$$

$$= \frac{(15 \times 2) - (15 \times 1)}{8} \qquad \text{(L.C.M. (4,8)=8)}$$

$$= \frac{30 - 15}{8}$$

$$= \frac{15}{8}$$

$= 1\frac{7}{8}$ Hours, i.e., Dinesh studies other subjects for $1\frac{7}{8}$ Hours.

Exercise 2.1

1. Find five equivalent fractions of each of the following:

(i) $\frac{2}{8}$ (ii) $\frac{6}{7}$ (iii) $\frac{7}{4}$ (iv) $\frac{100}{45}$

2. Use >, < and = sign for comparison of the following:

   (i) $\frac{3}{7}$ .... $\frac{2}{5}$ (ii) $\frac{6}{8}$ ..... . $\frac{12}{16}$

   (iii) $\frac{11}{15}$ ..... $\frac{12}{17}$ (iv) $\frac{3}{9}$ ..... . $\frac{15}{40}$

3. Arrange the following in increasing order

   (i) $\frac{1}{5}, \frac{3}{7}, \frac{7}{10}$ (ii)) $\frac{2}{9}, \frac{2}{3}, \frac{8}{21}$

4. Solve:

   (i) $2 + \frac{3}{5}$ (ii) $4 + \frac{7}{8}$ (iii) $\frac{3}{5} + \frac{2}{7}$

   (iv) $8\frac{1}{2} - 3\frac{5}{8}$ (v) $2\frac{2}{3} + 3\frac{1}{2}$ (vi) $\frac{7}{10} + \frac{2}{5} + \frac{3}{2}$

5. A rectangular photo whose length $2\frac{2}{3}$ inch and breadth $\frac{7}{6}$ inch. Find its perimeter.

6. Sheela took $3\frac{3}{5}$ Hours in whitewashing a shop and Neela completed whitewashing of similar shop in $3\frac{5}{7}$ Hours. Who took more time and how much?

7. Distributing the birthday cake among Reena, Teena and Meena; Reena was given $\frac{2}{5}$ part and Teena given $\frac{1}{3}$ part and remaining part was given to Meena. Find the Meena's share.

2.2   Product of Fractions

We know that the area of a rectangle = Length × Breadth. But if length and breadth are given in fractions then how will you calculate the area? Do you agree with this fact that we should know how the fractions are multiplied?

2.2.1 Multiplication of Fraction by a Whole Number
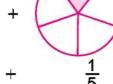
If we want to multiply the fraction $\frac{1}{5}$ by 3 then we add $\frac{1}{5}$ three times.
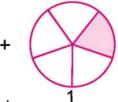
$$3 \times \frac{1}{5} = \frac{1}{5} + \frac{1}{5} + \frac{1}{5} = \frac{1+1+1}{5} = \frac{3 \times 1}{5} = \frac{3}{5}$$
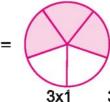
In graphical representation

> We know that multiplication means repeated addition. e.g. 3×4=4+4+4 =12
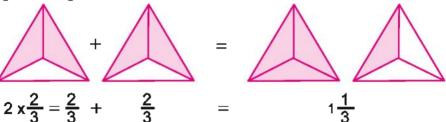
$3 \times \frac{1}{5} = \frac{1}{5}$ $+$ $\frac{1}{5}$ $+$ $\frac{1}{5}$ $=$ $\frac{3 \times 1}{5} = \frac{3}{5}$

Similarly

$$2 \times \frac{2}{3} = \frac{2}{3} + \frac{2}{3} = \frac{2+2}{3} = \frac{2 \times 2}{3} = \frac{4}{3} \text{ or } 1\frac{1}{3}$$

In graphical representation:

$$2 \times \frac{2}{3} = \frac{2}{3} + \qquad \frac{2}{3} \qquad = \qquad 1\frac{1}{3}$$
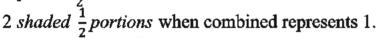
Similarly

$$\frac{2}{7} \times 3 = \frac{2 \times 3}{7} = \frac{6}{7}$$

Similarly for improper fraction, we have

$$4 \times \frac{5}{3} = \frac{4 \times 5}{3} = \frac{20}{3} = 6\frac{2}{3}$$

Figure shows two similar rectangles.

Each shaded portion represents $\frac{1}{2}$ of the rectangle.

Therefore, both the shaded portions together represent $\frac{1}{2}$ of 2.

2 *shaded* $\frac{1}{2}$ *portions* when combined represents 1.

Thus, we see that $\frac{1}{2}$ portion of 2 is 1.

We can also denote this as $\quad 2 \times \frac{1}{2} = 1$

Hence, $\frac{1}{2}$ of $2 = 2 \times \frac{1}{2} = 1$.

Similarly look at the rectangles given aside.

Each shaded portion represents $\frac{1}{2}$ of 1.

Hence, three shaded portion together represent $\frac{1}{2}$ part of 3.

3 shaded $\frac{1}{2}$ portions when combined

represents $1\frac{1}{2}$, i.e., $\frac{3}{2}$.

Therefore, $\frac{1}{2}$ of 3 is $\frac{3}{2}$ and $3 \times \frac{1}{2} = \frac{3}{2}$

Thus, we say that **"of "** represents the multiplication.

Do and learn ◆

Solve: (i) $3 \times \dfrac{8}{7}$ (ii) $\dfrac{9}{7} \times 6$ (iii) $4 \times \dfrac{7}{5}$ (iv) $4 \times \dfrac{4}{9}$

If the fraction is in mixed form then

$$7\dfrac{1}{2} \times 5 = \dfrac{15}{2} \times 5 = \dfrac{15 \times 5}{2} = \dfrac{75}{2} = 37\dfrac{1}{2}$$

$$3 \times 2\dfrac{5}{6} = 3 \times \dfrac{17}{6} = \dfrac{3 \times 17}{3 \times 2} = \dfrac{17}{2} = 8\dfrac{1}{2}$$

Do and learn ◆

Solve: (i) $5 \times 2\dfrac{3}{7} = ?$ (ii) $1\dfrac{4}{9} \times 6 = ?$

Now, what is the $\dfrac{1}{2}$ of 10?

Ramesh said "5",

because $\dfrac{1}{2}$ of $10 = 10 \times \dfrac{1}{2} = \dfrac{10}{2} = 5$.

Do and learn ◆

Can you find the value of : (i) $\dfrac{1}{2}$ of 5   (ii) $\dfrac{1}{4}$ of 16   (iii) $\dfrac{2}{5}$ of 25
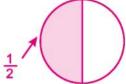
2.2.2 Multiplication of Fraction with Fraction

A tailor had 13 meters of cloth. In order to sew the cloth, he cuts the 13 meter cloth in 4 equal parts with each part now being $\dfrac{13}{4}$ meters in length. Now he divided one $\dfrac{13}{4}$ meters cloth into two equal parts from centre. Think, what will this one part out of the two parts represent?

This will represent $\dfrac{1}{2}$ of $\dfrac{13}{4}$, i.e., $\dfrac{13}{4} \times \dfrac{1}{2}$.
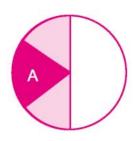
Let us understand the product by taking simple example before we solve it.

$\dfrac{1}{2} \times \dfrac{1}{3}$ means $\dfrac{1}{2}$ of $\dfrac{1}{3}$.

(i) Hence, we first find the $\dfrac{1}{2}$ of the total.
Figure aside shows $\dfrac{1}{2}$.

(ii) Now, how will you find $\dfrac{1}{3}$ of this shaded portion. Dividing the shaded portion ($\dfrac{1}{2}$ portion) in to three equal parts and taking one of them will represent $\dfrac{1}{3}$ of $\dfrac{1}{2}$. We know that $\dfrac{1}{3}$ of $\dfrac{1}{2} = \dfrac{1}{2} \times \dfrac{1}{3}$.

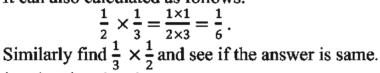In figure aside, part A represents $\frac{1}{3}$ of $\frac{1}{2}$.

(iii) How much is the part A of total? To find this we divide the unshaded portion in to the portions equal to A. This way we divide the entire unit in to six equal parts and the portion A is the sixth part of it. Hence,

$$\text{Part A} = \frac{1}{6}$$

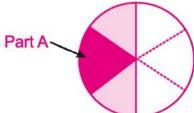Therefore, $\frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$

It can also calculated as follows:

$$\frac{1}{2} \times \frac{1}{3} = \frac{1 \times 1}{2 \times 3} = \frac{1}{6}.$$

Similarly find $\frac{1}{3} \times \frac{1}{2}$ and see if the answer is same.

$$\frac{1}{2} \times \frac{1}{3} = \frac{1}{3} \times \frac{1}{2} = \frac{1}{6}.$$

Similarly, $\frac{1}{3} \times \frac{1}{4} = \frac{1}{4} \times \frac{1}{3}$ and $\frac{1}{2} \times \frac{1}{5} = \frac{1}{5} \times \frac{1}{2}$.

Do and learn ◆

Find: (i) $\frac{1}{3} \times \frac{1}{7} = \frac{1 \times 1}{3 \times 7} = \boxed{}$ (ii) $\frac{3}{2} \times \frac{4}{7} = \boxed{} = \boxed{}$

          (iii) $\frac{1}{7} \times \frac{1}{5} = \frac{1 \times 1}{7 \times 5} = \boxed{}$ (iv) $\frac{3}{5} \times \frac{2}{3} = \boxed{} = \boxed{}$

Value of product of fractions

You have seen the product of two natural numbers is greater than or equal to both the numbers. Does this happen in case of fractions also? Let us see.

(i) Product of Proper Fractions

Complete the table:

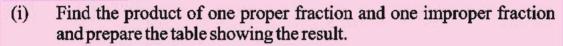|			
$\frac{1}{3} \times \frac{2}{5} = \frac{2}{15}$	$\frac{2}{15} < \frac{1}{3}$	$\frac{2}{15} < \frac{2}{5}$	**Product is less than each fraction**
$\frac{1}{5} \times \frac{2}{7} =$	....	....	....
$\frac{3}{5} \times \frac{7}{8} =$	....	....	....
$\frac{2}{5} \times \frac{4}{9} =$	....	....	....

After completing the table, do you agree that the product of two proper fractions is less than the given fractions?

(ii)   Let us find the product of two Improper Fractions

$\frac{7}{3} \times \frac{5}{2} = \frac{35}{6}$	$\frac{35}{6} > \frac{7}{3}$	$\frac{35}{6} > \frac{5}{2}$	**Product is greater than each fraction**
$\frac{6}{5} \times \frac{4}{3} =$	....	....	....
$\frac{9}{2} \times \frac{7}{4} =$	....	....	....
$\frac{3}{2} \times \frac{8}{7} =$	....	....	....

After completing the table, do you agree that the product of two improper fractions is more than the given fractions?

Do and learn ◆

(i)   Find the product of one proper fraction and one improper fraction and prepare the table showing the result.

Dheeraj has Rs. 25. If he spends $\frac{2}{5}$ of his money in buying notebook and pen then how much money has he spent?

We know that "of" represent the product. Therefore Dheeraj spent the money in buying the notebook and pen

$\frac{2}{5}$ of $25 = 25 \times \frac{2}{5} = \frac{25 \times 2}{5} = 5 \times 2 = $ Rs. 10.

Money left with Dheeraj $= 25 - 10 = 15$. Find, how much part of 25 this money is.

Example 5 $\frac{1}{5}$ of the total student in a class of 30 like studying English and $\frac{2}{5}$ of total like studying Mathematics and remaining like studying Science.

(i)   How many students like studying English?

(ii) How many students like studying Mathematics?

(iii) What fraction of total students like studying Science?

Solution   Total number of students in the class $= 30$

(i) $\frac{1}{5}$ of total like studying English.

Hence, the number of students who like studying English $= \frac{1}{5}$ of $30 = 30 \times \frac{1}{5} = 6$.

(i) Number of students who like studying Mathematics $= \frac{2}{5}$ of $30 = 30 \times \frac{2}{5} = 12$.

(ii) Number of students who like studying English and Mathematics $= 6 + 12 = 18$.
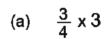
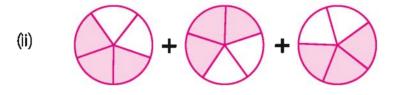Hence, the number of students who like Science $= 30 - 18 = 12$.

Thus, required fraction is $= \frac{12}{30}$, i.e., $\frac{2}{5}$ part likes studying Science.

Exercise 2.2

1. Match appropriate product with line diagrams :

(i)   (a) $\frac{3}{4} \times 3$

(ii)   (b) $\frac{1}{4} \times 2$

(iii)   (c) $\frac{3}{5} \times 3$

2. Show the following figures in terms of multiplication (repeated addition):

(i) $= \frac{1}{3} \times 2 =$ $=$ ....

(ii) $=$ .... $=$ ....

(iii)

$=$ .... $=$ $=$ ....
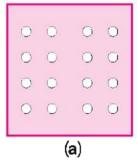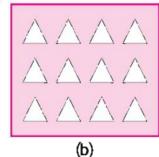
3. Multiply and express in the simplest form:
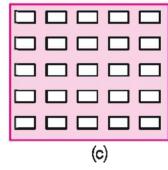
    (i) $8 \times \dfrac{3}{5}$ (ii) $\dfrac{2}{3} \times 4$ (iii) $\dfrac{5}{2} \times 6$ (iv) $15 \times \dfrac{3}{5}$ (v) $20 \times \dfrac{2}{3}$

    (vi) $18 \times \dfrac{1}{9}$ (vii) $2\dfrac{2}{3} \times \dfrac{6}{7}$ (viii) $12 \times \dfrac{5}{3}$ (ix) $\dfrac{3}{8} \times \dfrac{6}{4}$ (x) $\dfrac{4}{5} \times \dfrac{12}{7}$

4. Shade the following:

    (i)   Fill colour in $\dfrac{1}{2}$ of the circles in box (a)

    (ii)   Fill colour in $\dfrac{2}{3}$ of the triangles in box (b)

    (iii) Fill colour in $\dfrac{1}{5}$ of the rectangles in box (c)

(a)   (b)   (c)

5. Find the following:

    (i) $\dfrac{1}{3}$ of 27   (ii) $\dfrac{1}{3}$ of 18   (iii) $\dfrac{1}{5}$ of 50   (iv) $\dfrac{3}{4}$ of 24   (v) $\dfrac{5}{4}$ of 32   (vi) $\dfrac{3}{7}$ of 28

6. Find

    (i) $1\dfrac{3}{5}$ of 4   (ii) $\dfrac{2}{3}$ of $5\dfrac{1}{5}$ (iii) $\dfrac{8}{17}$ of $3\dfrac{2}{5}$ (iv) $\dfrac{3}{8}$ of $9\dfrac{2}{3}$ (v) $\dfrac{1}{5}$ of $\dfrac{3}{5}$ (vi) $\dfrac{1}{7}$ of $\dfrac{3}{10}$

7. Multiply the following fractions:

    (i) $3\dfrac{4}{5} \times \dfrac{1}{4}$ (ii) $\dfrac{3}{2} \times 6\dfrac{2}{5}$ (iii) $3\dfrac{4}{7} \times \dfrac{3}{5}$ (iv) $3\dfrac{2}{5} \times 4\dfrac{3}{8}$

8. Which is greater?

    (i) $\dfrac{2}{5}$ of $\dfrac{3}{4}$ or $\dfrac{3}{5}$ of $\dfrac{5}{8}$ (ii) $\dfrac{1}{2}$ of $\dfrac{6}{7}$ or $\dfrac{2}{3}$ of $\dfrac{3}{7}$

9. Manisha took 15 Litres Milk for selling it out. She sold $\dfrac{2}{5}$ part of milk to Kanchan and $\dfrac{1}{5}$ part of milk to Bhawna and remaining part to a hotel. Find

    (i)   How much milk she sold to Kanchan?

    (ii)   How much milk she sold to Bhawna?

    (iii) How much milk she sold to hotel?

10. 7 boys were placed each $\dfrac{3}{4}$ meters apart from the other for PT demonstration on Independence Day. What is the distance between the first and the last boy?

11. Rahul works on a painting $2\frac{3}{4}$ Hours daily. If he takes 8 days to complete this painting then calculate the number of hours he worked.

12. A car travel $23\frac{1}{5}$ Km using 1 Litre petrol. What distance will it travel using $2\frac{3}{4}$ Litre petrol?

13. (i) Write the number in the box so that $\frac{3}{4} \times \boxed{} = \frac{6}{40}$.

    (ii) The simplest form of number in the box is ………….

14. (i) Write the number in the box so that $\boxed{} \times \frac{5}{8} = \frac{10}{24}$.

    (ii) The simplest form of number in the box of (i) is ………….

2.3 Division of Fractions

Sumit has a paper strip of length 8 cm. He cuts this strip into 2 cm long small strips. We know that he will get $8 \div 2 = 4$ strips. If he cuts this strip in to small strips of length $\frac{3}{2}$ cm then how many strips will he get? He will get $8 \div \frac{3}{2}$ strips. Similarly a strip of length $\frac{15}{4}$ cm can be cut in to small strips of length $\frac{3}{2}$ cm. We will get $\frac{15}{4} \div \frac{3}{2}$ pieces.
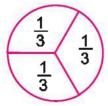
Therefore, we need to divide a whole number by a fraction and a fraction by another fraction. Let us learn this.
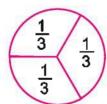
2.3.1 Dividing a Whole Number by a Fraction
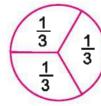
We find $1 \div \frac{1}{3}$. It means, how many times $\frac{1}{3}$ appears in 1? How many $\frac{1}{3}$ are seen in the figure aside.
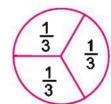
There are three such parts in '1' so that $1 \div \frac{1}{3} = 3$.

Similarly, $4 \div \frac{1}{3} =$ number of $\frac{1}{3}$ part obtained by dividing four such circles into similar $\frac{1}{3}$ parts $= 12$.

i.e., $4 \div \frac{1}{3} = 12$ such that $4 \div \frac{1}{3} = 4 \times \frac{3}{1} = 12$.

Find, using similar figures, $2 \div \frac{1}{5}$ and $5 \div \frac{1}{2}$.

2.3.2 Inverse of Fractions

Interchanging the numerator and denominators in $\frac{1}{3}$ we get $\frac{3}{1}$. Similarly

Interchange the numerator and denominator in $\frac{1}{5}$ and $\frac{2}{3}$.

$$\frac{1}{3} \times \frac{3}{1} = 1, \qquad\qquad \frac{1}{5} \times \frac{5}{1} = \ldots\ldots\ldots \qquad\qquad \frac{2}{3} \times \frac{3}{2} = \ldots\ldots\ldots$$

Two non-zero numbers ($\neq 0$) whose mutual product is 1 are known as reciprocal numbers of each other.

We have seen that

$$1 \div \frac{1}{3} = 1 \times \frac{3}{1} = 1 \times \left(\text{reciprocal of } \frac{1}{3}\right)$$

$$4 \div \frac{1}{3} = 4 \times \frac{3}{1} = 4 \times \left(\text{reciprocal of } \frac{1}{3}\right)$$

$$5 \div 1\frac{1}{2} = 5 \div \frac{3}{2} = 5 \times \frac{2}{3} = 5 \times \left(\text{reciprocal of } \frac{3}{2}\right)$$

$$2 \div \frac{3}{4} = 2 \ldots\ldots\ldots\ldots = \ldots\ldots\ldots\ldots$$

For dividing a whole number by a fraction, multiply the whole number by the inverse of given fraction.

Do and learn ◆

Solve   (i) $5 \div \frac{2}{3}$ (ii) $7 \div \frac{3}{4}$ (iii) $6 \div \frac{1}{5}$.

2.3.3 Dividing a Fraction by a Whole Number

What will be the value of $\frac{3}{5} \div 4$.

We can write this in the following manner

> Division of a number by 1 gives the same number again.

$$\frac{3}{5} \div \frac{4}{1} = \frac{3}{5} \times \frac{1}{4} = \frac{3}{5} \times \left(\text{reciprocal of } \frac{4}{1}\right) = \frac{3}{20}.$$

Similarly, $3\frac{2}{3} \div 5 = \frac{11}{3} \div \frac{5}{1} = \frac{11}{3} \times \frac{1}{5} = \frac{11}{15}$.

Do and learn: ◆ Fill in the blanks

(i) $2\frac{3}{5} \div 2 = \frac{13}{5} \div 2 = \ldots\ldots\ldots\ldots$ (ii) $\frac{8}{3} \div 5 = \ldots\ldots\ldots\ldots = \ldots\ldots\ldots$

(iii) $2\frac{2}{3} \div 3 = \ldots\ldots\ldots\ldots = \ldots\ldots\ldots$

2.3.4 Division of a Fraction by Another Fraction

$$\frac{1}{2} \div \frac{3}{5} = \frac{1}{2} \times \left(\text{reciprocal of } \frac{3}{5}\right) = \frac{1}{2} \times \frac{5}{3} = \frac{5}{6}.$$

Similarly, $2\frac{1}{3} \div 1\frac{1}{4} = \frac{7}{3} \div \frac{5}{4} = ?$

Do and learn: ◆

Solve   (i) $\frac{3}{5} \div \frac{1}{2}$ (ii) $2\frac{1}{2} \div \frac{3}{5}$ (iii) $5\frac{1}{6} \div \frac{9}{2}$

>∞◦◦── **Exercise 2.3** ──◦◦∞<

1. Find
   (i) $12 \div \frac{2}{3}$ (ii) $5 \div 3\frac{4}{7}$ (iii) $3 \div 1\frac{1}{3}$
   (iv) $4 \div \frac{8}{3}$ (v) $6 \div \frac{2}{3}$ (vi) $15 \div \frac{5}{7}$

2. Find the reciprocal of each of the following
   (i) $\frac{3}{7}$ (ii) $\frac{1}{8}$ (iii) $\frac{12}{7}$ (iv) $\frac{5}{8}$ (v) $\frac{9}{7}$

3. Find
   (i) $\frac{3}{7} \div 2$ (ii) $4\frac{3}{7} \div 7$ (iii) $\frac{6}{13} \div 5$
   (iv) $3\frac{1}{2} \div 4$ (v) $\frac{6}{5} \div 3$ (vi) $\frac{7}{3} \div 4$

4. Find
   (i) $\frac{7}{3} \div \frac{8}{7}$ (ii) $2\frac{1}{5} \div \frac{3}{5}$ (v) $\frac{2}{5} \div 1\frac{1}{2}$
   (iv) $3\frac{1}{5} \div 1\frac{1}{5}$ (v) $3\frac{1}{5} \div 2\frac{1}{3}$ (vi) $\frac{3}{5} \div \frac{5}{7}$

5. What will be number of $\frac{1}{4}$ parts of bread if each of 6 breads is divided into equal parts of $\frac{1}{4}$?

6. How many $\frac{1}{2}$ cm long pieces can be cut out of $11\frac{1}{2}$ cm long ribbon?

2.4   Review of Decimal Numbers

We have studied about decimal numbers in previous classes. Let us refresh them. How will we read following numbers?

(i)   24.2   = Twenty Four Decimal Two OR Twenty Four Point Two

(ii)   2.04   = Two Decimal Zero Four OR Two Point Zero Four

(iii)   325.52 = ...

(iv)   56.32   = ...

Study the following table and complete it:

Hundreds (100)	Tens (10)	Ones (1)	One Tenth $\left(\frac{1}{10}\right)$	One Hundredth $\left(\frac{1}{100}\right)$	One Thousandth $\left(\frac{1}{1000}\right)$	Number
4	2	1	2	5	8	421.258
6	0	8	5	0	7	608.507
---	0	3	2	1	0	303.210
8	---	6	---	7	0	876.170
7	8	---	---	3	---	784.035
0	1	2	3	4	5	---

We can also write these numbers in expanded forms as follows:

$$421.258 = 4 \times 100 + 2 \times 10 + 1 \times 1 + 2 \times \frac{1}{10} + 5 \times \frac{1}{100} + 8 \times \frac{1}{1000}$$

$5 \times \frac{1}{100} = \frac{5}{100}$ is the place value of 5 in the given number.

Similarly, we can write the remaining numbers given in the table.

2.4.1 Comparison, Addition and Subtraction of Decimal Numbers

Distance of a city A from city B is 38.750 Km and from city C is 38.075. From which city the distance of city A is more?

(i)   Number on the left of decimal are same. Therefore, we compare the digits on right hand side.

(ii)   Comparing the number by starting from One tenth place and moving to the right of the decimal we find that 7 > 0. Hence, 38.750 > 38.075.

Therefore, the distance of city A from city B is greater.

Do and learn: ◆   Which is the smaller number?
   (i) 35.37 or 35.07   (ii) 262.327 or 262.372

We use decimal numbers to convert the smaller units of Currency, length and weight in to bigger units. For example:

27 gm = $\frac{27}{1000}$ kg = 0.027 kg

550 Paisa = $\frac{550}{100}$ = Rs. 5.50

1 m 25 cm = 1m + $\frac{25}{100}$ m = 1.25 m

120 m = $\frac{120}{1000}$ km = ………….. km.

1 kg = 1000 gm
Rs. 1 = 100 Paisa
1 m   = 100 cm
1 km = 1000 m

Example 6 Ghisu puts 12 kg 400 gm Guava in a basket and 6 kg 750 gm Blackberry in another basket. How much weight will he carry while going to city?

Solution Weight of Guava in the basket = 12 kg 400 gm = 12.400 kg

Weight of Blackberry in the basket = 6 kg 750 gm = 6.750 kg

Total Weight = 19.150 kg.

Example 7 Durga and Vimla purchased 5 m 25 cm cloth for Salwar Suit. If 2 m 75 cm of cloth is required for Salwar Suit of Durga then how much cloth is left for the suit of Vimla?

Solution Total cloth purchased = 5 m 25 cm = 5.25 m

Cloth used for Durga = 2 m 75 cm = 2.75 m

Cloth left for Vimla = 5.25 – 2.75 = 2.50 m.

$\longmapsto\!\!\infty$ **Exercise 2.4** $\infty\!\!\longleftarrow$

1.   Compare the following pairs of numbers and identity the greater one:

(i)   0.7 and 0.07   (ii) 2.03 and 2.30   (iii) 7 and 0.7

(iv) 1.35 and 1.49   (v) 3.507 and 3.570   (vi) 85.2 and 85.02

2.   Covert the following small units in to bigger units:

(i) 7 paise to Rupees   (ii) 800 gm to Kg   (iii) 75 Meter to Km

(iv) 3470 Meter to Km   (v) 7 Kg 7 g to Kg

(vi) 47 Km 75 Meter to Km

3.   Write the following numbers in expanded form.

(i) 25.03   (ii) 2.503   (iii) 205.3   (iv) 2.053

4.   Find the place value of 3 in the following numbers:

(i) 34.82   (ii) 643.45   (iii) 547.03   (iv) 24.203

5.   Paras's father brought 7 kg 250 gm Green Chili, 15 kg 750 gm Tomatoes and 950 gm Green Coriander Leaves from the vegetable market. How much vegetable did he bring?

6.   Bhawna got Rs. 37.25 in her bank account towards interest and Anita got Rs. 25.50 in her bank account towards interest. Who got more interest amount and how much?

7.   How less 42.7 km is from 48 km?

8.   What value should be added to the sum of 24.57 and 36.3 to get 70?

2.4.2 Multiplication of Decimal Numbers

Manoj got 2.5 Litres petrol filled in his car. If the cost of petrol is Rs. 66.25 per litre then how much payment Manoj has to make for petrol?
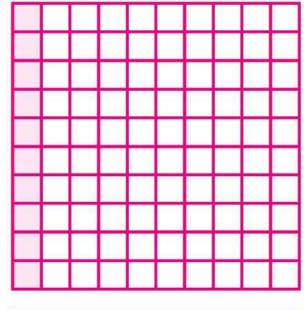
Here, both 66.25 and 2.5 are decimal numbers. Similarly we need to multiply decimal numbers in many cases. Let us learn the multiplication of two decimal numbers. First of all we find the value of 0.1×0.1. We know that

$$0.1 \times 0.1 = \frac{1}{10} \times \frac{1}{10} = \frac{1 \times 1}{100} = \frac{1}{100} = 0.01$$
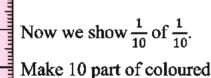
Look at its picture illustration.

$$0.1 \times 0.1 = \frac{1}{10} \times \frac{1}{10} = \frac{1}{10} \text{ of } \frac{1}{10}$$

Let us first show $\frac{1}{10}$ in the figure.

$\frac{1}{10}$

Now we show $\frac{1}{10}$ of $\frac{1}{10}$.

Make 10 part of coloured
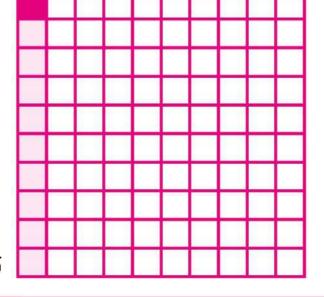
Portion and show one part of it.

$\frac{1}{10}$ of $\frac{1}{10}$

Hence, $\frac{1}{10} \times \frac{1}{10}$ or $\frac{1}{10}$ of $\frac{1}{10}$ shows $\frac{1}{100}$ of total unit, which is also written as 0.01.
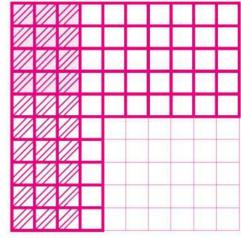
Thus, $0.1 \times 0.1 = 0.01$.

Similarly, $0.3 \times 0.4 = \frac{3}{10} \times \frac{4}{10}$ or $\frac{4}{10}$ of $\frac{3}{10}$.

Representing $\frac{3}{10} \times \frac{4}{10}$ by figure,

the shaded portion shows 12 cells of total 100 cells. Therefore :

$\frac{3}{10} \times \frac{4}{10} = \frac{12}{100}$ or $0.3 \times 0.4 = 0.12$.

This can also be done in the following manner. We first calculate $03 \times 04 = 12$ for 0.3×0.4 Then count the number of digits after decimals in the numbers to be multiplied and put a decimal in the product (here 12) after counting the same number of digits from the right hand side i.e. we will get 0.12.

Similarly, we will find $14 \times 2 = 28$ for 1.4×2 and put the decimal leaving one digit from the right of the resulting product. i.e., we get 2.8.

Do and learn: ◆

Find the value of (i) 2.3×3.5 (ii) 3.7×5 (iii) 2.4×7.35

Example 8 Ganeshi winnows (sorting) 7.5 kg Wheat every day. How much Wheat will she winnow in 10 days?

Solution Ganeshi winnows the Wheat in a day = 7.5 kg

She will winnow the Wheat in 10 days = 7.5×10

= 75.0 kg Ans.

Example 9 A rectangular photo frame has 2.25 m length and 1.5 m breadth. Find its area.

Solution Length of rectangular photo frame = 2.25 m

Breadth of the frame = 1.5 m

Area of the frame = Length breadth = 2.25×1.5

= 3.375 Square meter. Ans.

Also learn:

(i) 1.52 × 10   (ii) 1.52 × 100   (iii) 1.52 × 1000

Solution: (i) Similar to earlier activity, we have

$$152 \times 10 = 1520$$

Now, counting numbers after decimals

$$1.52 \times 10 = 15.20$$

(ii) Exactly in similar manner,

$$152 \times 100 = 15200$$

Now, counting numbers after decimals

$$1.52 \times 100 = 152.00$$

(iii) Similarly,

$$152 \times 1000 = 152000$$

$$1.52 \times 1000 = \dots\dots\dots.. \text{ Put decimals.}$$

> When a decimal number is multiplied by 10 or 100 or 1000, the decimal shifts towards right by the places equal to the number of zeros in 10 or 100 or 1000.

What do you conclude from above results? Are you satisfied with the pattern given in the box above?

Exercise 2.5

1. Find
    (i)   7 × 5.4   (ii)   80.1 × 2   (iii) 0.08 × 5
    (iv) 3 × 0.86   (v)   312.05 × 4   (vi)   6.08 × 8
2. Find
    (i)   3.72 × 10   (ii)   0.37 × 10   (iii) 0.5 × 10
    (iv)   1.08 × 100   (v)   73.8 × 10   (vi)   0.06 x 100
    (vii) 47.03 × 1000   (viii) 0.03 × 1000   (ix)   42.7 × 1000
3. Find
    (i)   4.2 × 3.5   (ii)   6.25 × 0.5   (iii) 11.2 × 0.15
    (iv)   0.08 × 0.5   (v)   101.01 × 0.01   (vi)   20.05 × 4.8
4. Find the area of a rectangle whose length is 6.4 cm and breadth is 3.2 cm.
5. A car covers a distance of 25.17 km in 1 Litre petrol. Find how much distance will it cover in 10.5 Litres petrol?
6. Prakash sells 2.500 kg of ghee to Raju every month. How much ghee Prakash would have sold to Raju in 10 months?
7. A side of an equilateral triangle is 4.5 cm. Find its perimeter.
8. Dipika buys a box of Tomatoes at the wholesale rate of Rs. 16.50 per kg from the vegetable market. If the Tomatoes weigh 22.5 kg then how much money will Dipika pay to the wholesaler?

2.5 Division of Decimal Numbers

Shakuntla has bought coloured strips each of length 8.5 cm for decorating her house. She wants to cut the strips in to the pieces of 1.7 cm for decoration. How many pieces can be cut from a strip?

For this we need to find $8.5 \div 1.7$. Let us try to learn with simple examples, how decimal numbers are divided?

2.5.1 Division of Decimal fraction by a Whole Number

Let us find $8.4 \div 2$. We know that 8.4 can be written as $\dfrac{84}{10}$ because the

expanded form of 8.4 is expressed as $\left(8 \times 1 + 4 \times \dfrac{1}{10}\right)$. Hence

$$8.4 \div 2 = \frac{84}{10} \div 2$$
$$= \frac{84}{10} \div \frac{2}{1}$$

We have learnt in division of fractions that we need to multiply by reciprocal of 2 for division.

$$= \frac{84}{10} \times \frac{1}{2}$$
$$= \frac{84 \times 1}{10 \times 2}$$
$$= \frac{42}{10} = 4.2$$

$$4.2 = 4 \times 1 + 2 \times \frac{1}{10}$$

$$4.2 = \frac{42}{10}$$

Also learn:

    (i) $45.32 \div 10$ (ii) $45.32 \div 100$ (iii) $73.25 \div 1000$

Solution: (i) $45.32 \div 10$

$$= \frac{4532}{100} \div \frac{10}{1}$$
$$= \frac{4532}{100} \times \frac{1}{10}$$
$$= \frac{4532}{1000} = 4.532$$

$$\left(\text{reciprocal of } \frac{10}{1} = \frac{1}{10}\right)$$

    (ii) $45.32 \div 100$

$$= \frac{4532}{100} \div \frac{100}{1}$$
$$= \frac{4532}{100} \times \frac{1}{100}$$
$$= \frac{4532}{10000}$$
$$= 0.4532$$

(iii)   73.25 ÷ 1000

$$= \frac{7325}{100} \div \frac{1000}{1}$$

$$= \frac{7325}{100} \times \frac{1}{1000}$$

$$= \frac{7325}{100000}$$

$$= 0.07325$$

Do you find any rule in the change of place of decimal in dividing decimal numbers by 10 or 100 or 1000?

Yes, digits in the number and the quotient (result) are same but **decimal displaces from its place towards left by the places equal to number of zeros attached with 1.**

Do and learn: ◆ Divide the given decimal numbers by 10, 100 and 1000.
(i) 132.4   (ii) 1.03   (iii) 40.033 (iv) 4.321

2.5.2   Division of any Whole Number by a Decimal Fraction

Let us see 32 ÷ 0.4

$$32 \div 0.4 = 32 \div \frac{4}{10} = 32 \times \frac{10}{4}$$

$$\left(\frac{4}{10} \text{ Reciprocal of } = \frac{10}{4} \right)$$

$$= 32 \times \frac{10}{4}$$

$$= \frac{(4 \times 8) \times 10}{4} = 8 \times 10 = 80 \text{ Ans.}$$

$$7 \div 1.6 = 7 \div \frac{16}{10} = 7 \times \frac{10}{16}$$

$$= 7 \times \frac{5}{8} = \frac{35}{8} = 4.375$$

Do and learn: ◆ Solve –
(i) 6 ÷ 1.2   (ii) 9 ÷ 4.5   (iii) 48 ÷ 0.8

2.5.3   Division of any Decimal Number by a Decimal Number

Consider 3.25 ÷ 0.5.

$$3.25 \div 0.5 = \frac{325}{100} \div \frac{5}{10}$$

$$= \frac{325}{100} \times \frac{10}{5} = \frac{325 \times 10}{100 \times 5} = \frac{65}{10} = 6.5 \text{ Ans.}$$

Similarly,

$$37.8 \div 0.14 = \frac{378}{10} \div \frac{14}{100} = \frac{378}{10} \times \frac{100}{14}$$

$$= \frac{378 \times 100}{10 \times 14} = 27 \times 10 = 270 \text{ Ans.}$$

Do and learn: ◆ Solve

(i) $7.75 \div 0.25$ (ii) $5.6 \div 1.4$ (iii) $42.8 \div 0.02$

Other interesting method:

$$2.73 \div 1.3 = \frac{2.73}{1.3}$$

$$= \frac{2.73}{1.30}$$

$$= \frac{273}{130}$$

$$= \frac{21}{10} = 2.1 \text{ Ans.}$$

$2.73 \div 1.3$ can be written as $\frac{2.73}{1.3}$
To make same number of digits after decimals, zeros can be placed after decimal and then decimal can be removed

(Leaving common factor 13)

Exercise 2.6

1.   Find

(i) $0.8 \div 4$ (ii) $0.42 \div 7$ (iii) $3.96 \div 6$ (iv) $842.4 \div 4$

(v) $14.49 \div 7$ (vi) $36 \div 0.2$ (vii) $7 \div 3.5$ (viii) $0.09 \div 3$

2.   Find

(i) $4.2 \div 10$ (ii) $98.6 \div 10$ (iii) $0.2 \div 10$

(iv) $143.2 \div 100$ (v) $86 \div 100$ (vi) $8.05 \div 100$

(vii) $44.32 \div 100$ (viii) $1.3 \div 1000$ (ix) $0.06 \div 1000$

3.   Find

(i) $1.2 \div 0.3$ (ii) $3.64 \div 0.4$ (iii) $9.6 \div 1.6$

(iv) $1.25 \div 2.5$ (v) $30.75 \div 1.5$ (vi) $4.08 \div 1.2$

(vii) $30.94 \div 0.7$ (viii) $76.5 \div 0.15$ (ix) $7.75 \div 0.25$

4.   A scooter covers a distance of 212.5 km in 5 Litres of petrol. How much distance will it cover in one Litre of petrol?

5.   The distances of houses of Gopal, Narayan and Krishna from school are 1.5 km, 0.7 km and 1.4 km respectively. Find the average of three distances.

$$\left(Average = \frac{Sum \ of \ Quantities}{Number \ of \ Quantities} \right)$$

6. A car covers 89.1 km distance in 2.2 Hours. Find the distance covered by the car in 1 Hour.

7. Find the area of the square whose perimeter is 44.08 m.

8. If the Area of a rectangle is 93.6 m and the width is 3.6 m. then find the perimeter of the rectangle.

Road Safety

Pedestrian must use Zebra Lines while crossing the road. It reduces the chances of any accident. Zebra lines are the rectangular strips made on road where vehicle drivers slow down the speed and go ahead. Along with this pedestrian use these lines during red signal and cross the road.

1. There are 8 black and 7 white lines at a zebra crossing. So tell what part of total strips is the number of white strips?

2. On one day 100 people crossed the road through zebra crossing of which there were 20 men, 30 women, 10 children and 40 students. Represent all data in decimal form.

We Learnt

1. We have studied about multiplication and division operations on fractions and decimal number in this chapter.

2. Product of fractions = $\dfrac{\text{Product of Numerators}}{\text{Product of Denomenators}}$

3. Product of two proper fractions is less than the fractions used for multiplication. Product of proper and improper fractions is greater than the proper fraction used for multiplication. Product of two improper fractions is greater than each of the fractions used for multiplication.

4. Mutual change of numerator and denominator in a fraction produces reciprocal fraction.

5. We have learnt how to divide fractions.

(i) Dividing a whole number by a fraction means multiplication of whole number by reciprocal fraction.

(ii) Dividing a fraction by a whole number means multiplication of fraction by reciprocal of whole number.

(iii) Dividing a fraction by another fraction means multiplication of a fraction by the reciprocal of another fraction.

6. When two decimal numbers are to be multiplied, we first multiply them as if they are whole numbers. Then we count the number of digits on the right of decimal and put decimal in the product after the total number of digits from the right.

7. While multiplying the decimal numbers by 10, 100, 1000 we shift ahead the decimal as many places as the numbers of zeros towards right.

8. We have also learnt the division of fractions.

(i) In dividing two decimal numbers we remove the decimals after equalizing the number of digits after decimal and then divide the number in usual manner.

(ii) In dividing decimal numbers by 10, 100, 1000 we shift the decimal towards left as many places as the number of zeros available on 1.

SQUARE AND SQUARE ROOT

3.1 Sonu and Dinu are making squares on a square sheet and writing their area by counting the squares.

Area of a square (Square 1) of size unity = 1 Square Unit

Area of a square of size 2 units = 4 Square Unit

Area of a square of size 3 units = 9 Square Units

You also make squares of 4, 5, 6, 7, 8, 9, 10 units and write their area by counting the squares.

Complete the table given below:

Side of Square	1	2	3	4	5	6				
Area of Square	1	4	9							

Table 3.1

What is special in the numbers 1, 4, 9, 16, 25, ………… and similar numbers?

Since we can express these numbers as $1 = 1 \times 1 = 1^2$; $4 = 2 \times 2 = 2^2$; $9 = 3 \times 3 = 3^2$, we find that these can be obtained by multiplying a number by itself. Such numbers are called as **Square Numbers.**

In general, for $s = r^2$, s is a square number. Is 24 a square number?

Think of following numbers and their squares and fill in the blanks:

Numbers	Square
1	$1 \times 1 = 1$
2	$2 \times 2 = 4$
3	$3 \times 3 = 9$
4	$4 \times 4 = 16$
5	$5 \times 5 = 25$
6	……………
7	……………
8	……………
9	……………
10	……………

Table 3.2

From the above table it is evident that there are only 10 numbers which are square between 1 to 100 which are square of a number and rest of the numbers are not squares.

Numbers 1, 4, 9, 16, 25, 36, 49, 64, 81 and 100 are square numbers and are called perfect square numbers.

Do and learn: ◆

Write the perfect square number lying between
(i) 20 and 30   (ii) 40 and 50

3.2 Properties of Square Numbers

Square numbers of the numbers from 1 to 20 are given in the table given below:

Numbers	Squares	Numbers	Squares
1	1	11	121
2	4	12	144
3	9	13	169
4	16	14	196
5	25	15	225
6	36	16	256
7	49	17	289
8	64	18	324
9	81	19	361
10	100	20	400

Table 3.3

Write the digits on ones place in square from above table in the form of set A:

A= {0, 1, 4, ...}

Write the numbers between 0 and 9 in B which are not in A:

B = {2, 3, ...}

On the basis of numbers in sets A and set B we can say that the numbers with 2, 3, 7, 8 lying at ones place cannot be square numbers.

Do and learn ◆
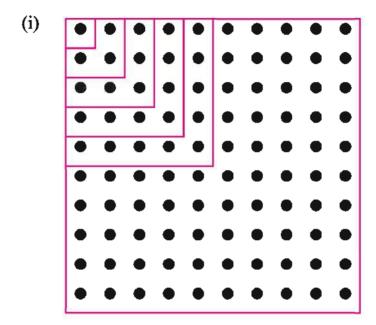
(1) Tell on the basis of digits at ones place, which numbers are not perfect square?
(i) 2304   (ii) 402   (iii) 3003   (iv) 100   (v) 1008
(2) Tell three numbers for which you can say with confidence that they cannot be perfect square.
(i) ....   (ii) ....   (iii) ....

Perfect squares of even and odd numbers given in table 3.3 are of which type?

Square of odd numbers – Even / Odd

Square of even numbers – Even / Odd

From the above activity, we can say that the **squares of even numbers are even and squares of odd numbers are odd**.

1. Interesting forms of square numbers:

(i)

In figure, squares are drawn starting from the North-West corner. Look at the squares carefully and write the number of black circles:

First square	1	=	1	=	1^2
Second square	1+3	=	4	=	2^2
Third square	1+3+5	=	9	=	3^2
Fourth square	1+3+5+7	=	…	=	…
Fifth square	1+3+5+7+9	=	…	=	…
Sixth square	……………..	=	…	=	…
Seventh square	……………..	=	…	=	…

We have seen that First square = First odd number $\quad= 1^2$

Second square = Sum of first two odd numbers $\quad= 2^2$

Third square = Sum of first three odd numbers $\quad= 3^2$

Proceeding in the similar fashion, we find that the sum of first eight odd numbers will be $= 8^2 = 64$.

2. Look at the squares of 1, 11, 111, ………
$$1^2 = 1$$
$$11^2 = 121$$
$$111^2 = 12321$$
$$1111^2 = 1234321$$
$$11111^2 = ...$$
$$111111^2 = ...$$

3. Write two adjacent numbers, like 4 and 5
   Square them $4^2 = 16$, $5^2 = 25$
   Difference of squares $= 25 - 16 = 9$
   Sum of the numbers $= 4 + 5 = 9$
   Write few more similar adjacent number.
   You will find that the difference of square of two adjacent numbers = sum of numbers

4. Pythagorean Triplets: $3^2 + 4^2$
   $$9 + 16 = 25 = (5)^2$$
   $$6^2 + 8^2$$
   $$36 + 64 = 100 = (10)^2$$

   You will see that in each example there is a triplet and the square of greatest number in the triplet is equal to the sum of squares of other two numbers. Such numbers are known as **Pythagorean Triplets**.
   In the above example 3, 4, 5 and 6, 8, 10 are Pythagorean triplets.

Example 1   Check whether 9, 40, 41 is a Pythagorean triplet or not?
Solution $(9)^2 + (40)^2$
                $= 81 + 1600$
                $= 1681 = (41)^2$
Hence, $(9)^2 + (40)^2 = (41)^2$ which proves that 9, 40, 41 is a Pythagorean triplet.

$\rightarrowtail\!\infty\!\infty\!\infty$ **Exercise 3.1** $\infty\!\infty\!\infty\!\leftarrowtail$

1. What will be unit place digit in the squares of following numbers:
   (i)   24   (ii)   17   (iii) 100   (iv)   55   (v)   111
   (vi)   1023 (vii)   5678   (viii)12796   (ix)   2412
2. Find the squares of numbers given below:
   (i)   18   (ii)   11   (iii) 107   (iv)   15 (v) 200 (vi) 27

3. Which of the following numbers has their square an even number:

    (i)   235   (ii)   395   (iv)   5508

    (iv)   2001   (v)   82003   (vi)   10224

4. Find the following sums without any operation:

    (i)   1+3+5+7

    (ii)   1+3+5+7+9+11+13

    (iii)   1+3+5+7+9+11+13+15+17+19

5. Write 64 as sum of eight odd numbers.

6. How many numbers are there between squares of following numbers?

    (i) 10 and 11   (ii) 17 and 18   (iii) 30 and 31

7. Check whether given three numbers are Pythagorean triplets or not?

    (i) 9, 12, 15   (ii) 7, 11, 13   (iii) 10, 24, 26

3.3   Square Root

Look at the squares of following numbers:

$$(4)^2 = 4 \times 4 = 16$$
$$(5)^2 = 5 \times 5 = 25$$
$$(6)^2 = 6 \times 6 = 36$$

We see in the above examples that square of 4 is 16. On the contrary we can say that square root of 16 is 4. Similarly, square of 5 is 25 and square root of 25 is 5. i.e., square root is inverse operation of square.

Square root is denoted by the sign "$\sqrt{}$"

For example: Square root of 81 = $\sqrt{81}$ = 9.

> **Do and learn** ◆   From the table 3.3, what will be the square roots of following?
> (i) 49   (ii) 64   (iii) 100

We have seen in the earlier example that sum of first 'n' odd numbers is 'n^2'.

For example: $5^2 = 1+3+5+7+9$

The method in which we add first five odd numbers and obtain the square of 5, we subtract odd numbers from 25 and obtain the square root of 25. Let us see

25−1=24   24−3=21   21−5=16   16−7=9   9−9=0

By successive subtraction of first five odd numbers from 25 we get remainder 0. This means square root of 25 is 5. i.e., $\sqrt{25} = 5$

You also try to find the square roots of some perfect square numbers using this process.

3.4   Finding Square Root by the Method of Prime Factorization
Given below are factors of some numbers and their squares

Numbers	Prime Factors of Numbers	Square Numbers	Prime Factors of Square Numbers
6	2 × 3	36	2 × 2 × 3 × 3
8	2 × 2 × 2	64	2 × 2 × 2 × 2 × 2 × 2
12	2 × 2 × 3	144	2 × 2 × 2 × 2 × 3 × 3

You will find that prime factors of a number repeats twice in the square of the number. For example: Prime factors of 6 are 2×3 then 2×2 and 3×3 comes in the prime factors of 36.

Contrary to this number of prime factors in the square root is half of the number of prime factors of square.

Let us find the square root of perfect square number 144.

We know that the prime factors of 144 are

$144 = \underline{2 \times 2} \times \underline{2 \times 2} \times \underline{3 \times 3}$

Making the pairs of prime factors we find that

$144 = (2 \times 2 \times 3)^2$

$144 = 2 \times 2 \times 3$

$144 = 12$

2	144
2	72
2	36
2	18
2	9
2	3
	1

Similarly, concentrate upon prime factors of 192.

$192 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3$

Here all the factors are not in pairs. Hence 192 is not a perfect square. If we want to make it perfect square, we need to multiply it by 3 or divide it by 3.

$\frac{192 \times 3}{} = \underline{2 \times 2} \times \underline{2 \times 2} \times \underline{2 \times 2} \times \underline{3 \times 3}$

$\sqrt{192 \times 3} = 2 \times 2 \times 2 \times 3$

$\sqrt{576} = 24$

Also, $\frac{192}{3} = \frac{2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3}{3}$

$\sqrt{\frac{192}{3}} = 2 \times 2 \times 2 = 8$

Example 3 Find the square root of 6400.
Solution

2	6400
2	3200
2	1600
2	800
2	400
2	200
2	100
2	50
5	25
5	5
	1

$$6400 = \underline{2 \times 2} \times \underline{2 \times 2} \times \underline{2 \times 2} \times \underline{2 \times 2} \times \underline{5 \times 5}$$
$$\sqrt{6400} = 2 \times 2 \times 2 \times 2 \times 5$$
$$= 80$$

Example 4 Is 60 a perfect square?
Solution

2	60
2	30
3	15
5	5
	1

$$60 = 2 \times 2 \times 3 \times 5$$

3 and 5 prime factors are not in pairs. Hence 60 is not a perfect square. We can see in the exact form that there is only one zero.

Example 5 Is 1800 a perfect square? If not then find the least multiple of 1800 which is a perfect square and find the square root of new number.

Solution We know that $1800 = \underline{2 \times 2} \times 2 \times \underline{3 \times 3} \times \underline{5 \times 5}$.

According to prime factors 2 does not exists in pairs. Hence 1800 is not a perfect square. If we make one more pair of 2 then it will become a perfect square. Therefore, multiplying 1800 by 2 we get

$$1800 \times 2 = \underline{2 \times 2} \times \underline{2 \times 2} \times \underline{3 \times 3} \times \underline{5 \times 5}.$$

Now every prime factor is in pair. Hence,

$$1800 \times 2 = 3600 \text{ a perfect square.}$$
$$\sqrt{3600} = 2 \times 2 \times 3 \times 5$$
$$= 60$$

Example 6 Find the smallest square number which is divisible by each 6, 9, and 15.

Solution We will solve it in two steps. First we will find LCM for the number divisible by 6, 9, and 15 and then find the multiple of LCM which is perfect square. LCM of 6, 9, 16 $= 2 \times 3 \times 3 \times 5$
$$= 90$$

2	6, 9, 15
3	3, 9, 15
3	1, 3, 15
5	1, 1, 5
	1, 1, 1

Since the factors of 90 are not in pairs we multiply it by 2 and 5 to complete pairs so that $90 \times 2 \times 5 = 2 \times \underline{2} \times 3 \times 3 \times 5 \times \underline{5}$.
Thus, 900 is the smallest perfect square divisible by 6, 9, and 15.

Exercise 3.2

1. What can be the unit place digit in the square root of following numbers
    (i) 9604   (ii) 65536   (iii) 998001 (iv) 60481729
2. Estimate and tell which numbers cannot be perfect squares?
    (i) 48   (ii) 81   (iii) 102   (iv) 24636
3. Find the square root by prime factorization method.
    (i) 1296   (ii) 729   (iii) 1764   (iv) 3969   (v) 4356 (vi) 1600
4. Numbers given below are not perfect squares. Find the smallest whole number to which when multiplied makes it perfect square.
    (i) 252   (ii) 396   (iii) 1620
5. Numbers given below are not perfect squares. Use prime factorization method to find the smallest number by which these can be divided to make them perfect squares.
    (i) 1000   (ii) 867   (iii) 4375
6. Rose plants are to be planted in a square shape garden. Number of plants in each row is equal to the number of rows. If 2401 plants are there in the garden then find the number of rows.
7. Find the smallest square number which is completely divisible by 4, 9 and 10

3.5   Finding Square Root by Division Method
When numbers are big then prime factorization method becomes lengthy and cumbersome. For this we use division method to find the square root.

Example 7 Focus on the following steps in finding the square root of 576.

Solution   **Step 1** Make pairs of digits by starting from unit place in the number.

e.g. in 576   5 $\overline{76}$

$$\begin{array}{r} 2 \\ 2\overline{\big)\,5\ \overline{76}} \\ \underline{-4} \\ 1 \end{array}$$

Step 2 Choose the greatest number whose square is either Equal or less than the extreme left digit or pair of digits. i.e., we need to choose a number whose square is less than 5, which is 2.

$$(2)^2 < 5 < (3)^2$$

Put this number as quotient at the top and its square below 5 and then subtract.

$$\begin{array}{r} 2 \\ 2\overline{\big)\,5\ \overline{76}} \\ +2\underline{-4} \\ 4\ 176 \end{array}$$

Step 3 Again, write the next pair after the remainder as we do in case of simple division. (Note that we write only one digit in case of simple division but a pair in the evaluation of square root.)

Step 4 Add the divisor with the same number and write it below.

Step 5 In our example we need to write a digit (any digit between 0 and 9) after the divisor 4 so that the new divisor can be (40, 41, 42, ……,, 49) and at the same time we write this digit after the quotient 2 at the top. The product of new divisor and this digit should be a number which is less than or equal to our dividend 176.

$$\begin{array}{r} 2\ 4 \\ 2\overline{\big)\,5\ \overline{76}} \\ +2\ 4 \\ \overline{44}\ 176 \\ 4\ \underline{176} \\ 0 \end{array}$$

Step 6 At this point 44 × 4 = 176. Now, since remainder is zero and no digit is left, we write the square root of 576 as 576 = 24

Example 8 Find the square root of 7056 by using division.

Solution   **Step 1** Make pairs of digits by starting from unit place in the number.

$$\overline{\big)\,\overline{70}\ \overline{56}}$$

Step 2 We choose the greatest number whose square is less than or equal to 70.

$$(8)^2 < 70 < (9)^2$$

We write this number as quotient and its square 64 below 70.

Step 3 – Divisor 8 is added again to 8 and we get a new divisor 16

$$\begin{array}{r} 8 \\ 8\overline{\big)\,\overline{70}\ \overline{56}} \\ +8\ \underline{64} \\ 16\ 6 \end{array}$$

Step 4 Now, we write next pair of digits 56. We get new dividend 656.

Step 5 Again we choose a digit from among (0 - 9) so that the divisor becomes (160, 161, ..., 169). We multiply this number by the same digit so that the product is less than or equal to 656.

```
        8
   8 | 70 56
  +8 | 64
  16 | 6
```

```
        84
   8 | 70 56
  +8 | 64
 164 | 6 56
   4 | -6 56
     | 0 00
```

This number is 4 in this example because $164 \times 4 = 656$.
Hence we find that $7056 = 84$

Example 9   Area of a square ground is 1089 m^2. Find the side of the ground.

Solution   Area of square ground $= \sqrt{1089}\,\text{m}^2$
Therefore, side of the ground $= \sqrt{1089}$
Hence, $= 1089 = 33\,\text{m}$.
Thus, the side of the ground $= 33\,\text{m}$.

```
        3 3
   3 | 10 89
   3 | 9
  63 | 1 89
   3 | 1 89
     | 0
```

Example 10   Find the smallest number which when subtracted from 1989 makes it a perfect square. Also find the square root of that number.

Solution   Let us try to find the square of the number 1989.
Here we see that 1989 is 53 more than a perfect square.
Therefore, by subtracting 53 from 1989 will make it a perfect square. $1989 - 53 = 1936$.

$$\therefore \quad \sqrt{1936} = 44$$

```
        4 4
   4 | 19 89
  +4 | 16
  84 | 3 89
  +4 | 3 36
     | 53
```

Similarly if we want to find that number which when added to 1989 makes it a perfect square then we will consider the square of 45 instead of 44. We have $45^2 = 2025$. Thus, we need to add $2025 - 1986 = 36$ in 1989 so as to make it a perfect square.

Example 11 Find the greatest four digit number which is a perfect square.

Solution We know that the greatest four digit number is 9999. We try to find the square root of it using division method. The remainder is 198, which shows that the square 99^2 is 198 less than 9999. Therefore, the required number is 9999-198 = 9801.

$$
\begin{array}{r|l}
 & 9\ 9 \\
\hline
9 & \overline{99}\ \overline{99} \\
+9 & 81 \\
\hline
189 & 18\ 99 \\
9 & 17\ 01 \\
\hline
 & 1\ 98
\end{array}
$$

3.6 Square Root of Decimal Numbers

Example 12 Consider the number $\sqrt{51.84}$

Solution **Step 1** - We follow making of pairs of digits in evaluating the square root of decimal numbers too. Since there are two parts in decimal numbers, namely integer part and decimal part. Pairing of integer part will be same as was in case of earlier examples i.e., starting from unit place. But, in case of decimal part pairing starts from one-tenth place. One-tenth and one-hundredth is one pair one-thousandth and ten-thousandth is another pair etc.

In this example the pairs will be $\overline{51}$ and $\overline{84}$

$$
\begin{array}{r|l}
 & 7 \\
\hline
7 & \overline{51}.\overline{84} \\
7 & 49 \\
\hline
 & 284
\end{array}
$$

Step 2 – As earlier, choose a number whose square is less than or equal to 51, i.e., $(7)^2 < 51 < (8)^2$ and we will write it at divisor column as well as at quotient place.

Step 3 – We will write the product of 7 by 7 below the divi dend and sum of 7 and 7 at the divisor column.

Step 4 – Remainder is 2. Next time we write 84 on the right of it so as to get 284 as dividend. Since 84 was the decimal part of the number we put decimal in quotient.

Step 5 – Now, as before, we choose a number from (0 – 9) and put after 4 in 14 so that the new divisor becomes either of (140, 141, 142, ….., 149) which when multiplied by the same number gives the number not greater than 284. This show our quotient. In this example, the number will be 2 so that $142 \times 2 = 284$.

$$
\begin{array}{r|l}
 & 7.2 \\
\hline
7 & \overline{51}.\overline{84} \\
7 & 49 \\
\hline
142 & 284 \\
2 & 284 \\
\hline
144 & 0\ 00
\end{array}
$$

Hence, $\sqrt{51.84} = 7.2$

Where to proceed?

Consider the number 176.341. Put the bars over integer part and decimal part. Now consider176; we start from the unit place near decimal and move towards left. First bar is over 76 and second bar is over 1. For 0.341, we start from the decimal and move towards right.

First bar is over 34 and in order to put second bar over 1 we put 0 so that it becomes $0.\overline{34}\overline{10}$

3.7 Estimating the Square Root
(i) Number of digits in square root
Consider following table

$1^2 = 1$	$99^2 = 9801$
$9^2 = 81$	$100^2 = 10000$
$10^2 = 100$	$999^2 = 9898001$

How many digits are there in the square of one-digit number? 1 or 2
How many digits are there in the square of two-digit number? 3 or 4
How many digits are there in the square of three-digit number? ....

Contrary to this, there will be one digit in the square root of one-digit numbers and 1 or 2 digits in the square root of two-digit numbers and so on.

> **Do and learn:** ◆
> How many digits will be there in the square of following numbers?
> (i)1369   (ii)15376   (iii) 6031936

We frequently need to evaluate square roots in our daily life.

There are 350 students in a school. They are to be arranged in square configuration and remaining students will look after general arrangements of Independence Day celebration. Here we need to estimate perfect square we know that $100 < 350 < 400$ and

$\sqrt{100} = 10$ and $\sqrt{400} = 20$

Therefore, $10 < \sqrt{350} < 20$ but still we are not near to square number.
We know that
$18^2 = 324$ and $19^2 = 361$.

Thus $18 < \sqrt{350} < 19$
Hence, we can make 18 rows and 18 columns of students out of 350 students
Remaining 16 students will look after general arrangements.

Exercise 3.3

1. Find the square root of the following number using division method:
   (i) 441   (ii) 576   (iii) 1225   (iv) 2916   (v) 4624   (vi) 7921

2. Find the square root of the following numbers without really calculating.
   (i) 121   (ii) 256   (iii) 4489   (iv) 60025

3. Find the square root of the following decimal numbers.
   (i) 6.25   (ii) 2.89   (iii) 32.49   (iv) 31.36   (v) 57.76

4. What is to be added in the following numbers so that they become perfect squares?
   (i) 420   (ii) 2000   (iii) 837   (iv) 3500

5. What is to be subtracted from the following numbers so that they become prefect square numbers?
   (i) 555   (ii) 252   (iii) 1650   (iv) 6410

6. Chairs are to be arranged for a wedding function in square configuration. 1000 chairs are available. How many additional chairs will be required for square configuration? At the same time, also find the number of chairs in each row.

7. Area of a square farm is 361 m². How much wire will be required for fencing the four sides?

8. Find the smallest number that can divide 2352 to make it a perfect square.

We Learnt
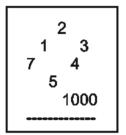
1. Normally if a number m is expressed as n^2(where m and n both are natural numbers) then m is a square number. For example: $n = 5$ and $m = 5^2 = 25$.

2. The number in which the unit place digit is 2, 3, 7, 8 can never be square numbers, i.e., unit place digit in all the square numbers is either of the numbers 0, 1, 4, 5, 6, or 9.

3. Number of zeros in the end in a square number is always even

4. Square root is the inverse operation of square.

5. A perfect square has two square roots one positive and other negative. Positive square root is denoted by $\sqrt{}$

Chapter 4

Rational Numbers
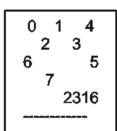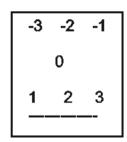
4.1 We started learning the numbers by counting the things around us. The numbers used for counting are called Natural numbers. We got the Whole numbers by including 0 in Natural numbers 1, 2, 3, 4, 5, ... Thereafter by including the negative of Natural numbers in Whole numbers 0, 1, 2, 3, 4 ...... we got Integers .... -3, -2, -1, 0, 1, 2, 3,...... Thus we have extended the number system up to Integers.

2 1 3 7 4 5 1000 -------------	0 1 4 2 3 6 5 7 2316 ----------	-3 -2 -1 0 1 2 3 _____
Natural Numbers	**Whole Numbers**	**Integers**

We have introduced fractions in previous chapters. We will further extend the number system in this chapter. We will learn the concepts of rational numbers, representation of rational numbers on number line, their comparison and finding the rational numbers lying between two rational numbers.

4.2 Need of Rational Numbers

We have studied that the integers are used for representing the opposite situations.

Example 1 If the profit of Rs. 250 is expressed as +250 then the loss of Rs. 250 is expressed as -250.

Example 2 If the height of any place 800 m above sea level is expressed as $\frac{4}{5}$ km then depth of 800 m below sea level can be expressed as $-\frac{4}{5}$ km. We could understand that $-\frac{4}{5}$ is neither an integer nor a fraction. In order to define such numbers we need to extend the number system.

4.3 What are Rational Numbers?

The word rational originated from ratio. We know that the ratio 2:5 can also be written as $\frac{2}{5}$, here 2 and 5 are natural numbers. But $\frac{-2}{5}$ cannot be expressed in -2:5. Any two integers p and q (where $q \neq 0$) can be written as $\frac{p}{q}$ Rational numbers are expressed in this form.

A rational number can be defined in the form of a number, which can be expressed as $\frac{p}{q}$ where p and q are integers and $q \neq 0$.

Thus, $\frac{3}{7}$ is a rational number, Here, $p = 3$ and $q = 7$.

Think and tell Is $\frac{-3}{7}$ a rational number?

4.4 Fractions and Rational Numbers

Write different fractions such as $\frac{3}{8}$, $\frac{7}{11}$, $\frac{4}{9}$, $1\frac{3}{5}$ etc.
Compare each fraction with $\frac{p}{q}$.

In $\frac{3}{8}$, $p = 3$ and $q = 8$

In $\frac{7}{11}$, $p = 7$ and $q = 11$.

Take other examples of fractions and compare with $\frac{p}{q}$ We find that every fraction is of the form $\frac{p}{q}$ where p and q are integers and $q \neq 0$. We could now say that all the fractions are rational numbers.

> **Do and learn** ◆
>
> Write the rational numbers in which
> 1. Numerator is a negative integer and denominator is a positive integer.
> 2. Numerator is a positive integer and denominator is a negative integer.
> 3. Both numerator and denominator are positive integers.
> 4. Both numerator and denominator are negative integers.

▪ Are all integers also rational numbers?

Any integer can be considered as a rational number. For example -3 is a rational number, because we can write it as $\frac{-3}{1}$. Integer 0 can also be written in $\frac{0}{1}$ or $\frac{0}{2}$ etc. Hence 0 is also a rational number.

- ▪ Zero is a rational number.
- ▪ Zero is neither a positive rational number nor a negative rational number.
- ▪ Rational numbers include integers and fractions.

Think! Is $\frac{-3}{-5}$ a rational number?

All rational numbers are not fractions but all fractions are rational numbers.

Rational number $\dfrac{-2}{-9}$ is not a fraction, whereas alternative form of $\dfrac{-2}{-9}$ is $\dfrac{2}{9}$, which is a fraction number.

4.5   Equivalent Rational Numbers

By multiplying or dividing the numerator and denominator of a rational number by same number it can be converted in to desired numerator or desired denominator.

Think of rational number $\dfrac{-5}{7}$

$$\dfrac{-5}{7} = \dfrac{(-5)\times 2}{7\times 2} = \dfrac{-10}{14}$$

$$\dfrac{-5}{7} = \dfrac{(-5)\times 3}{7\times 3} = \dfrac{-15}{21}$$

$$\dfrac{-5}{7} = \dfrac{(-5)\times(-2)}{7\times(-2)} = \dfrac{10}{-14}$$

$$\text{So} \quad \dfrac{-5}{7} = \dfrac{-10}{14} = \dfrac{-15}{21} = \dfrac{10}{-14}$$

The rational numbers which are mutually equal are called equivalent rational numbers.

$$\dfrac{10}{-15} = \dfrac{10 \div 5}{-15 \div 5} = \dfrac{2}{-3}$$

$$\dfrac{10}{-15} = \dfrac{10 + (-5)}{(-15) \div (-5)} = \dfrac{-2}{3}$$

Hence, $\dfrac{10}{-15} = \dfrac{2}{-3} = \dfrac{-2}{3}$ are equivalent.

Do and learn   Fill in the blanks.

$$\dfrac{2}{3} = \dfrac{4}{......} = \dfrac{......}{12} = \dfrac{10}{........} = \dfrac{......}{24}$$

$$\dfrac{5}{7} = \dfrac{......}{14} = \dfrac{25}{......} = \dfrac{........}{63} = \dfrac{100}{......}$$

$$\dfrac{25}{50} = \dfrac{......}{10} = \dfrac{1}{......} = \dfrac{......}{150} = \dfrac{250}{......}$$

4.6   Positive and Negative Rational Numbers

Both numerator and denominator in rational numbers $\frac{2}{3}, \frac{3}{7}, \frac{5}{8}$ and $\frac{2}{9}$ are positive. Such rational numbers are called positive rational numbers.

Rational numbers in which either numerator or denominator is a negative integer are called negative rational numbers. For example: $\frac{-3}{7}, \frac{4}{-5}, -\frac{1}{3}$ etc. What do you think about $\frac{-5}{-7}$

$$\frac{-5}{-7} = \frac{5 \times (-1)}{7 \times (-1)} = \frac{5}{7}$$

So, $\frac{-5}{-7}$ is a positive rational number.

Do and learn: ◆

1. Write three positive rational numbers.
2. Write two negative numbers.
3. Is $\frac{-15}{-1}$ a positive rational number? (Justify the answer)
4. Is -7 a negative rational number? (Justify the answer)
5. Which of the following numbers are positive rational number?

   (i) $\frac{-4}{5}$ (ii) $\frac{-7}{-9}$ (iii) $1\frac{2}{3}$ (iv) $\frac{3}{-7}$ (v) $\frac{1}{3}$
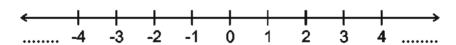
4.7   Rational Numbers on a Number Line

We have learnt to represented integers on a number line. Let us see such number line

.... -4   -3   -2   -1   0   1   2   3   4   ....

On number line positive integers lie on right hand side of 0 and are denoted by '+' sign. Negative integers lie on the left hand side of 0 and are denoted by '-' sign.
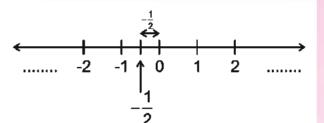
In previous classes, we have denoted fractions on number line.

Let us denote the rational number $-\frac{1}{2}$ on number line.
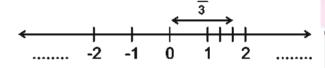
Since $-\dfrac{1}{2}$ is a negative rational

number, it will lie on the left of 0.

$-\dfrac{1}{2}$ lies between 0 and -1.

Divide space between 0 and -1 into two equal half and then locate $-\dfrac{1}{2}$ exactly in the middle of 0 and -1.
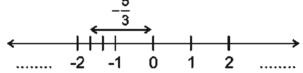
We know that how $\dfrac{5}{3}$ is denoted on number line.

We make three equal parts between 1 and 2 on the right of 0. Second part on the right of 1 denote $\dfrac{5}{3}$.

Let us now denote $\dfrac{-5}{3}$ on number line. It will be denoted at a distance on the left hand side of 0 equal to the point where $\dfrac{5}{3}$ is denoted on the right hand side of 0

Do and learn ◆

Denote the following rational numbers on number line:

(i) $-\dfrac{5}{4}$ (ii) $-\dfrac{7}{2}$ (iii) $-\dfrac{11}{3}$ (iv) $\dfrac{2}{5}$ (v) $\dfrac{4}{3}$

4.8   Simplest form of Rational Number

Look at the following rational numbers carefully.

$$\dfrac{1}{3}, \dfrac{3}{5}, \dfrac{-2}{7}, \dfrac{5}{8}, \dfrac{-9}{11}$$

In all the rational numbers given above

     (i) Denominator is positive integer, and

     (ii) The only one common factor between numerator and denominator is '1'.

Such rational numbers are called rational numbers in simplest form.

Every rational number can be expressed in simplest form.

Example 3   Express $\dfrac{-36}{24}$ in the simplest form.

Solution

$$\frac{-36}{24} = \frac{-36 \div 3}{24 \div 3} = \frac{-12}{8} = \frac{-12 \div 4}{8 \div 4} = \frac{-3}{2}$$

$$\frac{-36}{24} = \frac{-36 \div 12}{24 \div 12} = \frac{-3}{2}$$

So, $\dfrac{-36}{24}$ is the simplest form of $\dfrac{-3}{2}$

Do and learn ◆

Convert the following in to the simplest form

(i) $\dfrac{3}{15}$ (ii) $\dfrac{-6}{20}$ (iii) $\dfrac{10}{-35}$ (iv) $\dfrac{-45}{30}$ (v) $\dfrac{18}{-45}$

4.9   Comparison of Rational Numbers

We know how to compare two integers or two fractions and also the order relation, i.e., which of the given numbers is greater or smaller. Let us now compare two rational numbers.
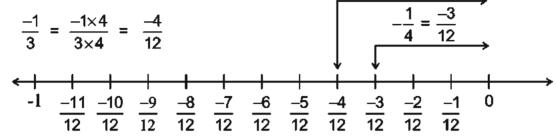
Two rational numbers $\dfrac{5}{7}$ and $\dfrac{7}{9}$ can be compared in the same manner as we compared fractions.

Let us compare two negative rational numbers $\dfrac{-1}{4}$ and $\dfrac{-1}{3}$ on the number line.

We have seen in reference to comparison of integers that integers on the right hand side are greater than the integer on the left hand side of the number line. Similarly $\dfrac{-1}{4}$ and $\dfrac{-1}{3}$ can also be compared by denoting them on number line. We take two equivalent rational numbers of both which has similar denominations.

For example -

$$\frac{-1}{4} = \frac{-1 \times 3}{4 \times 3} = \frac{-3}{12}$$

$$\frac{-1}{3} = \frac{-1 \times 4}{3 \times 4} = \frac{-4}{12}$$

$$-\frac{1}{3} = \frac{-4}{12}$$

$$-\frac{1}{4} = \frac{-3}{12}$$

Number line from -1 to 0 with markings: $\frac{-11}{12}, \frac{-10}{12}, \frac{-9}{12}, \frac{-8}{12}, \frac{-7}{12}, \frac{-6}{12}, \frac{-5}{12}, \frac{-4}{12}, \frac{-3}{12}, \frac{-2}{12}, \frac{-1}{12}, 0$

Since $\frac{-1}{4}$ is on the right hand side of $\frac{-1}{3}$, $\frac{-1}{4}$ will be greater than $\frac{-1}{3}$

$$-\frac{1}{4} > -\frac{1}{3}$$

But we have learnt from the study of fractions that

$$\frac{1}{4} < \frac{1}{3}$$

Do and learn ◆

Compare the numbers $\frac{-3}{4}$ & $-\frac{2}{3}$ and $-\frac{1}{3}$ & $-\frac{1}{5}$

Pairs of negative rational numbers are also treated in similar manner. In order to compare two negative rational numbers, we neglect the negative sign and compare them and then reverse the sign of inequality.

For example: In order to compare the numbers $-\frac{3}{7}$ & $-\frac{5}{9}$ we first compare the pair $\frac{3}{7}$ & $\frac{5}{9}$

$$\frac{3 \times 9}{7 \times 9} = \frac{27}{63}, \quad \frac{5 \times 7}{9 \times 7} = \frac{35}{63} \quad \therefore \quad \frac{27}{63} < \frac{35}{63}$$

Or $\frac{3}{7} < \frac{5}{9}$ From this we conclude that $-\frac{3}{7} > -\frac{5}{9}$

Do and learn ◆   Which rational number is greater?

1. $-\frac{3}{8}$ Or $-\frac{2}{7}$

2. $-\frac{7}{5}$ Or $-\frac{5}{3}$

3. $-\frac{5}{6}$ Or $-\frac{7}{8}$

Comparison of a negative rational number and a positive rational number is quite obvious. A negative rational number lies on the left hand side of zero and a positive rational number lies on the right hand side of zero. A negative rational number is always less than a positive rational number.

For example

$$-\frac{1}{2} < \frac{1}{2}$$

$$-\frac{3}{5} < \frac{1}{5}$$

$$-\frac{9}{4} < \frac{3}{2}$$

In order to compare the rational numbers $\frac{-4}{-7}$ and $\frac{-3}{-5}$ we first convert them in to standard form and then compare.

The standard forms of $\frac{-4}{-7}$ and $\frac{-3}{-5}$ are $\frac{4}{7}$ and $\frac{3}{5}$ respectively. This suggests that

$$\frac{4}{7} < \frac{3}{5}$$

> **Do and learn** ◆
>
> Do $\frac{4}{-9}$ and $\frac{-20}{45}$ denote same rational number?

4.10   Rational Numbers between two Rational Numbers

We know that the integers between 5 and 12 are 6, 7, 8, 9, 10, and 11. The integers between -3 and 3 are -2, -1, 0, 1, 2. This shows that the number of integers between two integers is finite.

Does this happen in case of rational numbers also? Let us see it with following example:

Kiran considered two rational numbers $-\frac{4}{3}$ and $-\frac{1}{2}$

She converted them in to the numbers with common denominator.

So $-\dfrac{4}{3} = -\dfrac{8}{6}$ and $-\dfrac{1}{2} = -\dfrac{3}{6}$

She wrote the rational numbers between $-\dfrac{8}{6}$ and $-\dfrac{3}{6}$ as follows:

$$-\dfrac{7}{6} < -\dfrac{6}{6} < -\dfrac{5}{6} < -\dfrac{4}{6}$$

Thus, she got the rational numbers $-\dfrac{7}{6}, -\dfrac{6}{6}, -\dfrac{5}{6}, -\dfrac{4}{6}$ between $-\dfrac{4}{3}$ and $-\dfrac{1}{2}$

Think! Are $-\dfrac{7}{6}, -\dfrac{1}{1}, -\dfrac{5}{6}, -\dfrac{2}{3}$ the only rational numbers between $-\dfrac{4}{3}$ and $-\dfrac{1}{2}$

Let us see.

$$-\dfrac{4}{3} = -\dfrac{8}{6} = -\dfrac{16}{12} \text{ and } -\dfrac{1}{2} = -\dfrac{3}{6} = -\dfrac{6}{12}$$

Now, the rational numbers between $-\dfrac{16}{12}$ and $-\dfrac{6}{12}$

$$-\dfrac{15}{12} < -\dfrac{14}{12} < -\dfrac{13}{12} < -\dfrac{12}{12} < -\dfrac{11}{12} < -\dfrac{10}{12} < -\dfrac{9}{12} < -\dfrac{8}{12} < -\dfrac{7}{12}$$

Or $-\dfrac{5}{4} < -\dfrac{7}{6} < -\dfrac{13}{12} < -\dfrac{1}{1} < -\dfrac{11}{12} < -\dfrac{5}{6} < -\dfrac{3}{4} < -\dfrac{2}{3} < -\dfrac{7}{12}$

Thus, we are successful in finding five more rational numbers $-\dfrac{5}{4}, -\dfrac{13}{12}, -\dfrac{11}{12}, -\dfrac{3}{4}$

$-\dfrac{7}{12}$ between $-\dfrac{4}{3}$ and $-\dfrac{1}{2}$

Using this method we can find as many (infinite) rational numbers as we want between two rational numbers.

Do and learn ◆

(i) Find five rational numbers between $-\dfrac{5}{7}$ and $-\dfrac{3}{8}$

(ii) Find five rational numbers between $-\dfrac{5}{3}$ and $-\dfrac{8}{7}$

Example 5 Write two rational numbers between the two rational numbers -2 and -1.

Solution First of all we write -2 and -1 in the form of rational numbers having common denominator.

$$-2 = -\frac{10}{5} \text{ and } -1 = -\frac{5}{5}$$

Now, the number of rational numbers between $-\frac{10}{5}$ and $-\frac{5}{5}$ are $-\frac{9}{5} < -\frac{8}{5} < -\frac{7}{5} < -\frac{6}{5}$

So, the rational numbers between -2 and -1 are $-\frac{8}{5}$ and $-\frac{7}{5}$

(We can take any two rational numbers from $-\frac{9}{5}, \ -\frac{8}{5}, \ -\frac{7}{5}, \ -\frac{6}{5}$)

Exercise 4

1. Write five rational numbers equivalent to the following rational numbers:

   (i) $-\frac{2}{3}$ (ii) $\frac{1}{5}$ (iii) $\frac{-5}{3}$ (iv) $\frac{4}{-9}$

2. Write three such rational numbers equivalent to $\frac{-5}{12}$ in which the denominator is 60, -96 and 108.

3. Write three such rational numbers equivalent to $\frac{-3}{7}$ in which the numerator is 24, -60 and 75.

4. Write the following rational numbers in the simplest form (standard forms):

   (i) $\frac{-18}{30}$ (ii) $\frac{44}{-72}$ (iii) $\frac{55}{22}$ (iv) $\frac{-16}{20}$

5. Represent the following numbers on number line:

   (i) $\frac{3}{5}$ (ii) $\frac{7}{8}$ (iii) $\frac{-8}{3}$ (iv) $-2\frac{1}{2}$ (v) $\frac{5}{7}$

6. Choose the correct sign from <, >, = and fill in the blanks:

   (i) $\frac{2}{3}$ ☐ $\frac{-5}{7}$ (ii) $\frac{-1}{4}$ ☐ $\frac{1}{-3}$ (iii) $\frac{-3}{5}$ ☐ $\frac{-1}{3}$

   (iv) $\frac{2}{7}$ ☐ $\frac{1}{2}$ (v) $\frac{-1}{2}$ ☐ $\frac{1}{-2}$ (vi) $\frac{-5}{4}$ ☐ $\frac{3}{5}$

7. Write five rational numbers between following rational numbers:

(i) -3 and -1 (ii) 0 and -1 (iii) $\dfrac{-4}{5}$ and $\dfrac{-5}{7}$

(iv) $\dfrac{1}{2}$ and $\dfrac{1}{4}$ (v) $\dfrac{2}{5}$ and $\dfrac{-4}{5}$ (vi) -2 and 0

8. Write three more rational numbers in each of the following:

(i) $\dfrac{-2}{5}, \dfrac{-4}{10}, \dfrac{-6}{15}$, — — —   (ii) $\dfrac{2}{-3}, \dfrac{4}{-6}, \dfrac{6}{-9}$, — — —

(iii) $\dfrac{1}{-3}, \dfrac{2}{-6}, \dfrac{3}{-9}$, — — —   (iv) $\dfrac{1}{-5}, \dfrac{2}{-10}, \dfrac{3}{-15}$, — — —

9. Write the following rational numbers in increasing order:

(i) $\dfrac{1}{2}, \dfrac{-1}{2}, \dfrac{-3}{4}, \dfrac{3}{4}$ (ii) $\dfrac{-3}{4}, \dfrac{-3}{7}, \dfrac{-3}{2}$ (iii) $\dfrac{-7}{11}, \dfrac{7}{15}, 0, -2, \dfrac{-2}{15}$ (iv) $\dfrac{2}{5}, \dfrac{4}{7}, \dfrac{1}{6}, \dfrac{5}{9}$

10. Write the following rational numbers in decreasing order:

(i) $\dfrac{9}{-24}, \dfrac{-3}{4}, \dfrac{5}{-12}, \dfrac{-7}{16}$ (ii) $\dfrac{-5}{6}, \dfrac{1}{6}, \dfrac{-8}{9}, \dfrac{-11}{12}$ (iii) $\dfrac{1}{3}, \dfrac{-2}{3}, \dfrac{-5}{6}, \dfrac{4}{-3}$ (iv) $\dfrac{3}{5}, \dfrac{-17}{-30}, \dfrac{-7}{10}, \dfrac{8}{-15}$

⋙ We Learnt ⋘

1. Rational numbers are denoted by $\dfrac{p}{q}$ where p and q are integers and $q \neq 0$.
2. All the fractions and whole numbers are rational numbers. $\dfrac{7}{8}, \dfrac{-2}{3}, 5$ are rational numbers.
3. If the numerator and denominator in a rational number are multiplied or divided by a same integer (other than zero) then the rational number so obtained is known as equivalent rational number. For example: $\dfrac{5}{6} = \dfrac{5 \times 3}{6 \times 3} = \dfrac{15}{18}$
4. Rational numbers are classified as positive and negative rational numbers. When both numerator and denominator are either positive or negative, the rational numbers is called as positive rational number. When either of the numerator and denominator are negative, then the rational number is called as negative rational number. For example: $\dfrac{2}{3}$ is a positive rational number and $-\dfrac{2}{3}$ a negative rational number.
5. Zero is a rational number but it is neither positive rational number nor negative rational number.
6. There lie infinite number of rational numbers between two rational numbers.

Powers and Exponents

5.1 Ravi asked a question to Mohan; what was the population of India in 2011? He replied; approximately 120 Crores. Ravi again asked; what is the distance between the Sun and the Earth? He immediately replied – Approximately 15 Crore Kilometers. Ravi again asked a question: What is the distance travelled by light covers in a second? He replied – 3 Crore Meters. Ravi once again asked a question: How much is the population of Rajasthan according to 2011 census?

Mohan replied – The population of Rajasthan is around 7 Crores in 2011. Now write them in terms of numbers; then Mohan said, "It is difficult to write in numbers". Can these numbers be read, written and understood easily. We can read and write such big numbers with the help of powers and exponents. In this chapter, we will study the numbers with base as an integer and exponent as whole number.

5.2 Exponent

Let us think of repeated numbers like

$4+4+4+4+4$, $5+5+5+5+5+5$, $7+7+7+7+7+7+7+7$

According to law of multiplication, the sum of repeated numbers can be expressed in the form of products $5 \times 4, 6 \times 5, 8 \times 7$.

Can we understand repetition of numbers by multiplication easily? Consider following numbers:

$4 = 2 \times 2$ $\qquad$ $8 = 2 \times 2 \times 2$ $\qquad$ $16 = 2 \times 2 \times 2 \times 2$

$32 = 2 \times 2 \times 2 \times 2 \times 2$

These could be written as

$2 \times 2 = 2^2$ $\qquad$ $2 \times 2 \times 2 = 2^3$ $\qquad$ $2 \times 2 \times 2 \times 2 = 2^4$
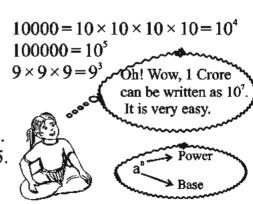
$2 \times 2 \times 2 \times 2 \times 2 = 2^5$

Similarly, $100 = 10 \times 10 = 10^2$ $\qquad\qquad$ $10000 = 10 \times 10 \times 10 \times 10 = 10^4$

$1000 = 10 \times 10 \times 10 = 10^3$ $\qquad\qquad$ $100000 = 10^5$

and $9 \times 9 = 9^2$ $\qquad\qquad$ $9 \times 9 \times 9 = 9^3$

$9 \times 9 \times 9 \times 9 \times 9 \times 9$ .... n factors.

Here, in 2^3 the base is 2 and the power is 3.

In 2^5, the base is 2 and the exponent is 5.

2^5 is read as "2 power 5".

Oh! Wow, 1 Crore can be written as 10^7. It is very easy.

Power

a^n

Base

Example 1 Write 64 in terms of exponents.

Solution

$64 = 2 \times 2 \times 2 \times 2 \times 2 \times 2$

$64 = 2^6$

Example 2 Which number is greater 3^4 or 4^3 and why?

Solution

$3^4 = 3 \times 3 \times 3 \times 3$

$\quad = 81$
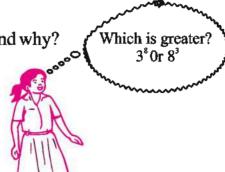
$4^3 = 4 \times 4 \times 4$

$\quad = 64$

you know $81 > 64$

$\therefore\ 3^4 > 4^3$

Therefore 3^4 is greater than 4^3

Which is greater? 3^8 Or 8^3

Example 3 Write the following numbers in terms of powers of prime factors:

(i) 36   (ii) 256   (iii) 1000

Solution

(i) 36

$= 2 \times 2 \times 3 \times 3$

$= 2^2 \times 3^2$

2	36
2	18
3	9
3	3
	1

(ii) 256

$= 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$

$= 2^8$

2	256
2	128
2	64
2	32
2	16
2	8
2	4
2	2
	1

(iii) 1000

$= 2 \times 2 \times 2 \times 5 \times 5 \times 5$

$= 2^3 \times 5^3$

2	1000
2	500
2	250
5	125
5	25
5	5
	1

Example 4 Simplify the following:
Solution

(i) 3×10^3 (ii) $5^2 \times 2^3$

(i) $3 \times 10^3 =$ $3 \times 10 \times 10 \times 10$

$=$ 3×1000

$=$ 3000

(ii) $5^2 \times 2^3 =$ $5 \times 5 \times 2 \times 2 \times 2$

$=$ 25×8

$=$ 200

Example 5 Find the values of following:

Solution (i) $(-1)^5$ (ii) $(-3)^4$

(i) $(-1)^5 =$ $(-1) \times (-1) \times (-1) \times (-1) \times (-1)$ $= -1$

(ii) $(-3)^4 =$ $(-3) \times (-3) \times (-3) \times (-3)$

$=$ 9×9 $=$ 81

Exercise 5.1

1. Express the following in terms of exponents:

(i) $7 \times 7 \times 7 \times 7 \times 7$ (ii) $3 \times 3 \times 3 \times 7 \times 7$

(iii) $a \times a \times a \times b \times b$ (iv) $5 \times 5 \times t \times t \times t$

2. Express each of the following numbers in the form of exponents:

(i) 32 (ii) 81 (iii) 343 (iv) 125

3. Identify the greater number in the following:

(i) 2^5 or 5^2 (ii) 3^5 or 5^3 (iii) 3^{10} or 10^3 (iv) 7^3 or 3^7

4. Express the following numbers in terms of the powers of prime factors:

(i) 324 (ii) 625 (iii) 1080 (iv) 1800

5. Simplify the following:

(i) 2×3^4 (ii) $7^3 \times 5$ (iii) $5^3 \times 2^2$ (iv) $3^2 \times 10^3$

(v) 0×10^4

6. Find the values of:

(i) $(-1)^3$ (ii) $(-5)^4$ (iii) $(-4)^2 \times (-2)^3$

5.3 Laws of Exponents

Rule 1 Multiplication of exponent numbers with same base.

Example 6 Find the value of $2^3 \times 2^4$.

Solution

$$2^3 \times 2^4 = (2 \times 2 \times 2) \times (2 \times 2 \times 2 \times 2)$$
$$= 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$$
$$= 2^7$$
$$2^3 \times 2^4 = 2^{(3+4)}$$
$$= 2^7$$

Note that here, the base in 2^3 and 2^4 is same and sum of exponents 3 and 4 is 7.

Example 7 Solve $(-5)^2 \times (-5)^3$.

Solution

$$(-5)^2 \times (-5)^3 = [(-5) \times (-5)] \times [(-5) \times (-5) \times (-5)]$$
$$= (-5) \times (-5) \times (-5) \times (-5) \times (-5)$$
$$= (-5)^5$$
$$(-5)^2 \times (-5)^3 = (-5)^{2+3}$$
$$= (-5)^5$$

In general we can say that if a is a non-zero number, where m and n are positive integers, then $a^m \times a^n = a^{m+n}$

Rule 2: Division of Exponent Numbers with Common Base

Let us divide the numbers with common base and different power.

Example 8 Solve $2^7 \div 2^3$.

Solution

$$2^7 \div 2^3 = \frac{2^7}{2^3} = \frac{2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2}{2 \times 2 \times 2}$$
$$= 2 \times 2 \times 2 \times 2$$
$$= 2^4$$

So, $2^7 \div 2^3 = \frac{2^7}{2^3} = 2^{7-3} = 2^4$

Hence, $2^7 \div 2^3 = 2^4$

Example 9 Find the value of $a^4 \div a^2$.

Solution $a^4 \div a^2 = \frac{a^4}{a^2} = \frac{a \times a \times a \times a}{a \times a}$

So, $a^4 \div a^2 = \dfrac{a^4}{a^2} = a^{4-2} = a^2$

If a is a non-zero number and m and n are two positive integers, where $m > n$, then

$a^m \div a^n = a^{m-n}$

Again,

Example 10   Simplify $3^3 \div 3^7$.

Solution $3^3 \div 3^7$

$= \dfrac{3^3}{3^7} = \dfrac{\cancel{3} \times \cancel{3} \times \cancel{3}}{\cancel{3} \times \cancel{3} \times \cancel{3} \times 3 \times 3 \times 3 \times 3}$

$= \dfrac{1}{3^4}$

$\dfrac{3^3}{3^7} = \dfrac{1}{3^{7-3}} = \dfrac{1}{3^4}$

If a is a non-zero number and m and n are two positive integers, where $m < n$, then

$a^m \div a^n = \dfrac{1}{a^{n-m}}$

Zero Exponent:

Look at the following action

$3^2 \div 3^2 = 3^{2-2} = 3^0$

But $3^2 \div 3^2 = \dfrac{3^2}{3^2} = \dfrac{3 \times 3}{3 \times 3} = 1$

So, $3^0 = 1$

We have got $3^0 = 1$ above. Similarly, if for any base the exponent is zero, then its value is 1

If a is a non-zero number then $a^0 = 1$.

Rule 3   Exponent of Exponent Number

Example 11   Find the value of $[(5)^3]^4$ ..

Solution $[(5)^3]^4 = (5^3) \times (5^3) \times (5^3) \times (5^3)$

$= 5^{3+3+3+3}$

$= 5^{(3 \times 4)}$

i.e., $[(5)^3]^4 = 5^{3 \times 4}$

It is concluded from above that

If a is a non-zero number and m and n are positive integers, then $(a^m)^n = a^{m \times n}$

Rule 4: Multiplication of Numbers with Different Base but Common Exponent

Example 12 Could you simplify $2^4 \times 3^4$?

Solution

$$2^4 \times 3^4$$

$$= (2 \times 2 \times 2 \times 2) \times (3 \times 3 \times 3 \times 3)$$

$$= 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 3$$

$$= (2 \times 3) \times (2 \times 3) \times (2 \times 3) \times (2 \times 3)$$

$$= (2 \times 3)^4$$

i.e., $2^4 \times 3^4 = (2 \times 3)^4$

> Note: Be aware that
> $$a^m + b^m \neq (a+b)^m$$
> $$a^m - b^m \neq (a-b)^m$$
> Example:
> $$2^3 + 5^3 \neq (2+5)^3$$
> $$2^3 - 5^3 \neq (2-5)^3$$

It is concluded from the above example that.

> If a and b are two non-zero number and m is a positive integers, then
> $$a^m \times b^m = (a \times b)^m$$

Rule 5   Division of Numbers with Different Base but Common Exponent

Example 13   Find the value of $8^5 \div 9^5$.

$$8^5 \div 9^5 \quad = \frac{8^5}{9^5} \quad = \frac{8 \times 8 \times 8 \times 8 \times 8}{9 \times 9 \times 9 \times 9 \times 9}$$

$$= \frac{8}{9} \times \frac{8}{9} \times \frac{8}{9} \times \frac{8}{9} \times \frac{8}{9}$$

$$= \left(\frac{8}{9}\right)^5$$

i.e., $\quad 8^5 \div 9^5 \quad = \frac{8^5}{9^5} \quad = \left(\frac{8}{9}\right)^5$

> If a and b are two non-zero number and m is a positive integer, then
> $$a^m \div b^m = \frac{a^m}{b^m} = \left(\frac{a}{b}\right)^m$$

<center>⟞⟝○⟨ Exercise 5.2 ⟩○⟞⟝</center>

1. Solve the following using laws of exponents.

(i) $\quad 3^7 \times 3^8$	(ii) $\quad (4)^7 \times (4)^2$	(iii) $\quad a^5 \times a^4$
(iv) $\quad 3^{15} \div 3^9$	(v) $\quad t^7 \div t^4$	(vi) $\quad (6^4 \times 6^2) \div 6^5$
(vii) $\quad (2^8)^3$	(viii) $\quad (a^5)^4$	(ix) $\quad 5^5 \times 8^5$

(x) $a^3 \times b^3$ (xi) $7^5 + 6^5$ (xii) $(25^3 \times 25^7) + 25^{10}$

(xiii) $7^5 \div 7^8$ (xiv) $(9^3)^0$

2 .Simplify the following:

(i) $\{(3^2)^3 \times 3^4\} \div 3^7$ (ii) $16^4 \div 4^2$ (iii) $\dfrac{5^7}{5^4 \times 5^3}$

(iv) $4^0 \times 5^0 \times 6^0$ (v) $\dfrac{3^9 \times a^6}{9^2 \times a^3}$ (vi) $(7^3 \times 7)^3$

(vii) $\dfrac{3^{10}}{3^5 \times 3^7}$ (viii) $\dfrac{a^9}{a^6} \times a^8$ (ix) $2^0 + 3^0 + 4^0$

3 .Simplify the following:

(i) $\dfrac{2^3 \times 7^2 \times 13^8}{56 \times 13^7}$ (ii) $\dfrac{(3^2)^3 \times 5^3}{9^2 \times 25}$ (iii) $\dfrac{2^5 \times 10^5 \times 5}{5^4 \times 4^3}$

5.4 Denoting Big Numbers in Exponents form

Look at the following:

54   = $\dfrac{54 \times 10}{10}$ = 5.4×10^1

540   = $\dfrac{540 \times 100}{100}$ = 5.4×10^2

5400   = $\dfrac{5400 \times 1000}{1000}$ = 5.4×10^3

54000 = $\dfrac{54000 \times 10000}{10000}$ = 5.4×10^4

Here we have expressed 54, 540, 5400, 54000 in standard forms.

The velocity of light is 300,000,000 m/s which can be expressed in the following standard form.

Standard form: 3×10^8 m/s.

When a number is expressed as a product of 1.0 or a decimal number greater than 1.0 and less than 10 and powers of 10, then such form of a number is known as its standard form.

5.5 Expressing Big Numbers in Standard Forms

We know that the big numbers can be expressed conveniently in standard forms by using exponents. Let us express the big numbers in to standard form using exponents.

We write 7465 in standard form.

7465 = 7.465×1000

= 7.465×10^3.

(Decimal is shifted three places to the left)

Mass of the Earth = 5976, 000, 000, 000, 000, 000, 000, 000 kg

$= 5.976 \times 10^{24}$ kg.

You would be agree with the fact that standard form of a number is easier than a 25 digit number from the point of view of expressing, comparing and understanding.

Example 14 Write the number 150,000,000,000 in to standard form.

Solution $150,000,000,000 = 1.5 \times 10^{11}$

(Decimal is shifted 11 places to the left)

Note: While adding the numbers, numbers should be written in the same powers of 10

Example 15 Write the following numbers in to standard form:

(i)	63000		(ii) 100000	(iii) 425000

Solution

(i)	63000	=	6.3×10000
		=	6.3×10^{4}
(ii)	100000	=	1×100000
		=	1×10^{5}
(iii)	425000	=	4.25×100000
		=	4.25×10^{5}

Example 16 According to census, the population of India in a year was 1,00,84,35,405. Write this in scientific notations.

Solution Population of India

=	1,00,84,35,405
=	$1.00,84,35,405 \times 1,00,00,00,000$
=	$1.008435405 \times 10^{9}$
=	1.008×10^{9} (approx.)

<center>⊱∘⟨ **Exercise 5.3** ⟩∘⊰</center>

1. Write the following numbers in to standard form:

(i)	50,0000	(ii)	48,30,000
(iii)	3,94,00,00,00,000	(iv)	30000000
(v)	180000		

2. Distance of the Sun from the Earth is approximately 15,00,00,000 km. Express this distance in scientific notations.

3. A person gets 3000 Calorie energy from his daily meal. Represent in scientific notations, how much energy he will get in one year.

4. According to an estimation, Indian railways transports approximately 1 Crore 30 Lakhs people from one place to another place daily. How many people travel by train in 30 days? Write your answer in standard form.

5. Write the following in simple form:

   (i) $2.5 \times (10)^4$ (ii) $1.75 \times (10)^6$

   (iii) $1.21 \times (10)^{-8}$ (iv) $4.50 \times (10)^{-5}$

⫸ We Learnt ⫷

1. Numbers can be expressed in terms of exponents. Use of exponents makes reading, understanding, comparing and performing operations upon very big and very small numbers easy.

2. Numbers follow certain rules in the exponent form: For non-zero number a and b and integers m & n.

   (i) $a^m \times a^n = a^{m+n}$

   (ii) $a^m \div a^n = a^{m-n}$ if m>n or $a^m \div a^n = \dfrac{1}{a^{n-m}}$ if n > m

   (iii) $(a^m)^n = a^{mn}$

   (iv) $a^m \times b^m = (ab)^m$

   (v) $a^m \div b^m = \left(\dfrac{a}{b}\right)^m$

   (vi) $a^0 = 1$

3. In order to express a number in scientific notation or standard form, we write the product of a decimal number lying between 1.0 and 10.0 (in which 1.0 is included but 10.0 is not included) and the powers of 10.

Vedic Mathematics

6.1 In previous classes, we have learnt the multiplication by using "EkadhikenPoorven", "EkNeunenPoorven", "Nikhilam".In this chapter, we will again study another methods of addition, subtraction, multiplication, division, fractions, squares and square roots. If the operations of these methods are carried out by verbal explanation then calculations become very easy and fast.

6.2 Sankalan-Vyavkalnabhyam

This method is used for making calculations easy in our daily life. Use of this method is based upon wholeness of base number which is 10 or multiple of 10. In this method, deviations are taken from the whole base number for making major calculations easy.

Example 1 Find the sum $8 + 11 + 7 + 12 + 9 + 13$.
Solution Looking at these numbers carefully we find that 8 is 2 less than 10 and 12 is 2 more than 10. Similarly 9 is 1 less than 10 and 11 is 1 more than 10.

$$(10-2) + (10+1) + (10-3) + (10+2) + (10-1) + (10+3)$$

Arranging the numbers by expressing them in terms of whole base numbers, we have

$$(10-\cancel{2}) + (10+\cancel{2}) + (10+\cancel{1}) + (10-\cancel{1}) + (10-\cancel{3}) + (10+\cancel{3})$$

$$= 20 + 20 + 20$$

$$= 60$$

Here, -2, 2; 1, -1 and -3, 3 are the pair whose sum -2+2; 1-1; and -3+3 is zero.

Example 2 Find the sum $26+48+107+63+13+44$.
Solution For making the given numbers the complete whole number we try to make in terms of 10 or multiple of 10.
$26+63+48+13+107+44$
By Sankalan-Vyavkalnabhyam

$$= 30 - 4 + 60 + 3 + 50 - 2 + 10 + 3 + 110 - 3 + 40 + 4$$

$$= 30 + 60 + 10 + 50 + 110 + 40 - 4 + 3 - 2 + 3 - 3 + 4$$

$= 90 + 10 + 50 + 150 + 1$

$= 100 + 200 + 1$

$= 300 + 1 = 301$

If we continue finding deviations and summing up the numbers in Sankalan-Vyavkalnabhyam, we can make addition easy.

6.3 Poornapoornabhyam

Make the pairs of numbers in such a way that the sum of each pair becomes a multiple of 10.

Example 3 Find the sum $27 + 58 + 392 + 68 + 32 + 23$
Solution

$= (27+23)+(58+392)+(68+32)$ (Try for making multiple of 10)

$= 50 + 450 + 100$

$= (50 + 450) + 100$

$= 500 + 100$

$= 600$

Example 4 Find the sum $45 + 67 + 38 + 55 + 62 + 33$.
Solution Arranging terms of pairs of multiples of 10, we get

$= (45 + 55) + (67 + 33) + 38 + 62$

$= 100 + 100 + 100$

$= 300$

Exercise 6.1

1. Find the following sum using Sankalan-Vyavkalnabhyam and Poornapornabhyam.

   (i) $282 + 718 + 796 + 524 + 804 + 376$

   (ii) $52 + 136 + 48 + 64$

   (iii) $135 + 248 + 322 + 65$

6.4 Subtraction (Nikhilam Sutra)

(We do subtraction by using Nikhilam Navatah Charmam Dashatah Sutra).

If we want to subtract 362 from 1000 then use of conventional method requires many stages to follow, takes more time and chances of getting wrong answer are still high. Let us use Vedic method:

Start from right hand side and do calculations towards left. Write 9 for each 0 on left and 10 for last zero. The digit on extreme left before 0 will be reduced by 1. Doing this we have.

1000	will become	0 9 9 10
− 362		0 3 6 2
		0 6 3 8

Example 5 Subtract 1837 from 70,000.
Solution 1 less than extreme left digit (7) = 6
Now   1 less from 9   = 8
               8 less from 9   = 1
               3 less from 9   = 6
Last digit   7 less from 10   = 3
i.e.,   Remainder will be 68163.
So, 70,000 − 1837 = 68163.

Example 6 Subtract 569 from 854.
Solution 854 − 569
Step 1 Here 4 < 9
So we take complement of 9 − 4 = 5.
Complement will be taken from 10. Therefore, complement of 5 is 5, which we write in unit place.
Step 2 Since 5 < 6 so difference of 5 and 6 is 1. Subtracting 1 from the complement 9 will give 8.
Step 3 1 less than 8, 8 − 1 = 7. Subtracting 5 from 7 gives 2 which we write at hundreds place.
854 − 569 = 285.

6.5   Interesting Multiplication Methods

You have learnt multiplication using Nikhilam method in class VI. In this class, we will study easy methods of multiplication.

6.5.1 Multiplication of any number by 10

For example $5 \times 10 = 50$ $10 \times 10 = 100$
                   $68 \times 10 = 680$.

Look at these three examples and discuss with your friends 'what difference do you visualize in original numbers (5, 10, 68) when multiplied by 10'? Perhaps, you will agree that 0 appears at unit place and original number shifts to tens place and ahead.

Do and learn ◆

1. If a number is multiplied by 100 and 1000 then what change do you observe in the product? Discuss with your friends.
2. Divide the class into two groups. One group should ask the question of multiplying the numbers by 10, 100 and 1000 and another group should answer and vice-versa. Play it like "ANTYAKSHARI".

6.5.2 Multiplication of a number by 5

You have learnt the multiplication of a number by 10. Now we will learn multiplication by 5 in a simple and interesting manner.

(i) 18×5

$$= 18 \times \frac{10}{2} \quad \text{(5 being the base of 10 and can be written as } 5 = \frac{10}{2}$$

$$= \frac{18}{2} \times 10 \ = 9 \times 10 \quad \left(\frac{18}{2} = 9\right)$$

$$= 90$$

(ii) 29×5

$$= 29 \times \frac{10}{2} \qquad \left(5 = \frac{10}{2}\right)$$

$$= \frac{29}{2} \times 10 \qquad \left(\frac{29}{2} = 14.5\right)$$

$$= 14.5 \times 10 \qquad \left(14.5 = \frac{145}{10}\right)$$

$$= \frac{145}{10} \times 10 \ = 145$$

So, while multiplying a number by 5 the result is obtained by taking half of the number and multiplying it by 10.

Do and learn ◆

1. Can we use the method of multiplication by 5 in the case of 50 and 500?
2. Can we multiply a number by 25 in the form of $\frac{100}{4}$ discuss in you class.

6.5.3 Multiplication of a number by 9 (Formula – EkNeunenPoorven Method)

Example 7 Multiply 6 by 9.

Solution

$$\begin{array}{r} 6 \\ \times\ 9 \\ \hline \dot{6}\,/9\text{-}\dot{6} \\ 5/4 \\ =54 \end{array}$$

(I) We use EkNeunenPoorven formula. So, we put the sign of Ek Neunen and write 6 on the left hand side of slash.

(ii) We subtract the ekneunen of multiplicand 6 from 9 on right hand side.

Example 8 Multiply 12 by 9.
Solution

$$12$$
$$\times 9$$
$$\overline{12/9-12}$$

$$11/9-11$$

$$11/-2 \text{ या } (\overline{2})$$

$$11\overline{2}$$

$$= 108$$

(i) Here the multiple is 9 but the multiplicand is greater than 9.

(ii) Using EkNeunenPoorven put one less than 12 = 11 on the left hand side of slash

(iii) Subtract 11 from 9 (one less than 12), i.e., 9-11 on the right hand side of slash

(iv) There is 11 on the left and -2 or $\overline{2}$ on the right of slash.

(v) Removing the slash in $11\overline{2}$ and converting in normal form we obtain 108.

6.5.4 Multiplication of a number by 99

You have learnt the multiplication of a number by 9. Let us now learn the multiplication by 99. The method of multiplication by 99 is same as that of 9. So, we understand by an example using EkNeunenPoorven method.

Example 9 Solve 18 × 99.
Solution

$$18$$
$$\times 99 \qquad \text{(Symbols as usual)}$$
$$\overline{18/99-18}$$

$$17/99-17$$

$$17/82$$

$$=1782$$

Example 10 Solve 99 × 99.
Solution

$$99$$
$$\times 99$$
$$\overline{99/99-99}$$

$$98/99-98$$

$$98/1$$

Is 99 × 99 = 981 correct? If not then could you discover the point of mistake? Yes you are correct. The base on the right hand side is 100 so, there should be two digit number but it is not so. Therefore we will write 1 as 01.

So the solution will be 9801 and not 981.
Can you multiply a number by 999 and 9999?

Do and learn ◆

Multiply any number by 999 and 9999 yourself and ask you teacher in case of any problem.

6.5.5 Multiplication of a number by 11

Let us learn a simple method of multiplication by 11.

Multiply 72 by 11.

$$\begin{array}{r} 72 \\ \times 11 \\ \hline 72 \end{array}$$

$$\begin{array}{r} 72\times \\ \hline =7(7+2)2 \end{array}$$

$$=7\,9\,2$$

Let us see one more method:

72×11

$72 \times (10+1)$

$720+72$

i.e., $7(7+2)\,2$

$= 792$

We find, in both the method, that there lies the sum of both the digits of multiplicand in between the digits of multiplicand.

Example 11   Multiply 81 by 11.
Solution

$$\begin{array}{r} 81 \\ \times 11 \\ \hline 8/(8+1)/1 \end{array}$$

$$891$$

Check if $81 \times 11 = 891$.

Example 12   Multiply 99 by 11.
Solution

$$\begin{array}{r} 99 \\ \times 11 \\ \hline 9/(9+9)/9 \end{array}$$

(in the number 18, 8 will remain at tens place and 1 will be added to the number at hundreds place)

$$9/18/9$$
$$=1089$$

Could we apply this rule in case of three digit numbers?
Discuss and practice such problems.

Exercise 6.2

1. Subtract using Nikhilam formula:

(i)   9000
     -3768
	(ii) 5872
	     -2987
		(iii) 4987
		     -1898

2. Multiply using appropriate formula:

    (i) 87×10 (ii) 53×100 (iii) 432×1000 (iv) 64×5

    (v) 72×50 (vi) 81×99 (vii) 99×999 (viii) 99×9

6.6   Fractions

You are familiar with fractions. We make fractions easy using Vedic Mathematics. Look at the following fractions carefully:

Example 13 $\dfrac{5}{8}, \dfrac{3}{8}, \dfrac{7}{8}, \dfrac{1}{8}$ Arrange in increasing order.

Solution   Denominators are same but numerators are different in thee fractions.

We can write them in increasing order.

$$\frac{1}{8}, \frac{3}{8}, \frac{5}{8}, \frac{7}{8}$$

The fraction in which the denominators are same, greater is the numerator greater would be the fraction. If numerators are same then a fraction having greater denominator would be smaller than that having smaller numerator.

Arrange $\dfrac{1}{3}, \dfrac{1}{5}, \dfrac{1}{4}, \dfrac{1}{2}$ in increasing order.

Here, the denominator 5 is the largest number. So, the smallest fraction would be $\dfrac{1}{5}$ and the largest fraction would be $\dfrac{1}{2}$ Arranging in increasing order

$$\frac{1}{5}, \frac{1}{4}, \frac{1}{3}, \frac{1}{2}$$
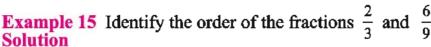
Example 14   Identify the greater fraction in $\dfrac{3}{4}$ and $\dfrac{4}{5}$

Solution

$$\frac{3}{4} \diagdown \diagup \frac{4}{5}$$
$$\overline{15 \quad 16}$$

(i)   Write the numerators and denominators of these fractions without lines.

(ii)   Cross multiplications are $3 \times 5 = 15$ and $4 \times 4 = 16$.

(iii)   Fraction lying on the side of larger product will be greater.

(iv)   Since $15 < 16$, so $\dfrac{3}{4} < \dfrac{4}{5}$

Example 15 Identify the order of the fractions $\frac{2}{3}$ and $\frac{6}{9}$

Solution

$$\frac{2}{3} \diagdown \frac{6}{9}$$
$$\overline{18 \quad 18}$$

(i) Cross multiplications are $3 \times 6 = 18$ and $2 \times 9 = 18$.
(ii) Products are same, so the fractions are equal.
(iii) Hence, these are equivalent fractions.

Exercise 6.3

1. Put appropriate sign between following fractions (any one of $<, =, >$):

   (i) $\frac{4}{9} \square \frac{3}{9}$ (ii) $\frac{4}{5} \square \frac{4}{10}$ (iii) $\frac{3}{5} \square \frac{6}{10}$

   (iv) $\frac{5}{7} \square \frac{6}{7}$ (v) $\frac{2}{3} \square \frac{3}{2}$

2. Arrange following fractions in ascending order:

   (i) $\frac{3}{7}, \frac{4}{7}, \frac{2}{7}, \frac{5}{7}$ (ii) $\frac{3}{5}, \frac{3}{7}, \frac{3}{4}, \frac{3}{8}$

3. Arrange following fractions in descending order:

   (i) $\frac{4}{5}, \frac{1}{5}, \frac{2}{5}, \frac{3}{5}$ (ii) $\frac{4}{6}, \frac{4}{7}, \frac{4}{8}, \frac{4}{5}$

6.6.1 Sum of fractions

If the denominators in the fractions are same then:

Example 16 Find the sum $\frac{1}{5} + \frac{2}{5}$

Solution

$$= \frac{1+2}{5} \quad \frac{\text{sum of numerators}}{\text{denominator}}$$

So, sum of fractions $= \dfrac{\text{sum of numerators}}{\text{denominator}}$

If the denominators in the fraction are not same then

Example 17 Find the sum of fractions $\frac{2}{3}$ and $\frac{4}{5}$

Solution

$$\frac{2}{3} + \frac{4}{5}$$

$$= \frac{2 \times 5 + 3 \times 4}{3 \times 5}$$

Cross multiplication values are $2 \times 5 = 10$ and $3 \times 4 = 12$
Product of denominator digits $3 \times 5 = 15$

$$=\frac{10+12}{15}\quad=\frac{22}{15}\quad=1\frac{7}{15}$$

Example 18 $\quad\frac{1}{2}+\frac{2}{3}+\frac{4}{5}$ Find the sum

Solution

$$\frac{1\times3\times5+2\times2\times5+4\times2\times3}{2\times3\times5}$$

Cross multiplication values are $1\times3\times5$, $2\times2\times5$ and $4\times2\times3$.

$$=\frac{15+20+24}{30}$$

Product of denominator digits $2\times3\times5=30$

$$=\frac{59}{30}\quad=1\frac{29}{30}$$

When given fractions do not have equal denominators and there exist common factors:

Example 19 Solve $\quad\frac{1}{4}+\frac{1}{10}$

Solution

$$\frac{1\times10+1\times4}{4\times10}\quad=\frac{10+4}{40}\quad=\frac{14}{40}$$

(To convert in to the simplest form, divide by same number)

$$=\frac{14\div2}{40\div2}\quad=\frac{7}{20}$$

(To be written in the simplest form)

6.6.2 Sum of mixed fractions (by Vilokanam formula and cross multiplication)

Product of mixed fractions could be evaluated easily by using Vilokanam formula and cross multiplication.

$$1\frac{3}{4}+2\frac{1}{3}\qquad\text{(Split mixed fractions using Vilokanam formula)}$$

$$1\frac{3}{4}=1+\frac{3}{4}\quad\text{and}\quad2\frac{1}{3}=2+\frac{1}{3}$$

$$=1+\frac{3}{4}+2+\frac{1}{3}$$

$$=(1+2)+\left(\frac{3}{4}+\frac{1}{3}\right)\qquad(\frac{3}{4}+\frac{1}{3}\ \text{By cross multiplication})$$

$$=3+\frac{3\times3+1\times4}{4\times3}\quad=3+\frac{9+4}{12}\quad=3+\frac{13}{12}\quad=3+1\frac{1}{12}\ \text{(using vilokanam)}$$

$$=(3+1)+\frac{1}{12}\quad=4+\frac{1}{12}\quad\text{or}\ 4\frac{1}{12}$$

6.7   Subtraction of fractions

Subtraction operation of fractions is similar to the addition operation of fractions. We would use (+) sign in addition operation and (-) sign in subtraction operation.

6.7.1   Subtraction of fractions when denominators are equal

Example 20   Solve $\dfrac{3}{5} - \dfrac{1}{5}$

Solution

$$\dfrac{3}{5} - \dfrac{1}{5} = \dfrac{3-1}{5} = \dfrac{2}{5}$$

6.7.2   Subtraction when denominators in fraction are not same and the common factors do not exist

Example 21   Solve $\dfrac{4}{5} - \dfrac{2}{3}$

Solution

$$\dfrac{4\times3-5\times2}{5\times3} = \dfrac{12-10}{15} = \dfrac{2}{15}$$

Example 22   Solve $\dfrac{1}{2} + \dfrac{1}{3} - \dfrac{1}{5}$

Solution

$$\dfrac{1\times3\times5+1\times2\times5-1\times2\times3}{2\times3\times5} \quad \text{(solution is like addition of fractions)}$$

$$= \dfrac{15+10-6}{30} = \dfrac{19}{30}$$

6.7.3 Subtraction of mixed fractions

Similar to addition operation, the subtraction of mixed fractions can be carried out using Vilokanam and cross multiplication.
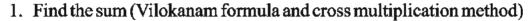
Example 23   Solve $3\dfrac{3}{4} - 3\dfrac{2}{5}$

Solution

$$\left(3+\dfrac{3}{4}\right)-\left(3+\dfrac{2}{5}\right)$$

$$(3-3)+\left(\dfrac{3}{4}-\dfrac{2}{5}\right)$$

$$= 0 + \dfrac{3\times5-4\times2}{4\times5}$$

$$= \dfrac{15-8}{20} = \dfrac{7}{20}$$

Exercise 6.4

1. Find the sum (Vilokanam formula and cross multiplication method)

(i) $\dfrac{1}{9} + \dfrac{4}{9}$ (ii) $\dfrac{7}{15} + \dfrac{2}{15}$ (iii) $\dfrac{1}{2} + \dfrac{3}{5}$

(iv) $\dfrac{4}{3} + \dfrac{2}{5}$ (v) $\dfrac{1}{3} + \dfrac{1}{4} + \dfrac{1}{5}$ (vi) $\dfrac{1}{2} + \dfrac{2}{3} + \dfrac{3}{5}$

2. Subtract the following (Vilokanam formula and cross multiplication method)

(i) $\dfrac{9}{10} - \dfrac{3}{10}$ (ii) $\dfrac{19}{5} - \dfrac{4}{5}$ (iii) $\dfrac{1}{2} + \dfrac{1}{3} - \dfrac{1}{6}$

(iv) $\dfrac{1}{3} + \dfrac{1}{4} - \dfrac{1}{5}$ (v) $3\dfrac{1}{2} - 1\dfrac{3}{4}$ (vi) $2\dfrac{5}{6} - 2\dfrac{1}{6}$

6.8   Product of fractions

Product of two fractions can be evaluated very easily. In this, the product of numerators is placed at numerator and the product of denominators is placed at denominator in the resulting fraction.

Multiply $\dfrac{1}{2}$ and $\dfrac{3}{4}$

$\dfrac{1}{2} \times \dfrac{3}{4}$ $\dfrac{1\times3}{2\times4} = \dfrac{3}{8}$

6.8.1 Product of two mixed fractions (use of 'EkadhikenPurven' formula)

If the sum of fractional parts of a mixed fraction is 1 and base and the Nikhilam digit is same then the product can be written in two parts as in the case of product of two simple numbers using "EkadhikenPoorven Formula".

Example 24   Solve $6\dfrac{1}{4} \times 6\dfrac{3}{4}$

Solution   (i) Sum of fractional parts $\dfrac{1}{4}, \dfrac{3}{4}$ $\dfrac{1}{4} + \dfrac{3}{4} = \dfrac{1+3}{4} = \dfrac{4}{4} = 1$

(ii)   Nikhilam digits are same $= 6$

(iii) Left Hand Side = First Part = Nikhilam Digit $\times$ Its Ekadhik (One more)

$6 \times (6+1) \Big/ \dfrac{1}{4} \times \dfrac{3}{4} = \dfrac{3}{16}$

(iv) Right Hand Side = Second Part = Product of fractional part

So $6\times(6+1)+\dfrac{1}{4}\times\dfrac{3}{4}$

$6\times7+\dfrac{3}{16}$

$42+\dfrac{3}{16}=42\dfrac{3}{16}$

Example 25   Multiply the fractions $15\dfrac{4}{7}\times15\dfrac{3}{7}$

Solution

$15\times(15+1)/\dfrac{4}{7}\times\dfrac{3}{7}$

$15\times16/\dfrac{12}{49}$

$240\dfrac{12}{49}$

6.8.2 Product of two fractions (using Vilokanam formula)

Example 26   Multiply the fractions $5\dfrac{1}{2}\times6$

Solution $\left(5+\dfrac{1}{2}\right)\times6$ (Vilokanam Formula)

$=5\times6+\dfrac{1}{2}\times6$ (Solution of bracket)

$=30+3$ (Half of $6=3$)

$=33$

Verification of answer: $5\dfrac{1}{2}\times6$

$=\dfrac{11}{2}\times6$ $\left(5\dfrac{1}{2}=\dfrac{11}{2}\right)$

$=11\times\dfrac{6}{2}$

$=11\times3$ (Half of $6=3$)

$=33$

Example 27   Multiply the mixed fraction $7\dfrac{1}{2}\times8\dfrac{1}{2}$
Solution

$\left(7+\dfrac{1}{2}\right)\times\left(8+\dfrac{1}{2}\right)$(By Vilokanam Formula)

$$7 \times 8 + 7 \times \frac{1}{2} + 8 \times \frac{1}{2} + \frac{1}{2} \times \frac{1}{2}$$

$$= 56 + 3\frac{1}{2} + 4 + \frac{1}{4}$$

$$= 56 + 3 + 4 + \frac{1}{2} + \frac{1}{4}$$

$$= 63 + \frac{6}{8} \qquad \left(\frac{6 \div 2}{8 \div 2} = \frac{3}{4} \right)$$

$$= 63\frac{3}{4}$$

Alternative Method $7 \times 8 + \frac{1}{2} \times \frac{1}{2} + (7 + 8)\frac{1}{2}$

$$= 56 + \frac{1}{4} + 15 \times \frac{1}{2}$$

$$= 56 + 7 + \left(\frac{1}{4} + \frac{1}{2} \right)$$

$$= 63 + \frac{3}{4}$$

$$= 63\frac{3}{4}$$

Exercise 6.5

Multiply the following numbers using Vilokanam formula.

(1) $\frac{1}{8} \times \frac{3}{5}$ (2) $5\frac{1}{2} \times 5\frac{1}{2}$ (3) $2\frac{3}{4} \times 2\frac{1}{4}$

(4) $3\frac{2}{5} \times 3\frac{3}{5}$ (5) $12\frac{1}{4} \times 12\frac{3}{4}$ (6) $8\frac{2}{7} \times 8\frac{5}{7}$

(7) $3\frac{1}{4} \times 4$ (8) $2\frac{1}{5} \times 5$ (9) $3\frac{1}{2} \times 4$

(10) $4\frac{1}{3} \times 6$

6.9   Square Numbers

Square numbers are those for which the prime factors exist in pairs. For example: 4 is a square number because its prime factors $= 2 \times 2$.

Here 2 is in a pair.

Is 100 a square number?

Let us find the prime factors of 100. These are $2 \times 2 \times 5 \times 5$. Here we have two pairs of 2 and 5. Hence both the numbers are square numbers.

Let us decide which numbers squares these are?

Prime factors of $4 = 2 \times 2$. Here we have a pair of 2, so it is a square of 2.

Prime factors of $100 = 2 \times 2 \times 5 \times 5$ (Pairs of 2 and 5)

So, $2 \times 5 = 10$, i.e., it is the square of 10.

To find the square of a number we multiply the number by itself. Let us discuss some easy method to find square of the numbers.

(1) **Finding the square of two/three digit numbers in which unit place digit is 5:**

(i) 15×15 $=$ $1 \times (1+1)/5 \times 5$ (Ekadhikenpoorven of tens digit)

$=$ $1 \times 2/25$

$=$ $2/25$

$=$ 225

(ii) 35×35 $=$ $3 \times (3+1)/5 \times 5$ (Ekadhikenpoorven of tens digit)

$=$ $3 \times 4/25$

$=$ 1225

(iii) 95×95 $=$ $9 \times (9+1)/5 \times 5$

$=$ $9 \times 10/25$

$=$ 9025

(iv) 105×105 $=$ $10(10+1)/5 \times 5$

$=$ $10 \times 11/25$

$=$ $110/25$ $=$ 11025

(v) 125×125 $=$ $12(12+1)/5 \times 5$

$=$ $12(13)/25$

$=$ 15625

It is clear from the examples that a number having 5 at ones place is multiplied by itself or upon squaring a number there appears 25 in the last essentially. We write the product of tens digit and its next digit (Ekadhiken) before it.

Square of numbers having 5 at tens place

The numbers 51 to 59 are the only numbers having 5 at tens place.

So, 51^2 $=$ 51×51

$=$ $\underline{26}\ \underline{01}$

$1 \times 1 = 01$ (Square of unit place digit)

$5 \times 5 + 1 = 26$ (Square of tens place digit + unit place digit)

53^2 = 53×53

28 09

$3 \times 3 = 09$ (Square of unit place digit)

$5 \times 5 + 3 = 28$ (Square of tens place digit + unit place digit)

59^2 = 59×59

34 81

$9 \times 9 = 81$

$5 \times 5 + 9 = 34$

Square of three digits numbers having 25 in the end

125^2 = 125×125

$(25 \times 25 = 625)$

$1 \times 15 = 15$ (Product of the number formed by unit and hundreds place in 125 with 1 at hundreds place)

So, 125^2 = 15625

325^2 = 325×325

$(25 \times 25 = 625)$

$3 \times 35 = 105$ (Product of the number formed by unit and hundreds place in 325 with 3 at hundredth place)

So, 325^2 = 105625

725^2 = 725×725

$(25 \times 25 = 625)$

$7 \times 75 = 525$

= 525625

625 always appears in the last in such cases. Product of the number formed by the digits at unit and hundredth place and the digit at hundredth place is put on the left hand side of 625.

Other methods of squaring the numbers

11×11 = 1 2 1 ⟵——— Square of digit at unit place

——— Double of the product of digits at tens place

——— Square of digit at tens place

31^2 = Square of digit at unit place in $31 \times 31 = 1 \times 1 = 1$

Double of the product of digits at tens place $(1 \times 3) \times 2 = 6$

Square of digit at tens place = $3 \times 3 = 9$

= 961.

12×12 = Square of digit at unit place $= 2 \times 2 = 4$

Double of the product of digits at ones and tens place $(1 \times 2)\ 2 = 4$

Square of digit at tens place $= 1 \times 1 = 1$

So square of $12 = 144$.

To find the square of three digit numbers we divide it into two parts and use Anurupyen method. Anurupyen means "Similarity and proportions".

For example: If we want to find the square of 152 then divide 152 in two parts 15 and 2.

$$152 \times 152 =$$

Square of digit at unit place $= 2^2 = 4$
Double of the product of digits at ones & tens place $= 2 \times 15 \times 2 = 60$
Square of digit at tens place $= 15^2 = 225$

225/60/4

225+6/04

$= \underline{23104}$

Here we observe the following for the number whose square is to be evaluated:

1. To find the square of the digit at tens place in the first part from the right.
2. Multiply the digits at middle part of main number and double it.
3. Square the second digit in the main number in third part.
4. Arrange the number.

Example 28 Find the square of 43.
Solution

$$43^2 = \overset{\text{III}}{4^2}\ \ \overset{\text{II}}{4 \times 3}\ \ \overset{\text{I}}{3^2}$$

$$\underline{\hspace{1cm} 4 \times 3 \hspace{1cm}}$$

16   12   9

$$\underline{\hspace{0.8cm} + 12 \hspace{1cm}}$$

16   24   9

16+2   49

1849

(Carry over number of middle part
(II) is added with 16)

Example 29 Find $(132)^2$
Solution

$$\overset{\text{III}}{(13)^2}\ \ \overset{\text{II}}{13 \times 2}\ \ \overset{\text{I}}{2^2}$$

$$\underline{\hspace{1cm} +13 \times 2 \hspace{1cm}}$$

169   26   4

$$\underline{\hspace{1.2cm} 26 \hspace{1cm}}$$

169   52   4

169+5   24

17424

<u>13 2</u>

Divide 132 in to two parts 13 and 2.

<p align="center">⟫⟪⚬◯⟪ **Exercise 6.6** ⟫◯⚬⟫⟪</p>

1. Find the square using above method:

(i) 18   (ii) 42   (iii) 83   (iv) 127   (v) 136

6.10 Square Root

When a number x is multiplied by itself then the value so obtained is x^2, called square number of the number x. Let us understand this way that x^2, is the pair $x \times x$. So, the square root of x^2 is x.
16 is a square number which is a pair of 4×4. So, the square root of 16 is 4.
Symbolic notation of square root is $\sqrt{}$

Digits of square root

Number of digits in a square of a number is double of the digits in it or one less than the double of the digits. Similarly, if the number of digits in the square root of a square number is even then it is half of it and one more than the half if odd. Let us look at the table given below:

No. of digits in a square number is odd				No. of digits in a square number is even			
Square Number	No. of Digits	Square Root	No. of Digits	Square Root	No. of Digits	Square Root	No. of Digits
1	1	1	$\dfrac{1+1}{2}=1$	16	2	4	$\dfrac{2}{2}=1$
100	3	10	$\dfrac{3+1}{2}=2$	81	2	9	$\dfrac{2}{2}=1$
961	3	31	$\dfrac{3+1}{2}=2$	1024	4	32	$\dfrac{4}{2}=2$
16641	5	129	$\dfrac{5+1}{2}=3$	108900	6	330	$\dfrac{6}{2}=3$

The number of digits in the square root of a perfect square number is equal to the number of pairs formed in it from the right hand side (i.e., from unit place) no matter the last pair contains only one digit.

Identification of perfect square number

1. Unit place of the perfect square number is either of 0, 1, 4, 5, 6 and 9, i.e., a number can't be a perfect square number if its unit place digit is either of 2, 3, 7 and 8.
2. Number of zeros in the last places of a perfect square number is even and the numbers on the left hand side of zeros should be a square number.
3. If sum of the digits in a number is 2, 3, 5, 6 and 8, then it can't be a perfect square.

Vedic method of finding the square root

1. First of all find if the number is a perfect square.
2. If it is a perfect square then find the number of digits.
3. Identify the unit place

Unit digit in the number	Unit digit in the square root
1	1 or 9
4	2 or 8
5	5
6	4 or 6
9	3 or 7

Now, find by using the Vilokanam method; what is the digit at tens place in the perfect square number?

Number Group	Tens Digit in Square root
1 − 3	1
4 − 8	2
9 − 15	3
16 − 24	4
25 − 35	5
36 − 48	6
49 − 63	7
64 − 80	8
81 − 99	9

Group 1 − 3 means it contains the numbers 1, 2, 3 and the possible square root of them could be considered as 1.

Vilokanam method of finding the square root is explained with following example:

Example 30   Find the square root of the number 361.
Solution   Following can be concluded by observing the number.

   (i)   Unit place of 361 is 1, so it could be a perfect square.
  (ii)   Sum of digits = 3+6+1 = 10, so the sum of digits is 1+0=1, it could be a perfect square.
 (iii)   There should be two digits in the square root of the number.

(iv) Making the pair in the number 361 we find that the second pair contains only 3. So tens place digit in the square root is 1.

(v) Ultimate digit in the number is 1, so, the ultimate digit in the square root would be either 1 or 9 and tens place digit would be 1 because 3 lies in the number group 1 – 3.

(vi) The square root of 361 could be 11 or 19.

(vii) Multiply the tens place digit by its successor.
Product = $1 \times 2 = 2$, 3 in second pair > product 2.
So, larger of the two is the square root, i.e., 19.

Example 31 Find the square root of 5184.
Solution (i) First pair = 84 and second pair = 51

(ii) Ultimate digit in first pair = 4 so, the ultimate digit in possible square root could be 2 or 8

(iii) The greatest square root included in 51 is 7, so the possible square root is 72 or 78. Product $7 \times 8 = 56$.

(iv) 51 < 56, so smaller of the two would be the square root. Square Root = 72.

Note: This method is applicable only up to four digit numbers

Exercise 6.7

Find the square root by using Vilokanam method:.

(1)	169	(2)	324	(3)	576	(4)	2025
(5)	3025	(6)	9025	(7)	1024	(8)	441

6.11 Division Operation

When a number is subtracted from a number repeatedly, then successive subtraction is called division operation. The number from which subtraction is carried out is called **dividend** and the number which is subtracted is called **divisor**. The number of times we subtract a given number is called **quotient**. The number left out by subtracting a number up to maximum time is called **remainder**. Remainder is always less than the divisor.

Example 32 Subtracting 2 successively from 10.
Solution $10-2 = 8$, $8-2 = 6$, $6-2 = 4$, $4-2 = 2$, $2-2 = 0$

Here 10 is dividend and 2 is divisor. Subtraction is made 5 times and the remainder is less than the divisor. So, quotient = 5 and remainder = 0.

6.11.1 Paravartya Yojyet Method (Transpose and Apply Method)

This method is used when the divisor is near to the base. In this method, the dividend is divided by the base of divisor and estimated quotient and remainder are calculated. This method has two categories:

(a)   When divisor is greater than base.

(b)   When divisor is less than base.

(a) When divisor is greater than base

(i)   Calculate the deviation of divisor from the base.

(ii)   Calculate the correction factor by transposing the deviation.(change the sign)

(iii)   Divide by correction factor leaving first digit in the dividend

(iv)   Divide the division operation in three parts. Learn by following example:

Example 33   Solve $4656 \div 11$.

Solution

Dividend	11	4 6 5	6
Base	10	$\overline{4}$ -	-
Deviation	1	$\overline{2}$	-
Correction	$\overline{1}$		$\overline{3}$
factor	Quotient	**4 2 3**	Reminder 3

Procedure

1. Make three parts for completing the division operation

2. Put divisor in first part, dividend in second part and in third part the digits equal to the number of zeros in the base.

3. Find base, deviation and correction factor.

4. Write below the first digit on the right of divisor.

5. Multiply this digit by correction factor write below the number after dividend

6. Subtract and write below and then multiply by correction factor. Repeat this process till we get digit in the third part.

Example 34   Solve $35984 \div 112$
Solution

Dividend	1 1 2	3 5 9	8 4
Base	1 0 0	$\overline{3}\ \overline{6}$	- -
Deviation	1 2		$\overline{2}$	$\overline{4}$
Correction factor	$\overline{1}\ \overline{2}$		$\overline{1}\ \overline{2}$
	Quotient	**3 2 1**	Reminder 32

(b)   When divisor is less than base:

Solve according to the procedure used earlier. It is explained with following example:

Example 35   Solve $30103 \div 9$
Solution

Dividend	9	3 0 1 0	3
Base	1 0	3 - -	-
Deviation	$\overline{1}$	3 -	-
Correction factor	1	4	4
	Quotient	3 3 4 4	Reminder 7

Remember, this time the correction factor is positive. So it will be added to next number.

In the given example we need to divide by 9 which is one less than the nearest base 10.

We will write the first digit 3 as it is in the quotient and then multiply by the correction factor (+1) and add to next number 0. Write the quotient 3 below the horizontal number. Again, multiply this by correction factor and add to the next number and write as quotient. Repeat these steps till the end.

Example 36   Solve $11022 \div 89$.
Solution

Dividend	8 9	1 1 0	2 2
Base	1 0 0	1 1	-
Deviation	$\overline{1}\ \overline{1}$	2	2
Correction factor	1 1		3 3
	Quotient	**1 2 3**	Reminder 75

Exercise 6.8

Solve following questions:

(1) 23244 ÷ 11   (2) 12064 ÷ 12   (3) 1234 ÷ 112

(4) 324842 ÷ 101   (5) 2012 ÷ 9   (6) 10321 ÷ 98

We Learnt
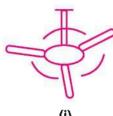
1. The addition and subtraction by calculating the deviation of numbers from 10 or multiples of 10 based upon Sankalan Vyavkalnabhyam formula

2. Learnt addition and subtraction by making two numbers near complete by using Poornapoornabhyam formula.

3. The subtraction by using NikhilamNavtahCharamdashatah formula.

4. Some interesting multiplication method of Vedic Mathematics, in which we learnt to multiply the numbers by 10, 100, 1000, 5, 50, 500 and 11 orally. Also learnt multiplication of numbers by 9, 99, 999 using EkNeunen method.

5. Learnt easy Vedic Mathematical methods for operations on fractions, square root using sub-formula Anurupen, Vilokanam and Nikhilam method respectively.
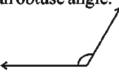
Lines and Angles
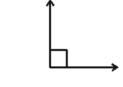
7.1 Look at the following figures carefully:

| (i) | (ii) | (iii) |

Look at the following angles and identify whether it is an acute angle, a right angle or an obtuse angle.

....
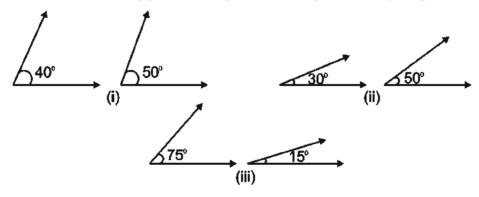
7.1.1 Supplementary Angle

When the sum of two angles is 90°, then they are mutually called **complementary angles**. For example: complementary angle of 30° is 60° and the complementary angle of 60° is 30° (30° + 60°= 90°). What will be the complementary angle of 45°?

Which of the following pairs of angles are complementary angles?
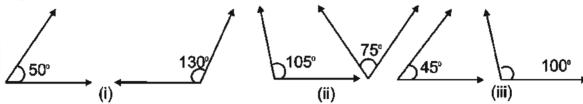
Do and learn: ◆
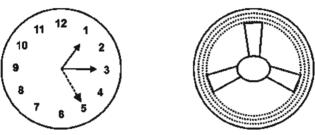
1.Can two acute angles be complementary angles of each other?
2.Can two obtuse angles be complementary angles of each other?
3.What is the complementary angle of a right angle?

7.1.2 Supplementary Angles

When the sum of two angles is $180°$, then they are called **Supplementary angles** of each other. Which of the following pairs of angles are Supplementary angles?
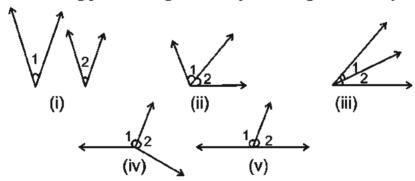
7.1.3 Adjacent Angles

In the above figures you can observe two angles placed next to each other. Where else you find two angles placed next to each other?

Such pairs of angles are called adjacent angles.

Adjacent angles have a common vertex; a common arm and both the angles are on the opposite sides of the common arm. It means they have no common interior points.

Which of the following pairs of angles are adjacent angles and why? Discuss.

Discussion in Mahak's class was as follows :

Mahak: Figure (i) and (iii) do not form adjacent angles because figure (i) does not have a common vertex and figure (iii) does not have a common arm in between them.
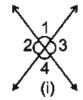
Chanda: Yes, rest of the three figures form adjacent angles and in figure (v) two arms which are not common form a straight line too.
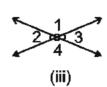
Mahak: But straight line forms an angle of $180°$.

Pair of linear angles: Those adjacent angles in which the sum of angles formed on both the sides of common arm is 180° is called pair of linear angles. These angles are Supplementary too.

7.1.4   Opposite Angles (Vertically Opposite Angles)
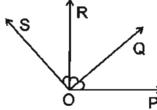
Trace the given figures on a paper with help of a trace paper. Cut off all the four angles from each figure with the help of scissors and separate them. Now put the angles on one another and see which angles are equal. You will find in each figure that ∠1 is equal to ∠4 and ∠2 is equal to ∠3. The pairs ∠1, ∠4 and ∠2, ∠3 are known as vertically opposite angles. **Vertically opposite angles** are formed when two lines intersect at a point.

⟩⟨∘∘⟨ Exercise 7.1 ⟩∘∘⟩⟨
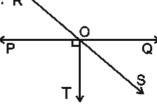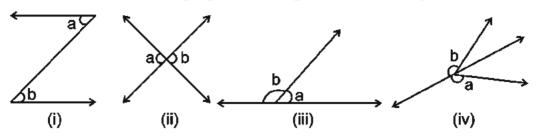
1. Separate out (Filter out) the supplementary and complementary angles from the following pairs of angles:

     (i)   140°, 40°   (ii)   170°, 10°   (iii)   75°, 15°

     (iv) 33°, 57°   (v)   115°, 65°   (vi)   25°, 65°

2. Find those pairs of angles that are complmentary to each other and also of same measure.

3. What will be the value of supplementary angle of a right angle?

4. Write the pairs of adjacent angles for the following figure:
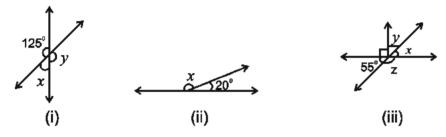
5. Find the following pairs of angles from the given figure:

     (I)   Similar Supplementary Angles

     (ii)   Dissimilar Supplementary Angles

     (iii)   Vertically Opposite Angles

     (iv)   Adjacent Angles which are not linear pair.

     (v)   Adjacent complementary Angles.

6. In which of the following figures the angles a and b are adjacent?

(i) (ii) (iii) (iv)
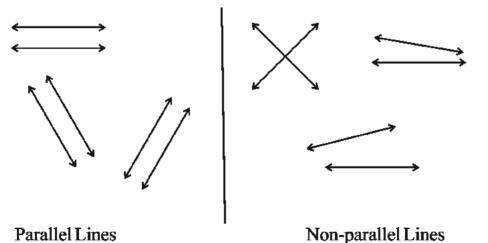
7. Find the value of unknown angle in the following figures:

(i) (ii) (iii)

8. Identify True or False.

   (i) Sum of the angles forming a linear pair is $180°$.

   (ii) Sum of vertically opposite angles is $90°$.

   (iii) If two angles are Supplementary then their sum is $180°$.

   (iv) If two adjacent angles are Supplementary then they are called linear pair.

7.2 Pair of Lines
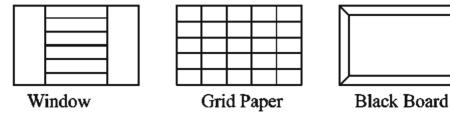
Parallel Lines   Non-parallel Lines

7.2.1 Parallel Lines

Two coplanar lines which do not intersect and the perpendicular distance between them always remain same are called parallel lines.
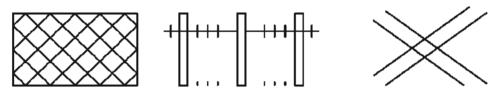
Look at the following figures carefully and identify the parallel lines:

| Window | Grid Paper | Black Board |
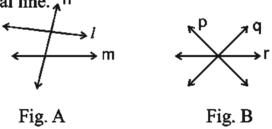
7.2.2 Intersecting Lines

Those lines which are not parallel, i.e., intersect each other are called intersecting lines.

Look at the figures given below carefully and identify the intersecting lines.

7.2.3 Transversal Lines

A straight line which intersects two or more lines at different points is called a transversal line.

Fig. A   Fig. B

In Fig. A, the pair of lines l and m are being cut by a transversal line n at different points. Is there any transversal line in Fig. B? We observe that all the lines in Fig. B intersect at a single point. So, this is not an example of transversal line.
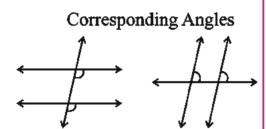
Do and learn: ◆

1. How many transversal lines can be drawn for a pair of lines?
2. If a transversal line is drawn on three lines, then how many points of intersection will we get?
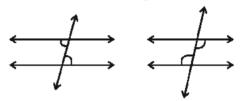
7.2.3.1   Angle formed by Transversal Line

When a transversal line (p) cuts the lines l and m then, 8 different angles are formed. Look at these 8 angles in the figure. The angles formed on the outer side such as $\angle 1$, $\angle 2$, $\angle 7$ and $\angle 8$ are called exterior angles. Similarly, the angle formed in the inner side such as $\angle 3$, $\angle 4$, $\angle 5$ and $\angle 6$ are called interior angles.

Corresponding Angles
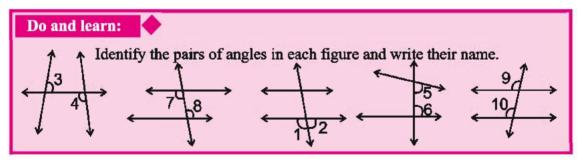
Corresponding angles form F shape.

Alternate Angles

Alternate angles form Z shape.

Pair of Corresponding Angles	$\angle 1$ & $\angle 5$, $\angle 2$ & $\angle 6$, $\angle 3$ & $\angle 7$, $\angle 4$ & $\angle 8$
Pair of Alternate Angles	$\angle 3$ & $\angle 6$, $\angle 4$ & $\angle 5$
Pairs of Alternate Exterior Angles	$\angle 1$ & $\angle 8$, $\angle 2$ & $\angle 7$
Pair of Interior Angles formed on the same side of Transversal Line	$\angle 3$ & $\angle 5$, $\angle 4$ & $\angle 6$
Pair of Exterior Angles formed on the same side of Transversal Line	$\angle 1$ & $\angle 7$, $\angle 2$ & $\angle 8$

Do and learn: ◆

Identify the pairs of angles in each figure and write their name.

7.2.3.2   Transversal of Parallel Lines

Draw the given figure on a paper. Now, cut off all these angles separately. Put $\angle 2$ on $\angle 6$ and see if they are equal? Similarly, put all the corresponding angles on each other and identify if they are equal?
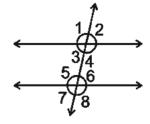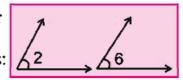
Fig 7.1

You will find that the corresponding angles of parallel lines are equal. Similarly, put all the angles on each other and check the following facts: Are alternate angles equal? Following results are obtained by these activities:

If a transversal line cuts two parallel lines then the alternate angle so formed are equal.
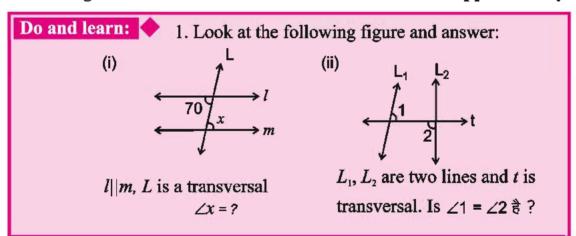
In Fig. 7.1, $\angle 3 + \angle 1 = 180°$ ($\angle 3$ & $\angle 1$ form linear pair)
But $\angle 1 = \angle 5$ (Corresponding Angles)
Similarly, $\angle 5 + \angle 3 = 180°$.
In this manner, following result is obtained:
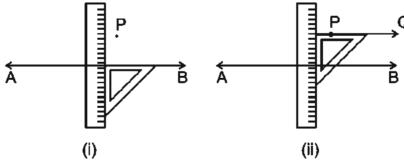
If two parallel lines are cut by a transversal line then each pair of internal angles formed on the same side of transversal are Supplementary.

Do and learn: ◆   1. Look at the following figure and answer:

(i)

70 x

$l\|m$, L is a transversal
$\angle x = ?$

(ii)

L_1 L_2
1
2

L_1, L_2 are two lines and t is transversal. Is $\angle 1 = \angle 2$ है ?

7.3.1 Drawing a line parallel to a given line through an external point
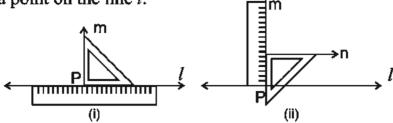
Given the line AB and P is an external point, we need to draw a line through P and parallel to AB.

(i)

(ii)

According to the figure, parallel lines can be drawn using a scale and a set-square.

7.3.2 Drawing a parallel line at a given distance from a given staraight line

P is a point on the line *l*.

(i)   (ii)
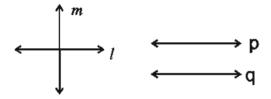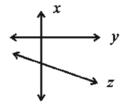
According to Fig. (i), put the right angled side of a set-square on the line *l* and draw a perpendicular line at P.

According to Fig. (ii) Rotate and put the set-square on point P and draw a line *n* (at given distance) parallel to *l* using the other side of set-square.

Exercise 7.2
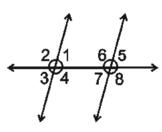
1. Write the names of parallel, intersecting and transversal lines in the following figures:
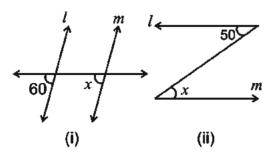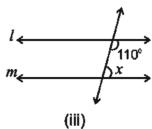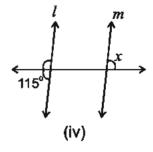
2. Identify the following from the given figure:
   (i)   Interior Alternate Angles
   (ii) Exterior Alternate Angles
   (iii) Corresponding Angles
   (iv) Interior angles on same side of transversal.
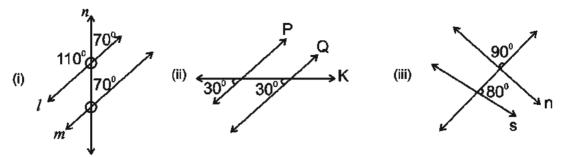
3. If $l \parallel m$, the find the value of x.

(i)   (ii)   (iii)   (iv)
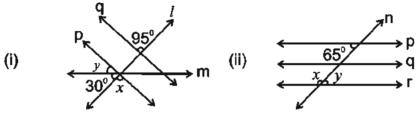
4. Which of the following are the pairs of parallel lines:

(i)

(ii)

(iii)

5. If $p \| q$ and $q \| r$ then find the value of x and y.

(i)

(ii)

6. Draw a line PQ and then draw line RS parallel to it.

7. Draw a line AB and a perpendicular from any point on it. Take a point C on this perpendicular line at 5 cm from AB. Draw a line parallel to AB through C.

⋙ We Learnt ⋘
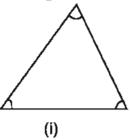
1. (i) When the sum of two angles is 90°, they are called complementary angles.
   (ii) Each angle is acute in a pair of complementary angles.

2. (i) If the sum of two angles is 180°, then they are called supplementary angles.
   (ii) In a pair of supplementary angles, an angle can be acute, right angle or obtuse.
   (iii) Two right angles are always supplementary to each other.

3. The angle formed on both the sides of common arm and common vertex are called adjacent angles.

4. When adjacent angles are supplementary they form linear pair.

5. (i) When two lines intersect on a point (vertex) then the angle formed on opposite to each other are called vertically opposite angles.
   (ii) Pairs of vertically opposite angles are always equal.

6. (i) A line which cuts two or more lines on different points is called transversal line.
   (ii) A transversal line intersecting two lines forms 8 angles, which are shown in figure.
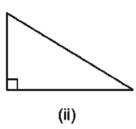
S. No.	Type of Angle	No. of pairs of Angles	Angles
1.	Interior Angles	–	$\angle s$, $\angle r$, $\angle a$, $\angle b$
2.	Exterior Angles	–	$\angle p$, $\angle q$, $\angle c$, $\angle d$
3.	Vertically Opposite Angles	4 pairs	$(\angle p, \angle r)(\angle q, \angle s)(\angle a, \angle c)(\angle b, \angle d)$
4.	Corresponding Angles	4 pairs	$(\angle a, \angle p)(\angle b, \angle q)(\angle c, \angle r)(\angle d, \angle s)$
5.	Interior Alternate Angles	2 pairs	$(\angle s, \angle b)(\angle a, \angle r)$
6.	Exterior Alternate Angles	2 pairs	$(\angle p, \angle c)(\angle q, \angle d)$
7.	Interior angles on the same side of transversal	2 pairs	$(\angle b, \angle r)(\angle a, \angle s)$
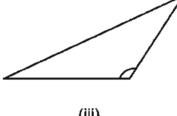
7. When a transversal line cuts two parallel lines then

    (i)   The corresponding angles are equal

    (ii) Alternate interior angles are equal

    (iii) Alternate exterior angles are equal

    (iv) Interior angles formed on same side of transversal are supplementary.

Chapter 8 — Triangle and its Properties

8.1 Triangle is a closed simple figure enclosed by three line segments, it has three sides, three angles and three vertices. Triangles are classified on the basis of sides and angles. Look at the triangles drawn hereunder.

(i)	(ii)	(iii)

What special do you find in these figures?
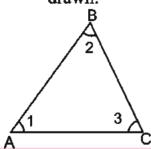
- All three angles of triangle (i) are acute, so it is called Acute Angled Triangle.
- In triangle (ii) one of the angles is right angle, so it is called Right Angled Triangle.
- In triangle (iii) one of the angles is obtuse angle, so it is called Obtuse Angled Triangle.
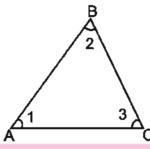
Does the measurement of other two angles change by changing the measurement of one of the three angles?
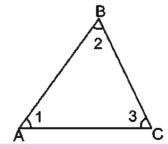
Try by drawing and testing various triangles and fill in the blanks.

Name of Triangle	Angles
$\triangle$ ABC	$\angle A = 50^0$, $\angle B = 60^0$, $\angle C = 70^0$
$\triangle$ ABC	$\angle A = 30^0$, $\angle B = \dots^0$, $\angle C = \dots^0$
$\triangle$ ABC	$\angle A = 100^0$, $\angle B = \dots^0$, $\angle C = \dots^0$
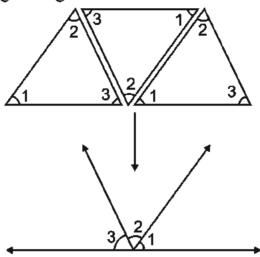
8.2 Property of Sum of Interior Angles of Triangle

1. Draw three triangles of same sides and angles and cut off the figures so drawn.
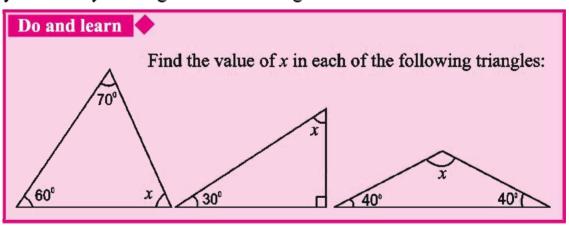
2. Arrange three triangles as given below

∠1, ∠2, ∠3 jointly form a straight angle so ∠1+ ∠2+ ∠3 = 180°.
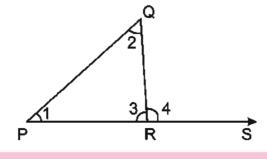
Sum of three interior angles of a triangle is 180°.

Try this fact by drawing some more triangles.

Do and learn ◆

Find the value of *x* in each of the following triangles:
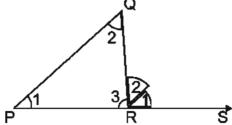
8.3   Exterior angles of a triangle and its properties

1.   Construct a triangle PQR and extend the side PR.

2. Construct one more triangle similar to Δ PQR, cut off the angles ∠1 and
   ∠2 and put them on the exterior angle∠QRS of Δ PQR as given in figure
   given below.
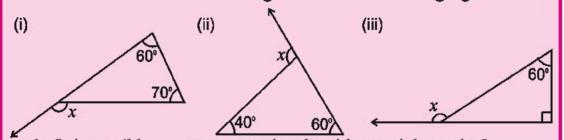
We observe that the angles ∠1 and ∠2, covers the exterior angle ∠QRS of
Δ PQR completely. So

$$\angle QRS = \angle P + \angle Q.$$

**Any exterior angle of a triangle is equal to the sum of two interior
angles on the opposite sides.**

Do and learn: ◆
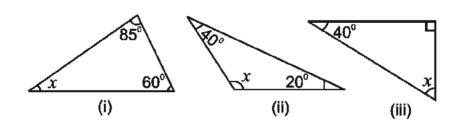
1. Find the value of exterior angle x from the following figures:

(i)   (ii)   (iii)
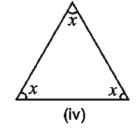
2. Is it possible to construct a triangle with two right angles?
3. Is it possible to construct a triangle whose all the three angles
   are greater than 60°?
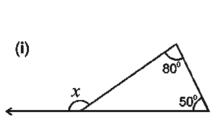
>∞∞∞< **Exercise 8.1** >∞∞∞<
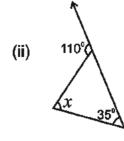
1.   Find the value of unknown angle x for the following figures:

(i)   (ii)   (iii)   (iv)
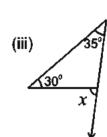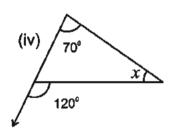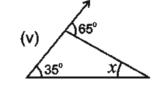
2. Find the unknown angle x for the following figures:

(i)

(ii)

(iii)

(iv)

(v)

(vi)
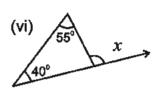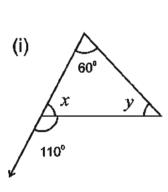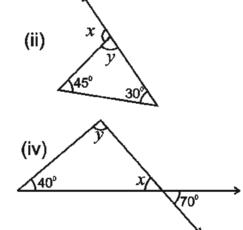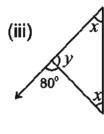
3. Find the values of the unknown angles x and y in the following figures:

(i)

(ii)

(iii)

(iv)

4. If one acute angle of a right angled triangle is 45°, then find another acute angle.

5. If two angles of a triangle are 50° each, then find the third angle.

6. If the angles of a triangle are in the ratio 1 : 2 : 3 then find each angle of the triangle.

7. Is it possible to construct a right angled triangle whose other two angles are 70° and 21°? If not, then why? Justify.

8. Given below are the triads. Which of the following triads represents the angles of a triangle?
    (i) 100°, 30°, 40°   (ii) 30°, 59°, 91°   (iii) 45°, 45°, 90°   (iv) 120°, 30°, 50°

8.4 Relationship among sides of a triangle
8.4.1 Sum of the length of two sides of a triangle
Construct the triangles according to given measurement
1. Δ XYZ having sides 5 cm, 4 cm, 6 cm
2. Δ MNO having sides 6.5 cm, 4.5 cm, 3 cm
3. Δ PQR having sides 5 cm, 6 cm, 12 cm
4. Δ UVW having sides 2cm, 3cm, 5 cm

Were you able to construct triangles from all the given measurements? If not, why? Discuss. Write the measurement of the sides of triangles you constructed in the following table:

Name of Triangle	Length of Side	Sum of Two Sides	Relationship of Sides	Sum of Two Sides is greater than the third	Triangle Exists Yes/No
ΔXYZ	$x = 5$ $y = 4$ $z = 6$	$x + y = 5 + 4$ $y + z = 4 + 6$ $z + x = 6 + 5$	$x + y > z$ $9 > 6$ $y + z > x$ $10 > 5$ $z + x > y$ $11 > 4$	Yes Yes Yes	
ΔMNO	m = n = o =	m + n = n + o = o + m =			
ΔPQR	p = q = r =	p + q = q + r = r + p =			
ΔUVW	u = v = w =	u + v = v + w = w + u =			

We conclude from the above table that **the sum of any two sides of a triangle is always greater than the third side**.

8.4.2   Difference of length of two sides of triangle
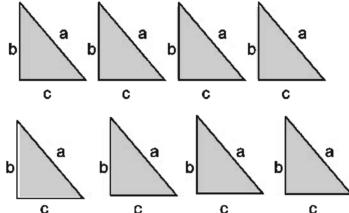
Consider the difference of length of two sides. What do you observe? Is the difference of length of any two sides of a triangle is less than, greater than or equal to third side. Observing few triangles you will find that the difference of length of any two sides of a triangle is less than the third side.

Do and learn: ◆
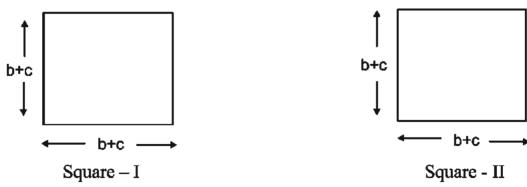
1. Construct a triangle with side lengths 3.5 cm, 4.5 cm and 6 cm.
2. Is it possible to construct a triangle with sides having length 4 cm, 5 cm and 9 cm?

8.5   Bodhayan Theorem (Pythagoras Theorem)
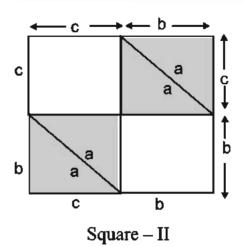
1. Construct a right angled triangle and copy it 8 times on a card sheet and cut them off. Suppose that the length of the side opposite to right angle (hypotenuse) is a and the length of other two sides are b and c.

2. Now find the sum of sides b and c and construct two squares of side measuring the length equal to $(b+c)$ on another card sheet.

Square – I   Square - II
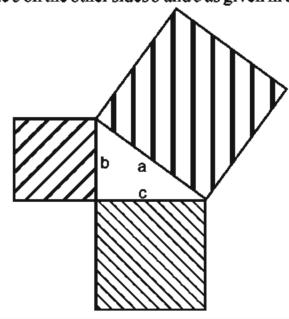
3. Now arrange four triangles in Square-I and four triangles in Square-II as given in following figure

Square – I   Square – II

4. Both the squares are similar and all the 8 triangles are similar. So Area of the blank portion of Square – I = Area of blank portion of Square – II or Area of the square formed in blank portion of Square – I = the sum of areas of the squares formed in the blank space in Square – II.

i.e., $a^2 = b^2 + c^2$.

This relationship of right angled triangles is known as Pythagoras Theorem. This was first derived by the Indian Mathematician Bodhayan and later Pythagoras gave its systematic proof in modern mathematics. This theorem can also be explored in following way. We arrange the square of side a on the longest side (hypotenuse) of Δ abc and squares of side b and side c on the other sides b and c as given in the following figure:
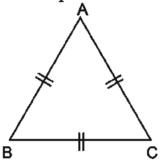
From this, we could say that
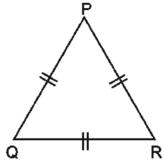
> In right angled triangle, the square of hypotenuse is the sum of squares of other two sides of the triangle. (i.e., base and perpendicular). Symbolically $a^2 = b^2 + c^2$.

8.6   Relation between sides and angles

Construct an equilateral triangle $\triangle ABC$ and its replica $\triangle PQR$ (with help of a trace paper) and cut them off. Now, put the $\angle P$ of $\triangle PQR$ on all three angles of $\triangle ABC$ one by one and observe:

When we put $\angle P$ on $\angle A$, then $\angle Q$ covers $\angle B$ and $\angle R$ covers $\angle C$.
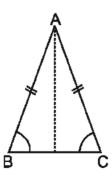
When we put equilateral triangles on each other they overlap completely. Thus, in an equilateral triangle all the sides as well as angles are equal.

Will two angles of a triangle be equal if its two sides are equal? If yes, then which one?

Construct an isosceles triangle $\triangle ABC$ on a card sheet/paper. Fold the triangle such that equal sides coincide with each other.

Are the angles opposite to equal sides also equal?

You would find that the angles opposite to equal sides are also equal.
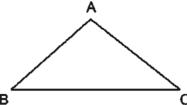
So, the sides AB and AC are called similar sides of the triangle and the angles $\angle B$ and $\angle C$ opposite to them are base angles and are mutually equal.

Therefore, in an isosceles triangle the angles opposite to the equal sides are equal.
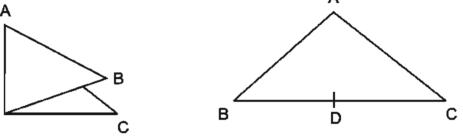
8.7 Medians of a triangle

1. Construct a triangle ΔABC on a card sheet and cut it off.

2. Fold the triangle so that the vertices B and C coincide. It will give middle point of BC. Name it D.
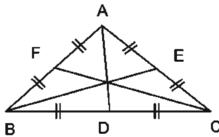
3. Now join the point A with the middle point of BC. AD is the median of the triangle ABC.

The line joining a vertex to the middle point of its opposite side of a triangle is known as median of the triangle.
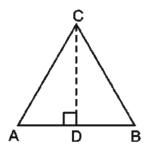
4. Similarly, median BE and CF can also be drawn as given below:
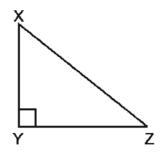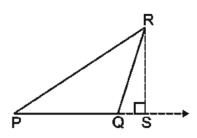
There can be maximum three medians in a triangle. The point of intersection of medians is called centroid of the triangle.

8.8 Altitude of triangle

An altitude of a triangle is a straight line drawn through the vertex and perpendicular to (i.e., forming a right angle) opposite side. Three such altitudes can be drawn for a triangle. The three altitudes intersect in a point called **Orthocentre** of the triangle. Identify the altitudes drawn in the following triangles:

$\triangle ABC$ is an acute angled triangle. All its altitudes lie inside triangle.

$\triangle XYZ$ is a right angled triangle. The two sides forming the right angle are themselves altitudes.
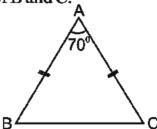
$\triangle PQR$ is an obtuse angled triangle. One of the altitudes lies outside the triangle.

The height of the triangle is equal to the altitude with base as the side on which it is drawn.

Exercise 8.2

1. Which of the following triads form a triangle:
   (i) 6, 5, 5   (ii) 2, 3, 5   (iii) 3, 4, 8   (iv) 3, 5, 6   (v) 4, 4, 8   (vi) 9, 2, 8

2. Find the value of all the angles of an equilateral triangle.

3. Fill in the blanks:
   (i)   At least two angles in a triangle are................
   (ii)   One altitude of an .... Tringle lies outside the triangle.
   (iii) Sum of any two sides of a tringle is .... than the third side.
   (iv)   Two angles of an .... triangle are equal.
   (v)   The line joining the vertex to the middle point of opposite side of a triangle is called ....
   (vi)   The point where three medians of a triangle meet is called ....
   (vii) All three .... of a triangle pass through the orthocentre of a triangle.

4. In $\triangle ABC$, $\angle A = 70°$ and $AB = AC$. Find the value of B and C.

5. Construct a triangle and show in it one median and one altitude.
6. The length of two sides of a triangle are 3 cm and 6 cm. What could be the minimum and maximum value of its third side?
7. Draw two equilateral triangular traffic symbols (warning symbols) that attracts your attention towards possible danger on the road.

We Learnt

1. Three sides and three angles in a triangle are known as its elements.
2. Sum of all the three angles in a triangle is 180°.
3. An exterior angle of a triangle is formed by extending a side in only one direction. Extension of a side in both the directions give two exterior angles.
4. Exterior angle is the sum of two opposite interior angles.
5. Properties of sides of triangle:
   (i) Sum of length of any two sides of a triangle is greater than the length of third side.
   (ii) Difference of length of sides is less than the length of its third side.
      Both the properties are useful in the possible construction of a triangle when its sides are given.
6. In a right angled triangle the side opposite to right angle is called hypotenuse and other two are called legs. In right angled triangle: Square of hypotenuse = sum of square of legs.
7. The line segment joining the vertex to the middle point of the opposite side in a triangle is known as median of the triangle. There are three medians in a triangle. Point of intersection of medians is called centroid of the triangle.
8. A perpendicular drawn from a vertex of triangle on the opposite side is known as altitude. A triangle can have three altitudes. Point of intersection of altitudes is known as orthocentre of the triangle.

9.1 Inder got many envelopes on his birthday as gift. He arranged the rupees which he got from those envelopes. He was arranging all the rupees according to different sizes. After arranging, he saw carefully those rupees and then said to his sister, "Didi all the rupees of 50-50 are quite equal in size. Likewise all the rupees of 100-100 are also equal in size". Didi replied, "Yes you are right. There are so many other things also which are equal in size and measure."
Which pair is equal in the diagram give below?
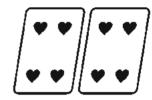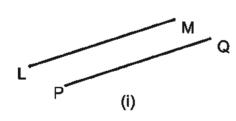
Fig. 9.1

- On which basis did you select like pairs?
- Which are the pairs which cover completely one another?

When one scale is placed over another scale then they cover completely one another because both are equal in shape and size.. Similarly when a playing card is placed over another card then it covers completely the other one. Such figures which cover completely the one another, are said to be congruent figures. Congruency is denoted by ≅ Are your Hindi and English books congruent to each other? Discuss with friends.

9.2 Congruence of Geometrical Figures
9.2.1 Congruence of line segments

(i)   **Fig. 9.2**   (ii)
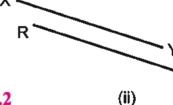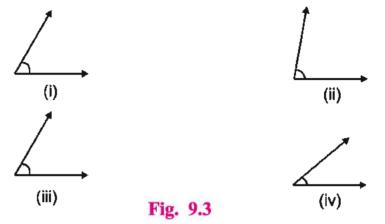
Measure both the pairs of line segments given in figure 9.2. What conclusion can be drawn about these line segments? Both the line segments are of equal length in diagram (i).Hence this pair is congruent. Both the line segments are not of equal length in diagram (ii). Hence this pair is not congruent.

Finally, we can say that two line segments are congruent when their lengths are equal.

9.2.2 Congruence of angles

Trace on a trace paper angle (i) of angles given in the figure 9.3. Now examine by placing it on (ii), (iii) and (iv) respectively.

(i)

(ii)

(iii)

(iv)

Fig. 9.3
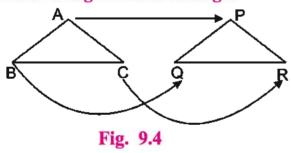
Which angle is covered by ∠ (i)?

Now measure each angle by protractor. Are measures of congruent angles equal ? We can draw the conclusion by this working that equal angles are congruent and measures of congruent angles are equal.
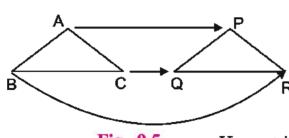
If two figures A and B are congruent then we'll write A ≅ B

If line segment AB and line segment ED are congruent then we'll write AB ≅ ED

Similarly if ∠1 and ∠2 are congruent then ∠1 ≅ ∠2

9.2.3 Congruence of Triangle

Fig. 9.4

If you superimpose ΔABC on ΔPQR in such a manner that A lies on P then will its remaining vertices also remain as they are? It is not necessary. While discussing congruency, not only measure of angles and lengths of sides matter, but matching of vertices also matter. There is a matching in the case given above. A ↔ P, B ↔ Q, C ↔ R

We can write the matching in this manner also. ΔABC↔ΔPQR But if A ↔ P, B ↔ R, C ↔ Q then we'll write ΔABC ↔ ΔPRQ .

Fig. 9.5

See the diagram carefully to understand the congruency in a better way –
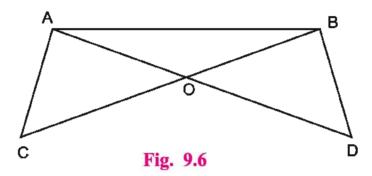
Here

AB ↔ PR, AC ↔ PQ, BC ↔ RQ

and

∠A ↔ ∠P, ∠B ↔ ∠R, ∠C ↔ ∠Q

Hence triangle ABC is congruent to triangle PRQ.

i.e. $\triangle ABC \cong \triangle PRQ$

Fig. 9.6

Here in $\triangle ABC$ and $\triangle BAD$

∠ABC = ∠BAD, ∠ACB = ∠BDA and ∠BAC = ∠ABD
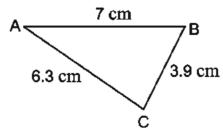
AB ↔ BA, BC ↔ AD, AC ↔ BD   i.e AB = BA, AC = BD and BC = AD
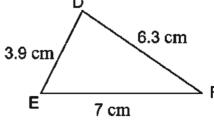
Hence $\triangle ABC \cong \triangle BAD$

Do and Learn ◆

1. When two triangles $\triangle ABC$ and $\triangle PQR$ are given, then six possible matching are there in these two. Find these matching by using cut-out of two triangles.
2. Do all matching show congruency? Identify it by superimposing.

Example 1   Is $\triangle ABC \cong \triangle DEF$?   Write their corresponding angles.
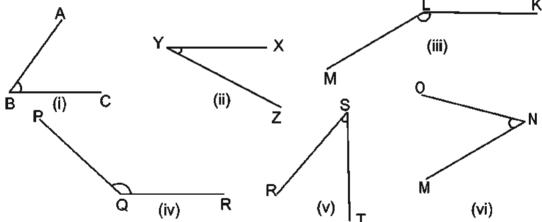
Solution   From $\triangle ABC$ and $\triangle DEF$ in the given figures.

AB = EF = 7 cm,   BC = DE = 3.9 cm,   AC = DF = 6.3 cm

Clearly Point A corresponds to F

Clearly Point B corresponds to E

Clearly Point C corresponds to D

Hence $\triangle ABC \cong \triangle FED$

$\left(\begin{array}{c} \text{Here pair of corresponding angles} \\ \angle A = \angle F \\ \angle B = \angle E \\ \angle C = \angle D \end{array}\right)$
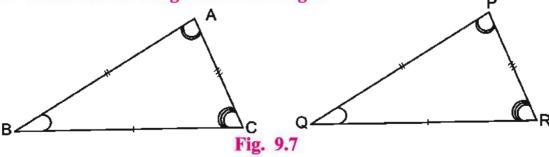
Exercise 9.1

1. If triangle ABC is congruent to triangle PQR then write all corresponding congruent parts of triangle.

2. If $\triangle LMN \cong \triangle XYZ$ then write those parts which corresponds to following –
   (i) $\angle N$ (ii) LM   (iii) $\angle M$ (iv) MN

3. Fill in the blanks:-

(i) Two line segments are congruent, if their .... are equal.

(ii) Two squares are congruent, if their .... are equal.

(iii) In two congruent triangles $\triangle PQR \cong \triangle ABC$, $\angle P$ measures 60^0 then $\angle A$ measures ....

4. Where you can be seen congruent figures in daily life? Write any two examples.

5. Select congruent angles in the diagrams given below (Identify by tracing the angles)

(Can you identify the congruence of angles with the compass? Do it.
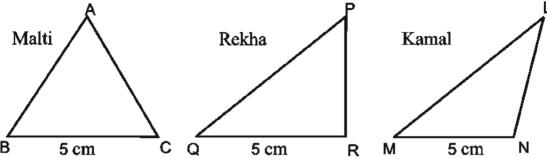
9.2.3 Condition for congruence of triangles

Fig. 9.7

In figure 9.7, both the triangles are equal in shape and size. Trace the △ABC with the help of tracing paper and put it on △ PQR Do the triangles ABC and PQR cover each other completely? Two triangles are congruent if corresponding parts of any two triangles are equal.
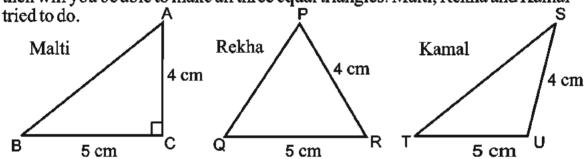
[A]   SSS (Side–Side–Side] Congruence

If you are given the measure of one side of any triangle as 5 cm. then how will you draw it. Malti, Rekha and Kamal made triangles as given below.
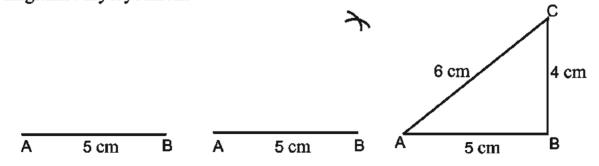
You will see that Malti made an equilateral triangle, Rekha made a right angled triangle and Kamal made an obtuse angled triangle.

Again if you are given the measures of two sides of a triangle as 4cm. and 5cm. then will you be able to make all three equal triangles. Malti, Rekha and Kamal tried to do.

You will find the different triangles formed in this case also.

If all three sides are known as 4cm, 5cm, and 6cm then can you draw three equal diagrams? Try it yourself.
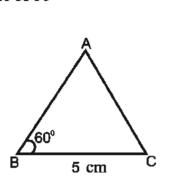
Thus the triangles made by Malti, Rekha and Kamal are equal and the corresponding sides of these triangles are equal in measure.
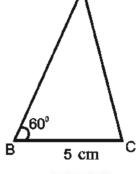
SSS rule – Two triangles are congruent if the three sides of the one are equal to the three corresponding sides of the other. This is known as the Side-Side Side rule of congruency.

[B]   SAS (Side–Angle–Side] Congruence

We saw that two congruent triangles cannot be formed with the help of one or two sides. If one angle and one side is given then can you make two congruent triangles? Malti, Rekha and Kamal Made triangles taking one side as 5cm and an angle of 60^0
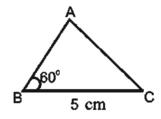
Fig. 9.8
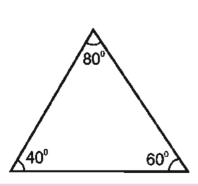
They all made triangles of different measures taking different length of side containing the angle.
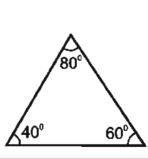
If we fix the length AB along with the base BC in the triangle i.e. AB=4 cm. then we'll find that all the triangles made will be congruent. This means, if we want to make $\triangle ABC$, congruent to $\triangle PQR$ then we should know the length of two sides and the angle included between them.
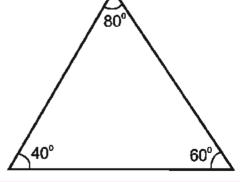
SAS rule – Two triangle are congruent if two sides and the angle included between them in one of the triangles are equal to the corresponding sides and the angle included between them of the other triangle.

[C] ASA (Angle–Side-Angle] Congruence

If one angle of a triangle is known than can you make a triangle? If all the angles of a triangle are known than can you make similar triangle? Malti, Rekha and Kamal draw diagram. Angle 40^0, 60^0, 80^0
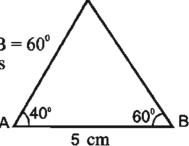
All the angles of a triangle are similar but sides are not equal hence we should know the lengths of sides. If two angles and a side containing these two angles are known to us then can we make congruent triangles?

All three children again made angles as $\angle A = 40^0$ $\angle B = 60^0$ and AB = 5 cm to draw the triangles. All the triangles comes out to be similar. This Means that to draw congruent triangle the measures of one side and two angles should be known.

ASA rule – Two triangles are congruent if two angles and the side included between them in one of the triangles is equal to the corresponding angles and the side included between them of the other triangle.

[D] Right Angle – Hypotenuse-Side (RHS) Congruence

We know that in the two right angled triangles the right angles are equal. What more should we know so that test of congruence of these can be done?
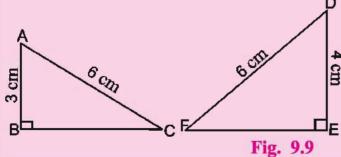
Following three cases are possible –
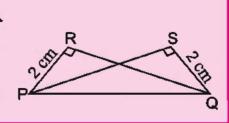
1. Remaining two corresponding angles are equal.

2. Two nearby sides of right angle are known.

3. Hypotenuse and one other side is known.

We see that in the first case by AAA, one cannot prove the congruence. In the second case Third side can be found if two sides are known. Hence congruence can be proved by SSS or SAS here. But third case is special for right angle triangle. This is known as Right Angle –Hypotenuse-Side (RHS) rule.
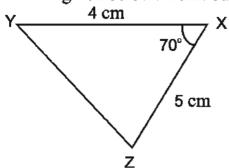
RHS rule - Two right angled triangles are congruent if the hypotenuse and one side of one of the triangle are equal to the hypotenuse and the corresponding side of the other triangle.
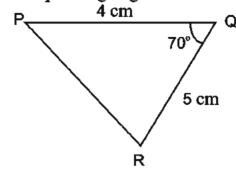
Do and Learn ◆

Some triangles are given in figure 9.9. Which triangles are congruent by RHS rule?

Fig. 9.9

Example 2   Are the triangles congruent on the basis of measures of triangle given below? Point out the corresponding angles?

Solution   In $\triangle$ XYZ and $\triangle$ PQR   XY = PQ   and   XZ = QR and $\angle$X = $\angle$Q

$$\therefore \triangle YXZ \cong \triangle PQR$$

Hence corresponding angles $\angle$X $\leftrightarrow$ $\angle$Q, $\angle$Y $\leftrightarrow$ $\angle$P, $\angle$Z $\leftrightarrow$ $\angle$R
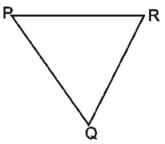
>──oo──< **Exercise 9.2** >──oo──<

1. Find the value of the following if $\triangle$ ABC $\cong$ $\triangle$ PRQ as given below-

   (I) Side PR   (ii) Side QR   (iii) Side PQ   (iv) $\angle$P   (v) $\angle$Q   (vi) $\angle$R
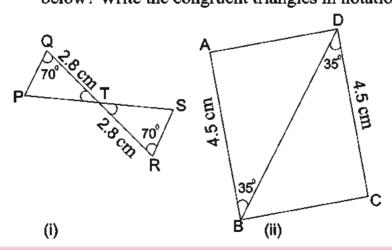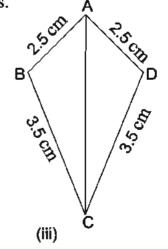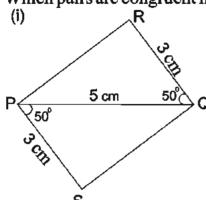
2. Which condition applies of congruence of triangles in the diagrams given below? Write the congruent triangles in notations.

      (i)   (ii)   (iii)
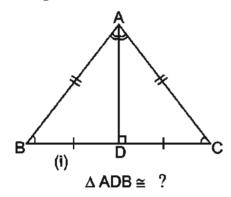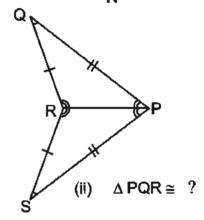
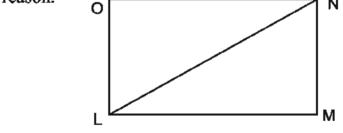3. Which pairs are congruent in the pairs of triangle given below.

(i)

(ii)

(iii)

4. Complete the statement.

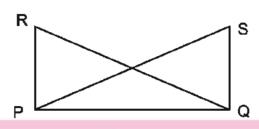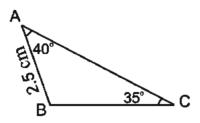(i)

$\triangle ADB \cong$?

(ii) $\triangle PQR \cong$?

5. In the diagram given below MNOL is a rectangle. Is $\triangle NOL \cong \triangle LMN$? If yes, then give reason.
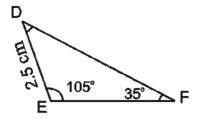
6. In the given diagram in $\triangle PQR$ and $\triangle PQS$, side PR=side QS and RQ=PS, then point out which statement is correct.

(i) $\triangle PQR \cong \triangle PQS$ (ii) $\triangle PQR \cong \triangle QPS$ (iii) $\triangle PQR \cong \triangle QSP$

7. In $\triangle ABC$ in given diagram $\angle A = 40^0$, $\angle C = 35^0$ and side $AB = 2.5$cm and in $\triangle DEF$ $\angle F = 35^0$, $\angle E = 105^0$ and side $DE = 2.5$ cm, is $\triangle ABC \cong \triangle DEF$?

$\bowtie$ **We Learnt** $\bowtie$

1. Congruent triangles are same in size and equal measures.

2. The method of superimposition of their replicas can be used to examine the congruence of triangles.

3. If all parts of a triangle are equal to the corresponding parts of another triangle then the triangles will be congruent.

4. Necessary and complete rules to show congruence of two triangles are as follows:-

    (i) SSS rule – Two triangles are congruent if the three sides of the one are equal to the three corresponding sides of the other.

    (ii) SAS rule – Two triangles are congruent if two sides and the angle included between them in one of the triangles are equal to the corresponding sides and the angle included between them of the other triangle.

    (iii) ASA rule – Two triangles are congruent if two angles and the side included between them in one of the triangles are equal to the corresponding two angles and the side included between them of the other triangle.

    (iv) RHS rule – Two right angled triangles are congruent if the hypotenuse and a side of one of the triangles are equal to the hypotenuse and the corresponding side of the other triangle.

10.1   Revise the chapters containing concept of triangle, its properties and congruence of triangles before studying this chapter.

We classify the triangles on the basis of sides and angles like as equilateral, isosceles and scalene triangles on the basis of sides and acute angled triangle, right angled triangle, obtuse angled triangle on the basis of angles etc.

We'll learn to construct different types of triangle in this chapter.

It is not necessary to know the measures of all the six elements (3 sides and 3 angles) to make a required triangle; the same has been studied in the chapter related with congruence of triangles. We can construct required triangle if we are given measures from any one of the groups given below. Here required is a unique triangle constructed on the basis of given measures.

     1.   Three sides.
     2.   Two sides and the angle between them.
     3.   Two angles and the side between them.
     4.   The hypotenuse and a leg in the case of a right-angled-triangle.

10.2   Construction of a triangle when the lengths of its three sides are given

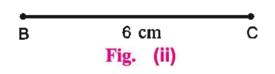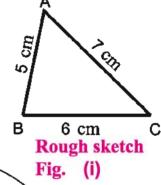**Example 1** Construct a triangle ABC, given that AB = 5 cm. BC = 6 cm. and AC = 7 cm.

Solution   **Step-1**

First, we draw a rough sketch with given measure.

Step-2

Draw a line segment BC of length 6 cm.

B   6 cm   C

Fig. (ii)

Rough sketch
Fig. (i)

Step- 3

From B, point A is at a distance of 5 cm. So, with B as centre, draw an arc of radius 5 cm.

B   6 cm   C

Fig. (iii)

Step-4
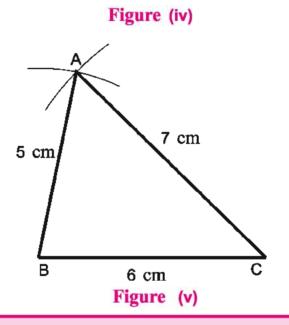
AC = 7cm, So, with C as centre, draw an arc of radius 7 cm. in such a way that this arc intersects the arc drawn from point B.

B —————— 6 cm —————— C

Figure (iv)

Step-5

A has to be on both these arcs drawn. So, it is the point of intersection of arcs. Mark the point of intersection of arcs as A. Join AB and AC. Triangle ABC is now ready.

5 cm   7 cm

B   6 cm   C

Figure (v)

Do and Learn ◆

1. Construct $\triangle XYZ$ in which XY = 4.5 cm. YZ = 5 cm. and ZX = 6 cm.
2. Construct an equilateral triangle of side 5.5 cm.
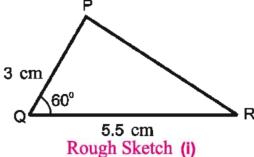3. Draw $\triangle PQR$ with PQ = 4 cm. QR = 3.5 cm. and PR = 4 cm. What type of triangle is this?

10.3 Construction of a triangle when the lengths of two sides and the measure of the angle between them are known

Here, we are given two sides and the angle between them. We first draw a rough sketch. The other steps follow according to example 2.
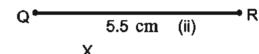
Example 2 Construct a triangle ΔPQR, given that PQ = 3 cm., QR = 5.5 cm. and ΔPQR = 60°.

Solution

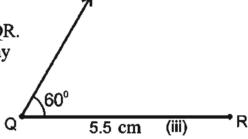**Step-1** First, we draw a rough sketch with given measures. (This helps us to determine the procedure in construction) figure (i)
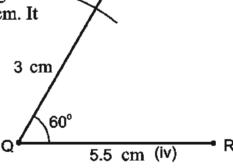
3 cm

60°

5.5 cm

Rough Sketch (i)

Step-2 Draw a line segment QR of length 5.5 cm.figure (ii)

Q •———————————• R
5.5 cm (ii)
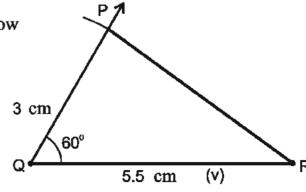
Step-3 At Q, draw QX making 60° with QR. (The point P must be somewhere on this ray of the angle.) figure (iii)

60°
Q 5.5 cm (iii) R

Step-4 To fix P, the distance QP has been given With Q as centre, draw an arc of radius 3 cm. It cuts QX at the point P.(figure (iv)).

3 cm
60°
Q 5.5 cm (iv) R

Step-5 Join PR. Triangle PQR is now obtained.(figure (v))

3 cm
60°
Q 5.5 cm (v) R

Think, and Discuss

Teacher- In a triangle ABC if the measures are AB = 3 cm, AC = 5 cm and, $\angle C = 30^0$ then. Can we draw this triangle?

Krishna, Vikram and Sarla try to construct it.

Krishna - We may draw AC = 5 cm. and draw $\angle C = 30^0$.

Vikram – CA is one arm of $\angle C$. Point B should be lying on the other arm of C.

Teacher – Observe that point B cannot be located uniquely. Thus, we can conclude that a unique triangle can be constructed only if the lengths of its two sides and the measure of the included angle between them is given.

Do and Learn ◆

(I) Construct ΔDEF such that DE = 5 cm., DF = 3 cm. and $\angle EDF = 90°$.

(ii) Construct an isosceles triangle in which the lengths of each of its equal sides is 6.5 cm. and the angle between them is 110°.

(iii) Construct ΔABC with BC = 7.5 cm., AC = 5 cm. and $\angle C = 60°$.
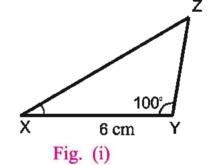

10.4 Construction of a triangle when the measures of two of its angles and the length of the side included between them is given

As before, draw a rough sketch. Now, draw the given line segment. Make angles on the two ends. See the example 3.

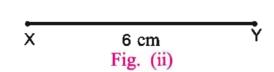**Example 3** Construct ΔXYZ if it is given that XY = 6 cm, $\angle ZXY = 30°$ and $\angle XYZ = 100°$.
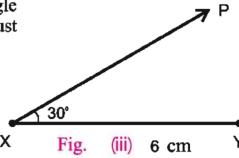
Solution

Step-1 Before actual construction, we draw a rough sketch with measures marked on it. (This is just to get an idea as how to proceed) (figure (I))
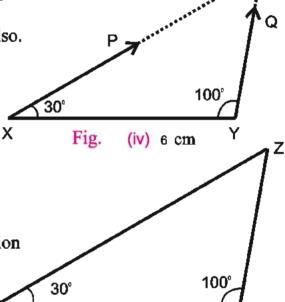

Fig. (i)

Step- 2 Draw a line segment XY of length 6 cm.(figure (ii))

Fig. (ii)

Step - 3 At X, draw a ray XP making an angle of 30° with XY. By the given condition Z must be somewhere on the XP.(figure (iii))

Fig.   (iii)   6 cm

Step-4 At Y, draw a ray YQ making an angle of 100° with YX. By the given condition, Z must be on the ray YQ also. (figure (iv))

Fig.   (iv) 6 cm

Step- 5 Z has to lie on both the rays XP and YQ. So, the point of intersection of the two rays is Z.(figure (v))

Fig.   (v)   6 cm

Do and Learn ◆

1. Construct $\triangle ABC$, given $\angle A = 60^0$, $\angle B = 30^0$ and AB = 5.8 cm

2. Construct $\triangle PQR$ if PQ =5 cm, $\angle PQR = 105^0$ and $\angle QRP = 40^0$

   (Hint: Recall angle- sum property of a triangle)

3. Examine whether you can construct $\triangle DEF$ such that

   EF = 7.2 cm, $\angle E = 110^0$ and $\angle F = 80^0$, Justify your answer.

10.5 Construction of a right angled triangle -
when the length of one leg and its hypotenuse are given. Here it is easy to make the sketch. Now, draw a line segment as per the given side. Make a right angle on one of its end. Use compasses to mark length of side and hypotenuse of the triangle. Complete the triangle. Consider the following example:
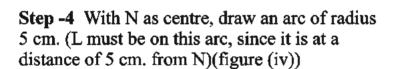
Example 4   Construct $\triangle$LMN, right- angled at M, given that LN = 5 cm and MN = 3 cm.
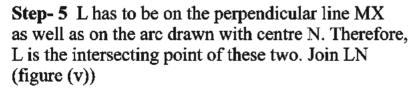
Solution   **Step-1** Draw a rough sketch and mark the measure. Remember to mark the right angle. (figure (I))

Fig. (i)
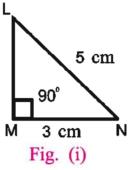
Step -2 Draw line segment MN of length 3 cm. (figure (ii))
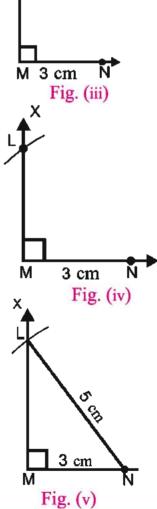
Fig. (ii)

Step -3 At M, draw MX $\perp$ MN. For this draw a 90° angle at M.(L should be somewhere on this perpendicular) (figure (iii))

Fig. (iii)

Step -4 With N as centre, draw an arc of radius 5 cm. (L must be on this arc, since it is at a distance of 5 cm. from N)(figure (iv))

Fig. (iv)

Step- 5 L has to be on the perpendicular line MX as well as on the arc drawn with centre N. Therefore, L is the intersecting point of these two. Join LN (figure (v))

Fig. (v)

> ### Do and Learn ◆
> 1. Construct the right angled triangle PQR, where $\angle Q = 90^0$, QR = 8 cm and PR = 10 cm
> 2. Construct a right- angled triangle whose hypotenuse is 6 cm. long and one of the legs is 4 cm. long.

Exercise 10

1. Construct $\triangle$PQR, when PQ = 4 cm, QR = 3 cm and RP = 5.5 cm
2. Construct $\triangle$ XYZ, when XZ = 6 cm, XY = 4.5 cm and $\angle X = 50°$.
3. Construct $\triangle$ ABC , when AB = 5 cm, $\angle A = 45°$ and $\angle B = 60°$.
4. Conbstruct $\triangle$ DEF, when hypotenuse DE = 5 cm, base DF = 3 cm and $\angle D = 90°$.
5. Construct an equilateral triangle of side 4 cm.
6. Construct $\triangle$ PQR, where PQ = 5 cm, $\angle P = 75°$ and $\angle R = 55°$
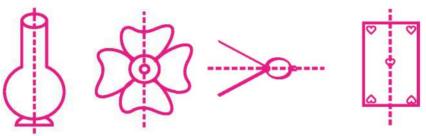
We Learnt

1. In this chapter, we studied the methods of constructions of triangles using ruler and compasses.
2. We studied the method of drawing a triangle, using indirectly the concept of congruence of triangles.
3. In this chapter we studied constructions of triangle using following sets of measures.
    - (i)   SSS: Given the three side lengths of a triangle.
    - (ii)   SAS : Given the lengths of any two sides and the measure of the angle between these sides.
    - (iii)   ASA : Given the measures of two angles and the length of side included between them.
    - (iv)   RHS : Given the length of hypotenuse of a right-angled triangle and the length of one of its legs.

SYMMETRY

11.1 We see many things, pictures etc. around us. Different types of geometry are seen in all these things.

If these figures may be cut or folded along the line drawn in the center then both the parts superimpose each other completely. Such figures are known as symmetric figures. We studied about symmetric figures, symmetry, and symmetrical axis in the previous classes. In this chapter we will study about identification and formation of symmetrical axis, reflection symmetry and rotational symmetry in the given figures.

> **Do and Learn** ◆ To show symmetry
> 1. Make a diagram showing symmetry.
> 2. Make some paper cut designs.
> 3. Make a rangoli.
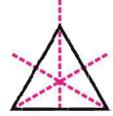
11.2 Linear Symmetry

The symmetry which we have discussed above is linear symmetry. In these diagrams there is such a straight line about which the figure may be folded so that the two parts of a figure will coincide (superimpose each other). Are you familiar with regular polygon? If no, then try to understand by discussing with your friends and teacher.

Regular polygons are symmetric figures. This is an interesting conclusion that each regular polygon has as many lines of symmetry as it has sides.
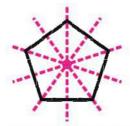
Three symmetric lines	Four symmetric lines	Five symmetric lines	Six symmetric lines
Equilateral Triangle	Square	Regular Pentagon	Regular Hexagon
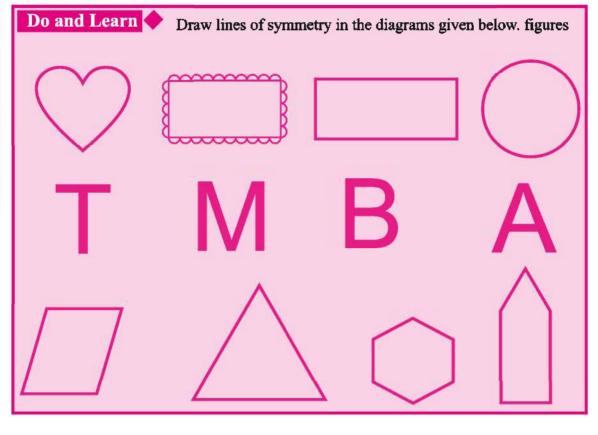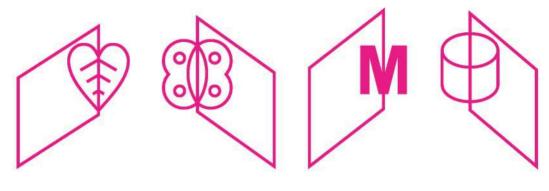
Do and Learn ◆ Draw lines of symmetry in the diagrams given below. figures

11.3 Reflection Symmetry
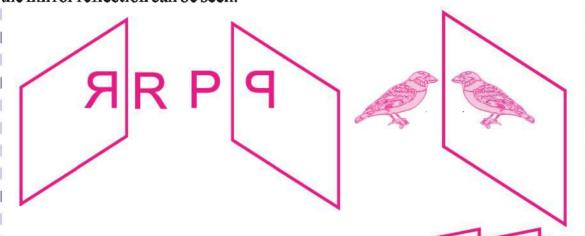
Take a plane mirror and see the different things opposite to it. We can see images of things in the mirror. Place the mirror on the diagrams given below such that half portion remains opposite to the mirror. We see that half of the portion lies opposite to the mirror and half of the portion lies in the mirror. On combining both the portions the diagram can be seen completely. This is reflection symmetry. Have a look at these images made in the mirror.
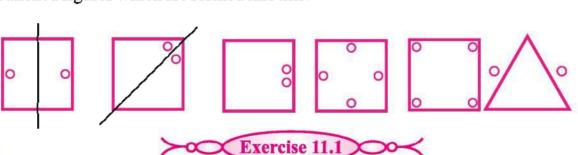
Mirror image of these diagrams is half portion of the diagram. The edge of the mirror is in the form of symmetric axis. Thus the concept of linear symmetry is closely related to reflection in a mirror. A mirror line, thus helps us to visualise a line of symmetry.

There is a mirror reflection of R,P and bird in the diagram given below. Here left-right changes in the orientations of the figure or its lateral reflection in the mirror reflection can be seen.
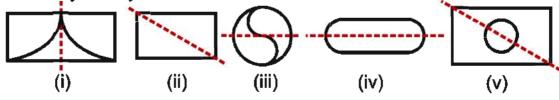
Activity

Take a simple quare paper. Fold this paper into two halves as shown in the figure. Now, punch a hole in the paper. Now open the paper. Fold of paper is a line of symmetry and the hole made in the paper is in the form of symmetric figure. Try to find out the line of symmetry in the punched figures which are formed like this.

><oOo< **Exercise 11.1** >oOo<
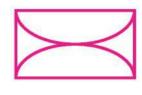
1. There is a dotted line in the figure (s) given below. Find whether this line is line of symmetry or not?

    (i)   (ii)   (iii)   (iv)   (v)

2. Draw line of symmetry in the figures given below.

| (i) | (ii) | (iii) | (iv) |
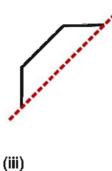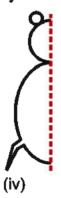
3. Complete the incomplete figure given below along the line of symmetry.

| (i) | (ii) | (iii) | (iv) |

11.4 Rotational Symmetry

Hands of a clock, the wheel of a bicycle and ceiling fans etc. are said to be in motion when they rotate. The rotation takes place on both the directions in some things, while the hands of a clock rotate in only one direction. The direction in which the hands of a clock move is called clockwise rotation.Otherwise it is said to be anticlockwise rotation. The wheel of a bicycle rotates in both the directions.
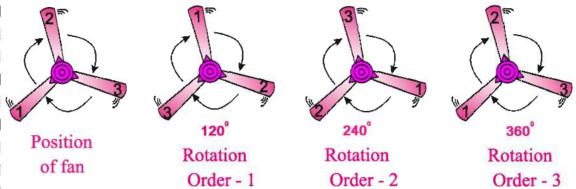
Do and Learn ◆

1. Give any two examples of clockwise rotation.

2. Give any two examples of anticlockwise rotation.

Think. Is there any change in size and shape of things like wheel of a bicycle, arms of a clock when they rotate? No. The rotation turns an object about a fixed point without change in size and shape. This fixed point is the center of rotation. The angle of turning during rotation is called the angle of rotation. The angle made on the centre formed by blades of ceiling fan is shown below.

| Position of fan | 120° Rotation Order - 1 | 240° Rotation Order - 2 | 360° Rotation Order - 3 |
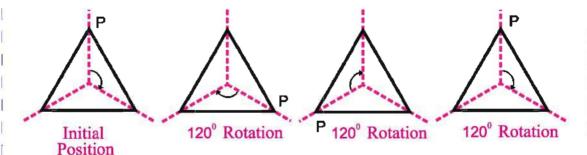
Here we see that on rotating the ceiling fan by 120° the blades look the same. Similarly the position remains same by rotation of 240° and 360°. Hence, we can say that ceiling fan has rotational symmetry and order of rotational symmetry is 3.

In a complete turn (360°) the number of times an object looks exactly the same is called the order of rotational symmetry e.g. In the example of ceiling fan given above the order of rotational symmetry comes out to be 3 as there are 3 same positions in a complete turn. Similarly order of rotational symmetry is 4 in a square.

Every object (shape) comes in its initial position after completing a complete turn means rotating 360°. Hence, order of rotational symmetry is definitely 1 in every object.
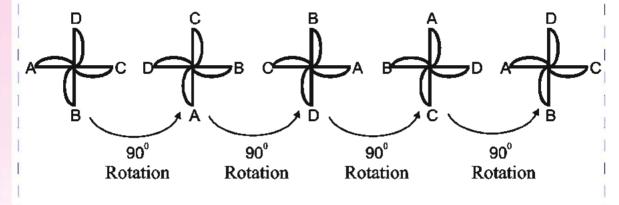
11.4.1 Some examples of rotation

An equilateral triangle comes in its initial position three times in a complete turn in clockwise direction. This is known as rotation of third order. The angle of rotation is 120° because after rotating 120° from its initial position, the triangle comes back in its former position, so its angle of rotation is 120°.

| Initial Position | 120° Rotation | 120° Rotation | 120° Rotation |
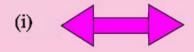
Rotation of disc

Have a look at the disc. The disc occupies its initial position 4 times in a complete turn. Hence the order of rotation of the disc is 4 and it comes back in its initial position on every 90°. Hence the angle of rotation of the disc is 90°.

| | 90⁰ Rotation | 90⁰ Rotation | 90⁰ Rotation | 90⁰ Rotation |
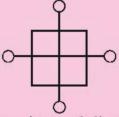
Do and Learn ◆
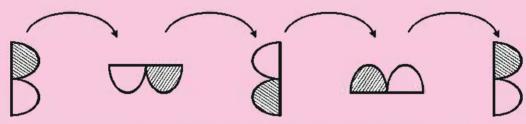
1. Determine the order of rotation and angle of rotation for the rotational symmetry in the figures given below.

(i)

(ii)

2. Determine the order of rotation, angle of rotation and direction of rotation of B.

3. Have a look of rotational symmetry in cross-section of fruits, traffic signals and wheel etc. Determine the order of rotation of all these.

Cross-section of fruits   **Traffic signal**   **Wheel**
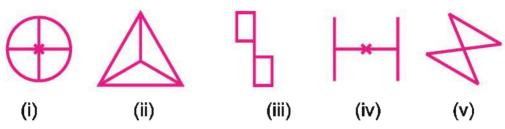
Exercise 11.2

1. Determine the order of rotational symmetry in the figures given below.

(i)   (ii)   (iii)   (iv)   (v)

2. Give names of 2 such figures which have linear symmetry and rotational symmetry of order more than 1.

3. Name the quadrilaterals which have both linear symmetry and rotational symmetry of order more than 1.

4. After rotating by 60° about its axis, a figure looks exactly the same as its original position. At what other angles will this happen for the figure.

We Learnt

1. A figure has line symmetry, if there is a line about which the figure may be folded so that the two parts of the figure will coincide.

2. Regular polygons have equal sides and equal angles. They have multiple (more than one) lines of symmetry.

3. Each regular polygon has as many lines of symmetry as it has sides.

Regular Polygon	Regular hexagon	Regular pentagon	Square	Equilateral triangle
No. of lines of symmetry	6	5	4	3

4. Mirror reflection leads to symmetry, under which the left-right orientation have to be taken care of.

5. Rotation turns an object about a fixed point. This fixed point is called as centre of rotation. The angle by which the object rotates is the angle of rotation.

6. If, after a rotation, an object looks exactly the same as it was before, we say that it has a rotational symmetry.

7. In a complete turn (of 360°), the number of times an object looks exactly the same as before is called the order of rotational symmetry.
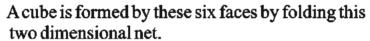
12.1 We had studied about solid shapes in class VI. We studied that solid shapes are known as three dimensional shapes because it has height or depth also in addition to length and breadth.

Cube, cuboid, cylinder, cone and sphere are three dimensional figures whereas square, rectangle, circle, etc. are two-dimensional figures.

We also studied that three dimensional shapes have faces, edges and vertices. Some shapes have plane surface, some have curved surface and some shapes have both. In this chapter we will learn visualizing surface of solid shapes.

12.2 Identification of two dimensional and three dimensional

Rekha cut a thin rectangular sheet of paper into a square net in such a way that it becomes a two dimensional net. It has six faces.

A cube is formed by these six faces by folding this two dimensional net.

This is a cube which is a three dimensional figure. Height is also included along with length and breadth in it. You can also discuss two dimensional and three dimensional figures. Match the given figures on the basis of examples.

Figure	Type of figure	Figure	Type of figure	Figure	Type of figure
	Two dimensional	Triangle			Two dimensional	Circle
	Three dimensional	Cylinder			Three dimensional	Cone
	Two dimensional	Cuboid			Three dimensional	Sphere
	Three dimensional	Square			Three dimensional	Cube

Two dimensional figures are said to be plane figures or 2-D figures and three dimensional figures are said to be solid shapes or 3-D figures.
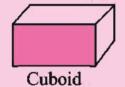
12.3   Representation of 3-D figures into 2-D figures

When solid shapes are drawn on a paper (plane) then their images are slanted so that they look like three dimensional. Two techniques of drawing 3-D figures on a plane surface (paper) are demonstrated below.

Look at the diagram of cube given below. It looks like as a cube when seen from the front while it is not possible to see all the surfaces actually. All the lengths are not equal in the drawn diagram while it must be equal in a cube. Then also, we identify that it is a cube.

Do and Learn ◆

1.   Some statements and diagrams of shapes are given below.
        Write down which statement is correct for every shape.
        (i)   I have six rectangular faces.
        (ii)   I have only one surface and which is also curved.
        (iii)   All my faces are square.
        (iv)   One of my faces is curved and two faces are plane.
        (v)   One of my faces is curved and one face is plane.

| Cuboid | Cube | Cylinder | Cone | Sphere |

2.   Fill the table.
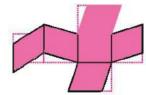
S.No.	Figure	Number of surface	Type of surfaces	
			Plane	Curve
1.	Cube			
2.	Cuboid			
3.	Cylinder	3	Plane-2	Curve-1
4.	Cone			
5.	Sphere			
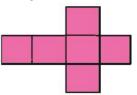
12.3.1   Representation of 3-dimensional shapes on the plane

Aditi is giving gift to her friend on her birthday. She wants to make a cuboidal box to pack that gift.She wants to see that how a cuboidal box is made, so she opens a tea box by cutting it.

Now she draws it on a card sheet and makes a box by cutting the card sheet.
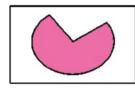
Closed Tea leaf box   Tea leaf box net

Similarly she draws the net on the card sheet for making birthday caps and makes the caps by cutting with scissors.
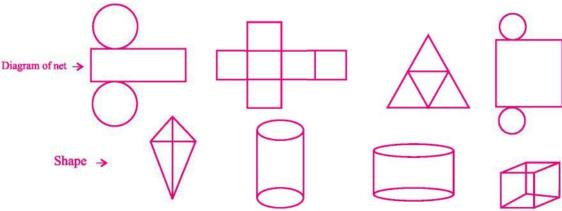
Do and Learn ◆

    (1) Cut similar type of net and make square boxes.

    (2) Make a lattice for making a cylindrical box.

Exercise 12.1

1. Net of some solid shapes are given below. Draw them on a thick sheet of paper and form 3 dimensional shapes by folding them at proper places and match them with correct shapes.

Diagram of net →

Shape →

2. Three nets are given for each figure. Select the proper lattice for each

Fig. (i)

(a)   (b)   (c)

Fig. (ii)

(a)   (b)   (c)
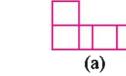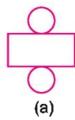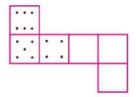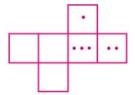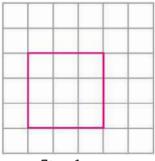
3. Playing dice is a cube on which dots are present on every face. The sum of dots of opposite faces is 7. Two nets of dice are given below. Make dots in proper number on blank surfaces.
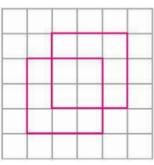
12.3.2   Oblique or slant sketches (Grid paper technique)
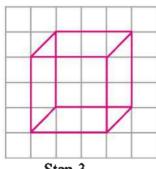
How can you draw such (figure) sketches? Let us attempt to learn such technique. You need a squared (grid paper) paper. Let us attempt to draw an oblique sketch of a 3×3×3 (each edge is 3 units) cube.
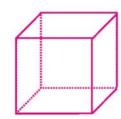
Step-1
Draw the front face

Step-2
Draw the opposite face.
Sizes of the faces have
to be same, but the sketch
is somewhat off-set from step-1.

Step-3
Join the corresponding
corners.
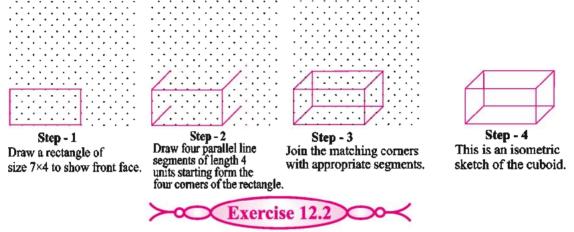
Step-4
Redraw using dotted lines for hidden
edges. The sketch is ready now.

12.3.3 Isometric sketches (Iso metric sheet technique)
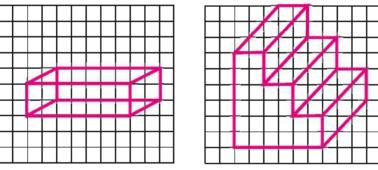
Have you seen an isometric sheet? Such a sheet divides the paper into small equilateral triangles made up of dotted or lines. In this sheet the points of one row are used to show the opposite plane and the points of next row are used to show the lateral plane. So that one can feel the depth or height of three dimensional object. A sample is given at the end of the book.

Let us attempt to draw an isometric sketch of a cuboid of dimensions 7×4×4 (which means the edges forming length, breadth and height or depth are 7,4,4 units respectively).

Step - 1
Draw a rectangle of size 7×4 to show front face.

Step - 2
Draw four parallel line segments of length 4 units starting form the four corners of the rectangle.

Step - 3
Join the matching corners with appropriate segments.

Step - 4
This is an isometric sketch of the cuboid.

Exercise 12.2

1. Make an oblique sketch for each one of the given isometric shapes on a grid paper.

2. Use isometric dot paper and make an isometric sketch for each one of the given shapes.

3. The dimensions of a cuboid are 5cm, 3cm and 2 cm. Draw three different isometric sketches of this cuboid.

Activity 1   Slicing (Cutting to pieces)Game

Here is a loaf of bread. It is like a cuboid with a square face. You 'slice' it with a knife. When you give a 'vertical' cut, you get several pieces. Each face of the piece is a square. We call this face a 'cross-section' of the whole bread. The cross-section is nearly a square in this case.

If your cut is not 'vertical' then you may get a different cross section. Think about it.
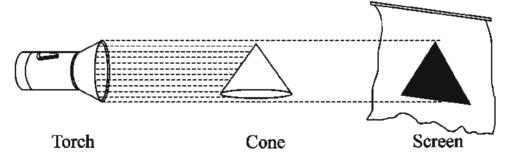
Similarly you noticed cross-sections of some vegetables when they are cut for the purpose of cooking in the kitchen. Observe the various slices and get aware of the shapes that result as cross-sections.

Activity 2   A shadow play
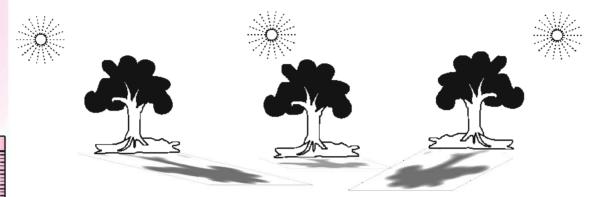
Shadows are a good way to illustrate how three-dimensional objects can be viewed in two dimensions. It is a form of entertainment using solid articulated figures in front of an illuminated back drop to create the illusion of moving images.

You will need an over-head projector or a torch and a few solid shapes to understand this activity. Keep a torchlight, right in front of a cone.

Torch   Cone   Screen
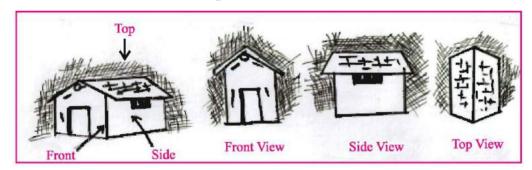
What type of shadow does it cast on the screen? If, instead of a cone, you place a cube in the above game, what type of shadow you will get?

Experiment with different positions of the source of light and with different positions of the solid object. Study their effects on the shapes and sizes of the shadow you get. Study the shadows of different shapes and sizes of trees, buildings etc. in relation to the position of the sun (morning, noon and evening).
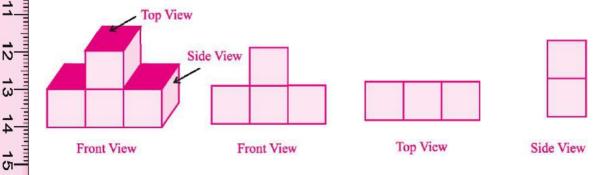
12.4 Looking at solid shapes from different angles (Front, Side and Top View)

One can look at an object standing in front of it or by the side of it or from above. Each time one will get a different view.

Consider front view, side view and top view by viewing the figure given below.

Exercise 12.3

1. Which type of cross-section is obtained on cutting horizontally and vertically the following solids?
    (i) A dice   (ii) A brick   (iii) A cylindrical trunk of wood
    (iv) A spherical apple   (v) An icecream cone

2. Here are the shadows of some solids, when seen under the lamp of an overhead projector. Estimate the probable solid(s) that match with each shadow.

    (i)   (ii)   (iii)   (iv)

3. The front, side and top views of shapes are given below. Identify and write them down.

    (i)

    --------   --------   --------
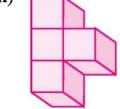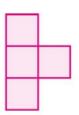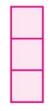
    (ii)

    --------   --------   --------
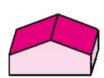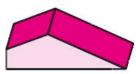
4. Draw the views of the solids given below from front, side and top.

    (i)   (ii)

5. Examine whether the given statements are true / false.
    1. The cross-section of a cucumber is almost circular when cut vertically.
    2. The shadow of the tent is triangular when the sun is just above the conical tent.
    3. Similar views of a cubical box are observed when seen from front, side and top.

⧓ We Learnt ⧓

1. Plane figures are of two-dimensions (2-D) and the solid shapes are of three-dimensions (3-D).

2. The corners of a solid shape are called its vertices; the line segments of its skeleton are its edges; and its flat surfaces are its faces.

3. A net is a skeleton- outline of a solid that can be folded to make it. The same solid can have several types of nets.

4. Solid shapes can be drawn on a flat surface (like paper) realistically. We call this 2-D representation of a 3-D solid.

5. Two types of sketches are possible:
   (i) On grid paper   (ii) On isometric sheet

6. Different sections of a solid can be viewed in many ways:
   (i) One way is to view by cutting or slicing the shape, which would result in the cross-section of the solid.
   (ii) Another way is by observing a 2-D shadow of a 3-D shape.
   (iii) One more way is to look at the shape from different angles; the front view, the side-view and the top-view can provide a lot of information about the shape observed.

13.1 We studied about the terms containing variables and constants e.g. $x, x+1, 2p-1, y-5, 3y+4$ in previous class. We have seen that by these terms, the problems can be expressed in a simple and general manner. Algebraic expressions can be represented in the form of general necessity in Algebra and by assuming this general concept as base, these are used in solving problems by operations with algebraic expressions.

13.2 Algebraic Expression

We made patterns from the game of matchsticks in the previous class.

Example 1 According to diagram place three sets of 2–2 matchsticks of shape ⌐ with one matchstick (|)

In this figure, number of matchsticks are 3, 5, 7 respectively which can be written as $2 \times 1 + 1, 2 \times 2 + 1, 2 \times 3 + 1$ etc.

If a set of matchsticks can be expressed by "n" then generally number of matchsticks can be expressed by $2 \times n + 1$ means $2n+1$.

In this way a combination of variables and constants is known as 'algebraic expression'. Look at some examples of algebraic expressions.

(1) Addition of 3 in any number can be expressed by $(x+3)$.

(2) Subtraction of 5 from four times of any number can be expressed by $(4x-5)$.

(3) One less from half of any number can be expressed by $\left(\frac{x}{2} - 1 \right)$.

Here unknown number is written by x.

On combining algebraic terms like this, one can get 'algebraic expression'. $(x+3), (4x-5) \left(\frac{x}{2}-1\right)$. Here we'll study about their properties.

> There is necessarily at least one variable in algebraic expression.

13.3 Terms of Algebraic Expression
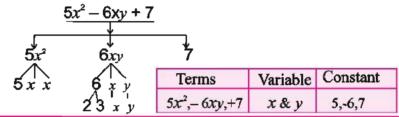
Any algebraic expression has smaller parts. Consider $5x + 3$ First we form $5x$ by multiplying 5 and x and add 3 to it. Similarly in $2x^2 + 3y$ we formed $2x^2$ by multiplying 2, x and x and then $3y$ by multiplying 3 and y separately. After forming $2x^2$ and $3y$ we add these two. Thus expression $2x^2 + 3y$ is formed.

These small – small parts of an expression, which are first formed separately and then added are known as terms of expression. Expression $9y^2-4xy$ contains two terms, first term is $9y^2$ which is the product of 9, y and y respectively. Second term $-4xy$ is the product of $-4, x$ and y respectively.

Then add these two terms as $9y^2 + (-4xy)$ and get an expression $9y^2-4xy$.

13.3.1   Factors of single term

Single term of algebraic expression can be the product of several variables and constants. We can represent the factors of expression and term by a tree diagram in a simple and attractive manner e.g.

Terms	Variable	Constant
$5x^2, -6xy, +7$	x & y	$5, -6, 7$

Do and Learn   Fill the table given below.

Expression	No. of terms	Term	Factor of Term	Variable	constant
$3x^2 + 6xy + 7y^2$	3	$3x^2, 6xy, 7y^2$	$3x^2 = 3 \times x \times x$	x, y	3, 6, 7
			$6xy = 2 \times 3 \times x \times y$		
			$7y^2 = 7 \times y \times y$		
$a^2 - b^2$	2				
$8p^2 - 3p + 7$					

13.4   Coefficient

Coefficients of factors of any term are equal to the product of remaining factors of that term. Coefficient can be of both types-algebraic and numerical.

> When coefficient of any term is $+1$ then we don't write it eg. Coefficient of $x^3 y^2$ in $x^3 y^2$ is $+1$. Similarly coefficient of $x^2 y^2$ in $-x^2 y^2$ is (-1)

Example 1   What is the coefficient of x in following expression?

$8x - 3y, \quad 5 - x + z, \quad y^2x - z^2, \quad 2z - 5xp$

Solution

	Expression	Terms with factor	Coefficient
(i)	$8x - 3y$	$8x$	8
(ii)	$5 - x + z$	$-x$	-1
(iii)	$y^2x - z^2$	y^2x	y^2
(iv)	$2z - 5xp$	$-5xp$	$-5p$

Do and Learn ◆

Match the coefficient in the following algebraic expression $4x^2y^2 - 3xy + 15$

Coefficient x^2y^2	x^2
Coefficient xy	$-3y$
Coefficient x^2	$-y$
Coefficient $4y^2$	-3
Coefficient x	$4y^2$
Coefficient $3x$	4

<center>❯❯❮◯❮ Exercise 13.1 ❯◯❯❮❮</center>

1. Find the factor of terms of expression by making tree diagram.

   (i) $9x^2-8$ (ii) $12x^2y + 8xy^2 - 15y^3$ (iii) $a^3 - b^3$

2. Find the coefficient in the given terms.

   (i) x in $4x$ (ii) y^2, x^2 and 9 in $9 x^2y^2$

   (iii) x^3, y^3 and x^3y^3 in $\frac{-8}{5} x^3y^3$

   (iv) a^2 and b^2 in $\frac{9a^2b^2}{13}$

13.5 Like and Unlike Terms

When algebraic factors of terms are same then they are known as like terms. When algebraic factors of terms are different then they are known as unlike terms eg. Consider $5xy$ and $3xy$ in $5xy - 6x + 3xy - 9$. Factors of $5xy$ are 5, x and y and factors of $3xy$ are $3, x$ and y. Thus their algebraic factors are same (by means of variables). Hence these are like terms.

> $3xy, 5yx$ are like terms as there is no effect on multiplication of variables in these terms because $xy = yx$.

Contrary to it, there are different algebraic factors in $5xy$ and $-6x$. These are unlike terms, similarly $5xy$ and -9 are unlike terms and $3xy$ and -9 are unlike terms.

Do and Learn ◆

Select the like terms in the following:

$3pq$, $-5p$, $6q + 5$, $- 8pq$, $p^2 + q$, qp

Example 2 Determine with reasons which pair are of like terms and which pair are of unlike terms in the given following pairs?

S. No.	Pair of term	Product	Algebraic term	Reason
1.	3ab 3b	3 x a x b 3 x b	Unlike	Variable a is not present in the second term.
2.	17a -6a	17 x a -6 x a	Like	Both the algebraic factors are equal.
3.	$5a^2b$ $5ab^2$	5 x a x a x b 5 x a x b x b	Unlike	Variables in both are same but their powers are different.
4.	-4ab 7ab	-4 x a x b 7 x b x a	Like	Both the algebraic factors are equal.

13.6   Polynomial Expression

Monomial	Which has only single terms like	$7xy$, $-3m$, y^2, x^2y^2
Binomial	Which has only two terms like	$x+y$, $x-5$, $pq+5$, m^2n^2+5m
Trinomial	Which has only three terms like	$x+y+2$, $3x^2-5x+7$, $ab+ab^2+b^2$

Expression containing one or more than one terms are known as polynomial expression.

Do and Learn ◆ Write in appropriate box by selecting monomial, binomial and trinomial expression from the following;

1. $2a^2 + b$
2. $4x^2y^3$
3. $3m -2n +1$
4. $2mn-3$
5. $\frac{7}{8} xy^2z$
6. $\frac{1}{3} x^2 + \frac{2}{3} xy + xy^2$
7. $ab+bc+ca$
8. $ax^2 + bx + c$
9. $5xy - 7 + 3n$
10. $3x + 1$
11. $\frac{9}{17} a^2 + b^2 - \frac{1}{2}$
12. $\frac{8}{19} p^2r^2q^2$

Monomial

Binomial

Trinomial

2.   Match the like terms.

(a) $4a^2b$

(b) $5nm$

(c) $\dfrac{3}{4}x^2y^2z^2$

(d) $\dfrac{-1}{5}\dfrac{a^3b^3}{c^3}$

(e) $\dfrac{-22}{7}\dfrac{p}{q}$

(f) $\dfrac{a^2}{b^2}$

(g) xyz

(h) $\dfrac{3}{x^2y^2z^2}$

(i) $\dfrac{8}{13}x^2y^2z^2$

(ii) $\dfrac{3p}{q}$

(iii) $\dfrac{5a^2}{7b^2}$

(iv) ga^2b

(v) nm

(vi) $\dfrac{a^3b^3}{c^3}$

(vii) $\dfrac{8}{x^2y^2z^2}$

(viii) $19xyz$

13.7 Addition and Subtractions of like terms

2 Pencils + 3 Pencils = 5 Pencils   2 Pencils + 3 Chalks

$$2x + 3x = (2+3)x$$
$$= 5x$$

We can add the pencils but cannot add pencils and chalks. Hence we can add or subtract the quantities of equal unit (equal variables).

eg. $5x^2y + 3x^2y = 8x^2y$

            $9a^2b^2 - 4a^2b^2 = 5a^2b^2$

Numerical coefficient of the term obtained by adding the like terms is equal to the sum of coefficients of all those terms. Similarly the result obtained by subtracting the two like terms is equal to the difference of numerical coefficients of those terms. It should be kept in mind that unlike terms cannot be added or subtracted in the way like terms are added or subtracted. For example on adding 5 to x the result comes out to be $x + 5$, similarly if $3xy$ is added to 7, the result is $3xy + 7$ and if 7 is subtracted from $3xy$, then the result is $3xy$-7.

Steps for addition, subtractions of algebraic expression

1. Identify the like and unlike terms.
2. Write down the like terms with their sign.
3. Add or subtract these like terms according to rules.
4. If there remains one or more unlike terms then write these by combining their signs.

Example 3 Add $3x + 8y$ and $8x + 5y$

Solutions $(3x + 8y) + (8x + 5y)$

$\quad = 3x + 8x + 8y + 5y$ (on keeping equal

$\quad = 11x + 13y$ algebraic terms altogether)

> We can add these two through column addition also.
>
> $3x + 8y$
> $8x + 5y$
> $\overline{11x + 13y}$

Example 4   Add $7ab + 4a$ and $2a + 5ba$

Solutions $(7ab + 4a) + (2a + 5ba)$

$\quad = 7ab + 4a + 2a + 5ab$ $\qquad 7ab + 4a$

$\quad = 7ab + 5ab + 4a + 2a$ $\qquad + 5ab + 2a$

$\quad = 12ab + 6a$ $\qquad \overline{12ab + 6a}$

Example 5   Subtract $3m^2 - 2xy$ from $11xy - 5m^2$

Solution $(11xy - 5m^2) - (3m^2 - 2xy)$

$\quad = 11xy - 5m^2 - 3m^2 + 2xy$

$\quad = 11xy + 2xy - 5m^2 - 3m^2$

$\quad = 13xy - 8m^2$

Example 6   Solve $(3m + 2n - 7) + (2m^2 + 5m + n^2)$

Solution $3m + 2n - 7 + 2m^2 + 5m + n^2$

$\quad = 3m + 5m + 2n - 7 + 2m^2 + n^2$

$\quad = 8m + 2n - 7 + 2m^2 + n^2$

$\quad = 2m^2 + n^2 + 8m + 2n - 7$

Do and Learn ◆

Addition and subtraction of algebraic expression.

(1) $m - n$ and $m + n$

(2) $mn - 5 + 2n$ and $nm + 2m - 3$

(3) $\dfrac{xy}{5} + \dfrac{x}{3}$ and $\dfrac{xy}{2} - \dfrac{x}{3}$

<div align="center">◆ ━○━ ∞ ━○━ **Exercise 13.2** ━○━ ∞ ━○━ ◆</div>

1.   Add the following algebraic expression.
      (i)   t - 4tz, 2t + 6tz   (ii) $7xy, 5xy, 3xy, -2xy$
      (iii) $5x - 7y, 3y - 4x + 2, 2x -3xy - 5$ (iv) $m^2 - n^2 - 1, n^2 - 1 - m^2, 1 - m^2 - n^2$
      (v) $3x + 11 + 8z, 5x - 7$ (vi) $a^2b + ab + ab^2, -a^2b + 2ba + 2a^2b^2$
      (vii) $x - y, y - z, z - x$

2.   Subtract the following algebraic expression.
      (i) $-5x^2$ from x^2 (ii) (a-b) from (a +b)
      (iii) $x^2 + 5x+4$ from $4x^2 -3xy + 8$ (iv) $5x^2 - 7xy + 5y^2$ from $3xy - 2x^2 - 2y^2$
      (v) $4pq - 5q^2 - 3p^2$ from $5p^2 + 2q^2 - pq^2$ (vi) $x^2 +10x - 5$ from $5x -10$

3.   What should be subtracted from $7x - 8y$ to get $x + y + z$?
4.   What should be added to $2p + 6$ to get $3p - q + 6$?

13.8   Find the value of algebraic expression.

The value of an algebraic expression depends on the values of variables which make that expression. In several cases we examine that whether it satisfies the equation formed on putting the value of variable in any expression.

Example 7 Find the values of following expression for $x = 3$.

(i) $x + 5$ (ii) $9x - 3$ (iii) $25 - 3x^2$ (iv) $4x^2 + 5x - 51$

Solutions

(i)   On putting 3 in place of x in $x + 5$
      = 3 + 5
      = 8

(ii) On putting 3 in place of x in $9x-3$
      = (9 × 3) - 3
      = 27 - 3 = 24

(iii) $25 - 3x^2$
      = $25 - 3 × (3)^2$
      = 25 - 3 × 3 × 3 = 25 - 27 = -2

(iv) $4x^2 + 5x -51$
      = $4 × (3)^2 + 5(3) - 51$
      = 4 × 9 + 5 × 3 - 51
      = 36 + 15 - 51 = 51 - 51 = 0

Example 8: Find the values of following expression for

(i) a + b (ii) 5a - 2b (iii) $a^2 - 2ab + b^2$ (iv) $a^3 - b^3$

Solutions: on putting a = 3 and b = 2 in given expressions

(i) a + b = 3 + 2 = 5 (ii) 5a - 2b = 5 x 3 - 2 x 2 = 15 - 4 = 11

(iii) $a^2 - 2ab + b^2$
= $(3)^2 - 2$ x 3 x 2 + $(2)^2$
= 9 - 12 + 4
= 13 - 12 = 1

(iv) $a^3 - b^3$
= $(3)^3 - (2)^3$
= 3 x 3 x 3 - 2x 2x 2
= 27 - 8
= 19

Exercise 13.3

1. Find the value of the following if $x = 2$.

(i) x - 3 (ii) 2x - 5 (iii) 9 - 6x (iv) $3x^2 - 4x - 7$ (v) $\frac{5x}{2} - 4$

2 Find the value of the following if $p = -1$.

(i) 4p + 5 (ii) $-3p^2 + 4p + 8$ (iii) 3(p - 2) + 6

3 Find the value of the following if a = 2 and b = -2.

(i) $a^2 - b^2$ (ii) $a^2 - ab + b^2$ (iii) $a^2 + b^2$

4 Find the value of the following if $x = 1$ and $y = 0$.

(i) 2x + 2y (ii) $2x^2 + y^2 + 1$ (iii) $2x^2y + 2x^2y^2 + y^2$ (iv) $x^2 + xy + 5$

We Learnt

1. Algebraic expressions are formed from variables and constants. Operations like +, -. ×, ÷ are to be carried out on variables and constants to make algebraic expressions.
2. Expression is composed from terms, by adding terms expression is formed.
3. Any term is a multiplication of its factors, factors of variables is said to be algebraic factor. Any one factor of term is known as coefficient of remainder of term.
4. An expression made up with one or more term is called polynomial It can be monomial (having single term), binomial (having two terms) and trinomial (having three terms).
5. The terms whose algebraic factors are same are known as like terms and the terms containing different algebraic factors are known as unlike terms.
6. Addition or subtraction of two like terms is again a like term, whose coefficient is equal to the sum or difference of coefficients of those like terms.
7. Two algebraic expression of like terms can be added or subtracted. The terms which are not like are left as such.
8. The value of algebraic expression depends on the value of variables.

Chapter 14 · SIMPLE EQUATION

14.1 We studied about algebraic term, algebraic expression and equation in previous classes. Some illustrations of these are as under.

Algebraic term $2x$, 3y, 5p etc.

Algebraic expression $3x + 5$, 2y - 3, 5p- 7 etc.

Equation $x = 2$, y = z + 1, p + 1 = 5 etc.

We studied to write mathematical statements in the form of equation by means of representation of equation of single variable and "trial and error method" to find their solutions and learnt that if solution of equation (value of variable) does not satisfy all the restrictions (conditions) of the statement then there is an error in either formation or solution of the equation. Hence there is a necessity of modification by consideration. The value of the variable for which this statement is true, is said to be root or solution of this equation. now we'll study other methods.

14.2 Solving an equation

An equation has two sides, first side is towards left which is known as left side or L.H.S. Second side is towards right which is known as right side or R.H.S. There is a sign of equality '=' in between both the sides. The numerical value of both the sides is equal. Both the sides of an equation are same as two balanced parts of a weighing balance.

If same mathematical operations (addition, subtraction, multiplication or division to any number) are performed in both the sides, then also the equation remains balanced. By doing so, the structure of the equation definitely changes.

To solve any equation $3x - 7 = 5$, we have to change the structure as $x = \dfrac{5 + 7}{3}$

there will be only variable on the L.H.S. and only numerical value on the R.H.S. To do so, we have to use one or more steps from the steps given below.

1. Addition of same number on both the sides.
2. Subtraction of same number from both the sides.
3. Multiplication of same nonzero number to both the sides.
4. Division by same nonzero number in both the sides.

The above method to solve an equation is said to be "Balance Method".

Let us understand this method by following example.

Example 1 Solve the equation $3x - 7 = 5$

Solution   Adding 7 to both the side

$$3x - 7 + 7 = 5 + 7$$

$$3x = 12$$

Dividing by 3 on both the sides

$$\frac{3x}{3} = \frac{12}{3}$$

$$x = 4$$

Verification of answer

We got the solution as $x = 4$ for the equation $3x - 7 = 5$, now find the

Value of both the sides by substituting x by 4.

$$\text{LHS} = 3x - 7$$
$$= 3 \times 4 - 7$$
$$= 12 - 7$$
$$= 5 = \text{RHS}$$
$$\text{LHS} = \text{RHS}$$

Transfer of a term from one side to another side by changing the sign or transfer of co-efficient of variable from one side to another side by division or multiplication is known as **transposition.**

Example 2   Solve the equation $5x + 2 = 17$ by transposition.

Solution

$$5x + 2 = 17$$
$$5x = 17 - 2 \quad \text{(From transposition of 2)}$$
$$5x = 15$$
$$x = \frac{15}{5} \quad \text{(From transposition of coefficient 5)}$$
$$x = 3$$
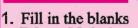
Verification of answer   LHS $= 5x + 2 = 5 \times 3 + 2 = 15 + 2 = 17 = $ RHS

Hence answer $x = 3$ is correct.

Do and Learn ◆

1. Fill in the blanks

Balance Method

$$7x + 6 = 34$$
$$7x + 6 - \ldots = 34 - \ldots$$
$$7x = \ldots\ldots$$
$$x = \ldots$$

Transposition Method

$$7x + 6 = 34$$
$$7x = 34 - \ldots$$
$$x = \frac{\ldots\ldots\ldots\ldots}{7}$$
$$x = 4$$

Verification of answer LHS $= 7x + 6$

$$= 7x \ldots\ldots +6$$
$$= \ldots\ldots +6$$
$$= \text{RHS}$$

2. Choose Correct/Incorrect

(i) $4x + x - 13 = 7$ में $x = 4$ (Correct/Incorrect)

(ii) $3x - 8 = 25$ में $x = 12$ (Correct/Incorrect)

(iii) $7x - 5 = 3x + 7$ में $x = 3$ (Correct/Incorrect)

(iv) $5x - 7 = 4x + 1$ में $x = 5$ (Correct/Incorrect)

To solve the equation containing rational coefficients, we have to find LCM (least common multiple) of denominators involved in the equation, and multiply both the sides of the equation by such LCM.

Example 3 Find the value of x in the equation $\frac{x}{3} - \frac{x}{4} = 1$ and verify the answer.

Solutions Here LCM of 3 and 4 is 12.

$$\text{or } \frac{x}{3} \times 12 - \frac{x}{4} \times 12 = 1 \times 12$$

$$\text{or } 4x - 3x = 12$$

$$\text{or } x = 12$$

Verification of answer LHS $= \frac{x}{3} - \frac{x}{4}$

$$= \frac{12}{3} - \frac{12}{4} = 4 - 3 = 1 = \text{RHS}$$

Hence answer $x = 12$ is Correct.

Example 4 Solve the equation $2(x + 4) = 12$

Solution
$$2x + 8 = 12$$
$$2x + 8 - 8 = 12 - 8 \quad \text{(On subtracting 8 from both sides)}$$
$$2x = 4$$
$$\frac{2x}{2} = \frac{4}{2} \quad \text{(Dividing by 2 on both sides)}$$
$$x = 2$$

Transposition Method
$$2(x + 4) = 12$$
$$2x + 8 = 12$$
$$2x = 12 - 8$$
$$2x = 4$$
$$2x = \frac{4}{2}$$
$$x = 2$$

⟩∞⟨ Exercise 14.1 ⟩∞⟨

Solve the equations given below and verify the answer.

1. $2x + 1 = 9$

2. $5x - 4 = 26$

3. $5x - 2x + 7 = 31$

4. $5x + 8 = 12 + 6$

5. $12x + 3x = 60$

6. $\dfrac{7x}{9} = 21$

7. $\dfrac{2x}{3} - \dfrac{x}{2} = 3$

8. $\dfrac{3x}{4} - \dfrac{2x}{5} = 7$

9. $\dfrac{7x + 1}{2} = 11$

10. $\dfrac{3l}{2} = \dfrac{2}{3}$

11. $7m + \dfrac{19}{2} = 13$

12. $6z + 10 = -2$

13. $\dfrac{q}{4} + 7 = 5$

14. $4(2 - x) = 8$

15. $3(n - 5) = 21$

16. $4 = 5(t - 2)$

17. $0 = 16 + 4(m - 6)$

14.3 Solving the word problems.

We use simple equation to solve simple problem for which the following steps are to be used in series-

1. Read the given problem carefully and write down "What is given" and what has to be found out.
2. Express unknown quantity with any variable.
3. Convert the given statements in the problem into mathematical statements by means of term or expression.
4. Write down the quantities (term or expression) which are equal according to condition of question in the form of an equation.
5. Find the value of variable by solving the equation and represent the solution of the problem.
6. Verify your answer.

Example 5   7 more than 4 times of a number is 43. Find the number
Solution   Let the unknown number be x
            Four times of the number $= 4x$
     7 more than 4 times of the number $= 4x + 7$

According to condition of question   Verification of answer
                                            7 more than four times the number

$$4x + 7 = 43$$
$$4x = 43 - 7$$
$$4x = 36$$
$$x = 9$$

$= 4 \times 9 + 7$

$= 36 + 7$

$= 43$ Hence the answer is correct

Example 6   An angle of a triangle is greater than 20^0 from another angle and smaller than 20^0 from the third. Find the values of all the three angles.

Solutions   Let us suppose first angle = x

Second angle = $x - 20^0$

Third angle = $x + 20^0$

According to the condition

$x + x - 20 + x + 20 = 180^0$ (Sum of all the three angles of a triangle

$x + x + x = 180^0$ is two right angles)

$3x = 180^0$

$\boxed{x = 60^0}$

First angle $x = 60^0$

Second angle $x - 20 = 60 - 20 = 40^0$

Second angle $x + 20 = 60 + 20 = 80^0$

Therefore all three angles are $= 60^0, 40^0, 80^0$

Verification of answer $= 60^0 + 40^0 + 80^0 = 180^0$

Hence the answer is correct.

⟩⟩⟩∞◯◯◯ Exercise 14.2 ◯◯◯∞⟨⟨⟨

1. By adding 12 in any number 43 is obtained . Find out that number.

2. On subtracting 5 from 4 times of any number, we get 27. Find out that number.

3. On adding double of a number in 5 times of that number, we get 42. Find out that number.

4. Sum of three consecutive numbers is 27. Find out that number.

5. Sum of three consecutive odd numbers is 39. Find out the numbers.

6. Sum of three consecutive even numbers is 48. Find out the numbers.

7. Ramu's age is 4 years more than three times of age of his son. If Ramu's age is 37 years, then find out his son's age.

8. Age of Ishu's father is 5 years more than three times of age the Ishu. Find out Ishu's age if her father's age is 44 years.

9. Riyaz thinks about a number in such a way that if he subtracts 7 from $2\frac{1}{2}$ times of that number, the result comes to be 23. Which number does Riyaz think

10. Age of Ramanjeet's father is 49 years. His father's age is 4 years more than 3 times of Ramanjeet's age. Find out Ramanjeet's age?

11. As compared to Jaipur, the road accidents per month in Jodhpur are 50 less than 3 times those occur in Jaipur. Road accidents in Jaipur are 400 per month. Then how many road accidents occurred in Jodhpur?

⋙= **We Learnt** =⋙

1. An equation is a condition on a variable such that value of terms on both the sides are same.
2. The value of the variable for which the equation is satisfied is called the solution of the equation.
3. An equation remains the same if the LHS and the RHS are interchanged.
4. In case of the balanced equation, if we
   (i)   Add the same number to both the sides, or
   (ii) Subtract the same number from both the sides, or
   (iii) Multiply both the sides by the same number,or
   (iv) Divide both sides by the same number, the balance remains undisturbed, i.e. the value of the LHS remains equal to the value of the RHS .
5. The above property gives a systematic method of solving an equation. We carry out a series of identical mathematical operations on the two sides of the equation in such a way that on one of the sides we get just the variable.
6. Transposing means moving to the other side. Transposition of a number has the same effect as adding same number to (or substracting the same number from) both sides of the equation. In case of addition or subtraction when you transpose a number from one side of the equation to the other side, you change its sign. In case of division, transposition results in multiplication and in case of multiplication transposition results in division.

15.1 Ratio – Proportion

Bhagat and Pratap started making map of Rajasthan. Bhagat told that to draw a large map on paper determine right ratio of measure (scale). They took 100 km = 1 cm and showed the road distance from Udaipur to Ajmer as 2.25 cm. At the same time, their classmates Keshav and Kalam came there and started finding out the actual distance between Udaipur and Ajmer.

Keshav's Method
Suppose distance =
D km then
$100 : D :: 1 : 2.25$
Or $\dfrac{100}{D} = \dfrac{1}{2.25}$

$100 \times 2.25 = 1 \times D$
$225 = D$
Actual distance = 225 Km

Kalam's Method

1 cm shows 100 km
2.25 cm will show
$= 100 \times 2.25$
$= 225$ Km
So actual distance
$= 225$ Km
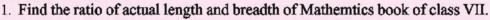
In actual life, proportions are applied for unitary rule, diagramatic presentation of map and proportional representations.

Do and Learn

1. Find the ratio of actual length and breadth of Mathemtics book of class VII.
2. Find the ratio of length and breadth of national flag by taking help from your teacher.
3. Find the ratio of length and breadth of your classroom by measuring it.
4. Measure your height and measure the length by expanding both the hand completely. Now find out the ratio between these two quantities.

Example 1 Find the ratio of 6 km with 400 m.

Solution Here both the quantities show the distance so we write this is one unit only 6 km. $= 6 \times 1000$
$= 6000$ m.

Hence required ratio 6 km : 400 m.
Means 6000 m : 400 m. Or 15 : 1

Example 2 Find out the value of x in following

$$3 : 25 :: x : 15$$

Solution $\dfrac{3}{25} = \dfrac{x}{15}$ (Express ratio form in terms of fraction)

$$x \times 25 = 3 \times 15$$

$$x = \dfrac{3 \times 15}{25} \text{ Or } x = 1.8$$

Therefore value of x = 1.8

Example 3   Farmer Balu required 25 litres of diesel in running a pump set for 15 hrs. If he has 45 litres of diesel then how many hours would he run the pump set?

Solution   Available quantity of diesel = 45 litres

From 25 litres of diesel pump set runs = 15 hrs.

From 1 litre of diesel pump set runs $= \dfrac{15}{25}$

From 45 litres of diesel pump set would run $= \dfrac{15}{25} \times 45$ hrs.= 27 hrs

⟩∞⟨ **Exercise 15.1** ⟩∞⟨

1. Find the ratio.

     (i) 60 paise with 3 rupees   (ii) 340 cm with 4m.

2. Write in simplest ratio.

     (i) 65 : 25   (ii) 72 : 64

3. Find out two equivalent ratios of the following ratios.

     (i) 3 : 5   (ii) 7 : 11

4. The length and breadth of carpet is 7 m. and 35 cm. respectively. Then find the following ratio.

     (i) Breadth with length   (ii) Length with breadth

5. If 12: x :: 14 : 21 then find the value of x.

6. Bhima halwai mixes 20 kg of sugar into 25 kg pulse for making halwa. While Bhikha halwai mixes 15 kg. of sugar into 12 kg of pulses. Find:

 (i)   How much sugar do both the halwais mix in per kg of pulse?

 (ii)   The prepared halwa of which halwai is sweeter?

7.   34 labourors are required for cleaning a 10.2 km long road. Then how many labourors are required for cleaning 7.5 km long road?

8.   The shadow of 7.5 mt. pole is 5 meter, then find out the height of a tree standing nearby whose shadow is 10 mt. long at the same time.

9.   Ramesh covers a distance of 10 km in 15 min by his motorcycle. How much time is required by Ramesh to cover a distance of 26 kms if the speed is same.

10.   3 kg of pulse is required in the midday meal of 60 students. How much quantity of pulse is enough on Saturday at the time of midday meal if 46 students were present?

15.2   Percentage

Pooja and Madhav after taking their exam result say to their mother on entering their house with joy.

Pooja – Mom, see I got 960 marks out of 1200 and I got first position in class.

Madhav – Mom, I got first position in class on getting 975 marks out of 1300. I got more marks than Pooja so I am more intelligent.

Pooja – Mom, how it can happen? Annual maximum marks are also more in Madhav's school?

Think, is Pooja right? Can you settle the dispute between them? At the same time father enters into the house and both of them approach their father to take his decision.

Papa explained in the following way.

For Pooja $\dfrac{\text{Marks obtained}}{\text{Maximum marks}}$ $= \dfrac{960}{1200} = \dfrac{96}{120} = \dfrac{8}{10} = \dfrac{4}{5}$

For Madhav $\dfrac{\text{Marks obtained}}{\text{Maximum marks}}$ $= \dfrac{975}{1300} = \dfrac{75}{100} = \dfrac{3}{4}$

Pooja   **Madhav**

$$\dfrac{4}{5} = \dfrac{16}{20} \qquad\qquad \dfrac{3}{4} = \dfrac{15}{20}$$

By Equivalent ratio $\dfrac{16 \times 5}{20 \times 5}$ By Equivalent ratio $\dfrac{15 \times 5}{20 \times 5}$

$$= \dfrac{80}{100} \qquad\qquad\qquad\qquad = \dfrac{75}{100}$$

$$\dfrac{80}{100} > \dfrac{75}{100}$$

Mom explained them that if maximum marks for both of them is 100 then Pooja gets 80 marks and Madhav gets 75 marks out of 100. Representation of a fraction with denominator 100 is known as how many percentage on each 100 or per hundreds. Percentage is represented by the symbol % which means hundredths. Percentage are fractions with denominator 100 ($\% = \frac{1}{100}$) and numerator of this fraction expresses rate of percentage.

Do and Learn ◆

1. 25 students of a class tell their interests about games.

    Kabaddi   -   4 students
    cricket   -   11 students
    chess   -   6 students
    tenis   -   3 students
    Other games -   1 student

   Express the number of students in percentage according to the interest in each game.

2. Following results are obtained on taking advice of 250 students on preference of menu of mid day meal in selected schools of Jalore Panchayat.

Menu	Students	Percentage
Chapati -vegetable	80	___%
Rice - pulse	70	___%
Porridge	35	___%
Chapati – pulse	60	___%

   Express the percentage of preference of each type of menu from the above results.

Abdul Uncle went out for a morning walk with his two grandsons. He met Khema and Pema, sons of farmer Deva on the way. Abdul Uncle discussed with them about their farming.

Pema –   "I sowed wheat in $\frac{3}{4}$ part of my farm and sowed mustard in rest of the part."

Khema –   "Uncle, I sowed wheat in $\frac{7}{10}$ part of my farm and in rest of the part I sowed mustard."

Uncle Abdul is returning home after the disussion. Uncle Abdul's grandson Karim said.

Karim –   "Grandpa, between Khema Tau and Pema Tau who sowed wheat in more part of his farm?"

We compare the part sown by them by percentage. For this we make such equivalent fraction of $\frac{3}{4}$ and $\frac{7}{10}$ with denominator as 100.

> If denominator of any fraction is 100, then the number at the numerator is the percentage.

Part sown by Pema Tau $\frac{3}{4} \times \frac{25}{25} = \frac{75}{100} = 75 \times \frac{1}{100} = 75\%$

Part sown by Khema Tau $\frac{7}{10} \times \frac{10}{10} = \frac{70}{100} = 70 \times \frac{1}{100} = 70\%$

So part sown by Pema Tau is more.

Second Method

We can also express percentage by multiplying the given fraction with $\frac{100}{100}$

$$\frac{3}{4} \times \frac{100}{100} = \frac{300}{4} \times \frac{1}{100} = 75 \times \frac{1}{100} = 75\%$$

$$\frac{7}{10} \times \frac{100}{100} = \frac{700}{10} \times \frac{1}{100} = 70 \times \frac{1}{100} = 70\%$$

15.2.1 Converting percentage into decimal fraction.

For this multiply by $\frac{1}{100}$ after removing %.

e.g. $-25\% = 25 \times \frac{1}{100} = \frac{25}{100} = \frac{1}{4} = 0.25$

15.2.2 Converting decimal fraction into percentage

For this decimal fraction is multiplied by 100%.

e.g. 0.6, 0.03, 0.75 will be converted into percentage in the following way

(i) when 0.6 is multiplied by 100% = 0.6 x 100%

$$= \frac{6}{10} \times 100\% = 60\%$$

(ii) when 0.03 is multiplied by 100% = 0.03 x 100%

$$= \frac{3}{100} \times 100\% = 3\%$$

(iii) when 0.75 is multiplied by 100% = 0.75 x 100%

$$= \frac{75}{100} \times 100\% = 75\%$$

Do and Learn ◆

1. Convert the following fractions into percentage.

    (i) $\frac{5}{8}$ (ii) $\frac{5}{3}$

2. Convert the decimal fractions into percentage.

    (i) 0.5   (ii) 0.08   (iii) 0.225   (iv) 6.5

3. Convert the percentage into simple fraction and decimal fraction.

    (i) 36%   (ii) $12\frac{1}{2}\%$ (iii) 3.6%

Example 4   Write the fraction $\frac{3}{25}$ into percentage.

Solution   Given number $= \frac{3}{25} \times 100\%$

$$= 12\%$$

Example 5 There are 44 boys in a class of 55 students. What is the percentage of boys ?

Solution There are 44 boys in a class of 55 students $= \dfrac{44}{55} \times 100\%$

Hence percentage of boys = 80%

Example 6 Convert following decimal numbers into percentage.

   (i) 0.9   (ii) 0.015

Solution

   (i) $0.9 \times 100\%$ (ii) $0.015 \times 100\% = \dfrac{15}{1000} \times 100\%$

       $= \dfrac{9}{10} \times 100\%$ $= \dfrac{15}{10}\% = 1.5\%$

       $= 90\%$

Example 7 22% girls are fond of making rangoli out of 50 girls in a class. Find out the number of girls who are fond of rangoli.

Solution Number of girls making rangoli $= 22\%$ of 50

                               $= 50 \times \dfrac{22}{100} = 11$ girls

Example 8 Convert the given percentage into simple decimal fraction.

   (i) $33\dfrac{1}{3}\%$ (ii) 150%

Solution   (i) $\dfrac{100}{3}\%$ (ii) 150%

          $= \dfrac{100}{3} \times \dfrac{1}{100}$ $= 150 \times \dfrac{1}{100}$

          $= \dfrac{1}{3}$ $= \dfrac{150}{100}$

          $= 0.33$ $= \dfrac{3}{2} = 1.5$

Exercise 15.2

1. Convert the given fraction numbers into percentage.

   (i) $\dfrac{3}{4}$ (ii) $\dfrac{7}{9}$ (iii) $\dfrac{14}{15}$ (iv) $3\dfrac{1}{3}$

2. Convert the given decimal fractions into percentage.
   (i) 0.84   (ii) 1.25   (iii) 0.875   (iv) 0.001

3. Convert the given percentage into simple fraction.
   (i) 52%   (ii) 125%   (iii) $6\dfrac{1}{4}\%$ (iv) $33\dfrac{1}{3}\%$

4. Find
   (i) 15% of 320   (ii) 35% of 875   (iii) 20% of 1250 gm. (iv) 16% of 32.5 m.

5. Find
   (i) 42% of what is 63.   (ii) 70% of what is 35.   (iii) 13% of what is 1170.

6. Convert the given percentage into decimals.
   (i) 7%   (ii) $1\frac{2}{5}$%   (iii) 0.03%   (iv) 16.7%

7. Out of 500 students 85% are girls in a school. Find the number of boys in the school.

8. During Green Rajasthan Campaign trees were planted in Akola village of Rajasthan out of which 10% of the trees dried. If 1800 trees are left here then how many trees were planted in the beginning?

9. 950 votes were casted at an election booth out of which 57 votes were rejected. If 1045 names of voters were registered in the voter list then what percentage of polling was done?

10. On the occasion of Shaheed Diwas, 28 members of out fo 35 members of Subhash Club donated blood. Similarly 38 out of 40 members of Tilak Club donated blood. which club members donated more percentage of blood?

15.3 Percentage Gain- Percentage Loss

In a town the rates of certain commodities in two years at Rohit Traders were as follows :

Cost Per Kg. Commodities	On 1.4.2014	On 1.4.2015
Sugar	30	27
Groundnut Oil	90	81
Wheat	13	15
Parmal Rice	28	32

Discuss the change in rates of commodities by seeing the above table. You will find that the rates of sugar and groundnut oil are reduced by Rs. 3 and Rs. 9 respectively. Whereas the rates of wheat and parmal rice have increased by Rs. 2 and Rs. 4 respectively. From these data you will feel that the rate of groundnut oil has decreased more and the rates of parmal rice increased more. If this type of change is expressed in percentage, then we can show the change in more correct way.

Commodity	Change in rate		Change	In percentage
	The Later value	The initial value		
Sugar	27	30	-3	-10%
Groundnut Oil	81	90	-9	-10%
Wheat	15	13	2	$15\frac{5}{13}$%
Parmal Rice	32	28	4	$14\frac{2}{7}$%

Change in rate in percentage $= \dfrac{\text{change}}{\text{Initial value}} \times 100$

For sugar $-$ $\dfrac{-3}{30} \times 100 = -10\%$

For groundnut oil $-$ $\dfrac{-9}{90} \times 100 = -10\%$

For wheat $-$ $\dfrac{2}{13} \times 100 = 15\dfrac{5}{13}\%$

For parmal rice $-$ $\dfrac{4}{28} \times 100 = 14\dfrac{2}{7}\%$

It is clear that loss/ reduction in rates of sugar and groundnut oil are same in percentage. Similarly growth in percentage of rates of wheat is more than parmal rice.

Do and Learn ◆

1. The population of a village is increased to 15000 from 12000 in last 10 years. Then what is the percentage growth of population?
2. Express rate of growth or loss in the form of percentage in the following:
   (1) The cost of electricity per unit has increased to Rs 6 from Rs. 3.50.
   (2) The cost of 100 envelopes has decreased to Rs. 80 from Rs 100.

15.4 Profit-Loss

Sumitra and Savitri bought 20 kgs. of bananas each from wholesale market and shopkeeper respectively at the rate of Rs. 20 and Rs. 25 respectively and both sell them at the rate of Rs. 22. Who will gain and who will bear loss.

Sumitra bought 20 kg. bananas for Rs. 400 at the rate of Rs. 20 per kg.

i.e C.P. of bananas bought by Sumitra $= 20 \times 20 = $ Rs.400

C.P. of bananas bought by Savitri $= 25 \times 20 = $ Rs.500

Both sold the bananas at the rate of Rs. 22

S.P. $= 22 \times 20 = $ Rs.440

Sumitra's selling price is more than her buying i.e. C.P.$<$ S.P. So she books profit.

Savitri sold the items in lesser price than her buying price i.e. S.P.$>$C.P. So she bears loss.

Sumitra earned Rs. 440-400 = Rs. 40

Therefore profit of Sumitra
$$= \text{S.P.-C.P.} = 440 - 400 = \text{Rs. } 40$$

And loss to Savitri = C.P. – S.P.
$$= 500 - 440 = \text{Rs. } 60$$

Now, see by expressing their profit / loss into percentages i.e. per hundred.
Sumitra earned profit on Rs. 400 = Rs. 40

Hence profit on Rs.1 $= \dfrac{40}{400}$

Or   profit on Rs.100 $= \dfrac{40}{400} \times 100$

Now, see by expressing their profit / loss into percentages i.e. per hundred.

Hence profit = 10%
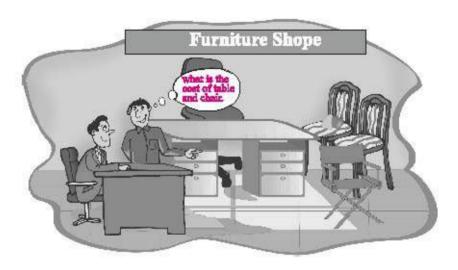
i.e. profit percentage $= \dfrac{\textbf{Profit}}{\textbf{C.P.}} \times \textbf{100}$

Savitri gets loss on Rs. 500 = Rs. 60

Hence loss on Rs. 1 $= \dfrac{60}{500}$

or   loss on Rs. 100 $= \dfrac{60}{500} \times 100 = 12\%$

i.e. loss percentage $= \dfrac{\textbf{Loss}}{\textbf{C.P}} \times \textbf{100}$

Furniture Shope

what is the cost of table and chair.

Customer   What is the price of table and stool?
Chhagan   On seeing the bill- Rs. 750
Shopkeeper Karma arrives after the customer goes. When he comes to know
about the matter, then.
Karma   Oh! You sold this item in loss.
Chhagan   No Dad, How can it be ? I have seen the bill , the price of one set
              was Rs. 700.

Karma - look, When I went to purchase 10 sets of these articles then I gave Rs. 200 as bus and taxi fare, Rs. 100 as labour for lifting, Rs. 250 as truck fare.

Chhagan Yes Daddy, it means we spent 200 +100 + 250 = Rs. 550 on these articles.

Karma That's why I am telling you, the price of these articles for us-
For 10 sets at the rate of Rs. 700 per set = Rs. 7000
And other overhead expenses = Rs. 550
Then total cost price = C.P. + overhead expenses
= Rs. 7000 + Rs. 550
= Rs. 7550

Chhagan It means that total price of a set is Rs.755, while I sold a set in Rs. 750 then there is a loss of Rs. 5 to us.

Karma If we want to earn Rs. 50 on one set then at what price it should be sold?

Chhagan Total C.P. Rs. 755 + profit Rs. 50 = Rs. 805 should have been the selling price.

Therefore to determine the S.P. of an article, some additional expenses like amount spent, labour charge,transportation etc. are included in the cost price.

Do and Learn ◆

1. Mahaveer bought 5 bags of sugar for Rs. 16000. He spent Rs. 200 for taxi fare, Rs. 120 for labour charges and Rs. 200 for transportation charges. Find the selling price of per kg. of sugar so as to earn a profit of Rs. 3 on each kg.

2. Manoj bought a second hand car for Rs. 1,50,000. He spent Rs. 60,000 on its engine repair and Rs. 15000 on replacing the tyre tubes. Manoj sold this car to Jitendra for Rs. 2,10,000. Calculate loss or gain in this business.

Example 9 Prem bought a sewing machine for Rs. 4800 and sold it for Rs. 5400. Find his gain percent.

Solution C.P. of sewing machine = Rs. 4800
S.P. of sewing machine = Rs. 5400
Profit = 5400-4800 = Rs. 600

$$\text{Profit percentage} = \frac{\text{Profit}}{\text{C.P.}} \times 100$$

$$\text{Profit percentage of Prem} = \frac{600}{4800} \times 100 = \frac{25}{2}$$

$$\text{Hence profit} = \frac{25}{2}\% = 12\frac{1}{2}\%$$

Example 10 Raheem bought a house for Rs. 1,40,000. He spent Rs. 14,000 on its registration, brokerage etc., Rs. 7000 in plumbing, Rs.1700 in repairing electric wires and Rs. 8300 for other repairing works. If he sold this house for Rs. 2,03,490 then find his gain percent.

Solution

Raheem bought the house	= Rs. 140000
Registration charges	= Rs. 14000
Plumbing cost	= Rs. 7000
Repairing electric wires	= Rs. 1700
Other repairs	= Rs. 8300

Total overhead expenses = 14000+ 7000+1700+8300= Rs.31,000 Real

C.P. of house = 140000+31000 = Rs. 171000

S.P. of house = Rs. 2,03,490

Profit = S.P.-C.P.

= 203490-171000

= Rs.32,490

$$\text{Profit percentage} = \frac{\text{Profit}}{\text{C.P.}} \times 100$$

$$\text{Profit percentage} = \frac{32490}{171000} \times 100 = \frac{3249}{171} = 19\%$$

Hence profit percentage = 19%

Example 11 A football club got victory in 12 matches this year whereas last year it won 15 matches. What is the loss or gain percent as compared to last year?

Solution Reduction in number of victories = 15-12 = 3

$$\text{Percentage reduction} = \frac{\text{Reduction}}{\text{Win in base year}} \times 100$$

$$= \frac{3}{15} \times 100$$

The number of victories reduce by 20%.

⤜∞⬯◯⬯⬯ Exercise 15.3 ⬯⬯◯⬯∞⤛

1. Kishor bought a chair for Rs. 450 and sold it for Rs. 500. Find Kishor's gain percentage.
2. Find gain or loss in following transactions of buying and selling. Find gain or loss percent in each case.
   (i) One bicycle was purchased for Rs. 3500 and sold for Rs. 3000.
   (ii) One washing machine was purchased for Rs. 15000 and sold for Rs. 15500.
   (iii) A toy car was purchased for Rs. 450 and sold for Rs. 540.

(iv)   Arvind bought a T.V. for Rs. 12000 and sold it for a profit of 15%. How much money is obtained on selling T.V.?

3. If the population of a town increases to 26500 from 25000 then find the percentage growth in population.

4. A businessman bought 50 kilograms cereal for Rs. 2000. He spent Rs. 400 on its cleaning. The value of cereal decreased in market due to high supply. He sells it for Rs. 41 per kg. Find his percentage gain or loss.

5. Shravan mechanic bought an old scooter for Rs. 5500. He spent Rs. 150 in its transportation and Rs. 550 on its repair. If he wants to earn 15% gain on it, then at what price should he sell the scooter?

15.5 Simple Interest

Ashok borrows Rs. 50,000 from an institute for the construction of his house. This borrowed amount is known as Principal. He repays Rs. 55,000 to that institute after one year.

Ashok paid Rs. 5,000 extra on Rs. 50,000. This extra amount is known as interest.

The amount of interest depends on following things:-

1. Amount borrowed (Principal)

2. Time (Period for which the amount is borrowed)

3. Rate (The extra amount paid on per hundred) which is determined on the basis of per month / per annum.

You can find the amount you have to pay at the end of the year by adding sum borrowed and the interest

i.e. Amount = Principal + Interest

Do and Learn

1. How much interest would Ashok have pay after 2 years if he was not able to return the money to the institute after1 year?

2. How much money in total has to be paid including interest?

The value of simple interest increases or decreases with increase or decrease in principal, time and rate of interest respectively.

The formula for simple interest can be expressed as follows

Simple interest = Principal × Time × Rate Per 100

Simple interest = principal × time × $\frac{Rate}{100}$

" mooll, ver aru kaal ka kanchan guna karay, ek sau se bhaag diye byaaj turant batlaya mathematician kanchan gave this 'shloka' for calculating simple interest.

Example 12 Ashok borrowed Rs. 20,000 for 3 years from a nationalized bank at the rate of 10% simple interest then what amount will he pay for interest and what total amount will he return?

Solution Amount borrowed (Principal) = Rs. 20,000

Rate of interest = 10%

Time = 3 years

Interest on Rs. 100 for 1 year = Rs. 10

Then interest on Rs. 1 for 1 year = Rs $\frac{10}{100}$

Interest on Rs. 20,000 for 1 year = $\frac{10}{100}$ × 20,000

Interest on Rs. 20,000 for 3 years = $\frac{10}{100}$ × 20,000 × 3

Simple interest = $\frac{10}{100}$ × 20,000×3 = Rs. 6,000

Amount returned with interest

Amount = amount borrowed + interest

Amount = principal + interest

= Rs. (20000 + 6000)

= Rs. 26,000

Example 13 Chhoga borrows a loan of Rs. 8,000 at 12% annual rate on simple interest. Find out how much amount he will repay after 1 year?

Solution

Amount borrowed = Rs. 8000

Rate of interest = 12% per year

If he borrows Rs. 100 then simple interest for 1 year = Rs. 12

If he borrows Rs. 1 then simple interest for 1 year = Rs.$\frac{12}{100}$

If he borrows Rs. 8000 then simple interest for 1 year = Rs.$\frac{12}{100} \times 8000$

$$= Rs. 960$$

amount (including simple interest after 1 year = principal + interest

$$= 8000 + 960$$
$$= Rs. 8960$$

$$\text{Or simple interest} = \frac{\text{principal} \times \text{time} \times \text{rate}}{100}$$

$$= \frac{8000 \times 1 \times 12}{100}$$

$$= Rs. 960$$

$$\text{Amount} = \text{Principal} + \text{Interest}$$
$$= 8000 + 960 = Rs. 8960$$

Example 14 If simple interest is Rs. 450 in 3 years at the rate of 10% then find the principal amount.

Solution

Given the rate = 10% , time = 3 years, interest = Rs. 450, principal = ?

$$\text{Simple Interest} = \frac{\text{Principal} \times \text{Time} \times \text{Rate}}{100}$$

$$450 = \frac{\text{principal} \times 3 \times 10}{100}$$

$$450 = \frac{\text{principal} \times 3}{10}$$

$$\text{Principal} \times 3 = 450 \times 10$$

$$\text{principal} = \frac{450 \times 10}{3}$$

$$\text{principal} = Rs. 1500$$

Exercise 15.4

1. Lalji took the loan of Rs. 1500 from a bank to buy a cow. He repaid the loan with interest Rs. 120 after 1 year. Find how much amount was paid by Lalji?

2. Rani takes a loan of Rs. 4000 at 12% annual rate of interest from Mahila Cooperative Bank for buying a sewing machine. Find that how much amount Rani has to pay in 1 year.

3. A sum of Rs. 3500 is borrowed at the 8% rate of interest on simple interest. After 2 years, how much amount due after 2 years?

4. At which rate of interest on Rs. 4500 after 2 years Rs. 360 will be due as simple interest?

5. Ravindra paid Rs. 320 after 1 year as simple interest at 8% annual rate. How much amount he borrow?

We Learnt

1. We are often required to compare 2 quantities in our daily life. They may be height, weight, salaries, marks etc.

2. A way of comparing quantities is percentage also. Percentages are numerator of fraction with denominator 100. Percentage means per 100.

3. Fractions can be converted to percentage and parentage can be converted into fractions.

4. Percentage is widely used in our daily life.

   (i) When part of a quantity is given, then we can find the value of whole quantity.

   (ii) When parts of a quantity are given to us as ratios, we have seen how to convert them to percentage.

   (iii) The increase or decrease in a quantity can also be expressed as percentage.

   (iv) The profit or loss incurred in a certain transaction can be expressed in terms of percentage.

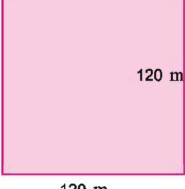   (v) While computing the interest on an amount borrowed the rate of interest is given in terms of percentage.

Chapter 16
PERIMETER AND AREA

16.1 Neelam and Rakesh made fencing of barbed wires around their field.

60 m

180 m
Neelam's Field
Fig. (i)

120 m

120 m
Rakesh's Field
Fig. (ii)

If cost of fencing is Rs.12 per meter then on whose field the cost of fencing would be maximum?

In which field, the cost of ploughing would be maximum at the rate of 100 rupees per square meter.

To find the total cost of fencing, we have to multiply the perimeter with the rate of fencing per meter.

Similarly to find the cost of ploughing we have to multiply the area (in square meter) with the rate of ploughing per square meter.

Since Neelam's field is rectangular.

Hence perimeter of Neeelam's field $= 2 \times (l + b.)$
$$= 2 \times (180 + 60)$$
$$= 2 \times 240$$
$$= 480 \text{ m}$$

While Rakesh's land is square

Hence perimeter of Rakesh's field $= 4 \times \text{Side}$
$$= 4 \times 120$$

Again area of Neelam's field $= l. \times b.$
$$= 180 \times 60$$
$$= 10800 \ sq.m.$$

Area of Rakesh's field $= (\text{Side})^2$
$$= (120)^2$$
$$= 120 \times 120$$
$$= 14400 \ sq.m.$$

As area of Rakesh's field is more, the cost of ploughing would be more.

Do and Learn ◆

1. The figures of plates showing registration number is given below. Calculate the perimeter by measuring length and breadth of plates of bus, taxi and private vehicles around

Bus	Taxi	Private vehicle
RJ19 PA 3807	RJ51 TA 1051	RJ271CO706

2. In the following case what do we require to find perimeter or Area?
   (i) Stitching the lace on the edges of dupatta.
   (ii) Putting the black soil in hockeys ground.
   (iii) Filling the ceiling of room.
   (iv) Fencing around the farm.

≫⊶⟨ Exercise 16.1 ⟩⊷≪

1. Radha takes 2 round daily along the sides of a square park of side 60 meter. Then find out that how much distance is covered by her daily?
2. Suresh has a ribbon of length 78 cm. He wants to apply it on the edges of a rectangular photo frame of length 26 cm. Find the breadth of frame.
3. Ranu wants to spread a carpet in the hall of the drawing room whose length is 50 m. Find the area of the carpet for this hall whose breadth is half of the length.
4. Gurmeet sowed the crop of Moong in 4200 square meter part of his land. He wants to fence around the fied. If the breadth of the land is 30 meter then how much wire (in length) is needed for it ?
5. Area of Playground of school is 38400 square meter. If the ratio of length and breadth of playground is 3 : 2 then find the perimeter of playground.
6. The perimeter of a rectangle and a square are same, the length and breadth of rectangle is 25 cm. and 15 cm. respectively. Which figure has greater area ?
7. Find the perimeter of following figures.
   (i) Triangle whose sides are 2 cm., 3 cm., and 4 cm.
   (ii) Equilateral triangle with side 8 cm.
   (iii) Isosceles triangle whose two equal sides are each equal to 10 cm. and the third side is 7 cm.
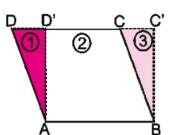
16.2 Area of a Parallelogram

We come across many shapes other than squares and rectangles around us. How will you find the area of a land which is parallelogram in shape?
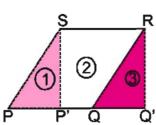
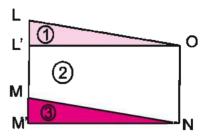Opposite sides of a parallelogram are equal and parallel.

Let us try

Draw three parallelograms of different measures.

Draw a perpendicular on the base from one vertex to the opposite side of the parallelogram.

Cut a triangle (1) formed by the vertex and perpendicular drawn from the vertex to the opposite side and join it to opposite parallel side shown as triangle (3). Both the triangles (1) and (3) are congruent by RHS rule. Hence area of Δ 1= area of Δ 3.

Parallel ogram	Base	Perpendic ular on base from vertex to the side opposite to the	Cut out the triangle figure (1)	Triangle added on opposite side (3)	Figure made in new positions (2)+(3)	Relation between area of parallelogram and rectangle (1)+(2)=(2)+(3)
ABCD	CD	AD'	ΔAD'D	ΔBC'C	ABC'D'	ABCD = ABC'D'
PQRS	PQ	SP'	ΔSP'P	ΔRQ'Q	P'Q'RS	PQRS= P'Q'RS
LMNO	LM	OL'	ΔOL'L	ΔNM'M	L'M'NO	LMNO = L'M'NO

It is clear from the table:

Area of {figure (1) + figure (2)}= Area of {figure (2) + figure (3)}

(Because area of figure (1) and figure (3) are equal by RHS rule of a right triangle)

Hence area of parallelogram   = Area of rectangle
                               = length × breadth
                               = base × perpendicular on base from the
vertex of opposite side.

Area of parallelogram = (base × height) square unit.

Activity

- Take transparent paper/sheet.
- Cut the parallelograms of different sizes on it.
- Find the area of these on putting squared block sheet or graph paper.

· Separate a triangular figure by cutting perpendicularly from vertex on the side opposite to the base of parallelogram.
· Make a rectangle by putting the separated figure on the other side.
· Find the area of rectangle thus formed from the graph paper/squared block sheet.
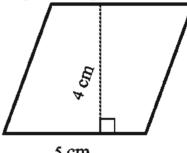· Compare the areas of parallelogram and rectangle.
· Here both the areas are same.

Example1   One side and corresponding height of a parallelogram is 5 cm. and 4 cm. respectively. Find the area of parallelogram.

Solution

Length of base = 5cm
height = 4cm
Area of parallelogram = base × height
= b × h
= 5 × 4 square cm.
= 20 square cm.

Example 2 If area of a parallelogram is 56 square cm. and its base is 7 cm. then find its height x ?

Solution

Area of parallelogram = base × height
$56 = 7 \times x$
Or $7 \times x = 56$

Or $x = \dfrac{56}{7}$

Or $x =$ 8 cm
Height of parallelogram is 8 cm.

Example 3 Length of two sides of parallelogram PQRS are 8 cm. and 5 cm. Corresponding height of base QR is 4 cm. Find:
(i)   Area of parallelogram PQRS.
(ii)   Corresponding height of base PQ.

Solution

(i)   Area of parallelogram PQRS = base × height
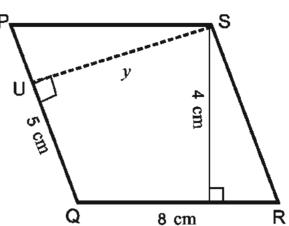= 8cm × 4 cm = 32 sq. cm.

(ii)   Base = 5cm
Height (SU) = y cm
Area = 32 sq. cm

Area of parallelogram = base × height

       or $32 = 5 \times y$

       or $5 \times y = 32$

$$y = \frac{32}{5} = 6.4 \text{ cm}$$
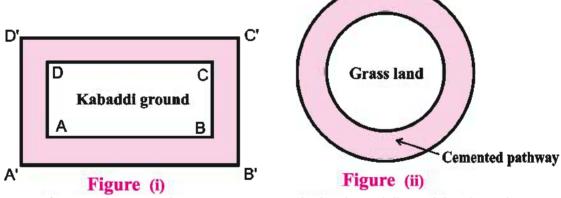
Hence corresponding height of base PQ = 6.4 cm

16.2.1 Path ways
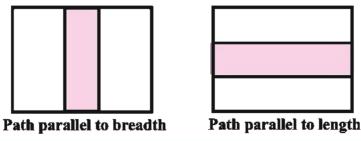
      Many situations are seen in our daily life in which path are present either in or out around a rectangular, square or circular park or path is made parallel to the length or breadth.

How to find area of path

1. Area of path made around rectangular, square or circular portion = Area of given portion along with path – Area of given portion without path.

Kabaddi ground

Grass land

Cemented pathway

Figure (i)   **Figure (ii)**
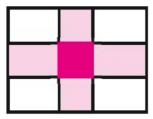
2. Area of path made on edges or centre parallel to length/breadth = length of parallel side × breadth of path.

Path parallel to breadth   **Path parallel to length**

3. Area of paths parallel to length and breadth intersecting mutually =
Area of path – Area of Common portion.
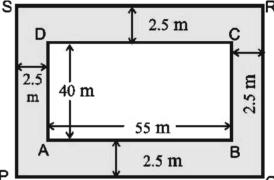
Path parallel to length and breadth.

Example 4 The length and breadth of a rectangular park is 55 m. and 40 m.
respectively. A 2.5 wide path is made around the park outside. Find
the area of the path.

Solution

In the figure, ABCD is a rectangular park
and shaded area shows 2.5 m. wide path.
We have to find.

Area of rectangular field PQRS included path – Area of rectangular park ABCD

Length of park with path (PQ) = Length of park (AB) + 2 × Breadth of path

$$= 55\,m + 2 \times 2.5\,m$$
$$= 55\,m + 5\,m = 60\,m$$

Breadth of park with path (PS)

$$= \text{Breadth of park (AD)} + 2 \times \text{Breadth of path}$$
$$= 40\,m + 2 \times 2.5\,m$$
$$= 40\,m + 5m = 45\,m$$

Area of rectangular park PQRS = Length × Breadth
$$= 60\,m \times 45\,m$$
$$= 2700\,sq.\,m$$

Area of rectangular park ABCD = Length × Breadth
$$= 55\,m \times 40\,m$$
$$= 2200\,sq.\,m$$

∴ Area of path = Area of rectangular park PQRS with path – Area of rectangular
park ABCD

$$= 2700\,sq.\,m. - 2200\,sq.\,m = 500\,sq.m$$

Example 5   A 5 meter wide path is present to the inner side of the boundary of a square park of side 80 meter. Find the area of this path. Find the expense of covering that path with red soil at the rate of Rs. 180 per square meter.
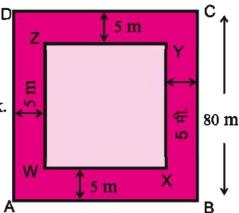
Solution

ABCD shows a square park of side 80 m. in the figure and the shaded part shows 5 m. wide path to the inner side of the park.

Area of path

$$= \begin{pmatrix} \text{Area of square} \\ \text{park ABCD} \end{pmatrix} - \begin{pmatrix} \text{Area of square park} \\ \text{WXYZ without path} \end{pmatrix}$$

Side of a park WX without path = Side AB of park -2×breadth of path

$$= 80m - 2 \times 5m$$
$$= 80m - 10m$$
$$= 70m$$

Area of square park ABCD $= (\text{side})^2 = (80\,m)^2 = 6400\,\text{sq. m.}$
Area of square park WXYZ $= (\text{side})^2 = (70\,m.)^2 = 4900\,\text{sq. m.}$
Area of path = Area of square park ABCD − Area of square park WXYZ
$$= 6400\,\text{sq. m.} - 4900\,\text{sq. m.} = 1500\,\text{sq. m.}$$

If cost of covering 1 sq. mt. with red soil   = Rs. 180
Cost of covering 1500 sq. mt. with red soil = 180×1500
$$= \text{Rs. } 2,70,000$$

Example 6   The length of a rectangular grassland is 75 m and breadth is 55 m In the centre two paths of 3 m wide parallel to the length and breadth of the ground are situated in such a way that they intersect each other at right angles. Find the area of the path?
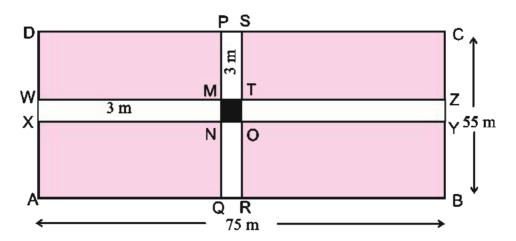
Solution

Area of path parallel to length (WXYZ) = Length × Breadth
$$= 75\,m \times 3\,m$$
$$= 225\,\text{sq. m}$$
Area of path parallel to width (PQRS) = Length × Breadth
$$= 55\,mt. \times 3\,m$$
$$= 165\,\text{sq. m}$$

Area of common path square MNOT (situated on both paths) = side×side

$$= 3 \text{ m.} \times 3 \text{ m.}$$

$$= 9 \text{ sq. m..}$$

Area of square MNOT i.e. 9 sq.mt. is included in both the paths.

Therefore area of complete path = Area of WXYZ + Area of PQRS − Area of square MNOT

$$= (225 + 165 - 9) \text{ sq. m.}$$
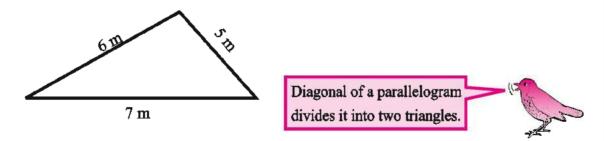
$$= (390 - 9) \text{ sq. m.}$$

$$= 381 \text{ sq. m.}$$

We have seen in the figure of above example that shaded part is located on both the paths. Therefore area of shaded path is subtracted.

16.3   Area of triangle
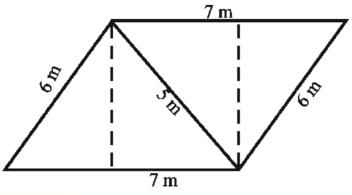
We have to find the expenditure of planting grass in a triangular park. How do we find the area of triangular park?

Diagonal of a parallelogram divides it into two triangles.

Let's think: Measurement of park is given in meters.

The scale taken as 1 m. = 1 cm. on the cardsheet by making two congruent triangles of 6 cm. , 7 cm. and 5 cm. Add both triangles in such a way that sides of equal measure reach near by and make a parallelogram.

∵ Area of triangle $= \frac{1}{2}$ (Area of parallelogram)

∴ **Area of triangle** $= \frac{1}{2}$ **(Base × height) square unit**
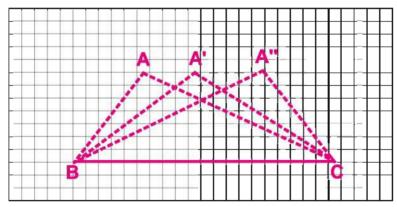
Do and Learn ◆

Draw parallelograms of different measures. Cut them along any diagonal and make two triangles.

- Are both the triangles congruent in every situation?
- Area of two congruent triangles always equal?
- Is the converse of it always true?
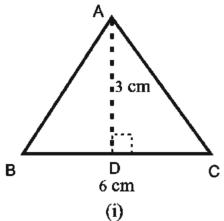
Let's try by doing:

Take a graph paper and draw different triangles on it by taking base and height of the same measure.

- Take three triangles of measure ABC, A'BC, and A"BC, and observe.
- Number of squares surrounded by three squares are same, which means that area of all three triangles are same.
- Can they completely cover each other? Cut and see them.

Example 7   Find the area of triangles shown in the figure.

A

3 cm

B   D   C
        6 cm
      (i)

P

2 cm

O   5 cm   R

(ii)

Solution   Shape (i) Area of triangle (ABC) = $\frac{1}{2}$ × Base × Height

$$= \frac{1}{2} \times BC \times AD$$

$$= \frac{1}{2} \times 6 \text{ cm} \times 3 \text{ cm} = 9 \text{ sq. cm}$$

Shape (ii) Area of triangle (PQR) = $\frac{1}{2}$ × Base × Height

$$= \frac{1}{2} \times QR \times PO$$

$$= \frac{1}{2} \times 5 \text{ cm} \times 2 \text{ cm} = 5 \text{ sq. cm}$$

Example 8   If area of triangle PQR is 52 square cm. and height PS = 8 cm. then find base QR.

Solution

Height of PS in the given shape = 8 cm
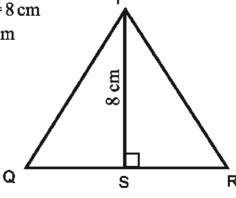
Area of triangle PQR = 52 sq. cm

Base QR of triangle PQR = ?

Area of triangle PQR = $\frac{1}{2}$ × Base × Height

$$= \frac{1}{2} \times QR \times PS$$

$$52 \text{ sq. cm} = \frac{1}{2} QR \times 8 \text{ cm}$$

P

8 cm

Q   S   R

$$QR = \frac{52 \times 2 \text{ cm}^2}{8 \text{ cm}}$$

$$= 13 \text{ cm}$$

Base QR   = 13 cm

Example 9 In a triangle ABC, AC = 10 cm., BC = 8 cm., and AE = 6 cm., then Find out

(i) Area of triangle ABC

(ii) Length of BD

Solution

(i) In triangle ABC, base BC = 8cm
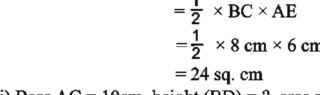
Height AE = 6cm

Area of triangle ABC $= \frac{1}{2} \times$ base $\times$ height

$= \frac{1}{2} \times BC \times AE$

$= \frac{1}{2} \times 8 \text{ cm} \times 6 \text{ cm}$

$= 24$ sq. cm

(ii) Base AC = 10cm, height (BD) = ?, area = 24 sq. cm

Area of triangle $= \frac{1}{2} \times$ base $\times$ height

Area of triangle $= \frac{1}{2} \times AC \times BD$

24 sq. cm $= \frac{1}{2} \times 10 \times BD$

$$BD = \frac{24 \times 2 \text{ cm}}{10} = 4.8 \text{ cm}$$

Example 10 Ratio of base and height of triangle PQR is 3:2. If its area is 108 square cm. then find out its base and height.

Solution According to figure ratio of base QR and height PQ, in the triangle PQR is 3: 2.

Base QR of triangle = $3 \times x$

Altitude PQ of triangle = $2 \times x$

Area = 108 sq. cm

Area of triangle $= \frac{1}{2} \times$ base $\times$ height $= \frac{1}{2} \times QR \times PQ$

108 sq. cm $= \frac{1}{2} \times 3x \times 2x$

108 sq. cm $= 3x^2$

or $3x^2 = 108$

or $x^2 = 36$
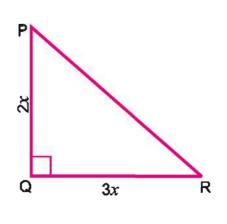
or $x = 6$

Base QR of triangle $= 3 \times x$
$$= 3 \times 6$$
$$= 18 \text{ cm}$$
Height PQ of triangle $= 2 \times x$
$$= 2 \times 6$$
$$= 12 \text{ cm}$$
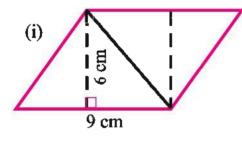
Exercise 16.2
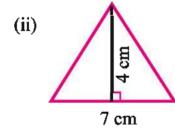
1. Find the area of parallelogram and triangle by observing the following figures.

(i)

9 cm, 6 cm

(ii)

5.5 cm, 3 cm

(ii)

4 cm, 7 cm

(iv)

6.5 cm, 3 cm
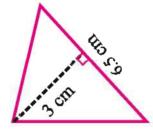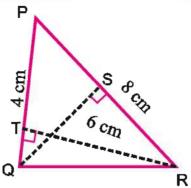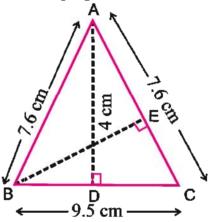
2.   The height of a parallelogram is one fourth of its base. If its area is 144 sq. cm. then find its base and height.

3.   The areas of triangular field of Kali and rectangular field of Hamida are same. The length and breadth of Hamida's field are 20 cm. and 15 cm. respectively. The length of base of Kali's field is 25 cm., then find out its height?

4.   If triangle PQR (attached Figure) PQ = 4 cm., PR = 8 cm., RT = 6 cm, then find out –
    (I)   Area of triangle PQR
    (ii)   Length of QS

5. Base of a triangle is 8 cm. If altitude of triangle is two times of its base, then find out the area of triangle.

6. ABC is an isosceles triangle in which AB=AC=7.6 cm. and BC=9.5 cm (attached fig.). The perpendicular AD from A on side BC is 4 cm. Find area of triangle ABC and measure of perpendicular BE from B on side AC.

7. Ratio of base and height of a parallelogram is 5 : 2. If area is 640 square cm. then find base and height.

8. Shyam has a rectangular garden of length 95 meter and breadth 80 meter. He wants to plant trees by digging 5 meter broad area outside the garden. Find out the area in which he will plant trees?

9. A 2 meter wide path is present to the inner side of a square ground of side 60 meter. Find out –
   (i) Area of path.
   (ii) Expenditure on cementing the path at the rate of Rs. 270 per square meter.

10. Two pathways parallel to the length and breadth have been constructed in the centre of a rectangular park. Whose length is 125 m. and breadth is 95 m. If width of each pathway is 10 meter. Find out –
    (i) Expenditure on putting soil on the pathway at the rate of Rs. 80 per square meter.
    (ii) Area of putting grass excluding the paths.

16.4.1   Circumference of Circle

Mamta wants to put a moulding frame of plastic around her tea table which is semicircular at both the ends.

Mamta told her sister Meena to bring a frame for this purpose. Meena wants to measure the length of edges of table. But she is facing problem in measuring the ends. Mamta guides her that to measure curved surface use different methods. Let's learn to measure the length of figure of curve. Mamta measured the distance with the help of thread by wraping it to circular bangle. This is known as "circumference" around the circular field.

> We can find out circumference by finding out the distance covered in one complete round on a plane surface by putting a mark on the circumference of a circular disc, wheel, bangle etc.

Meena – In all these situations there is problem in finding accurate circumference of a circular part. Lets determine a formula for it.

Mamta – Yes, I have seen that when blacksmith puts an iron rim around a wooden wheel he used to measure the length of diameter and on the basis of the length of diameter he estimates length of rim correctly and then puts it. Come. let's find the relationship between diameter and circumference. Mamta and Meena took 7 circular objects of different radius. They measured them with the help of thread and detected the ratio of circumference and diameter by filling the measurements in the following table.

Circule	Radius	Diameter	Perimeter	Perimeter ÷ Diameter
1	3.5 cm	7.0 cm	22.0 cm	$\frac{22}{7} = 3.14$
2	7.0 cm	14.0 cm	44.0 cm	$\frac{44}{14} = 3.14$
3	10.5 cm	21.0 cm	66.0 cm	$\frac{66}{21} = 3.14$
4	14.0 cm	28.0 cm	88.0 cm	$\frac{88}{28} = 3.14$
5	17.5 cm	35.0 cm	110.0 cm	$\frac{110}{35} = 3.14$

It is clear from the above table that the value of circumference/diameter is almost same for all shapes of different radii. This value is approximately 3.14. This constant is shown by "π" pie.

So $\dfrac{\text{circumference (c)}}{\text{diameter (d)}} = \pi$ or $\dfrac{\text{circumference (c)}}{2 \times \text{radius}} = \dfrac{c}{2r}$

or   c = πd   or   c = 2πr

So circumference of circular objects = πd = 2πr.

Example 11   Mohan wants to put golden rim on his mother's bangals. How long rim will he use when the radius of bangles is 3.5 cm. (without overlapped).

Solution   Radius of circular bangle = 3.5 cm

Circumference of circle = 2πr

$= 2 \times \dfrac{22}{7} \times \dfrac{35}{10}$ cm $\left| \pi = \dfrac{22}{7} \right.$

= 22 cm

Example 12   Diameter of a circular wheel is 11.2 cm. Find the circumference of wheel.

Solution

Diameter of wheel (d) = 11.2 cm

therefore radius (r) = 11.2 ÷ 2 cm = 5.6 cm

Circumference of circular wheel = 2πr

$= 2 \times \dfrac{22}{7} \times 5.6$ cm

= 35.2 cm

Example 13   What will be the distance covered by Banwari in rotating a wheel of radius 42 meter in 2 rotations?

Solution   Radius of circular wheel = 42 meter

Circumference of wheel = 2πr

$= 2 \times \dfrac{22}{7} \times 42$ meter

= 264 meter

∴   Distance covered by wheel in one round = 264 meter

∴   Distance covered by wheel in two round   = 264 × 2 meter

= 528 meter
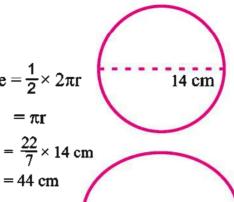
Example 14   Khusboo divides a circular paper plate of radius 14 cm. into two equal parts. Find out perimeter of each semi circular plate (use $\pi = \dfrac{22}{7}$)

Solution

Radius (r) of plate = 14 cm

Circumference of circle = $2\pi r$

Therefore, circumference of semicircle = $\frac{1}{2} \times 2\pi r$

$$= \pi r$$

$$= \frac{22}{7} \times 14 \text{ cm}$$

$$= 44 \text{ cm}$$

Diameter (d) of circle = 2 × radius

$$= 2 \times 14 \text{ cm}$$

$$= 28 \text{ cm}$$

So perimeter of each semicircular plate = Circumference of semicircle + diameter

$$= 44 \text{ cm} + 28 \text{ cm}$$

$$= 72 \text{ cm}$$
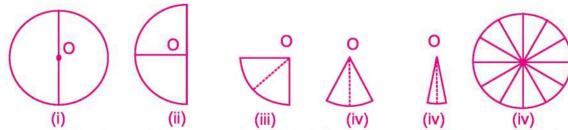
16.4.2 Area of Circle

Meena wants to put red soil on a circular ground of radius 28 meter, so she is calculating the expenditure for it, if for putting soil on 1 square meter, it costs Rs. 10. Meena's sister told her that here we have to find the area of that circular ground rather than its perimeter.

Both of them cut a circular sheet of transparent paper of radius 2.8 cm by taking scale of 10 meter = 1 cm to show circular part. They tried to determine the area by calculating the squares after putting the sheet on a graph paper. Due to the ends which were not straight they could find only a rough estimation of area. They thought of another method of calculating area:-

Meena folded the circle continuously according to the figure and cut it from crease.

(i)	(ii)	(iii)	(iv)	(iv)	(iv)

Mamta - We obtained 2 from one, 4 from 2, eight from 4, sixteen from 8 pieces of a circle.

Meena - If we fold like as onwards then we would get double pieces continuously

Mamta - One situation is like that obtained one piece will be triangular with height is equal to radius and base will be so small..

Meena - If we get n pieces then total area of n pieces would be equal to area of circle.

Mamta - Yes, In this case.

Area of circle = Area of {triangle 1 + triangle 2+ triangle 3 + triangle 4+ ------ +---------_+triangle n}
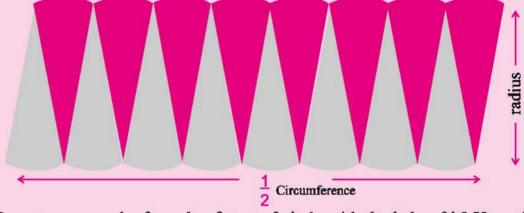
$$= \left[\frac{1}{2} b_1 r + \frac{1}{2} b_2 r + \frac{1}{2} b_3 r + \frac{1}{2} b_4 r + ... + ... + \frac{1}{2} b_n r\right]$$

$$= \frac{1}{2} r [b_1 + b_2 + b_3 + b_4 + ... + b_n] \quad (b_1, b_2, ... b_n = \text{base of all triangles})$$

$$= \frac{1}{2} r [b_1 + b_2 + b_3 + b_4 + ... + ... + b_n] = \frac{1}{2} r [2\pi r] \text{ square unit} \because (\text{Circumference} = 2\pi r)$$

$$= \frac{1}{2} (2\pi r^2) \text{ square unit} = \pi r^2 \text{ square unit}$$

Activity- Shade the half portion of a circle and folded it 6 times successively and obtained 64 parts by cutting along the crease. Arrange these parts according to the figure.

$\frac{1}{2}$ Circumference

Do you express the formula of area of circle with the help of it? You will find that this figure is same as rectangle. Its length and breadth are equal to circumference and radius 'r' respectively. If radius of circle is r then.

Area of rectangle = length × breadth

$$= \frac{1}{2} \times 2\pi r \times r \quad = \pi r^2$$

Hence area of required circle = πr^2

Example 15 Find the area of circular disc having 25 cm. as radius. (take $\pi = 3.14$)

Solution: Radius of disc , (r) = 25 cm.

Area of circular disc = πr^2

$$= 3.14 \times (25)^2$$
$$= 3.14 \times 25 \times 25$$
$$= 1962.50 \text{ sq.cm.}$$

Example 16 The diameter of a circular garden is 11.2 meter. Find out its area

Solution   Diameter d = 11.2 meter

Therefore, radius r = 11.2 ÷ 2 meter

= 5.6 meter

Area of circle = πr^2

$= \dfrac{22}{7}$ x $(5.6)^2$ square meter

$= \dfrac{22}{7}$ x 5.6 x 5.6 square meter

= 98.56 square meter

Example 17 Area of a circular plate is 2826 square cm. Find the radius of the plate. ($\pi = 3.14$)

Solution   Area = 2826 sq. cm

πr^2 = 2826 sq. cm

$3.14 \times r^2$ = 2826 sq. cm

$r^2 = \dfrac{2826}{3.14}$ sq.cm

r^2 = 900 sq.cm

r = 30 cm

Example 18 The adjoining figure shows two circles with the same centre. The radius of the larger circle is 12 cm and the radius of the smaller circle is 4cm. Find out –

(i)   The area of the larger circle.

(ii)   The area of the smaller circle.

(iii) The shaded area between the two circles (π=3.14)

Solution

(i) Diameter of larger circle r_2 = 12 cm

Area of larger circle = πr_2^2

= $3.14 \times (12)^2$ sq. cm

= $3.14 \times 12 \times 12$ sq. cm

= 452.16 sq. cm

(ii) Diameter of smaller circle r_1 = 8 cm

Area of smaller circle = πr_1^2

= $3.14 \times (8)^2$ sq. cm
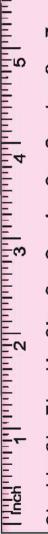
= $3.14 \times 8 \times 8$ sq. cm

= 200.96 sq. cm

(iii) Area of shaded portion = Area of larger circle – Area of smaller circle
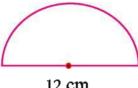
= 452.16 sq. cm – 200.96 sq. cm

= 251.20 sq. cm

Exercise 16.3
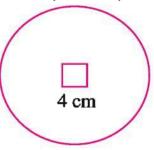
1. Find the circumference of circles having following radius$\left(\pi = \frac{22}{7}\right)$
   (i) 21 cm.   (ii) 28 cm   (iii) 10.5 cm

2. Find area of the following circles. Given
   (I) radius = 5cm   (ii) diameter = 42 meter   (iii) radius = 5.6 cm

3. Find the radius of a circular sheet whose perimeter is 132 meter .Also
   find its area. $\left(\pi = \frac{22}{7}\right)$

4. Circumference of a circle is 44 cm. Find the radius and area of triangle.$\left(\pi = \frac{22}{7}\right)$

5. The given figure is a semi circle with 12 cm. diameter. Find out its
   circumference.
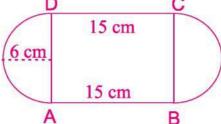
12 cm

6. The radius of a circular pond is 28 meter. A path of 1.4 meter width is
   present around it. Find the area of path.

7. Area of a circle is 616 square cm. The circle is surrounded by a path 2
   meter wide. What is the area of this path?

8. From a circular card sheet of radius 5 cm., a circular sheet of radius of 4
   cm. is removed. Find the area of the remaining sheet.$(\pi = 3.14)$

9. From a circular card sheet of radius 14 cm.,
   a square of 4 cm. is removed as shown in the
   adjoining figure. Find the area of the
   remaining sheet $\left(\pi = \frac{22}{7}\right)$

4 cm
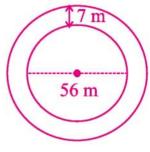
10. The ratio of diameter of two circle is 4 : 5. Find out ratio of their areas.

11. Durga wants to polish a circular table top of diameter 2.8 meter. Find
    the cost of polishing if the rate of polishing is Rs. 25 per square meter.

12. Gopi ties his horse with a rope of length 12 m. What area of grass can
    the horse graze upon?

13. In the given figure, two semicircular parts of diameter 12 cm. are added on both the ends of a rectangular part. Length of the part is 15 cm. Find out the area?

14. How many rounds does a wheel of radius 35 meter take in order to cover a distance of 880 meter? $\left(\text{Take } \pi = \frac{22}{7}\right)$

15. How much money does Parvat spend on putting soil on a 7 meter wide path around his circular park at the rate of Rs. 11 per square meter, when the diameter of park is 56 meter? $\left(\text{Take } \pi = \frac{22}{7}\right)$

16. The length of minute hand of a circular watch is 20 cm. How much distance does tip of minute hand cover in 1 hour? $(\pi = 3.14)$

Do and Learn ◆

To show traffic symbols of 5 circular discs are prepared by cutting and iron sheet. Radius of all discs is 21 cm.

Find out the meaning of all these symbols with the help of your teacher and find out circumference and area of discs.

⫷ We Learnt ⫸

1. Perimeter is the distance around a closed figure wheras area is the part of plane occupied by the closed figure.

2. We have learnt how to find perimeter and area of a square and rectangle in the earlier class. They are :

   (1) Perimeter of a square $= 4 \times$ side

   (2) Perimeter of a rectangle $= 2 \times$ (length + breadth)

   (3) Area of a square $=$ side $\times$ side

   (4) Area of a rectangle $=$ length $\times$ breadth

3. Area of a parallelogram $=$ base $\times$ height

4. Area of a triangle $= \frac{1}{2}$ (area of the parallelogram generated from it)

   $= \frac{1}{2} \times$ base $\times$ height

5. The distance around a circular region is known as its circumference.

   Circumference of a circle $= 2\pi r$ or Circumference $= \pi d$, where d is the diameter of a circle and $\pi = \frac{22}{7}$ or 3.14 (approximately).

6. Area of a circle $= \pi r^2$, where r is the radius of the circle.

DATA HANDLING

17.1 We studied in our previous class that data are those numerical facts which are collected for definite purposes.

We studied types of data, collection of data, arranging the data and tabulation of data with the help of tally marks along with the reading and making of pictograms and bar diagrams. Collection of data, graphing and representation help us organize our experiences and draw inferences from them.

In this chapter, we will study reading and making of double bar graphs and central tendencies, arithmetic mean, median and mode of non-classified data.

In daily life, we face different data which we observe in newspapers, magazines, television or other sources. Let us look at some common forms of data that we come across.

Daily routine of Amar	
Work	Time Spent
Food and routine	2 hours
School	6 hours
Games and entertainment	3 hours
Cooperation in house works	2 hours
Studies	3 hours
Sleeping	8 hours

Table 17.1

Number of patients on Monday in a Primary Health Centre	
Name of disease	Number of patients
Fever	22
Cold and Cough	26
Eye diseases	08
Skin diseases	12
Accidental injury	07
Dental diseases	05

Table 17.2

Performance of Arti in first two tests		
Subject	First Test	Second test
Hindi	5	8
English	6	8
Mathematics	3	9
Science	6	9
Social Science	5	8
Sanskrit	8	7

Table 17.3
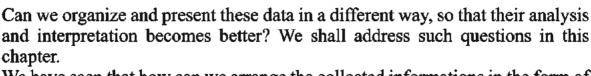
What do these collections of data tell us? For example we can say that Amar spends 6 hours in school and 3 hours at home on studies from his daily routine. (Table 17.1)

Similarly Arti performed better in second test as compared to first test in all the subjects and improved the best in Mathematics.

Can we organize and present these data in a different way, so that their analysis and interpretation becomes better? We shall address such questions in this chapter.

We have seen that how can we arrange the collected informations in the form of frequency distribution table and then represent it in the form of pictographs or bar graphs. We can say that tallest bar is mode if bar represents frequency.
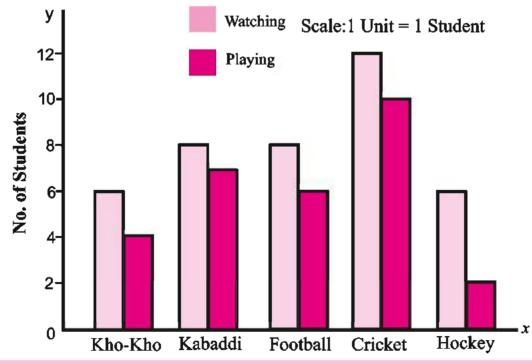
17.2   Draw double bar graphs

Consider the following data which were obtained from a survey executed in a class.

Favourite game	Kho-Kho	Kabaddi	Football	Cricket	Hockey
Watching	6	8	8	12	6
Playing	4	7	6	10	2

Above data show the interest of playing and watching a game by different students of a class. By seeing these data we can tell that which game is played by the maximum students and which game is viewed by the least students.

But in order to find difference in the students who like watching and playing a particular game, we have to compare the number of students watching and playing. To do this, we'll learn to draw the graphs which are said to be double bar graphs. The comparison of both the interests is given by bar graphs side by side.

Example1 The number of C.F.L. tubes and L.E.D. bulbs sold by a seller every year from the year 2011 to2015 are given below.

Year	2011	2012	2013	2014	2015
CFL Tube	1200	1400	1100	900	600
LED Bulb	100	400	700	1000	1400
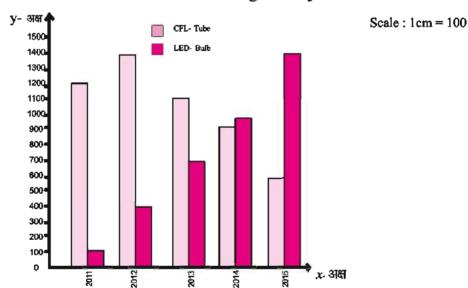
Draw a double bar graph and answer the following questions.
1. The sale of which type of light equipment's increased continuously?
2. Increased or decreased in light equipment's in 2015 as compared to 2011?
3. In which year the difference in the sale of light equipment's of both the types stood highest?

Solution

Steps of construction of double bar graph
1. Make x-axis (horizontal) and y-axis (vertical) on the graph paper. Both of these meet at the origin (0,0).
2. Write down the year 2011 to 2015 on the x-axis.
3. Mention the number of C.F.L. tubes and L.E.D. bulbs on y- axis.
4. Take proper scale on y-axis so that number of both the light equipment's can be written easily. We can take 1 cm. = 100 on y- axis.
5. Find the length of each column by dividing by 100 to the number.
6. Exhibit C.F.L. tube and L.E.D. bulb altogether by columns.

(1) It is obvious after seeing double bar graph that the sale of LED bulb increased continuously.
(2) There is an increase in the total light equipment's in the year 2015 as compared to 2011.
(3) It is obvious after seeing double bar graph that the difference in the sale of both the light equipment's in the year 2011 stood highest.

Do and learn ◆

1. The marks of the subjects Mathematics and Science of five students of class 7 are given in the table. Exhibit these by the vertical double bar graph.

Name of Student	Mathematics	Science
Arti	65	75
Varsha	70	75
Simran	55	70
Radha	75	80
Jyoti	50	60
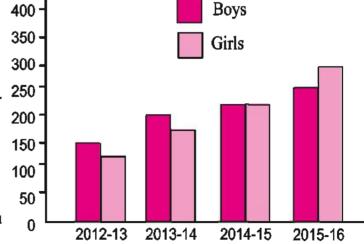
2. The details of different expenditures occurring in one month of two families are shown in the table. Make a double bar graph on the basis of this table and answer the following questions.

Expenditure Head	Family 1	Family 2
House rent	2000	2500
Electricity, Water, Telephone	800	600
Food Items	8000	7000
Children Education	2000	3000
Savings	2200	1900

(i) Under which head the expenditure is maximum?

(ii) Under which head the expenditure is minimum?

(iii) If monthly income of both the families is Rs.15000 then what percentage of expenditure is used on children education in both the families?

⟩⟩⟩∞⟨ Exercise 17.1 ⟩∞⟨⟨⟨

1. In the following graph, number of students of a school are shown below according to session. Answer the questions on the basis of this graph.

(i) In which session, number of girls were more than number of boys in the school?

(ii) In which session, number of both girls and boys in the school were equal?

(iii) What was the total number of students in the school in the session 2015-16?

2. Under the free textbook distribution scheme from the year 2011 to 2015, the distribution of books of the subjects Mathematics and Hindi of class 7 in a district were as follows:

Subject/Year	2011	2012	2013	2014	2015
Mathematics	8000	8500	9500	11000	13000
Hindi	9000	10000	10500	11500	14000

Draw a double bar graph and answer the following questions-
(i)   The demand of book of which subject is maximum always?
(ii) In which year the difference in the demand of both the books is minimum?
(iii) In which year the difference in the demand of both the books is maximum?

3. Estimated distance of following cities of Rajasthan from Udaipur by road and by train are given in the following table. Draw a double bar graph on the basis of table and answer the following questions:-

Distance from Udaipur	By Road (in km.)	By train (in km.)
Ajmer	290	310
Jaipur	410	440
Bikaner	530	580
Jodhpur	270	300
Kota	360	570

(i)   Which city is at a maximum distance from Udaipur by road?
(ii) Which city has least difference in the distance by road and by train?
(iii) Which city has highest difference in the distance by road and by train?

17.3   Collection of Data

In our daily life, expression of facts through data plays very important role For example a better way to express, "India's population is enough" is to say, "the population of India according to census 2011 is 1 Arab, 21 crore 8 lakh." Similarly it would be appropriate to say that number of students in our school is 867 rather than number of students in our school is enough. Hence we can say that we can express our thoughts very clearly via data. As we need to collect stone, lime, cement, bricks etc. before constructing a building, similarly it is very much necessary to collect data to draw conclusion and analyse it. We can be able to find the solution logically to understand the complex situations by the proper use of these data. But it is very much necessary that the data taken should be pure, general and authenticate.

We can classify data into two parts on the basis of its sources of collection of data.

         (a) Primary Data   **(b) Secondary Data**

(a) Primary data - The data which is collected first time by self or by help are known as primary data. For example, if you have to study the family position of students of your class then you'll have to collect information regarding monthly income-expenditure, number of brothers-sisters, source of income etc. These data shall be classified as primary data.

(b) Secondary Data - The data which has already been collected by any person or institute, which can be in either published or unpublished form are called secondary data. For example one can obtain the data related with census or literacy from the Census Department of India authorized by Government of India.

17.4 Organisation of Data

When we collect data, we have to record and organize it. Why do we need to do that?

Consider the following example:-

The heights of 8 students under health test in a school were as follows:-

Vijay –	140 cm	Kishor –	138 cm	Vidhya –	130 cm
Tabassum –	135 cm	Ramesh –	145 cm	Sarika –	125 cm
Divyanshi –	131 cm	Mohit –	144 cm		

It was not an easy task to draw conclusions from these data in this form. Sarika wrote these heights in ascending order in the tabular form.

Name of student	Height (in m.)	Name of student	Height (in cm)
Sarika	125	Kishor	138
Vidhya	130	Vijay	140
Divyanshi	131	Mohit	144
Tabassum	135	Ramesh	145

Answer following questions:-

     1. What is the name of the tallest student?

     2. What is the name of the shortest student?

     3. What is the difference in heights of Kishor and Tabassum?

When we put data in a proper tabular form it becomes easy to understand and interpret it. Many kinds of data we come across are put in tabular form. Our school rolls, progress report, temperature record, daily attendance record and many other are all in tabular form.

Can you think of a few more data that you come across in tabular form?

> ### Do and Learn ◆
>
>   Determine the weight of students of your class by weighing machine. Tabulate these by arranging data. Write these data in ascending or descending order. Then answer the following questions:-
>
> 1. The weight of which student is maximum in the class?
> 2. How many students have weight more than 25 kg.in the class?
> 3. How many students have weight in between 20 to 30 kg.?

17.5 Measures of Central tendency

   You must have studied or heard following statements in your daily life.

      1. The average age of students of class 7 is 13 years.

      2. The lunch taken by each student is 150 grams in mid-day meal.

      3. The average temperature is $30^0 C$ of last 10 days.

      4. Lakshya studies 5 hours every day.

Consider the following statements:-

Can you say that age of each and every student of class 7 is 13 years according to first statement or each and every student takes exactly 150 gm. food according to the second statement?

   Clearly the answers of these questions are "no".

   Then what do these statements mean?

   We understand from "average" that age of maximum students of class 7 is near about13 years. The age of some students may be below 13 years or some may be above 13 years.

   Similarly the average temperature of last few days 1s 32° which means that the temperature remained around 32°. The temperature may be below 32° or may be above 32°.

   Similarly we can say that "average" is a number which shows or represent central tendency of a group of data or observations. As average is measure of central tendency of a group of highest and lowest data. There is a necessity of central values or different representatives to interpret different types of data.

   Out of these one value of representation is algebraic or arithmetic mean.

17.6 Arithmetic Mean

   For a group of data the commonly used value of representation is arithmetic mean. In brief, it is known as mean. Consider the following example:

Example 2   The net income of per day in a week of a fruit seller is Rs. 500, Rs. 650, Rs. 400, Rs.425, Rs. 450, Rs.600 and Rs.475 respectively. Find the average income of fruit seller?

Solution

$$\text{Average income of fruit seller} = \frac{\text{Total income of the week}}{\text{Number of days in the week}}$$

$$= \frac{500 + 650 + 400 + 425 + 450 + 600 + 475}{7}$$

$$= \frac{3500}{7} = \text{Rs. } 500$$

Average income of fruit seller would be Rs. 500 daily.

Example 3 Find arithmetic mean of first six even numbers.

Solution We know that first six even numbers are 2, 4, 6, 8, 10, 12. To find arithmetic mean, we have to add all the observations and then divide it by total number of observations. Hence in this case

$$\text{Arithmetic mean} = \frac{\text{Sum of all the observations}}{\text{Number of observations}}$$

$$= \frac{2 + 4 + 6 + 8 + 10 + 12}{6}$$

$$= \frac{42}{6} = 7$$

Thus arithmetic mean of first six even numbers is 7.

17.7 Range

Consider the following example:

Example 4 The salaries of five teachers working in a school are Rs. 25000, Rs. 18000, Rs. 20000, Rs. 22000 and Rs. 23000 per month.

1. What is the salary of the teacher who gets highest salary?
2. What is the difference of salaries of teachers who gets highest and lowest salary?
3. Find the mean of salaries of these teachers.

Solution Arranging salaries of teachers in ascending order we have 18000, 20000, 22000, 23000, and 25000.

The information's we draw from this are:

1. The salary of the teacher who gets highest salary is Rs. 25000.
2. Highest and lowest salaries are respectively Rs. 25000 and Rs. 18000. Difference between these two = 25000-18000 = Rs. 7000

3. Mean of salaries of teachers $= \dfrac{18000 + 20000 + 22000 + 23000 + 25000}{5}$

$$= \frac{108000}{5} = \text{Rs. } 21600$$

It is obvious from above example that we can estimate the expansion of observations in the form of difference in between highest and lowest observations. We call this result as **range** of observations of data.

> **Do and Learn** ◆
> 1. Find the mean heights of your family members.
> 2. Find the mean age of your family members.

<div align="center">∞⊂⊂⊂ Exercise 17.2 ⊃⊃⊃∞</div>

1. Following are the number of students of a school from class 6 to 12. 78,72,67,59,54,49,48 Then find:
   (i) In which class the number of students is maximum?
   (ii) In which class the number of students is minimum?
   (iii) What is the range of these data?
   (iv) Find the mean of these data.
2. Find the mean of first 10 whole numbers.
3. A cricketer scored the runs in 6 innings as follows:
   68, 03,17,78,12,104 Find the arithmetic mean of runs scored.
4. The number of passengers travelled in the bus running from Bikaner to Udaipur from Monday to Friday are as follows:
   45, 48,32,40,30 What is the mean of passengers in each day?
5. Following crops were raised up to five years in a village. The profit (in rupees) per acre on the crop were as follows:-

Crop	2011	2012	2013	2014	2015
Barley	6000	8000	5000	6500	8500
Gwar	7000	8000	12000	9000	8500
Groundnut	9000	7000	10000	8000	13000

   Answer the following questions on the basis of above table.
   (i) Find the mean profit of each crop in five years.
   (ii) Which crop should be raised next year on the basis of above answer?
6. If arithmetic mean of digits 3, 4, 8, 5, x, 3 is 4 then find the value of x.
7. The number of books given to students in 10 days from a library are as follows:-
   40, 57, 32, 59, 72, 66, 40, 62, 72, 60
   Find the mean of books given to students every day?
8. The average of five numbers is 18. If four numbers are 22, 20, 14 and 13 then find the fifth number.

9. The temperature of a particular week in a city is noted as below:

Day	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday
Temperature (in °C)	37	37.5	40	36.5	37.5	35	35.5

   (i)   Find the range of temperature by using the data.
   (ii) Find the mean temperature of this week.
   (iii) How many days did the temperature remain more than average?

10. In a singing competition in a school, three judges gave marks out of 100 to
    four singer contestants as follows:

Name of contestant	Judge I	Judge II	Judge III
Rashi	78	75	72
Suman	82	75	83
Poonam	68	64	69
Khushboo	49	56	51

   1. What is the range of marks given by the judges?
   2. Find the mean of total marks.
   3. Point out the name of winner.
   4. What is the difference in the means of winner and the contestant
      placed at the fourth place.

17.8 Mode

   Second type of representative value is mode. Consider the example.

Example 5   The shoes of different sizes are available on the shoe shop. The
               shopkeeper recorded the sale of shoes to find weekly demand of
               shoes given in the table below.

Shoe number	5	6	7	8	9	10
Sale	12	27	40	45	26	18

               If we find the mean of shoes sold by the shopkeeper, then

Solution

$$\text{Mean} = \frac{\text{Total Number of shoes sold}}{\text{Total types of number of shoes}} = \frac{168}{6} = 28$$

   Then should the shopkeeper keep 28 pairs of shoes per week of each size?
Definitely the shopkeeper will keep a greater stock of the shoes of number 7,8
as compared to other sizes on the basis of above record. Because the sale of
shoes of size 7, 8 is comparatively more.

   Of all, the sale of shoes of size 8 is the highest. This is another value of
representation of data. This value of representation is said to be mode.

> The term which repeats maximum number of times in a given
> data is called as mode. It means the term which has maximum
> frequency is called as mode.

Example 6   Find the mode of the following numbers.

5, 4, 4, 2, 5, 7, 5, 6, 5, 4, 3, 5

Solution   On arranging the numbers in ascending order

2, 3, 4, 4, 4, 5, 5, 5, 5, 5, 6, 7

It is obvious from the observations that number 5 repeats maximum number of times.

Hence mode will be 5.

17.8.1   Mode of large and unclassified data

If number of data is large then it is not easy to count after writing them either in ascending or descending order. In this situation, we tabulate the data with the help of tally marks. We had learnt how to tabulate the data in previous classes.

Example 7   30 athletes participated in a 100 meter race. The time taken (in seconds) by them in completing the race is as follows:

14, 12, 13, 12, 10, 12, 14, 13, 12, 11, 12, 13, 14, 12, 14, 12, 13, 14, 14, 11, 10, 11, 12, 14, 13, 12, 12, 11, 12, 14 Find the mode of these data.

Solution   On tabulating the data

Time (in seconds)	Tally Marks	Number of athletes
10	‖	2
11	‖‖	4
12	‖‖‖ ‖‖‖ ‖	11
13	‖‖‖ ‖	6
14	‖‖‖ ‖	7
	Total	30

By seeing this table we can immediately say that the mode of these data is 12 because maximum number of athletes completed their race in 12 seconds.

Think and Discuss

- Is it possible to have 2 modes in a group of numbers?
- Can mode be found out just by observation?

Do and Learn ◆

1. 40 students of class 7 wrote their number of family members together. This number is shown below:

   4, 3, 5, 4, 7, 3, 5, 6, 4, 4, 4, 7, 6, 4, 5, 4, 3, 4, 5, 6, 7, 4, 4, 5, 3, 4, 6, 4, 5, 5, 4, 3, 4, 7, 6, 4, 3, 5, 4, 5   Find the mode of these data.

2. Find mode of following data.

   21, 22, 25, 24, 22, 23, 23, 24, 25, 24, 22, 24, 23, 24, 23, 24, 22, 21, 25, 23

We saw that mean provides us average of all the observations of the data and mode shows the observations which occurs maximum number of times.
Think over following examples.

1. You have to calculate the daily consumption of electricity in your house.
2. The seller of ready made garments has to fulfil supplies of his stall.
3. We have to find the height of the door for our home.
4. Have to select a sweet in the form of a favourite sweet for students of a class. Then which sweet could be selected?

When we think about first statement then in order to find out consumption of electricity daily, we will find out the weekly electricity consumption from electric meter, then by taking its mean, we can find out daily electric consumption.

Can we use the same method for second statement?

From the example of shoes we can see that for supply of clothes, mean is not a suitable representative value. Mode is suitable for it.

Similarly for third statement both median and mode are not suitable values. Here the height of door has to be decided according to the tallest member of the family. Similarly examine the fourth statement by thinking and find suitable representative value for it.

17.9 Median

We have seen that in some cases arithmetic mean is a proper measure of central tendency and in some cases mode is a proper measure of central tendency.

Think over another example.

The salaries of 9 employees in a factory are as follows:

3300, 4200, 5000, 3500, 4300, 3500, 4400, 3500, 5500

If we want to distribute the employees into two groups according to the payment, then what would be better representative in this case either arithmetic mean or mode? By arranging these data into ascending order

3300, 3500, 3500, 3500, 4200, 4300, 4400, 5000, 5500

In the above data we see that 4200 is such a number which has groups of 4-4 numbers on both of its side. It means that the payment of 4 employees is less than 4200 and payment of 4 employees is more than 4200. In this way on arranging numbers in ascending or descending order the number lying exactly in the middle is known as median.

> If data are arranged in either ascending or descending order then the term lying in the middle is known as median.

Example 8 Find median of following data.

$$0,47,35,20,30,40,50$$

Solution   On arranging data in ascending order we get

$$0,20,30,35,40,47,50$$

In the above data there are 7 terms. To find middle term of the data it is divided by 2 after adding 1 to the number of terms. (When number of terms are odd).

This means that median term of above data is the fourth term which is 35. Hence the median of above data is 35.

Similarly if number of terms are even then after arranging in ascending order the mean of two middle terms is median.

$$\text{Median Term} = \frac{7+1}{2} = \frac{8}{2} = 4 \text{ fourth term.}$$

Do and Learn ◆

1. The data arranged in ascending order are as follows:
   8, 11, 12, 16, 16 + x, 20, 25, 30
   If median is 18 then find x.
2. Jyoti scored the following marks (out of 10) in different subjects
   5,7,0,3,5,8
   Jyoti calculated mean, median and mode from the remaining numbers ignoring 0. Did she do it correctly?

Exercise 17.3

1.   Find the mode from the following data:
     7, 6 ,4, 5, 6, 4, 6, 3, 2, 7, 8, 6, 4, 6, 5
2.   Vandana took a dice. She tossed the dice 20 times and noted the obtained number each time:
     3, 4, 6, 3, 5, 2, 2, 3, 5, 4
     5, 6, 6, 1, 5, 6, 3, 5, 2, 4
     Find the median and mode with the help of above data.
3.   The weights (in kgs.) of 30 labourors working in a factory is as follows:
     60, 65, 70, 65, 60, 70, 65, 70, 75, 80, 75, 60, 65, 70, 65, 65
     70, 65, 60, 70, 65, 75, 80, 75, 80, 65, 60, 65, 70, 80
     Find the median and mode with the help of above data.

4. Find the median of following variables:

37, 31, 42, 43, 46, 25, 39, 45, 32

5. The heights of 21 persons of a class are as follows:

147, 149, 150, 152, 148, 151, 148, 150, 151, 149

152, 151, 152, 151, 150, 148, 149, 152, 153, 151, 152

(i) Find the median and mode of the above data.

(ii) Are there more than one mode in the above data?

6. The runs scored by the players in a cricket match are as follows:

105, 47, 0, 36, 50, 16, 7, 70, 65, 36, 52

Find the mean, median and mode from the above data. Are they all equal?

<div align="center">⋈ **We Learnt** ⋈</div>

1. The data collected can be shown in the form of bar graphs with the help of frequency distribution table.

2. With the help of a double bar graph we can compare the two groups of observations in single inspection.

3. The collection, recording and presentation of data help us organize our experiences and draw inferences from them.

4. Before collecting data we need to know what we will use it for.

5. The data that is collected needs to be organized in a proper table, so that it becomes easy to understand and interpret.

6. Arithmetic mean is one of the representative values of data.

7. The mean can be obtained by dividing the sum of group of data by the number of data, which lies in between the lowest and highest value.

8. The term that occurs most often in a group of data is known as mode. A set of data can have more than one mode.

9. If data are arranged either in ascending or descending order, then the term which lies exactly in the middle is known as median.

ANSWER SHEET

Exersice 1.1

1. (i) (a) -9^0 C (b) -5^0 C (c) 6^0 C (d) 20^0 C (ii) 27^0c (iii) 11^0C (iv) yes
2. Positive 1300 Rs. 3. (i) -7 (ii) -2 (iii) -1100 (iv) -29 (v) 0
4. (i) > (ii) = (iii) > (iv) < (v) =
6. (i) (-3) (ii) 0 (iii) (+ 5) (iv) (+4) 7. (i) b (ii) c (iii) a

Exersice 1.2

1. (i) -12 (ii) -24 (iii) 720 (iv) 0 (v) 630 (vi) 140 (vii) 30
3. 9 minute 4. 9 balls 5. (i) -5 (ii) -5 (iii) 1 (iv) -25 (v) 0
   (vi) -15 (vii) 3 6. (i) 100 Pencil (ii) 140 Pencil
8. (i) +60 (ii) -15 (iii) -5 7.

x	2	3	-4	-2	1
3	6	9	-12	-6	3
-2	-4	-6	+8	-4	-2
-1	-2	-3	4	2	-1
4	8	12	-16	-8	4
2	4	6	-8	-4	2

Exersice 1.3

1. (i) (b) (ii) (c) (iii) (d) (iv) (a)
2. (i) Commutativity (ii) -1 Distributivity
   (iii) -4 Associativity
3. (i) 1440 (ii) -78000 (iii) 432 (iv) 44955
4. (i) True (ii) False (iii) True (iv) True
   (v) False

Exersice 2.1

1. (i) $\dfrac{1}{4}$, $\dfrac{3}{12}$, $\dfrac{4}{16}$, $\dfrac{5}{20}$, $\dfrac{6}{24}$ (ii) $\dfrac{6}{7}$, $\dfrac{12}{14}$, $\dfrac{18}{21}$, $\dfrac{24}{28}$, $\dfrac{30}{35}$

   (iii) $\dfrac{7}{4}$, $\dfrac{14}{8}$, $\dfrac{21}{12}$, $\dfrac{28}{16}$, $\dfrac{35}{20}$, $\dfrac{42}{24}$ (iv) $\dfrac{20}{9}$, $\dfrac{40}{18}$, $\dfrac{60}{27}$, $\dfrac{80}{36}$, $\dfrac{120}{54}$

2. (i) > (ii) = (iii) > (iv) <   3. (i) 1/5 < 3/7 < 7/10 (ii) 2/9 < 8/21 < 2/3
4. (i) 13/5 (ii) 39/8 (iii) 31/35 (iv) 39/8 (v) 37/6 (vi) 13/5
5. 47/6   6. Neela took more time and $\dfrac{4}{35}$ hours   7. Meena's share = $\dfrac{4}{15}$

Exersice 2.2

1. (i) (b) (ii) (c) (iii) (a) 2. (i) 2/3 (ii) 2 x 1/2, 1 (iii) 9/4 3.(i) 24/5 (ii) 8/3 (iii) 15
   (iv) 9 (v) 40/3 (vi) 2 (vii) 16/7 (viii) 20 (ix) 9/16 (x) 48/35
5. (i) 9 (ii) 6 (iii) 10 (iv) 18 (v) 40 (vi) 12
6. (i) 32/5 (ii) 52/15 (iii) 8/5 (iv) 29/8 (v) 3/25 (vi) 3/70
7. (i)19/20 (ii) 48/5 (iii)15/7 (iv) 119/8 8. (i) 3/5 of 5/8 (ii) 1/2 of 6/7
9. (i) 6 liter (ii) 3 liter (iii) 6 liter 10. 9/2 meter (11) 22 hours (12) 319/5 Km
13.(i) 2/10 (ii) 1/5
14.(i) 2/3 (ii) 2/3

Exercise 2.3

1. (i) 18   (ii) $\dfrac{7}{5}$ (iii) $\dfrac{9}{4}$ (iv) $\dfrac{3}{2}$ (v) 9   (vi) 21

2. (i) $\dfrac{7}{3}$ (ii) $\dfrac{8}{1}$ (iii) $\dfrac{7}{12}$ (iv) $\dfrac{8}{5}$ (v) $\dfrac{7}{9}$ (vi) $\dfrac{1}{5}$

3. (i) $\dfrac{3}{14}$ (ii) $\dfrac{31}{49}$ (iii) $\dfrac{6}{65}$ (iv) $\dfrac{7}{8}$ (v) $\dfrac{2}{5}$ (vi) $\dfrac{7}{12}$

4. (i) $\dfrac{49}{24}$ (ii) $\dfrac{11}{3}$ (iii) $\dfrac{4}{15}$ (iv) $\dfrac{8}{3}$ (v) $\dfrac{48}{35}$ (vi) $\dfrac{21}{25}$

5. 24 parts   6. 23 pieces

Exercise 2.4

1. (i) 0.7   (ii) 2.30   (iii) 7   (iv) 1.49   (v) 3.570   (vi) 85.2
2. (i) .07 Rs. (ii) .800 kg. (iii) .075 km. (iv) 3.470 km. (v) 7.007 kg. (vi) 47.075 km.

3. (i) $2 \times 10 + 5 \times 1 + 0 \times \dfrac{1}{10} + 3 \times \dfrac{1}{100}$ (ii) $2 \times 1 + 5 \times \dfrac{1}{10} + 0 \times \dfrac{1}{100} + 3 \times \dfrac{1}{1000}$

   (iii) $2 \times 100 + 0 \times 10 + 5 \times 1 + 3 \times \dfrac{1}{10}$ (iv) $2 \times 1 + 0 \times \dfrac{1}{10} + 5 \times \dfrac{1}{100} + 3 \times \dfrac{1}{1000}$

4. (i) 30   (ii) 3   (iii) $\dfrac{3}{100}$ (iv) $\dfrac{3}{1000}$

5. 23.950 kg.   6. Bhavana, 11.75 Rs.   7. 5.3 km   8. 9.13

Exercise 2.5

1. (i) 37.8   (ii) 160.2   (iii) 0.4   (iv) 2.58   (v) 1248.2   (vi) 48.64
2. (i) 37.2   (ii) 3.7   (iii) 5   (iv) 108   (v) 738   (vi) 6
 (vii) 47030   (viii) 30   (ix) 42700   3.(i) 14.7   (ii) 3.125   (iii) 1.68
 (iv) 0.04   (v)1.0101   (vi) 96.24
4. 20.48 cm^2 5. 264.285   6. 25 kg   7. 13.5 cm   8. 371.25 Rs.

Exercise 2.6

1. (i) 0.2   (ii) 0.06   (iii) 0.66   (iv)210.6   (v) 2.07   (vi)180
 (vii) 2   (viii) 0.03
2. (i) 0.42   (ii) 9.86   (iii) 0.02   (iv)1.432   (v) 0.86   (vi) 0.0805
 (vii) 0.4432   (viii) 0.0013   (ix) 0.00006
3.(i) 4   (ii) 9.1   (iii) 6   (iv) 0.5   (v) 20.5   (vi) 3.4
(vii) 44.2   (viii) 510   (ix) 31
4.   42.5   5.   1.2 km   6. 40.5 km
7.121.4404   8.   59.2 meter

Exercise 3.1

1. (i) 6   (ii) 9   (iii) 0   (iv) 5   (v) 1   (vi) 9
(vii) 4   (viii) 6   (ix) 4

2. (i) 324   (ii) 121   (iii) 11449   (iv) 225   (v) 40000   (vi) 729

3. (ii), (iii),(vi)   4. (i) 16   (ii) 49   (iii) 100

5. 1 + 3 + 5 + 7 + 9 + 11 + 13 + 15   6. (i) 21   (ii) 35   (iii) 61

7. (i), (iii) are pythogorian triplets

Exercise 3.2

1. (i) 2, 8   (ii) 4, 6   (iii) 1, 9   (iv) 3, 7

2. (i) and (iii) can not be perfect square

3. (i) 36   (ii) 27   (iii) 42
(iv) 63   (v) 66   (vi) 40

4. (i) 7   (ii) 11   (iii) 5

5. (i) 10   (ii) 3   (iii) 7   6. 49   7. 900

Exercise 3.3

1. (i) 21   (ii) 26   (iii) 35   (iv) 54   (v) 68   (vi) 89

2. (i) 11   (ii) 16   (iii) 67   (iv) 245

3. (i) 2.5   (ii) 1.7   (iii) 5.7   (iv) 5.6   (v) 7.6

4. (i) 21   (ii) 25   (iii) 4   (iv) 100

5. (i) 26   (ii) 27   (iii) 50   (iv) 10

6. 24 chairs, numbers of chairs in each row = 32   7. 76 meter   8. 3

Exercise 4

1. (i) $\dfrac{-4}{6}, \dfrac{-6}{9}, \dfrac{-8}{12}, \dfrac{-10}{15}, \dfrac{-12}{18}$ (ii) $\dfrac{2}{10}, \dfrac{3}{15}, \dfrac{4}{20}, \dfrac{5}{25}, \dfrac{6}{30}$

(iii) $\dfrac{-10}{6}, \dfrac{-15}{9}, \dfrac{-20}{12}, \dfrac{-25}{15}, \dfrac{-30}{18}$ (iv) $\dfrac{8}{-18}, \dfrac{12}{-27}, \dfrac{16}{-36}, \dfrac{20}{-45}, \dfrac{24}{-54}$

2. $\dfrac{-25}{60}, \dfrac{40}{-96}, \dfrac{-45}{108}$ 3. $\dfrac{24}{-56}, \dfrac{-60}{140}, \dfrac{75}{-175}$

4. (i) $\dfrac{-3}{5}$ (ii) $\dfrac{11}{-18}$ (iii) $\dfrac{5}{2}$ (iv) $\dfrac{-4}{5}$

6. (i) $\dfrac{2}{3}$ $\boxed{>}$ $\dfrac{-5}{7}$ (ii) $\dfrac{-1}{4}$ $\boxed{>}$ $\dfrac{1}{-3}$ (iii) $\dfrac{-3}{5}$ $\boxed{<}$ $\dfrac{-1}{3}$

  (iv) $\dfrac{2}{7}$ $\boxed{<}$ $\dfrac{1}{2}$ (v) $\dfrac{-1}{2}$ $\boxed{=}$ $\dfrac{1}{-2}$ (vi) $\dfrac{-5}{4}$ $\boxed{<}$ $\dfrac{3}{5}$

7. (i) $\dfrac{-8}{3}, \dfrac{-7}{3}, -2, \dfrac{-5}{3}, \dfrac{-4}{3}$ etc.   (ii) $\dfrac{-5}{6}, \dfrac{-4}{6}, \dfrac{-3}{6}, \dfrac{-2}{6}, \dfrac{-1}{6}$ etc.

  (iii) $\dfrac{-55}{70}, \dfrac{-54}{70}, \dfrac{-53}{70}, \dfrac{-52}{70}, \dfrac{-51}{70}$ etc.   (iv) $\dfrac{6}{40}, \dfrac{7}{40}, \dfrac{8}{40}, \dfrac{9}{40}, \dfrac{11}{40}$ etc.

  (v) $\dfrac{-3}{5}, \dfrac{-2}{5}, \dfrac{-1}{5}, \dfrac{0}{5}, \dfrac{1}{5}$ etc.   (vi) $\dfrac{-9}{5}, \dfrac{-8}{5}, \dfrac{-7}{5}, \dfrac{-6}{5}, \dfrac{-4}{5}$ etc.

8.   (i) $\dfrac{-8}{20}, \dfrac{-10}{25}, \dfrac{-12}{30}$ (ii) $\dfrac{8}{-12}, \dfrac{10}{-15}, \dfrac{12}{-18}$

     (iii) $\dfrac{4}{12}, \dfrac{5}{15}, \dfrac{6}{18}$ (iv) $\dfrac{4}{-20}, \dfrac{5}{-25}, \dfrac{6}{-30}$

9.   (i) $\dfrac{-3}{4}, \dfrac{-1}{2}, \dfrac{1}{2}, \dfrac{3}{4}$ (ii) $\dfrac{-3}{2}, \dfrac{-3}{4}, \dfrac{-3}{7}$

     (iii) $-2, \dfrac{-2}{15}, \dfrac{-7}{11}, 0, \dfrac{7}{15}$ (iv) $\dfrac{1}{6}, \dfrac{2}{5}, \dfrac{5}{9}, \dfrac{4}{7}$

10.   (i) $\dfrac{-9}{24}, \dfrac{5}{-12}, \dfrac{-7}{16}, \dfrac{-3}{4}$ (ii) $\dfrac{1}{6}, \dfrac{-5}{6}, \dfrac{-8}{9}, \dfrac{-11}{12}$

     (iii) $\dfrac{1}{3}, \dfrac{-2}{3}, \dfrac{-5}{6}, \dfrac{4}{-3}$ (iv) $\dfrac{3}{5}, \dfrac{-17}{-30}, \dfrac{8}{-15}, \dfrac{-7}{10}$

Exercise 5.1

1. (i) 7^5 (ii) $3^3 \times 7^2$ (iii) $a^3 \times b^2$ (iv) $5^2 \times t^3$

2. (i) $2 \times 2 \times 2 \times 2 \times 2 = 2^5$ (ii) $3 \times 3 \times 3 \times 3 = 3^4$

  (iii) $7 \times 7 \times 7 = 7^3$ (iv) $5 \times 5 \times 5 = 5^3$

3. (i) $2^5 > 5^2$ (ii) $3^5 > 5^3$

  (iii) $3^{10} > 10^3$ (iv) $3^7 > 7^3$

4. (i) $2^2 \times 3^4$ (ii) 5^4 (iii) $2^3 \times 3^3 \times 5$ (iv) $2^3 \times 3^2 \times 5^2$

5. (i) 162   (ii) 1715   (iii) 500   (iv) 9000

  (v) 0

6. (i) -1   (ii) 625   (iii) -128

Exercise 5.2

1-   (i) 3^{15} (ii) $(4)^9$ (iii) a^9 (iv) 3^6

    (v) t^3 (vi)   6   (vii) 2^{18} (viii) a^{20}

    (ix) 40^5 (x) $(ab)^3$ (xi) $\left(\dfrac{7}{6}\right)^5$ (xii)   1

    (xiii) $\dfrac{1}{7^3}$ (xiv)   **1**

2.   (i) 3^3 (ii) 4^5 (iii)   1   (iv)   1

    (v) $3^5 \times a^3$ (vi) 7^{12} (vii) $\dfrac{1}{3^2}$ (viii) a^{11}

    (ix)   3

3.   (i)   91   (ii)   45   (iii)   400

Exercise 5.3

1.   (i) 5.0×10^5 (ii) 4.83×10^6 (iii) 3.94×10^{11} (iv) 3.0×10^7 (v) 1.8×10^5
2. 1.5×10^8 3. 1.095×10^6 4. 3.9×10^8
5.   (i) 25000   (ii) 1750000   (iii) 121000000   (iv) 450000

Exercise 6.1

1. 3500   2. 300   3. 770

Exercise 6.2

1.(i) 5232 (ii) 2885   (iii) 3089
2.(i) 870 (ii) 5300   (iii) 432000   (iv) 320
 (v) 3600 (vi) 8019   (vii) 98901   (viii) 891

Exercise 6.3

1. (1) > (2) > (3) =   (4) <   (5) <

Exercise 6.4

1. (i) 5/9   (ii) 3/5   (iii) 11/10 (iv) 26/15 (v) 47/60 (vi) 53/30
2. (i) 3/5   (ii) 3   (iii) 2/3   (iv) 23/60 (v) 7/4   (vi) 2/3

Exercise 6.5

(1) 3/40   (2) $30\dfrac{1}{4}$ (3) $6\dfrac{3}{16}$ (4) $12\dfrac{6}{25}$ (5) $156\dfrac{3}{16}$ (6) $72\dfrac{10}{49}$ (7) 13   (8) 11   (9) 14
(10) 26

Exercise 6.6

1. (i) 324   (ii) 1764   (iii) 6889   (iv) 16129   (v) 18496

Exercise 6.7

(1) 13   (2) 18   (3) 24   (4) 45   (5) 55   (6) 95

(7) 32   (8) 21

Exercise 6.8

(1) quotient = 2113   (2) quotient = 1005   (3) quotient = 11   (4) quotient = 3216

reminder = 01   reminder = 04   reminder = 02   reminder = 26

(5) quotient = 223   (6) quotient = 105

reminder = 05   reminder = 31

Exercise 7.1

1. complimentary angles (iii), (iv), (vi), supplimentary angles (i), (ii), (v)   2. $45^0, 45^0$ 3. 90^0

4. $\angle POQ$ and $\angle QOR$, $\angle POQ$ and $\angle QOS$, $\angle POR$ and $\angle ROS$, $\angle QOR$ and $\angle ROS$

5.   (i) $\angle QOT$ and $\angle TOP$

(ii) $\angle QOS$ and $\angle SOP$, $\angle POR$ and $\angle ROQ$

(iii) $\angle ROQ$ and $\angle SOP$

(iv) $\angle QOS$ and $\angle SOT$, $\angle SOT$ and $\angle TOP$, $\angle TOP$ and $\angle POR$

(v) $\angle QOS$ and $\angle SOT$

6.   (iii), (iv)   7. (i) $x = 55^0, y = 125^0$ (ii) $x = 160^0$ (iii) $x = 55^0, y = 35^0, z = 125^0$

8.   1. True   2. False   3. True   4. True

Exercise 7.2

1. (i) Intersecting lines-l,m   (ii) Parallel lines - p,q   (iii) Intersecting lines-y, z, Transversal line - x

2.   (i) $\angle 1$ and $\angle 7$, $\angle 4$ and $\angle 6$

(ii) $\angle 2$ and $\angle 8$, $\angle 3$ and $\angle 5$

(iii) $\angle 1$ and $\angle 5$, $\angle 2$ and $\angle 6$, $\angle 4$ and $\angle 8$, $\angle 3$ and $\angle 7$

(iv) $\angle 1$ and $\angle 6$, $\angle 4$ and $\angle 7$

3.   (i) $x = 60^0$ (ii) $x = 50^0$ (iii) $x = 70^0$ (iv) $x = 115^0$ 4. (i) and (ii)

5.(i) x = 95^0 , y = 55^0 (ii) x =115^0 , y = 65^0

Exercise 8.1

1. (i) $x = 35^0$ (ii)$x = 120^0$ (iii)$x = 50^0$ (iv)$x = 60^0$

2. (i) $x = 130^0$ (ii) $x = 75^0$ (iii)$x = 65^0$ (iv) $x = 50^0$ (v) $x = 30^0$ (vi) $x = 95^0$

3. (i) $x = 70^0$ y = 50^0 (ii) x = 75^0 y = 105^0 (iii) x= 40^0 y = 100^0 (iv) $x = 70^0$ y = 70^0

4. 45^0 5. 80^0 6. 30^0 , 60^0 , 90^0

7. No. Two angles of triangle are 70^0 and 21^0 so the third angle will be 89^0, which is not right angle.

8. (ii) , (iii)

Exercise 8.2

1.(i), (iv), (vi)   2.60^0

3. (i) acute angle   (ii) obtuse angle   (iii) obtuse angle   (iv) isosceles
   (v) Median   (vi) centroid   (vii) altitude

4- B = 55^0, C = 55^0 6. Minimum = 4 cm, Maximum = 8 cm

Exercise 9.1

1. AB ↔ PQ, BC ↔ QR, CA ↔ RP,∠A =∠P,∠B =∠Q व ∠C =∠R

2. (i)∠z   (ii) xy   (iii)∠y   (iv) yz

3. (i) Length (ii) Sides   (iii) 60^0

5. ∠ABC =∠ONM,∠PQR =∠MLK, ∠XYZ = ∠RST

Exercise 9.2

1. (i) 3.5 cm   (ii) 2 cm   (iii) 2.9 cm   (iv) 45^0 (v) 70^0 (vi) 65^0
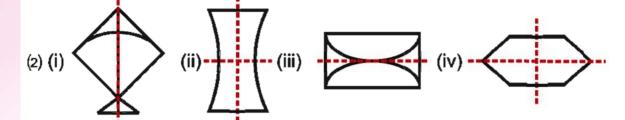
2. (i) ASA, Δ TQP ≅ Δ TRS   (ii) SAS, Δ ABD ≅ CDB   (iii) SSS, Δ ABC ≅ Δ ADC

3. (i) Δ PRQ ≅ Δ PSQ   (ii) Δ MXZ ≅ Δ NYZ   (iii) Δ QRP ≅ Δ TSQ
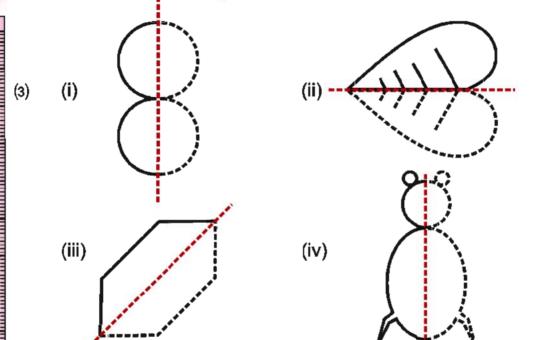
4. (i) Δ ADC   (ii) Δ PSR

5. Yes, RHS and SSS   6. (ii) Δ PQR ≅ Δ QPS   7.Yes

Exercise 11.1

(1)   (i),   (iii),   (iv)

(2) (i)   (ii)   (iii)   (iv)

(3)   (i)   (ii)

(iii)   (iv)

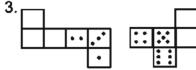## Exercise 11.2

(1)   (i)4,   (ii)3,   (iii)2   (iv) 2   (v) 2

(2)   Circle, Square

(3)   Rectangle, Parallelogram

(4) 120^0, 180^0, 240^0, 300^0, 360^0

Exercise 12.1

2. (i) c (ii) a

3.

Exercise 12.3

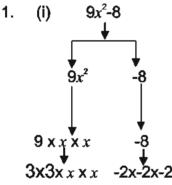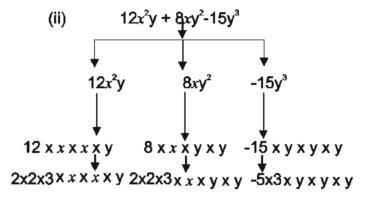2. (i) Sphere   (ii) Cone   (iii) Cuboid   (iv) Cube

3. (i) Top, Side, Front   (ii) Front, Side, Top

5. (i) True   (ii) False   (iii) True

Exercise 13.1

1. (i) $9x^2-8$ (ii) $12x^2y + 8xy^2 - 15y^3$

(iii) $a^3 - b^3$

2. (i) 4   (ii) coefficient of $y^2 = 9x^2$, coefficient of $x^2 = 9y^2$, coefficient of $9 = x^2y^2$

(iii) coefficient of $x^3 = \dfrac{-8y^3}{5}$, coefficient of $y^3 = \dfrac{-8x^3}{5}$, coefficient of $x^3y^3 = \dfrac{-8}{5}$

(iv) coefficient of $a^2 = \dfrac{9}{13}b^2$,   coefficient of $b^2 = \dfrac{9}{13}a^2$,

Exercise 13.2

1. (i) $3t + 2tz$
   (ii) $13xy$
   (iii) $3x-4y-3xy-3$
   (iv) $-m^2-n^2-1$
   (v) $8x+8z+4$
   (vi) $3ba+ab^2+2a^2b^2$
   (vii) 0

2. (i) $6x^2$
   (ii) $2b$
   (iii) $3x^2-8xy+4$
   (iv) $10xy-7x^2-7y^2$
   (v) $8p^2+7q^2-5pq$
   (vi) x^2-5x-5

3. $6x - 9y - z$ 4. $p- q$

Exercise 13.3

1. (i) -1 (ii) -1 (iii) -3 (iv) -3 (v) 1
2. (i) 1 (ii) 1 (iii) -3
3. (i) 0 (ii) 16 (iii) 8
4. (i) 2 (ii) 3 (iii) 0 (iv) 6

Exercise 14.1

(1.) $x = 4$ (2.) $x = 6$ (3.) $x = 8$ (4.) $x = 2$ (5.) $x = 4$

(6.) $x = 27$ (7.) $x = 18$ (8.) $x = 20$ (9.) $x = 3$ (10.) $l = \dfrac{4}{9}$

(11.) $m = \dfrac{17}{14}$ (12.) $z = -2$ (13.) $q = -8$ (14.) $x = 0$ (15.) $n = 12$

(16.) $t = \dfrac{14}{5}$ (17.) $m = 2$

Exercise 14.2

(1.) 31 (2.) 8 (3.) 6 (4.) $8,9,10$ (5.) $11,13,15$

(6.) $14,16,18$ (7.) 11 year (8.) 13 year (9.) 12 (10.) 15 year (11.) 150

Exercise 15.1

(1) (i) $1 : 5$ (ii) $17 : 20$ (2) (i) $13 : 19$ (ii) $9 : 8$
(3) (i) $6 : 10, 9 : 15$ (ii) $14 : 22, 21 : 33$ (4) (i) $5: 1$ (ii) $1 : 5$
(5) $x = 18$

6. (i) Bheema 800 gm., Bheekha 1 kg. 250 gm.   (ii) Bheekha Halwai   7. 25 worker

8. height of tree (h) = 15 m.   9. 39 minute   10. 2 kg 300 gm.

Exercise 15.2

(1) (i) 75% (ii) 77   %(iii) 99   % (iv) 333   %

(2) (i) 84% (ii) 125%   (iii) 8.75% (iv) 0.1%

(3) (i) $\frac{13}{25}$ (ii) $\frac{5}{4}$ (iii) $\frac{1}{16}$ (iv) $\frac{1}{3}$

(4) (i) 45   (ii)   306.25 (iii) 93.75 (iv) 5.20

(5) (i) 150   (ii) 50   (iii) 9000

(6) (i) 0.07 (ii) 0.014 (iii) 0.0003   (iv) 0.167

(7) 75 boys   (8)   2000 plants   (9)   90%

(10) martyrs day =80% , Tilak club 95%

Exercise 15.3

(1) 11 $\frac{1}{9}$% profit

(2) (i) 14$\frac{2}{7}$% Loss (ii) 3$\frac{1}{3}$% profit (iii) 20% profit (iv) 13800 Rs.

(3) Increased by 6 %   (4) 14$\frac{7}{12}$%   (5) 7130 Rs.

Exercise 15.4

(1) 1620 Rs.

(2) 4480 Rs.

(3) 560 Rs., 4060 Rs.

(4) 4% Principal

(5) 4000 Rs. Principal

Exercise 16.1

(1) 480 meter (2) 13 cm

(3) 1250 m^2 (4) 340 meter

(5) 800 meter (6) area of square = 25 cm^2 more

(7) (i) 9 cm (ii) 24 cm (iii) 27 cm

Exercise 16.2

(1) (i) 54 cm^2 (ii) 16.5 cm^2 (iii) 14 cm^2 (iv) 9.75 cm^2

(2) 24 cm , 6 cm (3) 24 cm (4) (i) 12 cm^2 (ii) 3 cm

(5) 64 cm^2

(6) 19 cm^2 , BE = 5 cm (7) 40 cm , 16 cm (8) 1850 m^2

(9) (i) 464 m^2 (ii) Rs.1,25,280

(10) (i) Rs.1,68,000 (ii) 9775 m^2

Exercise 16.3

(1) (i)132 cm (ii)176 mm (iii) 66 cm

(2) (i) 78.57 cm^2 (ii) 1386 m^2 (iii) 98.56 cm^2

(3) Radius = 21cm, 1386 m^2 (4) Radius = 7 cm, 154 cm^2

(5) 30.84 cm (6) 252.56 m^2 (7) 188.57 cm^2

(8) 28.26 cm^2 (9) 600 cm^2 (10) 16 : 25

(11) Rs.154 (12) 452.16 cm^2 (13) 253.14 cm^2

(14) 4 round (15) Rs.15,246 (16) 125.6 cm

Exercise 17.1

(1) (i) in 2015 - 16   (ii) 2014 - 15   (iii) 550
(2) (i) Hindi   (ii) 2014   (iii) 2012
(5) (i) Bikaner   (ii) Ajmer   (iii) Kota

Exercise 17.2

(1) (i) Class VI   (ii) Class 12   (iii) 30   (iv) Mean = 61
(2) 4.5   (3) 44   (4) 39
(5) (i) Millet - Rs. 6800, Guar - Rs. 8900, Groundnut - Rs. 9400   (ii) Groundnut
(6) 1   (7) 56 (8) 21   (9) (i) Range -5   (ii) 37℃   (iii) 3 day
(10) (i) 34   (ii) 68.5   (iii) Suman   (iv) 28

Exercise 17.3

(1) 6   (2) mean = 4 mode = 5
(3) median = 65, mode= 65   (4) 39
(5) (i) median - 151, mode - 151, 152
(ii) Yes, 151 and 152 are two modes
(6) mean - 44, median - 47, mode - 36,
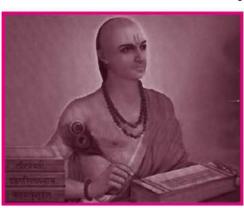(7) (i) True   (ii) False   (iii) False   (iv) False

Great Mathematician Bhaskaracharya

Bhaskaracharya was born in 1114 AD on Vizzadveek. Some people indicated that this place belongs to Bijapur (Karnataka) where as some people believed that it belongs to Jalgaon (Maharashtra). There are three well known famous works of Bhaskaracharya on Astronomy and Mathematics.

Siddanta Shiromani : There are four major parts of this standard work on Astronomy -
(i)   Lilavathi
(ii)   Bijganitam
(iii)   Grahganita
(iv)   Golaadhyay
These four works are also considered as separate books. The importance of these can be seen from the fact that more than four thousand tikas are also available along with translation in various local and foreign languages.

Lilavathi

It contains 9 chapters and 261 shlokas. He emphasized the importance of women education by giving the name of his daughter to this work.

Bijganitam

It consists of 12 chapters and 213 shlokas as in which the operations of algebra had been described. In other two works, Sphere and spherical Trigonometry, Grahanganita and Panchangganita are included.

Contributions of Bhaskaracharya

1.   The name "Khahar" given to a number obtained from a positive number after dividing by zero. "Kh" means zero therefore Khahar means the number whose denominator is zero.

2.   The description of permutations and combination in Lilavathi had been given in Ankpash Section. Some triangles are given under the title of Khandmeru which are known as "Pascal Triangle".

3.   The solution of quadratic equation of algebra $61 x^2 + 1 = y^2$ and other similar equations had been obtained by Chakraval's method which could be solved after 500 years by the western mathematicians.

4.   The stationary value and lowest value of π had been given by him respectively as $\frac{22}{7}$ and 3.1416.

5.   He revealed the simple proof of Pythagoras theorem.

6.   He gave the formulae to find volume of cylinder and cube in the Mensuration.

7.   He described the method to make magic squares in his book. India named Bhaskar to a satellite due to his incredible contribution in the Mathematic and Astronomy.