

MATHEMATICS

CLASS - 9



**BOARD OF SECONDARY EDUCATION, RAJASTHAN
AJMER**

Text Book Translation Committee

MATHEMATICS

Class - IX

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MATHEMATICS

Class - IX

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PREFACE

This book has been written in accordance with the new syllabus for class IX prescribed by the Board of Secondary Education, Rajasthan, Ajmer. In presenting the book the basic object of the syllabus has been fully kept in mind and an attempt has been made to acquaint the students with the contribution of Indian Mathematician towards the development of scientific traditions. The contribution of Indian Mathematicians have been mentioned at appropriate places. Every effort has been made to present the subject in simple and lucid manner Important principal have been explained in detail.

In the interest of the students sufficient number the illustrative examples have been given. At the end of each chapter a summary of the chapter is given in the form of important points, which will help the students in revision. In each chapter objective, short and essay type questions have been given in sufficient number in the miscellaneous exercise.

We hope the book will be useful to students. Students, teachers and reviewers are requested to send their comments, suggestions and to point out any shortcoming in the book, so that the desired improvement in the book can be made in the subsequent edition.

Authors

SYLLABUS

MATHEMATICS

Class-IX

Time- 3.15 hours

Subject code-09

Max. Marks-100

S.N.	Name of Unit	Name of Chapter	Marks	Total Marks
1.	Vedic Mathematics	1. Vedic Mathematics	08	08
2.	Number System	1. Number System	06	06
3.	Algebra	1. Polynomials	10	20
		2. Linear Equations in Two Variable	10	
4.	Geometry	1. Introduction with geometry and Angle	02	26
		Plane Geometry line and angle		
		2. Rectilinear Figures	06	
		3. Congruence and Inequalities of Triangles	04	
		4. Construction of Triangle	04	
		5. Quadrilateral	06	
		6. Area of triangle and quadrilateral	04	
5.	Mensuration	1. Area of plane figures	07	15
		2. Surface area and volume of cube and cuboid	08	
6.	Trigonometry	1. Angle and their measurement	03	10
		2. Trigonometric ratios of acute angle	07	
7.	Statistics	1. Statistics	10	10
8.	Road Safety Education	1. Road Safety Education	05	05

Details of the Syllabus

	Marks
Unit 1 Vedic Mathematics	8
1. Fundamental concept of Vedic Mathematics (Part-I)	
The meaning and applications of the sutras Ekadhikena Purvena, Ekanyunena Purvena, Vinculum (Negative) number, base, sub-base, deviation, The meaning & application of Sutra Nikhilam. Navtah Charamam dastah, Navanka and Ekadashanka methods of checking the results for addition, subtraction and multiplication operations.	
Unit 2 Number system	6
Review of Rational numbers on Number line, Irrational number, Real number and their Decimal expansions, representation of irrational number on the number line, successive magnification, Geometrical representation of a real number, operation on real numbers, laws of exponent for real numbers	
Unit 3 Algebra	20
1. Polynomials	10
Definition of a polynomial with one variable, its coefficient, constant polynomials, zero polynomial, linear polynomial, zeros of a polynomial, remainder, theorem, factorization of polynomials, Algebraic identities :	
$(x+y)^2 = x^2 + 2xy + y^2$, $(x-y)^2 = x^2 - 2xy + y^2$	
$x^2 - y^2 = (x+y)(x-y)$, $(x+a)(x+b) = x^2 + (a+b)x + ab$	
$(x+y+z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$	
$(x \pm y)^3 = x^3 \pm y^3 \pm 3xy(x \pm y)$	
$x^3 + y^3 + z^3 - 3xyz = (x+y+z)(x^2 + y^2 + z^2 - xy - yz - zx)$	
2. Linear Equation in two variables	10
Introduction, linear equation in two variables, Rectangular co-ordinate system, graph of linear equation of two variable. Algebraic methods of solving simultaneous linear equations :- (i) Method of Elimination (by substitution, by equating the co efficient) (ii) cross-multiplication method, condition for solvability, applications of linear equations in two variables.	
Unit 4 Geometry	26
1. Introduction of Plane Geometry, Line and Angle	02
Fundamental concepts, theorems and geometric construction, geometric symbols, Angle and its measurements, transversal line and parallel line, basic	

constructions. Construction draw a bisector of a given line segment, draw a bisector of a given angle, construction of various angle 60° , 120° , 30° , 90° , 45° , 135° with the help of ruler and compass, draw an equivalent angle of an angle on a point of a line, draw the various angles of any measurement with the help of ruler and compass. Draw a perpendicular from a point which is out side the line on any line. Draw a perpendicular at any point of the line.

2. Rectilinear Figures 6

Triangle and its angle, classification of triangle, rectilinear figures.

3. Congruence and Inequalities of Triangle 4

Theorem- Angle-Side-Angle some properties of triangle, some other concepts for the congruence of triangle, Side-Side-Side-rule, RHS-rule, inequalities of triangle perpendicular distance of a line from a external point.

4. Construction of Triangles 4

Construction of triangles when three sides are given, two sides and angle between them is given, two angles and one side is given. Construction a right-angled triangles when two sides and an angle opposite to one side is given, some difficult constructions of triangles.

5. Quadrilateral 6

Types of quadrilateral, properties of parallelogram, mid point theorem, construction of quadrilaterals when four sides and a diagonal are given, three sides and two angles between them are given, two consecutive side and angle between them and other two angles are given, construction of parallelogram and trapezium.

6. Area of triangles and Quadrilaterals 4

Introduction, area, figures made on same base and between pair of same parallel lines.

- Parallelograms made on same base and between same parallel lines are equal.
- Area of triangle between same base and same parallel lines are equal.
- If the area of two triangle are equal and one side of a triangle is equal to one side of other triangle, then their corresponding altitudes are equal.
- Baudhayan theorem - In a right angled triangle, square made on hypotenuse, is equal to the sum of the squares made on other two sides.

Converse of Baudhayan theorem- In a triangle, if square of a longest side is equal to the sum of the squares of other two sides, angle opposite to this side is a right angle.

Unit 5 Mensuration	15
1. Area of plane figures	7
Introduction, area of triangle, Heron's formula, area of an isosceles triangle, area of an equilateral triangle, area of right angled triangle, area of quadrilateral, area of parallelogram, to find the area of different quadrilateral (cyclic quadrilateral, rhombus, trapezium) and their uses.	
2. Surface area and Volume of cube and cuboid	8
Introduction, cube, cuboid, diagonal of cube and cuboid. Surface area and volume of cube and cuboid.	
Unit 6 Trigonometry	10
1. Angle and their measurement	3
Trigonometry, positive and negative distance, angle, positive and negative angles, angles of any magnitude and their measurement, sexagesimal system, centesimal system, circular system . Value of π , value of 1 radian.	
2. Trigonometric ratios of Acute Angles	7
Right-angled triangle, trigonometric ratios of acute angle, relation between trigonometric ratios, trigonometric identities.	
Unit 7 Statistics	10
1. Statistics	10
Introduction, Primary data, Secondary data, presentation of data, graphical representation of data, bargraph, histograms (According to change in base and height), frequency polygon, measures of central tendency. Mean, mode, median.	
Unit 8 Road Safety Education	5
Percentage (objective, content, activity) circle (objective, content, activity), statistics (objective, content, activity) quadrilaterals (objective, content, activity), road signs, probability (objective), data.	

Prescribed book :

Mathematics, Board of Secondary Education Rajasthan, Ajmer

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Vedic Mathematics

1.01 Introduction

Shankaracharya of Puri Swami Bharati Krishna ji Tirtha was the research scholar and inspiration in the field of Vedic Mathematics. He meditated hard for about eight years in the Sringeri math. After attaining the highest state of perfect siddhi he realized internally the Aphorisms and Mathematical Sutras described in ancient Indian literature Vedas, Brahamans', Sahintas', Vedangas etc. and reconstructed them in Sanskrit the divine language. Vedic Mathematics profounded by Swami ji is based on these Mathematical sixteen Sutras and thirteen Upsutras. These Sutras are very easy, effective, useful and correspond one to many interpretations in general. Problems of any branch of Mathematics can be solved by these Sutras easily.

1.02 Usefulness of Vedic Mathematics

By these Vedic mathematical Sutras calculations become easy and short. Less time is consumed. Students feel less mental stress. By Sutra's based methods the results or answers of the questions can be verified, hence giving the student more confidence. By these formulae the possibility of committing error by students become negligible.

By these Sutras more interest in mathematics is developed in students consequently they attain excellent achievement in subject mathematics. They can mentally solve the hard exercises. That is by Vedic Mathematics is called mental mathematics in the mathematical world. According to Swami ji the capacity and speed of calculations of students increases to five fold by the practice of Vedic Mathematics. By its practice there is an unbelievable development in their intellect and intelligence. Neocortex (mind) of the Vedic Mathematic students evolves very fast.

List of Sutras and Subsutras

Sutras	Subsutras
1. Ekadhikena Purvena (also a corollary)	1. Anurupyena
2. Nikhilam Navata- charamam Dasatah	2. Sisyate Sesamjnah
3. Urdhva-tiryagbhyam	3. Adyamadyenantya-mantyaena
4. Paravartya Yojayet	4. Kevalaih Saptakam Gunyat
5. Sunyama Samyasamuchaye	5. Vestanam
6. (Anurupye) Sunyamanyat	6. Yavadunam Tavadunam
7. Sankalana-vyavakalanabhyam (also a corollary)	7. Yavadunam Tavadunikrtya Varganca Yojayet
8. Puranapurabhyam	8. Antyayoradaskae'pi
9. Chalana-Kalanabhyam	9. Antyayoreva
10. Yavadunam	10. Samuccayagunita
11. Vyastisamastih	11. Lopanasthapanabhyam
12. Sesanyankena Caramena	12. Vilokanam
13. Sopantyadvayamantyam	13. Gunitasamuccayah Samuccayagunita
14. Ekanyunena Purvena	
15. Gunitasamuccayah	
16. Gunakasamuccayah	

Meaning and Applications of special Sutras

1.03 Sutra Ekadhikeana Purvena :

(a) Meaning : Sutra is composed of two words 'Ekadhika' and 'Purva', its meaning being "By one more than the previous one". Doing Ekadhika on a number means adding one to that number or marking Ekadhika dot (·) on its unit digit i.e.,

$$\text{Ekadhika of } 1\dot{2} = 12 + 1 = 13$$

In case if a digit of a number is marked the Ekadhika dot, its new value is to be found out e.g. Ekadhika of digit 3 of a number 1534

$$\text{yields the new number } = 15\dot{3}4 = 1544$$

Purvena (Purva) means 'By the previous one'. In case of digits of a number it points out the previous digit and in case of two numbers it points out the previous number e.g. In a number 685 the previous digit of five is 8 and in a multiplication 62×99 the previous number is 62. All the above operations can be mentally done.

(b) Applications :

(i) Addition : Sutra based method is applicable in all types of questions of addition.

Method : Write the numbers of a question columnwise. Start adding from top unit digit. At a digit where the sum equals to ten or more than ten. mark an Ekadhika dot on its previous digit. Start adding again with the unit digit of that sum. Repeat the process. Write the end sum at answer's place. Add other columns in the same way. The method is clarified by the following examples.

Example 1. Add the following :

Steps

- | | |
|-------------|--|
| 3 7 9 9 5 ↓ | (i) Column, I $5+6=11$ |
| 0 6 8 9 8 6 | (ii) Mark Ekadhika dot (·) on 8 |
| 7 5 4 3 8 | (iii) Start adding again with 1 |
| 0 5 8 9 0 9 | (iv) $1+8=9$, Again $9+9=18$ |
| 2 4 1 3 2 8 | (v) Mark Ekadhika dot on 8 |
| | (vi) Write 8 below at the answer's place |
| | (vii) Add other columns in the same way. |

Example 2. Add the following:

km.	m.	Steps
2 8	0 8 4	(i) Three columns in a metre.
$\dot{3}$ 2	$\dot{3}$ 6 5	(ii) No columns should remain vacant hence write 084 for 84.
$\dot{0}$ 6 $\dot{5}$	7 $\dot{2}$ 5	(iii) Now add columnwise by the above method.
$\dot{3}$ 8	$\dot{2}$ 5 0	
<hr/> 1 6 4	<hr/> 4 2 4	

(ii) Subtraction :

In Vedic Mathematics subtraction can be performed by four or five methods. The best method is based on Sutra Ekadhikena Purvena and Param mitra unka (the best friend). The two digits or numbers are said to be the best friend of each other if their sum is equal to ten i.e. 2 is the best friend of 8, 6 the best friend of 4 and 0 the best friend of 9.

Method : When lower digit does not subtract from the upper digit. Then add the param mitr unka of lower digit to the upper digit and write the sum in place of answer. Also write the mark of Ekadhikena on the previous digit of lower digit. By the repetition the process we get the remainder. If upper digit is greater than or equal to the lower digit, then there is no need to add the param mitra unka. Subtract by normal process.

The method is clarified by the following examples

Example 3. Subtract

$$\begin{array}{r} 5\ 7\ 6\ 2\ 5 \\ -\ 2\ \dot{9}\ \dot{8}\ 4\ 3 \\ \hline 2\ 7\ 7\ 8\ 2 \end{array}$$

Steps

- (i) $5 - 3 = 2$ write it on the answer's place
- (ii) 4 is not subtracted from 2. So the best friend of 4 is 6 is to be added to 2. Write the sum = 8 at answer's place.
- (iii) Also mark an Ekadhikena dot at digit 8 prior to digit 4.
- (iv) $\dot{8} = 9$ is not subtracted from 6 so add the best friend of 9 i.e. 1 to 6 i.e. $1 + 6 = 7$ Write it on answer's place,
- (v) Mark Ekadhika dot on 9.
- (vi) Similarly $0 + 7 = 7$ and $5 - \dot{2} = 2$ will give the answer.

Example 4. Subtract

Steps

- (i) If time unit columnwise bases are different.
- (ii) In unit column of second and minute Base = 10

hour	min.	sec.
24	12	15
$\dot{0}\dot{6}$	$\dot{2}\dot{4}$	30
17	47	45

- (iii) In tens's place column of second and minute, Base = 6
- (iv) In Hour's column Base = 10.
- (v) In all columns best friend of a digit is calculated by these bases.

Note : In Decimal Number System the base is taken as 10.

(iii) Multiplication :

In Vedic Mathematics for multiplication there are variance methods based on Sutrats. Sutra Ekadhikena Purvena based method is universal and effective. Some of its special sub methods are very easy and attractive. Here with let us define some new item . The unit digit of a number is known as Charamam digit and all other digits of a number are called Nikhilam digits. e.g. In a number 723 the Charamam digit is 3 and all the digits 7, 2, are called Nikhilam digits.

By the sutra 'Ekadhikena Purvena' when the sum of the Charamam digits is equal to 10 or power of 10 and the remainder Nikhilam digits are equal to each other, we can find the multiplication of two numbers easily.

Method :

- (1) There are two sides of product one is L.H.S. and another is R.H.S.
- (2) Write the product of charamam or last digits in L.H.S.
- (3) In R.H.S. write the product of nikhilam digit and its Ekadhikena.
- (4) In the R.H.S. keep the number of digits double of the number of zeroes in the sum of charamam digit i.e. two digits on R.H.S. if sum = 10.
- (5) If number of digit in R.H.S. is less or more then we adjus the digit.

The method is explained by the following examples :

Example 5. Multiply : (Sum = 10)

Steps

- | | |
|-----------------------------|---|
| 83×87 | (i) Sum of Charamam digits $3 + 7 = 10$. |
| $= 8 \times 9 / 3 \times 7$ | (ii) remainder Nikhilam digits are equal = 8. |
| $= 7221$ | (iii) Two digits in R.H.S. = 21 |

Example 6. Multiply : (Sum = 100) **Steps**

$$\begin{array}{l}
 586 \times 514 \\
 = 5 \times 6 / 86 \times 14 \\
 = 30 / 1204 \\
 = 301204
 \end{array}
 \quad
 \begin{array}{l}
 \text{(i) Sum of last two digits} = 86 + 14 = 100 \\
 \text{(ii) remainder Nikhila digits are equal} = 5 \\
 \text{(iii) Four digits in R.H.S.} = 1204
 \end{array}$$

Example 7. Multiply : (Sum = 1000). **Steps**

$$\begin{array}{l}
 3993 \times 3007 \\
 = 3 \times 4 / 993 \times 007 \\
 = 12 / 006951 \\
 = 12006951
 \end{array}
 \quad
 \begin{array}{l}
 \text{(i) Sum of last three digits} = 993 + 007 = 1000 \\
 \text{(ii) Six digits in R.H.S.} = 006951 \\
 \text{(add two zeros before 6 by adjustment)}
 \end{array}$$

Example 8. Multiply : (Sum = 1)

$$\begin{array}{l}
 9\frac{5}{11} \times 9\frac{6}{11} \\
 = 9 \times 10 / \frac{5}{11} \times \frac{6}{11} \\
 = 90\frac{30}{121}
 \end{array}
 \quad
 \begin{array}{l}
 \text{Steps} \\
 \text{(i) Sum of fractions} = \frac{5}{11} + \frac{6}{11} = 1 \\
 \text{(ii) remainder nikhila digits equal to each other} = 9
 \end{array}$$

Example 9. Multiply : (Sum = 1)

$$\begin{array}{l}
 11.7 \times 11.3 \\
 = 11 \times 12 / .7 \times .3 \\
 = 132.21
 \end{array}
 \quad
 \begin{array}{l}
 \text{Steps} \\
 \text{(i) Sum of decimal fractions} = .7 + .3 = 1 \\
 \text{(ii) remainder Nikhila digits equal to each other} = 11.
 \end{array}$$

Note: By Sutra based method all questions can be solved orally. Their products can be written in a line.

1.04 Sutra Ekanyunena Purvena :

(a) Meaning : Sutra is composed of two words - 'Eka nyunena' and 'Purvena'. Its meaning is "By one less than the previous one". Place a dot beneath the unit digit of a number which is to be less by one. This dot is known as Eka nyunena mark e.g. Eka nyunena of 7 in $57 = 5\dot{7} = 57 - 1 = 56$.

Like the previous Sutra Ekadhikena Purvena in this sutra we can find the value of new number by subtract one from any digit of the number. e.g. In a number 124. We get a new number 024 by taking the nyunena of digit 1. i.e. new number = $1\dot{2}4 = 024 = 24$.

(b) Applications:

(i) Subtraction (Sutra Ekanyunena Purvena + Param mitra unka)

By this sutra every question of subtraction can be solved easily.

Method:

When the lower digit is greater than the upper number digit add the best friend of lower number digit and keep this sum digit at the answer's place. Also place an Eka nyunena dot beneath the digit which is prior to the upper number digit. The repetition of this process will yield the remainder result. The method is clarified by the following examples :

Example 10. Subtract :

$$\begin{array}{r} 5 \quad 6 \quad 0 \\ - 3 \quad 7 \quad 4 \\ \hline 1 \quad 8 \quad 6 \end{array}$$

Steps

- (i) The whole process is same as Ekadhikena Purvena method.
- (ii) Difference is only one that in this process we place an Ekanyunena dot beneath the digit which is prior to the upper digit instead of Ekadhikena mark.

Example 11. Subtract:

kg.		g.
1 2 5		0 9 5
7 8		2 2 8
<hr/>		<hr/>
0 4 6		8 6 7

Steps

- (i) 95 g is written as 095.
- (ii) $5 + 2$ (best freind of 8) = 7, write 7 at the answer's place. Also mark Eka nyunena dot beneath the digit 9.
- (iii) $8 - 2 = 6$
- (iv) $0 + 8$ (best freind of 2) = 8, write 8 at the answer's place. Also mark Eka nyunena dot beneath the digit 5.
- (v) $4 + 2$ (best freind of 8) = 6, write 6 at the answer's place. Also mark Eka nyunena dot beneath the digit 2.
- (vi) $1 + 3$ (best freind of 7) = 4, write 4 at the answer's place. Also mark Eka nyunena dot beneath the digit 1.
- (viii) 0

(ii) Multiplication :

By Sutra Eka nyunena Purvena multiplication of two numbers can be easily performed if one of these numbers consists of digit nine only. For convenience the number having every digit 9 will be termed as multiplier and the other number as multiplicand.

Method :

There are two sides of the product L.H.S. = multiplicand – 1

R.H.S. = multiplier – L.H.S.

Hence Multiplier \times Multiplicand = Multiplicand – 1 / Multiplier – L.H.S.

Three situations arises in this multiplication :

(1) No. of digits of multiplier = No. of digits of multiplicand

(2) No. of digit of multiplier > No. of digits of multiplicand

(3) No. of digit of multiplier < No. of digits of multiplicand

I Situation :

(No. of digits of multiplier = No. of digits of multiplicand)

Let us see the following examples :

1. 8×9

L.H.S. = $8 - 1 = 7$

R.H.S. = $9 - 7 = 2$

$\therefore 8 \times 9 = 8 - 1 / 9 - 7 = 72$

2. 8567×9999

= $8567 - 1 / 9999 - 8566$

= 85661433

II Situation :

(No. of digits of multiplier > No. of digits of multiplicand)

Let us see the following examples :

3. 68×999

= 068×999

= $067 / 999 - 067$

= 67932

4. 4523×999999

= 004523×999999

= $004522 / 9995477$

= 4522995477

Note: (1) Greater the no. of digits of multiplier so many 9 digits appear in the middle of the answer e.g. see question no. 3 and 4.

(2) Sum of respective digits of L.H.S. and R.H.S. of the product will always be equal to 9, i.e. first digit of LHS + first digit of RHS = 9.

III Situation :

(No. of digits of multiplier < No. of digits of multiplicand)

The method is explained by the following examples:

5. 43×9

$$= 42/9 - 42$$

$$= 429 - 42 = 387$$

6. 512×99

$$= 511/99 - 511$$

$$= 51199 - 511 = 50688$$

Exercise 1.1

By Sutra Ekadhikena Purvena add the following:

1.
$$\begin{array}{r} 98765 \\ 63217 \\ 89522 \\ 60543 \\ \hline \end{array}$$

2.
$$\begin{array}{r} 89789 \\ 97686 \\ 76978 \\ 86798 \\ \hline \end{array}$$

3.
$$\begin{array}{r} \text{kg.} \quad \text{g.} \\ 178 \quad 45 \\ 246 \quad 725 \\ 569 \quad 188 \\ 45 \quad 894 \\ \hline \end{array}$$

4.
$$\begin{array}{r} \text{km.} \quad \text{m.} \quad \text{cm.} \\ 25 \quad 510 \quad 36 \\ 47 \quad 85 \quad 52 \\ 18 \quad 123 \quad 75 \\ 53 \quad 805 \quad 28 \\ \hline \end{array}$$

By vedic method subtract the following:

5.
$$\begin{array}{r} 746 \\ - 389 \\ \hline \end{array}$$

6.
$$\begin{array}{r} 4032 \\ - 3543 \\ \hline \end{array}$$

7.
$$\begin{array}{r} 6007 \\ - 1852 \\ \hline \end{array}$$

8.
$$\begin{array}{r} 8317 \\ - 6454 \\ \hline \end{array}$$

By Sutra Ekadhikena Purvena multiply the following.

9. 42×48 10. 103×107

11. 294×206 12. 413×487

Multiply with the help of Sutra Ekanyunena Purvena

13. 54×99 14. 214×999

15. 47×999 16. 342×99999

17. 73×9 18. 467×99

By Vedic method multiply the following:

19. $15\frac{5}{7} \times 15\frac{2}{7}$ 20. $24\frac{10}{13} \times 24\frac{3}{13}$

21. 4.5×4.5 22. 9.85×9.15

1.05 Vinculum (Negative) Numbers :

Concept of Vinculum operation is the contribution of Vedic mathematics. By Vinculum operation calculations become easy, short and sometimes oral. By this process numbers of big digits like (6,7,8,9) can be converted into numbers of small digits like (0,1,2,3,4,5). These days Vinculum operation has been adopted by computer science also. Digits with bars $\bar{2}$, $\bar{4}$ etc. are called Vinculum digits. Their respective values are -2 and -4 . This bar (line) is called Vinculum line or Vinculum mark. Positive and Vinculum digits can be placed together to represent a number. e.g. $2\bar{3}$ or $\bar{2}\bar{4}$. The number $1\bar{2}\bar{4}$ is spoken as one Vinculum two Vinculum four. It will not be called one Vinculum twenty four.

1.06 Base, Sub Base, Deviation :

Base : Here Base means a Number Base. Any number greater than 1 can be a base. In Vedic mathematics to make the calculation easy and convenient bases are usually chosen as 10 or 100 or any power of 10. In metric system of numbers base is always taken as 10.

Sub-Base : Sub-Base is the multiple of base. Mostly this number always ends at zero. If Base = 10, then Sub Base = $10 \times x$, x being a whole number, if Base = 100, then Sub Base = $100 \times x$, x being a whole number. In case of Sub Base calculations become easy if Sub-Base are used in place of base but necessary adjustment is to be done, in the left hand side of the answer. The reasons will be explained in the coming examples.

Deviation : When Base or Sub-Base is subtracted from the given number the remainder is called the deviation.

$$\text{Deviation} = \text{Number} - \text{Base}$$

or
$$\text{Deviation} = \text{Number} - \text{Sub-Base}$$

If the number is greater than the Base or Sub-Base, the deviation is positive. If number is smaller than the Base or Sub-base, the deviation is negative. The deviation consists so many digits as there are the number of zero to the Base.

Deviation of 18 w.r.t. Base 10 = +8 (Number of Zero in the Base is one)

and Deviation of 94 w.r.t. Base 100 = -06. (Number of Zero in the Base is two)

1.07 Sutra Nikhilam Navtah Charamam dastah

(a) Meaning : Sutra Nikhilam means "All from nine and last from ten". In ancient Indian Mathematics digit nine is supposed to be very creative and an apex number. The number 10 is taken to be the whole number but here the sutra indicates simply the subtraction process. Applications like Vinculum, Subtraction, Multiplication, Squaring, Cubing, Divisions etc. are based on this sutra.

(b) Applications:

(i) To convert ordinary number into Vinculum number

(Sutra Ekadhikena Purvena + Sutra Nikhilam)

When an ordinary number consists of digit 5 or more than 5, it can be converted into Vinculum number by Nikhilam method.

- Method:**
- (1) Subtract Charamam (unit) digit from 10.
 - (2) Subtract the remainder digits from the number 9.
 - (3) Draw Vinculum line on each digit so obtained.
 - (4) Mark the Ekadhikena dot on the previous digit 0 or less than 5 of the remainder.

The method is explained by the following examples:

Example 12 : Convert the following ordinary number into Vinculum number:

- | | |
|---|--|
| | Steps |
| $\begin{aligned} & 8 \ 9 \ 8 \\ & = 0 \ 8 \ 9 \ 8 \\ 1. \quad & = \overset{\cdot}{0} \bar{1} \ 0 \ \bar{2} \\ & = 1 \ \bar{1} \ 0 \ \bar{2} \end{aligned}$ | <ol style="list-style-type: none"> (i) Subtract unit digit 8 from 10. (ii) Subtract remaining digits 9 and 8 from 9. (iii) Draw Vinculum Bar on 2 and 1. (iv) Place an Ekadhikena dot on 0 prior to $\bar{1}$ |
| $\begin{aligned} 2. \quad & 1 \ 8 \ 4 \ 6 \ 9 \\ & = \overset{\cdot}{1} \ \bar{2} \ 4 \ \bar{3} \ \bar{1} \\ & = 2 \ \bar{2} \ 5 \ \bar{3} \ \bar{1} \end{aligned}$ | <p style="text-align: center;">Steps</p> <ol style="list-style-type: none"> (i) Subtract 9 from 10 and 6 from 9 respectively. (ii) Draw Vinculum bar on 1 and 3. (iii) Place Ekadhikena dot on 4. |

(iv) Restart the process from digit 8.

Note : (1) When in between the bigger digit of a normal number occurs a digit equal or less than 5 than we repeat this process.

(2) No vinculum bar is drawn on zero.

(ii) To convert Vinculum number into ordinary number

(Sutra Ekanyunena Purvena + Sutra Nikhilam)

- Method:** (1) Subtract the positive value of Charmam digit from 10.
(2) Subtract the positive values of remaining Nikhilam digits from 9 respectively.
(3) Place an Ekanyunena dot beneath the non Vinculum digit.
(4) Repeat the process if necessary.

The method is clarified by the following examples:

Example 13 : Change the following into ordinary number:

$$\begin{aligned} 1. \quad & 2 \overline{4} \overline{3} \\ & = 2 \ 5 \ 7 \\ & = 1 \ 5 \ 7 \end{aligned}$$

$$\begin{aligned} 2. \quad & 6 \overline{2} \overline{4} \ 5 \ \overline{3} \ \overline{2} \\ & = 6 \ 7 \ 6 \ 5 \ 6 \ 8 \\ & = 5 \ 7 \ 6 \ 4 \ 6 \ 8 \end{aligned}$$

(iii) Multiplication of two numbers :

(Sutra Nikhilam – Base)

When two numbers are nearer to the base = 10 or 100 or power of 10, their multiplication can be found out easily by Sutra Nikhilam - Base.

- Method:** (1) Choose the base = 10 or 100 etc. nearer to the numbers.
(2) Find the deviations of the numbers with reference to base and write them before their numbers.
(3) Divide the product place in two parts by an oblique line.
(4) Write the product of deviations on R.H.S.
(5) On L.H.S. write one number plus the deviation of the other number.
(6) Adjust the number of digits of R.H.S. according to the number of zeroes of the base.
(7) The deficiency of digit number is fulfilled by writing zero. If the product digit number is more, take it to the left hand side.
(8) If the product of deviations is negative, change it into positive by taking one or two etc. from L.H.S. Kindly note that the value of one from L.H.S. is equivalent to the base value.

The method is explained by the following examples:

Example 14 : Multiply the following by Nikhilam – Base method:

1. 12×14 , Base = 10

$$\begin{array}{r} = 12 \quad + 2 \\ \quad 14 \quad + 4 \\ \hline = 14 + 2/2 \times 4 \\ \\ = 168 \end{array}$$

Steps

- (i) Deviations = +2, +4
- (ii) One digit in R.H.S.

2. 92×87 , Base = 100

$$\begin{array}{r} = 92 \quad - 08 \\ \quad 87 \quad - 13 \\ \hline = 92 - 13/(-08)(-13) \\ \\ = 79/104 = 8004 \end{array}$$

Steps

- (i) Deviations = -08, -13
- (ii) Two digits in R.H.S. so digit 1 or 104 to L.H.S.

3. 7×18 , Base = 10

$$\begin{array}{r} = 7 \quad - 3 \\ \quad 18 \quad + 8 \\ \hline = 7 + 8/(-3) \times 8 \\ = 15/-24 \\ = 15 - 3/30 - 24 \\ = 12/30 - 24 \\ = 126 \end{array}$$

Steps

- (i) Product = 15/-24
- (ii) Bring 3 from L.H.S. to R.H.S.
- (iii) In R.H.S. value of 3 will be equal = 30

4. 1007×1012

$$\begin{array}{r} = 1007 \quad + 007 \\ \quad 1012 \quad + 012 \\ \hline = 1012 + 7/084 \\ \\ = 1019084 \end{array}$$

Steps

- (i) Base = 1000
- (ii) d = +007, +012
- (iii) Three digits in R.H.S. so write zero before 84.

(iv) Multiplication of two numbers

(Sutra Nikhilam – Sub-Base)

Sometimes it becomes difficult to multiply two big deviations. In this case concept of Sub-Base is helpful. The method is similar to the previous Sutra Nikhilam Base method except that for adjustment in L.H.S. product is multiplied by the Sub-Base digit (Upadhar Unka) and R.H.S. remains as usual. The method is explained by the following examples:

Example 15 : Multiply the following by Sutra Nikhilam – Sub-Base.

1. 32×33

$$\begin{array}{r} = 32 \quad +2 \\ \quad 33 \quad +3 \\ \hline = 35 \times 3/6 \\ = 1056 \end{array}$$

Steps

- (i) Base = 10, Sub-Base = $10 \times 3 = 30$.
- (ii) Upadhar unka = 3
- (iii) deviation from Sub-Base = +2, +3
- (iv) Adjustment in L.H.S. = 35×3

2. 54×56

$$\begin{array}{r} = 54 \quad +4 \\ \quad 56 \quad +6 \\ \hline = 60 \times 5/24 \\ = 300/24 \\ = 3024 \end{array}$$

Steps

- (i) Base = 10, Sub-Base = $10 \times 5 = 50$
- (ii) Upadhar unka = 3
- (iii) deviation from Sub-Base = +4, +6
- (iv) L.H.S. = 60×5
- (v) Adjustment in R.H.S. will be done in the end.

3. 54×56

$$\begin{array}{r} = 54 \quad +4 \\ \quad 56 \quad +6 \\ \hline = 60 \times \frac{1}{2}/24 \\ = 3024 \end{array}$$

Steps

- (i) Base = 100, Sub-Base = $100 \times \frac{1}{2} = 50$
- (ii) Updhar unka = $\frac{1}{2}$
- (iii) d = +04 and 06
- (iv) Two digits in the R.H.S.

4. 206×212

$$\begin{array}{r} = 206 \quad +06 \\ \quad 212 \quad +12 \\ \hline = 218 \times 2/72 \\ = 43672 \end{array}$$

Steps

- (i) Base = 100, Sub-Base = 100×2
- (ii) Upadhar unka = 2
- (iii) d = +06, +12
- (iv) Two digits in R.H.S.

(v) Multiplication of three numbers:

(Sutra Nikhilam – Base)

Multiplication process is divided into three steps :

Step I : Any number + deviations of other two numbers.

Step II : Sum of the products of two deviations.

Step III : Product of three deviations.

The method is explained by the following examples:

Example 16 : Multiply the following by Sutra Nikhilam – Base.

1. $91 \times 93 \times 96$, Base = 100

Number	Deviation
91	-09
93	-07
96	-04

$$= 93 - 09 - 04 / 36 + 28 + 63 / (-9)(-4)(-7)$$

or $91 - 07 - 04$

or $96 - 09 - 07$

$$= 80 / 127 / (-252)$$

$$= 81 / 27 - 3 / 300 - 252$$

$$= 81 / 24 / 48$$

$$= 812448$$

Steps

- (i) deviation = -09, -07, -04
- (ii) In the third part $(-09)(-07)(-04) = -252$
- (iii) In the middle part $(-09)(-04) + (-07)(-04) + (-09)(-07) = 127$
- (iv) 3 taken from II part to III part place value being 300
- (v) $300 - 252 = 48$

2. $103 \times 105 \times 106$,

$$= 106 + 03 + 05 / 15 + 30 + 18 / 90$$

$$= 114 / 63 / 90$$

$$= 1146390$$

Steps

- (i) Base = 100
- (ii) deviation = +06, +03, 05.
- (iii) rest process as above.

3. $12 \times 13 \times 15$

$$= 12 + 3 + 5 / 6 + 15 + 10 / 30$$

$$= 20 / 31 / 30$$

$$= 2340$$

Steps

- (i) Base = 10
- (ii) deviation = 2, 3, 5.

Note: So as many digits are to be kept in II and III parts as there are number of zeroes in the Base.

(vi) Multiplication of three numbers:

(Sutra Nikhilam – Sub-Base)

In Nikhilam Sub-Base method the first part (from the left) and middle part of the product is multiplied by (Sub-Base digit)² or (Upadhar Unka)² and (Sub Base digit) or (Upadhar Unka) respectively. This is the only difference between Nikhilam Base and Sub-Base methods.

The method is explained by the following examples:

Example 17 : Multiply the following Nikhilam Sub-Base method.

- | | |
|--|---|
| <p>1. $21 \times 24 \times 25$</p> <p>21 +1</p> <p>24 +4</p> <p>25 +5</p> <p>$= 2^2 (21 + 4 + 5) / 2 (4 + 20 + 5) / 1 \times 4 \times 5$</p> <p>$= 4 \times 30 / 2 \times 29 / 20$</p> <p>$= 120 / 5 \cdot 8 / 2 \cdot 0$</p> <p>$= 12600$</p> | <p style="text-align: center;">Steps</p> <p>(i) Base = 10, Sub-Base = $10 \times 2 = 20$.</p> <p>(ii) Upadhar Unka = 2</p> <p>(iii) deviation = +1, +4, +5</p> <p>(iv) One digit each in II and III part.</p> |
| <p>2. $502 \times 503 \times 504$</p> <p>$= 5^2 (502 + 03 + 04) / 5 (6 + 12 + 8) / 2 \times 3 \times 4$</p> <p>$= 25 \times 509 / 5 \times 26 / 24$</p> <p>$= 12725 / 130 / 24$</p> <p>$= 127263024$</p> | <p style="text-align: center;">Steps</p> <p>(i) Base = 100,</p> <p style="padding-left: 20px;">Sub-Base = 100×5</p> <p>(ii) Upadhar Unka = 5</p> |

Exercise 1.2

Change into Vinculum numbers :

- | | |
|---------|---------|
| 1. 89 | 2. 878 |
| 3. 9687 | 4. 6578 |

Change into ordinary numbers :

- | | |
|-----------------------|-----------------------------|
| 5. $3\bar{2}1$ | 6. $2\bar{4}\bar{3}\bar{2}$ |
| 7. $4\bar{3}0\bar{2}$ | 8. $450\bar{4}\bar{9}$ |

Multiply the following by Sutra Nikhilam

- | | |
|---------------------------------|--------------------------------|
| 9. 102×107 | 10. 94×92 |
| 11. 72×73 | 12. 203×204 |
| 13. $11 \times 12 \times 13$ | 14. $97 \times 98 \times 99$ |
| 15. $102 \times 103 \times 104$ | 16. $99 \times 101 \times 103$ |

Methods of checking the results

There are two methods of checking the results received from any operation.

- (a) Navanka method
- (b) Ekadashanka Method

(a) Navanka method :

In the navanka method we find the beejanka of any number by taking base as digit 9. After subtracting 9 from the digits of a number or sum of the digits of a number remaining single digit is know as Beejanka of that number e.g. Beejanka of 947 = 2.

In the different operation the application of navanka method is explained by the following example.

Example 18 :

- (1) Checking of Addition results:

Beejanka		
5 3 8 9	7	<p style="text-align: center;">Steps</p> <p>(i) Sum of Beejanka (row-wise) = 7 + 1 + 5 + 1 = 5</p> <p>(ii) Beejanka of the sum = 2 + 1 + 9 + 6 + 5 = 5</p>
6 4 7 2	1	
5 9 3 6	5	
4 1 6 8	1	
2 1 9 6 5	5	

Both are equal, hence answer is correct.

- (2) Checking of the Subtraction results:

$\begin{array}{r} 8134 \\ - 5678 \\ \hline 2456 \end{array}$	<p style="text-align: center;">Steps</p> <p>(i) Beejanka of the minuend = 7 = 16</p> <p>(ii) Beejanka of the subtrahend = 8 or -8</p> <p>(iii) Beejanka of the remainder = 8</p>
--	---

- (3) Checking of the Multiplication results:

$$73 \times 77 = 5621$$

- (i) Beejanka of the multiplicand = 7 + 3 = 10 = 1
- (ii) Beejanka of the multiplier = 7 + 7 = 14 = 5
- (iii) Beejanka of the multiplicand \times multiplier = 1 \times 5 = 5

(iv) Beejanka of the product $= 5 + 6 + 2 + 1 = 5$

Beejanka of L.H.S. = Beejanka of R.H.S.

Note: 1. If two digits of any row or two digits of any column interchange their places the error cannot be spotted by Navanka method.

2. In Vedic Mathematics there are several methods to solve a question. The result can be verified by Ekadashanka Method.

(b) Ekadashanka Method or Difference Method:

In this method the difference of the sum of digits at odd places and the sum of digits at even places is called the Beejanka of the number e.g. Beejanka of 63254

$$= 4 - 5 + 2 - 3 + 6 = 4$$

The use of the Ekadashanka method for different operations is explained here with by the following examples:

(i) Checking of Addition results :

Row-wise Beejanka

$$63254 \quad 4 - 5 + 2 - 3 + 6 = 4$$

$$54327 \quad 7 - 2 + 3 - 4 + 5 = 9$$

$$89325 \quad 5 - 2 + 3 - 9 + 8 = 5$$

$$206906 \quad \text{Sum} = 18$$

$$\text{Beejanka of the sum } 18 = 8 - 1 = 7$$

$$\text{Beejanka of the sum } 206906 = 6 - 0 + 9 - 6 + 0 - 2 = 7$$

Both are equal, hence answer is correct.

(ii) Checking of the Subtraction results:

Row-wise Beejanka

$$7348 \quad 8 - 4 + 3 - 7 = 0$$

$$-5249 \quad 9 - 4 + 2 - 5 = 2$$

$$2099$$

$$\text{Beejanka of difference} = (i) - (ii) = 0 - 2 = -2$$

$$\text{Beejanka of the Remainder} = 9 - 9 + 0 - 2 = -2$$

Since both are equal, the answer is correct.

(iii) Checking of the Multiplication results:

$$54 \times 56 = 3024$$

$$\text{Beejanka of } 54 = 4 - 5 = -1$$

$$\text{Beejanka of } 56 = 6 - 5 = +1$$

$$\text{Multiplication of Beejanka } (-1) \times (+1) = -1$$

$$\begin{aligned} \text{Beejanka of the product} &= 4 - 2 + 0 - 3 \\ &= -1 \end{aligned}$$

Both are equal, hence answer is correct.



Answers

Exercise 1.1

- | | |
|------------------------|--------------------------|
| 1. 312047 | 2. 351251 |
| 3. 1039 kg., 852 gm. | 4. 144 Km. 524 m. 91 cm. |
| 5. 357 | 6. 489 |
| 7. 4155 | 8. 1863 |
| 9. 2016 | 10. 11021 |
| 11. 60564 | 12. 201131 |
| 13. 5346 | 14. 213786 |
| 15. 46953 | 16. 34199658 |
| 17. 657 | 18. 46233 |
| 19. $240\frac{10}{49}$ | 20. $600\frac{30}{169}$ |
| 21. $20 \cdot 25$ | 22. $90 \cdot 1275$ |

Exercise 1.2

- | | |
|------------------------------|------------------------------------|
| 1. $1\bar{1}\bar{1}$ | 2. $1\bar{1}\bar{2}\bar{2}$ |
| 3. $10\bar{3}\bar{1}\bar{3}$ | 4. $1\bar{3}\bar{4}\bar{2}\bar{2}$ |
| 5. 281 | 6. 1568 |
| 7. 3698 | 8. 44951 |
| 9. 10914 | 10. 8648 |
| 11. 5256 | 12. 41412 |
| 13. 1716 | 14. 941094 |
| 15. 1092624 | 16. 1029897 |



Number System

2.01 Review of Rational Numbers on the Number Line

Numbers have great importance in our daily life. We have studied about different numbers beginning from natural numbers to rational numbers. Now, we review them on the number line.

(i) Natural numbers

This line increases towards the right hand side from 1 to infinity.



Fig. 2.01

(ii) Whole numbers

This line increases towards the right hand side from 0 to infinity.



Fig. 2.02

(iii) Integers

This line increases towards both sides starting from 0 to infinity.



Fig. 2.03

(iv) Rational numbers

This line increases towards both sides starting from 0 to infinity.



Fig. 2.04

But there are many numbers found between -1 , 0 and 0 , 1 . We can use the concept of mean to find out the rational numbers between two rational numbers. There are infinitely rational numbers between two rational numbers.

With the course of changing time, numbers also developed. First of all, natural numbers were developed. Sum and multiplication of two natural numbers is also a natural number. The set of natural numbers is denoted by N , *i.e.*,

$$N = \{1, 2, 3, 4, 5, 6, 7, \dots\}$$

If we want to solve the question $x + 7 = 7$, then we get the value of x is 0 . So we can not solve this equation with the help of natural numbers so the number zero is included to the set of natural numbers and this new set is known as set of whole numbers which is denoted by W , *i.e.*,

$$W = \{0, 1, 2, 3, 4, 5, 6, 7, \dots\}$$

If we solve of the equation $x + 15 = 6$. and found the value of x , then we need a number $x = -9$ which is not a whole number. Here, again we need to develop an another set of numbers. If we include negative numbers to the set of whole numbers, we get another set of numbers which is known as set of integers. The set of integers is denoted by I or Z , *i.e.*,

$$Z = \{-3, -2, -1, 0, 1, 2, 3, \dots\}$$

Now, we notice that some more numbers also lie between two integers which come out when integer is divided by another integer for example $\frac{1}{2}, \frac{1}{4}, \frac{1}{3}, \frac{2}{5}, \dots$

A number which can be represented in the form of $\frac{p}{q}$ is called a rational number,

where p and q both are integer but $q \neq 0$ either $\frac{p}{q}$ is terminating or it is recurring. In a set of rational number, natural numbers, whole numbers and integers are included.

There are infinitely rational numbers between two rational numbers.

2.02 Irrational Numbers

Let us see the number line again and think whether all the numbers have been included on this number line or some number have been still remaining. Let us discuss the remaining numbers that are not rational number. Such numbers that cannot be expressed in the form

of $\frac{p}{q}$, where p and q are integers and $q \neq 0$, is called an irrational number. As we know that there are infinitely rational numbers. In the same way there are infinitely irrational numbers, for which some examples are given below :

$$\sqrt{2}, \sqrt{3}, \sqrt{5}, \sqrt{7}, \pi, 0.15150015000150000\dots$$

Recall that when we use the symbol " $\sqrt{\quad}$ ", we assume that it is the square root of a

positive number. So $\sqrt{25} = 5$, though 5 and -5 both are square root of 25.

The set of all rational and irrational numbers is named as the set of real numbers which is denoted by R .

2.03. Real Numbers and their Decimal Expansions

Now, we will discuss decimal expansion to distinguish between rational and irrational numbers. We will also explain how real numbers can be represented on the number line using their decimal expansions. Let us start with the rational numbers.

Let us see three examples: $\frac{3}{8}$, $\frac{8}{9}$, $\frac{6}{7}$

$$\frac{3}{8} = 0.375$$

$$\frac{8}{9} = 0.88888\dots$$

$$\frac{6}{7} = 0.857142857142\dots$$

In all of the above examples, $\frac{p}{q}$, ($q \neq 0$) is applied on rational number, then we

get the various situation after dividing p by q which are following,

I Condition : Remainder becomes zero

In the example of $\frac{3}{8}$ the remainder becomes 0 after some steps. Decimal expansion of $\frac{3}{8}$

is 0.375. Let us consider some other example like $\frac{1}{4} = 0.25$, $\frac{456}{125} = 3.648$. In these examples, the decimal expansion terminates after a finite number of steps. The expansion of such numbers is called terminating decimal expansion.

II Condition : The remainder never becomes zero but we get a repetition of digits in the quotient.

For example $\frac{8}{9} = 0.88888\dots$ and $\frac{6}{7} = 0.857142857142\dots$ The expansion of such numbers is non-terminating recurring.

In the decimal expansion of $\frac{8}{9}$, 8 repeats. So the usual way of showing repetition of 8 is

to write it as $0.\overline{8}$. Similarly, in decimal expansion of $\frac{6}{7}$ the block 857142 repeats,

So we write $\frac{6}{7}$ in the form $0.\overline{857142}$, where the bar above the digits indicates that block

of digits that repeats. Similarly, 2.67474 can also be written as $2.\overline{674}$. From above examples, we get non-terminating (repeating) decimal expansion. In this way we found that there are only two cases of decimal expansion in rational numbers, they can either be terminating or non-terminating repeating (recurring) decimals.

Example 1. Show that 2.152786 is a rational number or express 2.152786 in the form of $\frac{p}{q}$, where p and q are integers and $q \neq 0$.

Solution : We have $2.152786 = \frac{2152786}{1000000}$, so it is a rational number.

Example 2. Show that $0.8888\dots = 0.\overline{8}$ can be expressed as $\frac{p}{q}$, where p and q are integers and $q \neq 0$.

Solution : Let

$$x = 0.\overline{8}$$

or $x = 0.8888\dots \dots (i)$

Multiplying both the sides of equation 10, we get

$$10x = 10 \times (.8888\dots) = 8.888\dots$$

$$10x = 8.888\dots \dots (ii)$$

Subtracting (i) from (ii), we have

$$9x = 8.000\dots = 8$$

$$\Rightarrow x = \frac{8}{9}$$

Example 3. Show that $0.\overline{47} = 0.474747\dots$ can be expressed in the form of $\frac{p}{q}$,

where p and q are integers and $q \neq 0$.

Solution : Let $x = 0.4747\dots \dots (i)$

Since two digits are repeating, so multiplying equation (i) by 100, we get

$$100x = 47.474747\dots \dots (ii)$$

Subtracting (i) from equation (ii), we get

$$99x = 47.000\dots = 47$$

$$\therefore x = \frac{47}{99}$$

Example 4. Show that $0.12\bar{3} = 0.123333\dots$ can be expressed as $\frac{p}{q}$, where p and q are integers and $q \neq 0$

Solution : Let $x = 0.12333$... (i)

Here, 1 and 2 do not repeat, but 3 repeats, since first two digits are non-repeating.

$$100x = 12.333 \dots \text{ (ii)}$$

Again, multiplying the equation (ii) by 10, we get

$$1000x = 123.333 \dots \text{ (iii)}$$

Now subtracting equation (ii) from equation (iii), we get

$$900x = 111.000$$

$$x = \frac{111}{900} = \frac{37}{300}$$

So, every number with a non-terminating recurring decimal expansion can be expressed in the form of $\frac{p}{q}$, where p and q are integers and $p \neq 0$.

The decimal expansion of a rational number is either terminating or non-terminating recurring. Now, we think a number like $x = 0.150120015000150000\dots$ and we find that this number

can not be changed in the form of $\frac{p}{q}$, (where p and q are integers and $q \neq 0$.)

So, due to specific property of number like this, it is called irrational number.

Hence, the number whose decimal expansion is non-terminating, non-recurring is called irrational number.

Infinitely many irrational numbers can be generated equivalent to x.

Decimal expansion of some irrational number $\sqrt{2}$, $\sqrt{3}$ are given here.

$$\sqrt{2} = 1.41421356237\dots$$

$$\sqrt{3} = 1.73205080756\dots$$

For over the years, mathematicians have developed various techniques to produce more and more digits in the decimal expansion of irrational numbers. For example, to find digits in the decimal expansion of $\sqrt{2}$ by the division method. From the sulbasutras (rules of chords), a mathematical book of vedic period (800 B.C.- 500 B.C.). We get the approximate value.

Similarly the history of finding the decimal expansion of π is also quite interesting

Greek's famous scientist Archimedes calculated the value of π in decimal expansion. He showed $3.140845 < \pi < 3.142857$. Aryabhata (476-550AD), the great Indian mathematician and astronomer, found the value of π exact to four decimal places (3.1416). Using high speed computers and advanced algorithms, the value of π has been computed to many decimal places.

Example 5. Find an irrational number between $\frac{1}{7}$ and $\frac{2}{7}$.

Solution : We can easily calculate that $\frac{1}{7} = 0.142857142857 \dots = 0.\overline{142857}$

and $\frac{2}{7} = 0.2857142857142 \dots = 0.\overline{2857142}$

To find an irrational number between $\frac{1}{7}$ and $\frac{2}{7}$, we find a number which is non-terminating, non-recurring lying between them. Thus, we can find infinitely many numbers. An example of a number like this is $0.150150015000150000 \dots$.

Exercise 2.1

- Classify the following numbers as rational or irrational :
 (i) $\sqrt{23}$ (ii) $\sqrt{225}$ (iii) 0.3797 (iv) $7.4784478 \dots$
 (v) $1.101001000100001 \dots$
- Write three numbers whose decimal expansions are non-terminating non-recurring.
- Write the following in decimal form and show the type of decimal expansion
 (i) $\frac{36}{100}$ (ii) $\frac{1}{11}$ (iii) $4\frac{1}{8}$ (iv) $\frac{3}{13}$
 (v) $\frac{2}{11}$ (vi) $\frac{329}{400}$
- Express the following in the form of $\frac{p}{q}$, where p and q are the integers and $q \neq 0$:
 (i) 0.3 (ii) $0.\overline{47}$ (iii) $1.\overline{27}$ (iv) $1.\overline{235}$
- Find three irrational numbers between the rational numbers $\frac{5}{7}$ and $\frac{9}{11}$.

Representation of Real Numbers on the Number Line

As we have studied in the previous section that a real number can be either a rational or an irrational number. So we can say that any real number can be represented on the number line in its unique point and every point on the number line represents a unique real number. Due to this, the number line is called the real number line. With the help of some examples

we shall study the representation method of irrational numbers on the number line.

Example 6. Represent $\sqrt{2}$ on the real number line.

Solution : It can be done easily. Take a square $OABC$ with the side of unit length (See the Fig. 2.05)

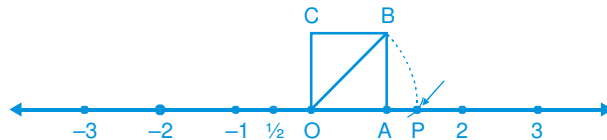


Fig. 2.05

Then using the Bodhyan theorem you can see that $OB = \sqrt{1^2 + 1^2} = \sqrt{2}$. Taking a distance of OB in a compass and with centre O , draw an arc on the number line that intersects it at the point P . Then point P on number line corresponds to $\sqrt{2}$.

Example 7. Represent $\sqrt{3}$ on the number line.

Solution : Let us see the Fig. 2.06.

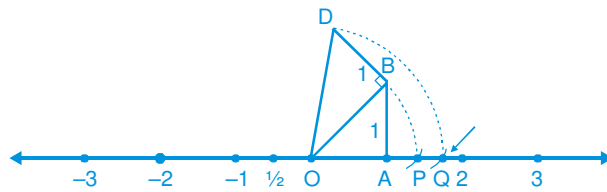


Fig. 2.06

Draw a perpendicular BD on the side OB with unit length. (As shown in the Fig. 2.06).

Then according to Bodhyan theorem $OD = \sqrt{(\sqrt{2})^2 + 1^2} = \sqrt{3}$. Taking O as centre and with the radius OD , draw an arc which intersect the number line at Q then Q corresponds to $\sqrt{3}$.

Similarly, after determining the place of $\sqrt{n-1}$ on the number line, we can easily determine the place of \sqrt{n} , where n is a positive integer.

Process of Successive Magnification

In the previous section, we have studied that a real number has its decimal expansion. We can represent the given number on a number line with the help of decimal expansion. Let us locate 2.05 on the number line as we see it on a scale.

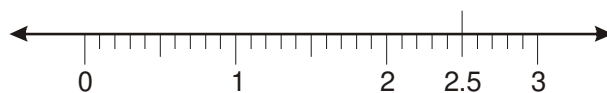


Fig. 2.07

For example, we want to determine the place of 2.775 on a number line. We have to consider the Fig 2.08 given below. We notice that 2.775 is located somewhere between 2 and 3. Divide this distance into ten equal parts. After that divide the distance between 2.7 and 2.8 in ten equal parts again. We divide the distance between 2.77 and 2.78 into ten equal parts. The point 2.775 is fifth point of this division. This process of visualisation of numbers in the number line through a magnifying glass is called the process of successive magnification. So, we have seen that it is possible by sufficient successive magnifications to visualise the position of a real number with a terminating decimal expansion on the number line.

In the same way we can see the position of non-terminating non recurring real number on the number line with the help of this process.

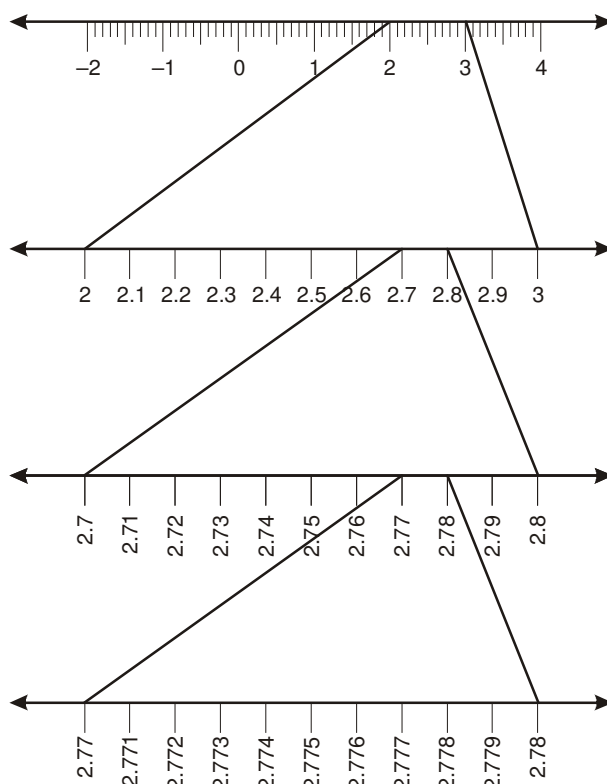


Fig. 2.08

Based on imagination of successive magnification form given in the above examples, we can again say that every real number is represented by a unique point on the number line. Further, every point on the number line represents one and only one real number.

2.4. Geometrically Representation of a Real Number

If a is a natural number, then $\sqrt{a} = b$ means $b^2 = a$ and $b > 0$. The same definition can be extended for positive real numbers. Let $a > 0$ be a real number, then $\sqrt{a} = b$ means $b^2 = a$ and $b > 0$.

Now we shall show how the value of \sqrt{x} can be found out on the number line geometrically, where x is a positive integer.

To find \sqrt{x} , for any positive real number x , we mark B so that $AB = x$ and as in Fig. 2.09 mark C so that $BC = 1$.

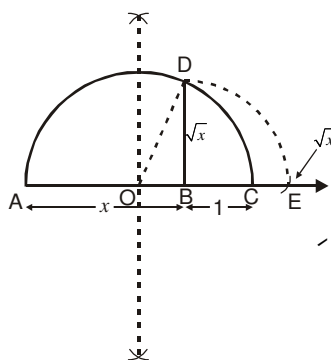


Fig. 2.09

In a right angled ΔOBD (Fig. 2.09), we have

$$\text{So, } OC = OD = OA = \frac{AB + BC}{2} = \frac{x+1}{2} \text{ unit}$$

$$OB = AB - OA = x - \left(\frac{x+1}{2}\right) = \frac{x-1}{2} \text{ unit}$$

By bodhyan theorem, we get

$$BD^2 = OD^2 - OB^2 = \left(\frac{x+1}{2}\right)^2 - \left(\frac{x-1}{2}\right)^2 = \frac{4x}{4} = x$$

$$\Rightarrow BD^2 = x$$

$$\Rightarrow BD = \sqrt{x}$$

The representation of diagrametic and geometric method can be obtained by this construction which shows that \sqrt{x} exists for all real numbers $x > 0$. If you want to know the position of \sqrt{x} on the number line, then let us treat the line BC as the number line, with

B as zero, C as 1, and so on. Draw an arc with centre B and radius BD, which intersects the number line E (see the Fig. 2.09). Then E represents \sqrt{x} .

We can extend the idea of square roots to cube roots, fourth roots, fifth roots and in general n^{th} roots, where n is a positive integer. Recall your understanding of square roots and cube roots from earlier classes.

What is $\sqrt[3]{27}$? We know that $\sqrt[3]{27} = 3$. Find the value of $\sqrt[5]{243}$. If $b = \sqrt[5]{243}$ then $b^5 = 243$, $b^5 = (3)^5 \Rightarrow b = 3$. So, that value of $\sqrt[5]{243} = 3$. In the same way $\sqrt[n]{a}$ can be defined, where $a > 0$ and n is a positive integer.

Let $a > 0$ be real number and n is a positive integer, then $\sqrt[n]{a} = b$, if $b^n = a$ where $b > 0$.

Symbol " $\sqrt[n]{\quad}$ " is called the radical sign. $\sqrt[n]{a}$ can be expressed as $a^{1/n}$.

2.05. Operation on Real Numbers

We have studied in the previous classes, that rational numbers satisfy the commutative, associative and distributive laws for addition and multiplication. We have also studied that if we add, subtract, multiply or divide (except zero) two rational numbers, we still get a rational number. That is rational numbers are closed with respect to addition, subtraction, multiplication and division. We also notice that irrational number also satisfy the commutative, associative, distributive laws for addition and multiplication. However the sum, difference, quotients and products of irrational are not always irrational. For example

$(\sqrt{5}) - (\sqrt{5})$, $(\sqrt{7}) \cdot (\sqrt{7})$ and $\frac{\sqrt{13}}{\sqrt{13}}$ are rational numbers.

Let us see what happens when a rational number is added with an irrational number and a rational number is multiplied by an irrational number?

For an example, $\sqrt{5}$ is an irrational number, then the number $2 + \sqrt{5}$ and $2\sqrt{5}$ are of what kind? Undoubtedly these are irrational number because these numbers give the non-terminating non-recurring decimal expansion. Non-terminating non-recurring decimal expansion of irrational numbers can be understood more by the following examples.

Example 8. Check whether $7\sqrt{5}$, $\frac{7}{\sqrt{5}}$, $\sqrt{2}$, $\sqrt{2} + 24$, $\pi - 3$ are irrational or not.

Solution : We know that, $\sqrt{5} = 2.236\dots$, $\sqrt{2} = 1.4142\dots$, $\pi = 3.1415$

$$\text{Then } 7\sqrt{5} = 15.652\dots, \frac{7}{\sqrt{5}} = \frac{7\sqrt{5}}{\sqrt{5}\sqrt{5}} = \frac{7\sqrt{5}}{5} = 3.1304\dots,$$

$$\sqrt{2} + 24 = 25.4142\dots, \quad \pi - 3 = 0.1415$$

All of these are non-terminating non-recurring decimals. Hence, all of these are irrational numbers.

Example 9. Add : $2\sqrt{3} + 3\sqrt{5}$ and $\sqrt{3} - \sqrt{5}$.

Solution :

$$\begin{aligned} & (2\sqrt{3} + 3\sqrt{5}) + (\sqrt{3} - \sqrt{5}) \\ &= (2\sqrt{3} + \sqrt{3}) + (3\sqrt{5} - \sqrt{5}) \\ &= (2+1)\sqrt{3} + (3-1)\sqrt{5} \\ &= 3\sqrt{3} + 2\sqrt{5} \end{aligned}$$

Example 10. Multiply : $6\sqrt{7}$ by $2\sqrt{7}$.

Solution :

$$\begin{aligned} 6\sqrt{7} \times 2\sqrt{7} &= 6 \times 2 \times \sqrt{7} \times \sqrt{7} \\ &= 12 \times 7 = 84 \end{aligned}$$

Example 11. Divide : $8\sqrt{15}$ by $2\sqrt{5}$.

Solution :

$$8\sqrt{15} \div 2\sqrt{5} = \frac{8\sqrt{3 \times 5}}{2\sqrt{5}} = \frac{8\sqrt{3}\sqrt{5}}{2\sqrt{5}} = 4\sqrt{3}$$

We can derive the following conclusions from these examples.

- (i) The sum or difference of a rational number and an irrational number is irrational.
- (ii) The product or quotient of a non-zero rational number with an irrational number is irrational.
- (iii) If we add, subtract, multiply or divide two irrational numbers, the result may be rational or irrational.

Here, some identities related to square root, which are useful for rationalizing have been given.

Let a and b are positive real numbers, then

(i) $\sqrt{ab} = \sqrt{a} \sqrt{b}$

(ii) $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$

(iii) $(\sqrt{a} + \sqrt{b})(\sqrt{a} - \sqrt{b}) = a - b$

(iv) $(\sqrt{a} + \sqrt{b})(\sqrt{c} - \sqrt{d}) = \sqrt{ac} - \sqrt{ad} + \sqrt{bc} - \sqrt{bd}$

(v) $(\sqrt{a} + \sqrt{b})^2 = a + 2\sqrt{ab} + b$

(vi) $\frac{1}{a + \sqrt{b}} = \frac{a - \sqrt{b}}{a^2 - b}$

$$(vii) \frac{1}{a+b\sqrt{x}} = \frac{a-b\sqrt{x}}{a^2-b^2x}, \text{ where } x \text{ is a natural number.}$$

$$(viii) \frac{1}{\sqrt{x}+\sqrt{y}} = \frac{\sqrt{x}-\sqrt{y}}{x-y}, \text{ where } x \text{ and } y \text{ are natural numbers.}$$

Above identities will be used to rationalise the denominator. When the denominator of an expression contains a term with a square root (or a number under a radical sign), the process of converting it to an equivalent expression whose denominator is a rational number is called rationalising the denominator.

Example 12. Rationalise the denominator of $\frac{1}{\sqrt{2}}$.

Solution : We know that $\sqrt{2} \cdot \sqrt{2} = 2$, which is rational number.

Multiply the numerator and denominator by $\sqrt{2}$ we have

$$\frac{1}{\sqrt{2}} = \frac{1}{2} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

In this form, it is easy to locate $\frac{1}{\sqrt{2}}$ on the number line. It is half way between 0 and $\sqrt{2}$.

Example 13. Rationalise the denominator of $\frac{1}{2+\sqrt{3}}$.

Solution : Here, the conjugate of the denominator $(2+\sqrt{3})$ is $(2-\sqrt{3})$. Multiply the numerator and denominator by $(2-\sqrt{3})$, we have

$$\frac{1}{2+\sqrt{3}} = \frac{1}{(2+\sqrt{3})} \times \frac{(2-\sqrt{3})}{(2-\sqrt{3})} = \frac{2-\sqrt{3}}{4-3} = 2-\sqrt{3}$$

Example 14. Rationalise the denominator $\frac{5}{\sqrt{3}-\sqrt{5}}$

Solution : By applying the identity (iii).

$$\text{We have } \frac{5}{\sqrt{3}-\sqrt{5}} = \frac{5}{\sqrt{3}-\sqrt{5}} \times \frac{\sqrt{3}+\sqrt{5}}{\sqrt{3}+\sqrt{5}} = \frac{5(\sqrt{3}+\sqrt{5})}{3-5} = \left(-\frac{5}{2}\right)(\sqrt{3}+\sqrt{5})$$

Example 15. Rationalise the denominator $\frac{1}{7+3\sqrt{2}}$

Solution : Here, conjugate of the denominator $(7+3\sqrt{2})$ is $(7-3\sqrt{2})$. Multiply the numerator and denominator by $(7-3\sqrt{2})$, we have

$$\frac{1}{7+3\sqrt{2}} = \frac{1}{7+3\sqrt{2}} \times \left(\frac{7-3\sqrt{2}}{7-3\sqrt{2}} \right) = \frac{7-3\sqrt{2}}{49-18} = \frac{7-3\sqrt{2}}{31}$$

Exercise 2.2

1. Classify the following numbers as rational or irrational :

(i) $2 - \sqrt{5}$

(ii) $(3 + \sqrt{23}) - \sqrt{23}$

(iii) $\frac{2\sqrt{11}}{7\sqrt{11}}$

(iv) $\frac{1}{\sqrt{3}}$

(v) 5π

2. Rationalise the denominator of the following :

(i) $\frac{1}{5+3\sqrt{7}}$

(ii) $\frac{1}{\sqrt{2} + \sqrt{3}}$

(iii) $\frac{1}{\sqrt{7}-2}$

3. If $\frac{3+2\sqrt{2}}{3-\sqrt{2}} = a+b\sqrt{2}$, then find the values of a and b . When a and b are rational numbers.

2.6. Laws of Exponent for Real Numbers

As we have studied in the previous classes about the laws of exponents. Here a , m and n are natural numbers a is called the base and m and n are the exponents.

Here a , m , n and b are natural numbers.

(i) $a^m a^n = a^{m+n}$

(ii) $(a^m)^n = a^{mn}$

(iii) $\frac{a^m}{a^n} = a^{m-n}, m > n$

(iv) $a^m b^m = (ab)^m$

What is $(a)^0$? Its value is 1. Therefore, $(a)^0 = 1$

The value of $\frac{1}{a^n} = a^{-n}$. Now we can use these laws on negative exponents. For

example :

$$(i) 7^2 \cdot 7^{-5} = 7^{2-5} = 7^{-3} = \frac{1}{7^3}$$

$$(ii) (5^3)^{-7} = 5^{-21}$$

$$(iii) \frac{23^{-15}}{23^7} = 23^{-15-7} = 23^{-22}$$

$$(iv) (6)^{-3} (7)^{-3} = (42)^{-3}$$

We can extend the laws of exponents that we have studied earlier, even when base is a positive real number and the exponents are rational numbers. In the previous section, we have defined $\sqrt[n]{a}$ for real number, where $a > 0$.

$$x^n = a \Rightarrow x = a^{1/n} = \sqrt[n]{a}$$

$$4^{3/2} = (4^{1/2})^3 = 2^3 = 8$$

$$4^{3/2} = (4^3)^{1/2} = (64)^{1/2} = (8^2)^{1/2} = 8$$

Let $a > 0$ be a real number. Let m and n be integers such that m and n have no common factors other than 1, and $n > 0$, then :

$$(i) a^{m/n} = (\sqrt[n]{a})^m = \sqrt[n]{a^m}$$

$$(ii) \sqrt[n]{a} = \sqrt[n \times p]{a^p}$$

Let $a > 0$ be a real number and p and q are rational numbers, then :

$$(i) a^p \cdot a^q = a^{p+q}$$

$$(ii) (a^p)^q = a^{pq}$$

$$(iii) \frac{a^p}{a^q} = a^{p-q}$$

$$(iv) a^p b^p = (ab)^p$$

Simplify :

$$(i) 4^{2/3} \cdot 4^{1/3} = 4^{2/3+1/3} = 4^{3/3} = 4^1 = 4 \quad (ii) (3^{1/5})^8 = 3^{8/5}$$

$$(iii) \frac{9^{1/5}}{9^{1/3}} = 9^{1/5-1/3} = 9^{(3-5)/15} = 9^{-2/15}$$

Exercise 2.3

1. Find the value of :

(i) $81^{1/2}$ (ii) $64^{1/6}$ (iii) $(125)^{1/3}$

2. Find the value of :

(i) $(4)^{3/2}$ (ii) $32^{2/5}$ (iii) $16^{3/4}$

3. Simplify :

(i) $2^{2/3} \cdot 2^{1/7}$ (ii) $\left(\frac{1}{3^3}\right)^7$

4. Find the value of x :

$$\left(\frac{3}{5}\right)^x \left(\frac{5}{3}\right)^{2x} = \frac{125}{27}$$

Answer Sheet

Exercise 2.1

- (i), (iv) and (v) irrational
(ii) and
(iii) rational
- 0.01001000100001 . . . ; 0.2020020000200002 . . . , 0.00300030000 . . .
- (i) 0.36 terminating ;
(ii) $0.\overline{09}$ non-terminating recurring;
(iii) 4.125 terminating ;
(iv) $0.\overline{230769}$ non-terminating recurring
(v) $0.\overline{18}$ non-terminating recurring ;
(vi) 0.8225 terminating
- (i) $\frac{1}{3}$ (ii) $\frac{47}{99}$ (iii) $\frac{14}{11}$ (iv) $\frac{233}{990}$
- 0.7507500750007500075 . . .
0.767076700767000767 . . .
0.80800800080008 . . .

Exercise 2.2

- (i) irrational ; (ii) rational ; (iii) rational ; (iv) irrational ; (v) irrational
- (i) $-\frac{1}{38}(5-3\sqrt{7})$; (ii) $-(\sqrt{2}-\sqrt{3})$; (iii) $\frac{1}{3}(\sqrt{7}+2)$
- $a = \frac{13}{7}, b = \frac{9}{7}$

Exercise 2.3

- (i) 9 ; (ii) 2 ; (iii) 5
- (i) 8 ; (ii) 4 ; (iii) 8
- (i) $2^{\frac{2}{21}}$; (ii) 3^{-21}
- $x = 3$

Polynomials

3.01. Introduction

In earlier classes, we have studied various operation of algebraic expressions in which algebraic expressions are included in these classes, we have used the following algebraic identities for factorisation.

$$(x + y)^2 = x^2 + 2xy + y^2.$$

$$(x - y)^2 = x^2 - 2xy + y^2 \text{ and}$$

$$x^2 - y^2 = (x + y)(x - y).$$

In this chapter we shall study a particular type of algebraic expressions, called *polynomials*, and the terminology related to it. We shall also study some algebraic identities to factorization of polynomials.

3.02. Polynomials

We know that a variable is denoted by the symbols x, y, z , etc. A variable can take any real value. When any constant and variables are expressed with four main operations, then the combination is called the algebraic expression. For examples, $3x, 5x, -x, -\frac{3}{2}x$ etc., are algebraic expressions. General form of an algebraic expression is ax , where a is constant and x is variable. $3x, x^2 + 3x, x^3 + 2x^2 - 4x + 5$ etc. are algebraic expressions. In all of these the exponents of variable x is a whole number. These type of expressions are called the polynomials in one variable. In all of above examples x is a variable. A polynomial is denoted as $p(x), g(x), q(y)$ etc.

For example

$$p(x) = 3x^2 + 4x - 5$$

$$g(x) = x^3 + 1$$

$$q(y) = y^3 + 2y - 1$$

$$s(t) = 3 - t - 2t^2 + 5t^3$$

A polynomial can have a lot of but finite number of terms

$$p(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_2 x^2 + a_1 x + a_0$$

where $a_n, a_{n-1}, \dots, a_2, a_1, a_0$ are the constants and $a_n \neq 0$. In polynomial $x^2 + 3x$; x^2 and $3x$ are terms of the polynomial. In a polynomial every term has a coefficient.

In polynomial $5x^3 - 2x^2 + x + 3$

coefficient of $x^3 = 5$, coefficient of $x^2 = -2$

coefficient of $x = 1$, coefficient of $x^0 = 3$

Is 3 is a polynomial?

3, -7, 9 etc., are called **constant polynomials**.

0 is called zero polynomial.

Polynomials having only one term are called **monomials**.

For example, $3x, 5x^2, -3x^3, 2, t^2, y$ etc.

Polynomials having two terms are called **binomials**.

For example, $x+2, x^2-2x, y^n+2, t^{30}-t^3$ etc.

Similarly, polynomials having three terms are called trinomials.

For example, $p(x) = x^2 + x + 1$

$$g(x) = x - x^2 + \sqrt{3}$$

$$t(y) = y^3 + y + 3$$

$$s(t) = t^4 + t^2 - 2$$

The highest power of variable in a polynomial is called **the degree of the polynomial**.

In $p(x) = 4x^3 - 2x^2 + 8x - 21$; the highest power of the term $4x^3 = 3$.

In $q(y) = 3y^7 - 4y^6 + y + 9$ the highest power of the term $3y^7 = 7$.

So, the degree of polynomials $p(x)$ and $q(y)$ are 3 and 7 respectively.

In the constant monomial $g(x) = 2$, the term 2 with highest power $= 2x^0$. So its degree = 0

Conclusion : The degree of a non-zero constant polynomial is zero.

Now analyse the following $p(x) = 5x + 4$, $g(y) = 12y$, $r(t) = 4 - 2t$ and

$$s(u) = \sqrt{3} + 2u.$$

Degree of all these polynomials is 1 (one).

Polynomials having degree 1 are called the linear polynomials.

Generally, a linear polynomial is expressed as :

$$p(x) = ax + b, a \neq 0$$

The maximum terms of a linear polynomial is two. It means a linear polynomial is either a binomial or a monomial.

Observe the following polynomial :

$$p(x) = 2x^2 - 3x + 15, \quad g(x) = 5x^2 + 3 \quad \text{and} \quad g(y) = y^2 + 2y$$

These polynomials have degree 2 are called the quadratic polynomials.

Generally, a quadratic polynomial is expressed as :

$$p(x) = ax^2 + bx + c, a \neq 0$$

A quadratic polynomial in one variable has at most three terms. It means a quadratic polynomial can be monomial, binomial or trinomial.

A polynomial of degree three is called a cubic polynomial. A cubic polynomial is expressed as $p(x) = ax^3 + bx^2 + cx + d, a \neq 0$ can have at most four terms. A polynomial in one variable x of degree n is an expression of the form :

$$p(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x + a_0 \quad \text{where } a_n \neq 0, a_n, a_{n-1}, \dots, a_1, a_0 \text{ are constants.}$$

In particular if $a_0 = a_1 = a_2 = a_3 = \dots = a_n = 0$ (all the constants are zero), we get the zero polynomial, that is denoted by 0. The degree of the zero polynomial is not defined.

Consider an algebraic expression $x + \frac{1}{x}$.

$$x + \frac{1}{x} = x + x^{-1}$$

The exponent of 2nd term of expression is -1 which is not a whole number.

$$\sqrt{x} + 5 = x^{1/2} + 5$$

The exponent of $x^{1/2}$ is $\frac{1}{2}$, which is not a whole number.

$$\sqrt[3]{y} + y^3 = y^{1/3} + y^3$$

Here, exponent of $y^{1/3}$ is $\frac{1}{3}$, which is not a whole number.

All the above expressions are not polynomials because as none of them has exponent as a whole number.

We have studied polynomials having one variables. There are polynomials having more than one variable for example, $x^2 + y^2 + xyz$, $p^2 + 8q^3 + r^4$, $t^2 + s^3$. These polynomials have 3, 3 and 2 variables respectively. We shall study these polynomials later on.

Exercise 3.1

1. Which of the following expressions are polynomials in one variable. Find the number of terms also :?

(i) $3x^2 - 5x + 13$	(ii) $y^2 + 2\sqrt{3}$	(iii) $y + \frac{3}{y}$
(iv) 3	(v) $2\sqrt{x} + \sqrt{3}x$	(vi) $x^{12} + y^3 + t^{20}$

2. Write the coefficient of x^2 in following expressions :

(i) $12 + 3x + 5x^2$	(ii) $7 - 11x + x^3$	(iii) $\sqrt{3}x - 7$	(iv) $\frac{\pi}{2}x^2 + x$
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3. Write an example of binomial of degree 45.
 4. Write an example of monomial of degree 120.
 5. Write an example of trinomial of degree 8.
 6. Can you give some examples other than questions number 3, 4, and 5 have been given? If yes, then give two more examples for each.
 7. Write the degree of each of the following polynomials :

(i) $12 - 3x + 2x^3$	(ii) $5y - \sqrt{2}$	(iii) 9	(iv) $3 + 4t^2$
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3.03. Zeros of a Polynomial

Consider a polynomial

$$p(x) = 2x^3 - 3x^2 + 4x - 2$$

If we replace x by 2 everywhere in $p(x)$, we get

$$\begin{aligned} p(2) &= 2 \times (2)^3 - 3 \times (2)^2 + 4 \times 2 - 2 \\ &= 2 \times 8 - 3 \times 4 + 4 \times 2 - 2 \\ &= 16 - 12 + 8 - 2 = 10 \end{aligned}$$

So, we can say that the value of $p(x)$ at $x = 2$ is 10.

Similarly
$$p(0) = 2 \times (0)^3 - 3 \times (0)^2 + 4 \times 0 - 2 = -2$$

and
$$\begin{aligned} p(-1) &= 2(-1)^3 - 3(-1)^2 + 4 \times (-1) - 2 \\ &= 2 \times -1 - 3 \times 1 - 4 \times 1 - 2 = -11 \end{aligned}$$

We can say that the value of polynomial $p(x)$ can be obtained by replacing the $x = \alpha$ in $p(\alpha)$.

Example 1. Find the value of the polynomial $p(x) = 8x^2 - 3x + 7$, at $x = -1$ and $x = 2$.

Solution : $p(x) = 8x^2 - 3x + 7$

The value of the polynomial $p(x)$ at $x = -1$ is :

$$\begin{aligned} p(-1) &= 8(-1)^2 - 3(-1) + 7 \\ &= 8 + 3 + 7 = 18 \end{aligned}$$

Again, the value of the polynomial $p(x)$ at $x = 2$ is :

$$\begin{aligned} p(2) &= 8(2)^2 - 3(2) + 7 \\ &= 32 - 6 + 7 = 33 \end{aligned}$$

Example 2. Find the value of the polynomial $p(x) = 2x^3 - 13x^2 + 17x + 12$, at $x = -\frac{1}{2}$.

Solution : $p(x) = 2x^3 - 13x^2 + 17x + 12$

Put $x = -\frac{1}{2}$

$$\begin{aligned} p\left(-\frac{1}{2}\right) &= 2\left(-\frac{1}{2}\right)^3 - 13\left(-\frac{1}{2}\right)^2 + 17\left(-\frac{1}{2}\right) + 12 \\ &= 2 \times \frac{-1}{8} - 13 \times \frac{1}{4} + 17 \times \frac{-1}{2} + 12 \\ &= -\frac{1}{4} - \frac{13}{4} - \frac{17}{2} + 12 = 0 \end{aligned}$$

Example 3. Find the value of the polynomial $p(x) = x^3 - 6x^2 + 11x - 6$ at $x = 1$.

Solution : $p(x) = x^3 - 6x^2 + 11x - 6$

$$\begin{aligned} p(1) &= (1)^3 - 6(1)^2 + 11(1) - 6 \\ &= 1 - 6 + 11 - 6 = -12 + 12 = 0 \end{aligned}$$

In the above example since $p(1) = 0$, so we say that 1 is a zero of polynomial $p(x)$.

In general we can say that a zero of a polynomial $p(x)$ is a number “ α ” (alpha) such that $p(\alpha) = 0$.

What is the value of $p(1) = 0$, for polynomial $p(x) = x - 1$

$$p(1) = 1 - 1 = 0$$

We observed that the zero of the polynomial $p(x) = x - 1$ is obtained by equating it to 0 i.e., $x - 1 = 0$, which gives $x = 1$. We say $p(x) = 0$ is a polynomial equation and 1 is the root of the polynomial equation $p(x) = 0$. So we can say that 1 is the zero of the polynomial $x - 1$, or a root of the polynomial equation $x - 1 = 0$.

What is the zero of the constant polynomial 7?

It has no zero because replacing x by any number in $7x^0$ still gives us 7. In fact, a non-zero constant polynomial has no zero.

What about the zero of the zero polynomial? By convention, every real number is a zero of the zero polynomial.

Examples 4. Check whether 3 and -3 are zeroes of the polynomial $p(x) = x + 3$.

Solution : $p(x) = x + 3$

$$\therefore p(3) = 3 + 3 = 6$$

$$p(-3) = -3 + 3 = 0$$

Therefore, -3 is a zero of the polynomial $p(x) = x + 3$ but 3 is not a zero.

Example 5. Find a zero of the polynomial $p(x) = 3x + 2$.

Solution : Finding a zero of $p(x)$ is the same solving the equation $p(x) = 0$

$$\therefore p(x) = 3x + 2 = 0 \Rightarrow 0 = 3x + 2$$

$$\Rightarrow 3x = -2 \Rightarrow x = \frac{-2}{3}$$

Now, if $p(x) = ax + b$, $a \neq 0$, is a linear polynomial, then we can find a zero of $p(x)$ from above examples.

It means finding a zero of the polynomial $p(x)$ is to solve the polynomial equation $p(x) = 0$.

Now, $p(x) = 0 \Rightarrow ax + b = 0$, $a \neq 0$

$$ax = -b$$

$$x = -\frac{b}{a}$$

So, $x = -\frac{b}{a}$ is the only one zero of $p(x)$ i.e., a linear polynomial has one and only one zero.

Example 6. Verify that 3 and 0 are the zeroes of the polynomial $x^2 - 3x$.

Solution : Let $p(x) = x^2 - 3x$

Then $p(3) = (3)^2 - 3(3) = 9 - 9 = 0$

and $p(0) = (0)^2 - 3(0) = 0 - 0 = 0$

Hence, 3 and 0 are both zero of the polynomial $x^2 - 3x$.

We can get the following conclusion.

1. A zero of a polynomial need not to be a 0.
2. 0 may be a zero of a polynomial.
3. Every linear polynomial has one and only one zero.
4. A polynomial may have more than one zero.

Exercise 3.2

1. Find the value of the polynomial $2x^3 - 13x^2 + 17x + 12$ at the following value of x :
 - (i) $x = 2$ (ii) $x = -3$ (iii) $x = 0$ (iv) $x = -1$
2. Find the $P(2), P(1)$ and $P(0)$ for each of the following polynomials :
 - (i) $p(x) = x^2 - x + 1$ (ii) $p(y) = (y + 1)(y - 1)$
 - (iii) $p(x) = x^3$ (iv) $p(t) = 2 + t + t^2 - t^3$
3. Verify whether the followings are zeroes of the polynomial indicated against them :
 - (i) $p(x) = x^2 - 1$; $x = 1, -1$ (ii) $p(x) = 2x + 1$; $x = -\frac{1}{2}$
 - (iii) $p(x) = 4x + 5$; $x = \frac{-5}{4}$ (iv) $p(x) = 3x^2$; $x = 0$
 - (v) $p(x) = (x - 3)(x + 5)$; $x = 3, -5$ (vi) $p(x) = ax + b$; $x = -\frac{b}{a}$
 - (vii) $p(x) = 3x^2 - 1$; $x = -\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}$ (viii) $p(x) = 3x + 2$; $x = \frac{-2}{3}$

4. Find the zeros of the following polynomials :

(i) $p(x) = x - 4$ (ii) $p(x) = 4x$

(iii) $p(x) = bx, b \neq 0$ (iv) $p(x) = x + 3$

(v) $p(x) = 2x - 1$ (vi) $p(x) = 3x + 7$

(vii) $p(x) = cx + d, c \neq 0, c, d$ are real numbers.

Remainder Theorem :

We know that when we divide 25 by 7, we get quotient 3 and remainder 4. Mathematically we express it as

$$25 = (3 \times 7) + 4$$

Similarly if we divide 48 by 8, we get

$$48 = (6 \times 8) + 0$$

Here, remainder is (0) and we say that 8 is a factor of 48 or 48 is a multiple of 8. In the same way we can divide a polynomial by another polynomial. In first case if divisor is monomial. For example on divide polynomial $3x^3 + 2x^2 + x$ by monomial x .

$$(3x^3 + 2x^2 + x) \div x = \frac{3x^3}{x} + \frac{2x^2}{x} + \frac{x}{x} = 3x^2 + 2x + 1,$$

Here, you have noticed that x is common to each term of $(3x^3 + 2x^2 + x)$ $3x^3 + 2x^2 + x = x(3x^2 + 2x + 1)$. We say that x and $(3x^2 + 2x + 1)$ are factors of $3x^3 + 2x^2 + x$ and $3x^3 + 2x^2 + x$ is a multiple of x as well as a multiple of $3x^2 + 2x + 1$.

Now divide $5x^2 + x + 1$ by x .

$$(5x^2 + x + 1) \div x = (5x^2 \div x) + (x \div x) + (1 \div x)$$

Here, when 1 is divided by x , we don't get a polynomial term. So, in this case we stop here, and note that 1 is the remainder. Thus, we have

$$5x^2 + x + 1 = [(5x + 1) \times x] + 1$$

Here, we get $(5x + 1)$ as a quotient and 1 as a remainder. So $(5x - 1)$ is not a factor of $5x^2 + x + 1$. Since, the remainder is not zero.

$$\text{Dividend} = (\text{Divisor} \times \text{Quotient}) + \text{Remainder.}$$

In general, if $p(x)$ and $g(x)$ are two such polynomials that the degree of $p(x)$ is greater than $g(x)$ and $g(x) \neq 0$, then we get two polynomials $q(x)$ and $r(x)$.

$$p(x) = g(x) \cdot q(x) + r(x) \text{ where } r(x) = 0$$

Or the degree of $r(x)$ is smaller than that of degree of $g(x)$;

When $p(x)$ is divided by $g(x)$, we get quotient $q(x)$ and remainder $r(x)$.

Example 7. Divide $p(x)$ by $g(x)$; where $p(x) = 7x + 5x^2 + 3$ and $g(x) = x + 1$.

Solution :

$$\begin{array}{r} \overline{5x+2} \\ x+1 \overline{) 5x^2 + 7x + 3} \\ \underline{5x^2 + 5x} \\ 2x + 3 \\ \underline{2x + 2} \\ 1 \end{array}$$

Note : We take the following steps for above division operation.

Step I : We write the dividend $7x + 5x^2 + 3$ and divisor $x + 1$ in the standard form arranging the terms in the descending order i.e. dividend as $5x^2 + 7x + 3$ and divisor as $x + 1$.

Step II : We divide the first term of the dividend by the first term of the divisor; i.e., we divide $5x^2$ by x get $5x$. This gives us the first term of the quotient.

Step III : We multiply the divisor by the first term of quotient $5x$ and subtract this product $5x^2 + 5x$ from the dividend. This gives us the remainder as $2x + 3$.

Step IV : We take this remainder $2x + 3$ as the new dividend. We repeat the step II to get 2 is the second term of the quotient.

Step V : Similarly as step III. We multiply the divisor $x + 1$ by the second terms of quotient 2 and subtract the product $2x + 2$ from the dividend $2x + 3$. This gives us 1 as remainder.

This process continues till the remainder is 0 or the degree of the new dividend is less than the degree of the divisor. At the last stage, dividend becomes the remainder and the sum of the quotients gives us the whole quotient.

In this example divisor is a linear polynomial. Let us see the relation between the remainder and certain values of dividend.

In $p(x) = 5x^2 + 7x + 3$, substituting -1 in place of x , we get

$$p(-1) = 5(-1)^2 + 7(-1) + 3 = 5 - 7 + 3 = 1$$

Hence the remainder obtained on dividing $p(x) = 5x^2 + 7x + 3$ by $(x + 1)$ is the same as the value of the polynomial $p(x)$ at the zero of the polynomial $(x + 1)$ i.e., -1 .

Let us consider some more examples.

Example 8. Divide the polynomial $2x^4 - 3x^3 + 3x + 1$ by $x + 1$.

Solution :

$$\begin{array}{r}
 2x^3 - 5x^2 + 5x - 2 \\
 x+1 \overline{) 2x^4 - 3x^3 + 3x + 1} \\
 \underline{2x^4 + 2x^3} \\
 -5x^3 \\
 \underline{-5x^3 - 5x^2} \\
 5x^2 + 3x + 1 \\
 \underline{5x^2 + 5x} \\
 -2x + 1 \\
 \underline{-2x - 2} \\
 3
 \end{array}$$

Remainder = 3

Here the zero of divisor $x + 1$ is -1 . So on putting $x = -1$ in $p(x)$.

$$\begin{aligned}
 p(-1) &= 2(-1)^4 - 3(-1)^3 + 3(-1) + 1 \\
 &= 2 + 3 - 3 + 1 = 3 \\
 &= \text{Remainder}
 \end{aligned}$$

Example 9. Find the remainder obtained on dividing $p(x) = x - 1$ by $x^3 - 1$.

Solution :

$$\begin{array}{r}
 x^2 + x + 1 \\
 x-1 \overline{) x^3 - 1} \\
 \underline{x^3 - x^2} \\
 x^2 - 1 \\
 \underline{x^2 - x} \\
 x - 1 \\
 \underline{x - 1} \\
 0
 \end{array}$$

Now the remainder is 0.

The root of divisor $x - 1$ is $x = 1$ and $p(x) = x^3 - 1$

$$\therefore p(1) = (1)^3 - 1 = 1 - 1 = 0$$

So $p(1) = 0$ is equal to the remainder obtained by actual division.

In this way, it is a simple method to find the remainder obtained on dividing a polynomial by a linear polynomial. We shall now generalise this fact in the form of a theorem.

Remainder Theorem

Let $p(x)$ be any polynomial of degree greater than or equal to one and let a be any real number. If $p(x)$ is divided by the linear polynomial $x - a$, then the remainder is $p(a)$.

Proof: Let $p(x)$ be any polynomial with degree greater than or equal to 1. Suppose that when $p(x)$ is divided by $x - a$, the quotient is $q(x)$ and the remainder is $r(x)$, i.e.,

$$p(x) = (x - a)q(x) + r(x)$$

Since, the degree of $x - a$ is 1 and the degree of $r(x)$ is less than degree of $(x - a)$, the degree of $r(x) = 0$. It means that $r(x)$ is a constant, say r .

So for every value of x , $r(x) = r$

Therefore $p(x) = (x - a)q(x) + r$

In particular, if $x = a$, this equation gives us

$$\begin{aligned} p(a) &= (a - a)q(x) + r \\ &= 0 \times q(x) + r = r \\ &= r \end{aligned}$$

Hence Proved.

Example 10. Find the remainder when $x^4 - 4x^2 + x^3 + 2x + 1$ is divided by $x - 1$.

Solution: Let $p(x) = x^4 - 4x^2 + x^3 + 2x + 1$

The zero of $x - 1$ is 1.

$$\begin{aligned} \text{Thus, } p(1) &= (1)^4 - 4(1)^2 + (1)^3 + 2(1) + 1 \\ &= 1 - 4 + 1 + 2 + 1 = 5 - 4 = 1 \end{aligned}$$

Thus, remainder = 1

Example 11. Verify whether the polynomial $p(x) = 4x^3 - 12x^2 + 13x - 4$ is a multiple of $g(x) = 2x - 1$.

Solution: As we know, $p(x)$ will be a multiple of $g(x)$ only when $g(x)$ divides $p(x)$ completely i.e., remainder is zero.

So, $g(x) = 2x - 1 = 0$

$$x = \frac{1}{2}$$

$$\begin{aligned} \therefore p\left(\frac{1}{2}\right) &= 4\left(\frac{1}{2}\right)^3 - 12\left(\frac{1}{2}\right)^2 + 13\left(\frac{1}{2}\right) - 4 \\ &= \frac{1}{2} - 3 + \frac{13}{2} - 4 = 0 \end{aligned}$$

Hence, $g(x)$ is a factor of $p(x)$, means $p(x)$ is multiple of $g(x)$.

Exercise 3.3

1. Find the remainder, if the polynomial $x^4 + x^3 - 3x^2 + 3x + 1$ is divided by following linear expression :
 (i) $x - 1$ (ii) $x - \frac{1}{2}$ (iii) $x + \pi$ (iv) $3 + 2x$ (v) x
2. Find the remainder, when $2x^3 + 2ax^2 - 5x + a$ is divided by $x + a$.
3. Check whether $x + 1$ is factor of $x^3 + 3x^2 + 3x + 1$ or not.
4. We get the same remainder if polynomials $x^3 + x^2 - 4x + a$ and $2x^3 + ax^2 + 3x - 3$ are divided by $x - 2$. Find the value of a .

Factorisation of Polynomials :

It is found by observing the example 11. Since, the remainder $p\left(\frac{1}{2}\right) = 0$,

therefore, $g(x) = (2x - 1)$, is factor of $p(x)$. So for a polynomial $p(x)$

$$p(x) = (2x - 1)q(x)$$

This is the particular case of the theorem that is given below.

Factor Theorem : If $p(x)$ is a polynomial of degree $n \geq 1$ and a is any real number such that $p(a) = 0$ then $(x - a)$ is a factor of $p(x)$, i.e. if $(x - a)$ is a factor of $p(x)$ then $p(a) = 0$.

Example 12. Examine whether $x - 3$ is a factor of polynomials

$$x^3 - 3x^2 + 4x - 12 \text{ and } 3x - 9.$$

Solution : Given, $p(x) = x^3 - 3x^2 + 4x - 12$, $q(x) = 3x - 9$

According to factor theorem if $(x - 3)$ is a factor of $p(x)$ and $q(x)$, then :

$$p(3) = q(3) = 0$$

The zero of $(x - 3)$ is 3.

$$\begin{aligned} p(3) &= (3)^3 - 3(3)^2 + 4(3) - 12 \\ &= 27 - 27 + 12 - 12 = 0 \end{aligned}$$

Thus, $x - 3$ is a factor of $p(x)$

Similarly, $q(3) = 3 \times 3 - 9 = 0$

Thus, $x - 3$ is a factor of $q(x)$ also.

Example 13. Find the value of a if $x - 5$ is a factor of the polynomial $x^3 - 3x^2 + ax - 10$.

Solution : As $x - 5$ is a factor of $p(x) = x^3 - 3x^2 + ax - 10$

$$\therefore p(5) = 0$$

$$\text{Now, } p(5) = (5)^3 - 3(5)^2 + a(5) - 10 = 0$$

$$\Rightarrow 125 - 75 + 5a - 10 = 0$$

$$\Rightarrow 40 + 5a = 0$$

$$\text{Thus, } a = -\frac{40}{5} = -8$$

The factor theorem is used to factorise some polynomials of degree 2 and 3. We are already familiar with the factorisation of quadratic polynomials like $ax^2 + bx + c$ where $a \neq 0$ and a, b, c are constants by splitting the middle term.

$$\text{Let } ax^2 + bx + c = (px + q)(rx + s)$$

$$= prx^2 + (ps + qr)x + qs$$

Comparing the coefficients of both side, we get

$$a = pr$$

$$b = ps + qr$$

$$c = qs$$

Where b is the sum of two numbers ps and qr ; whose product is

$$(ps)(qr) = (pr)(qs) = a \cdot c$$

Therefore, we can say that to factorise $ax^2 + bx + c$, we have to write b as the sum of those two numbers whose product is ac .

Example 14. Factorise $6x^2 + 17x + 5$ by splitting the middle term and using the factor theorem.

Solution : 1. By splitting the middle term :

We have to split middle term 17 into such numbers whose sum is 17 and product is $6 \times 5 = 30$

$$\text{Factors of 30 are, } 1 \times 30 = 30$$

$$2 \times 15 = 30$$

$$3 \times 10 = 30$$

$$5 \times 6 = 30$$

The sum of the pair of 2 and 15 is 17. So

$$\begin{aligned} 6x^2 + 17x + 5 &= 6x^2 + (2 + 15)x + 5 \\ &= 6x^2 + 2x + 15x + 5 \\ &= 2x(3x + 1) + 5(3x + 1) \\ &= (3x + 1)(2x + 5) \end{aligned}$$

2. By factorisation theorem

$$6x^2 + 17x + 5 = 6\left(x^2 + \frac{17}{6}x + \frac{5}{6}\right) = 6 \cdot p(x)$$

Let the zeroes of $p(x)$ are a and b , then

$$6x^2 + 17x + 5 = 6(x - a)(x - b)$$

So,
$$ab = \frac{5}{6}$$

Now, possible values of a and $b = \pm \frac{1}{2}, \pm \frac{1}{3}, \pm \frac{5}{2}, \pm \frac{5}{3}, \pm 1$

Putting the values respectively, we have

$$p\left(\frac{1}{2}\right) = \frac{1}{4} + \frac{17}{12} + \frac{5}{6} \neq 0$$

$$p\left(-\frac{1}{3}\right) = \frac{1}{9} + \frac{17}{6} \times \frac{-1}{3} + \frac{5}{6}$$

$$= \frac{1}{9} - \frac{17}{18} + \frac{5}{6} = 0$$

So $\left(x + \frac{1}{3}\right)$ is a factor of $p(x)$

Similarly, by trial, we can find that $\left(x + \frac{5}{2}\right)$ is a factor of $p(x)$

$$\begin{aligned}\text{Therefore, } 6x^2 + 17x + 5 &= 6\left(x + \frac{1}{3}\right)\left(x + \frac{5}{2}\right) \\ &= 6\left(\frac{3x+1}{3}\right)\left(\frac{2x+5}{2}\right) \\ &= (3x+1)(2x+5)\end{aligned}$$

Example 15. Factorise $x^2 - 7x + 12$ with the help of factor theorem.

Solution : Let $p(x) = x^2 - 7x + 12$

Now, if $p(x) = (x - a)(x - b)$, then

Here, constant term $ab = 12$

So, to look for the factors of $p(x)$ we find the factors of 12.

Factors of 12 = 1, 2, 3, 4, 6

$$p(3) = (3)^2 - 7(3) + 12 = 0$$

So, $(x - 3)$ is factor of $p(x)$

$$\text{Similarly, } p(4) = (4)^2 - 7(4) + 12 = 0$$

So, $(x - 4)$ is factor of $p(x)$

$$\text{Thus, } x^2 - 7x + 12 = (x - 3)(x - 4)$$

Example 16. Using the factor theorem, factorise the polynomial

$$x^4 + x^3 - 7x^2 - x + 6 .$$

Solution : Let $p(x) = x^4 + x^3 - 7x^2 - x + 6$

The factors of constant term $6 = \pm 1, \pm 2, \pm 3$ and ± 6

$$p(1) = (1)^4 + (1)^3 - 7(1)^2 - 1 + 6 = 8 - 8 = 0$$

So, $(x - 1)$ is factor of $p(x)$.

$$\text{Similarly, } p(-1) = (-1)^4 + (-1)^3 - 7(-1)^2 - (-1) + 6 = 8 - 8 = 0$$

So, $(x+1)$ is also a factor of $p(x)$

$$p(2) = (2)^4 + (2)^3 - 7(2)^2 - (2) + 6 = 30 - 30 = 0$$

Therefore, $(x-2)$ is another factor of $p(x)$

$$p(-2) = (-2)^4 + (-2)^3 - 7(-2)^2 - (-2) + 6 = 24 - 36 \neq 0$$

So, $(x+2)$ is not a factor of $p(x)$

$$p(-3) = (-3)^4 + (-3)^3 - 7(-3)^2 - (-3) + 6 = 90 - 90 = 0$$

So, $(x+3)$ is a factor of $p(x)$ |

Since, $p(x)$ is a polynomial of degree 4, therefore it can not have the factors more than 4.

$$\therefore p(x) = k(x-1)(x+1)(x-2)(x+3)$$

$$\Rightarrow x^4 + x^3 - 7x^2 - x + 6 = k(x-1)(x+1)(x-2)(x+3) \dots (1)$$

put $x=0$, we get

$$0+0+0-0+6 = k(-1)(1)(-2)(3)$$

$$\Rightarrow 6 = 6k$$

$$\Rightarrow k = 1$$

Replacing 1 for k in the equation (i) we get

$$x^4 + x^3 - 7x^2 - x + 6 = (x-1)(x+1)(x-2)(x+3)$$

Exercise 3.4

1. Determine which of the following polynomials has $(x-1)$ as a factor :

(i) $x^4 - 2x^3 - 3x^2 + 2x + 2$ (ii) $x^4 + x^3 + x^2 + x + 1$

(iii) $x^4 + 3x^3 - 3x^2 + x - 2$ (iv) $x^3 - x^2 - (2 + \sqrt{3})x + \sqrt{3}$

2. Using the factor theorem, find if $g(x)$ is a factor of $p(x)$?

(i) $p(x) = 3x^3 - x^2 - 3x + 1$; $g(x) = x + 1$

(ii) $p(x) = 2x^4 - 7x^3 - 13x^2 + 63x - 45$; $g(x) = x - 1$

(iii) $p(x) = 3x^3 + 3x^2 + 3x + 1$; $g(x) = x + 2$

(iv) $p(x) = 2x^3 + x^2 - 2x - 1$; $g(x) = 2x + 1$

3. Find the value of k , when $(x - 5)$ is a factor of the polynomial $x^3 - 3x^2 + kx - 10$.
4. Find the value of k , when $(x - 1)$ is a factor of the polynomial $2x^2 + kx + \sqrt{2}$.
5. Find the value of a and b , if $(x + 1)$ and $(x - 1)$ are the factors of the polynomial $x^4 + ax^3 - 3x^2 + 2x + b$.
6. Factorise :

(i) $3x^2 + 7x + 2$	(ii) $4x^2 - x - 3$
(iii) $12x^2 - 7x + 1$	(iv) $6x^3 + 5x - 6$
7. Find the zeroes of the polynomials :

(i) $x^3 + 6x^2 + 11x + 6$	(ii) $x^3 + 2x^2 - x - 2$
(iii) $x^4 - 2x^3 - 7x^2 + 8x + 12$	(iv) $x^3 - 2x^2 - x + 2$
(v) $x^3 - 3x^2 - 9x - 5$	(vi) $x^3 - 23x^2 + 142x - 120$

Algebraic Identities :

In our earlier classes, we studied that an algebraic identity is an algebraic equation that is true for all values of the variables occurring in it. We have already studied the identities given below in our previous classes.

Identity I : $(x + y)^2 = x^2 + 2xy + y^2$

Identity II : $(x - y)^2 = x^2 - 2xy + y^2$

Identity III : $x^2 - y^2 = (x + y)(x - y)$

Identity IV : $(x + a)(x + b) = x^2 + (a + b)x + ab$

All of above identities involved product of binomials. Let us extend the Identity I to a trinomial $x + y + z$. We shall compute $(x + y + z)^2$.

Let $x + y = t$, then

$$\begin{aligned} \therefore (x + y + z)^2 &= (t + z)^2 \\ &= t^2 + 2tz + z^2 \quad (\text{Using Identity I}) \\ &= (x + y)^2 + 2(x + y)z + z^2 \end{aligned}$$

Substituting the value of t , we have

$$\begin{aligned} &= x^2 + 2xy + y^2 + 2xz + 2yz + z^2 \\ &= x^2 + y^2 + z^2 + 2xy + 2yz + 2zx \end{aligned}$$

So, we get the following identity :

Identity V : $(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$

Example 17. Expand : $(2x + 4y + 3z)^2$

Solution : On comparing with identity V.

$$x = 2x, y = 4y, z = 3z$$

On using the Identity V.

$$\begin{aligned}(2x + 4y + 3z)^2 &= (2x)^2 + (4y)^2 + (3z)^2 + 2(2x)(4y) + 2(4y)(3z) + 2(3z)(2x) \\ &= 4x^2 + 16y^2 + 9z^2 + 16xy + 24yz + 12zx\end{aligned}$$

Example 18. Expand : $(2a - 3b - 4c)^2$

Solution : Using Identity V

$$\begin{aligned}(2a - 3b - 4c)^2 &= [2a + (-3b) + (-4c)]^2 \\ &= (2a)^2 + (-3b)^2 + (-4c)^2 + 2(2a)(-3b) + 2(-3b)(-4c) + 2(-4c)(2a) \\ &= 4a^2 + 9b^2 + 16c^2 - 12ab + 24bc - 16ac\end{aligned}$$

Example 19. Factorise : $4x^2 + y^2 + z^2 - 4xy - 2yz + 4xz$

Solution : $4x^2 + y^2 + z^2 - 4xy - 2yz + 4xz$

$$\begin{aligned}&= (2x)^2 + (-y)^2 + (z)^2 + 2(2x)(-y) + 2(-y)(z) + 2(z)(2x) \\ &= (2x - y + z)^2 \quad (\text{Using Identity V}) \\ &= (2x - y + z)(2x - y + z)\end{aligned}$$

Now, let us extend identity I to compute $(x + y)^3$.

$$\begin{aligned}\text{Here, } (x + y)^3 &= (x + y)(x + y)^2 \\ &= (x + y)(x^2 + 2xy + y^2) \\ &= x(x^2 + 2xy + y^2) + y(x^2 + 2xy + y^2) \\ &= x^3 + 2x^2y + xy^2 + x^2y + 2xy^2 + y^3 \\ &= x^3 + 3x^2y + 3xy^2 + y^3 \\ &= x^3 + y^3 + 3xy(x + y)\end{aligned}$$

So, we get the following identities :

Identity VI : $(x + y)^3 = x^3 + y^3 + 3xy(x + y)$

Replacing y by $-y$ in the above identity, we get

$$\text{Identity VII : } (x - y)^3 = x^3 - y^3 - 3xy(x - y) = x^3 - y^3 - 3x^2y + 3xy^2$$

Example 20. Using the identities expand the following expressions :

$$(i) (4a + 3b)^3 \qquad (ii) (3x - 5y)^3$$

Solution : (i) Comparing the expression with $(x + y)^3$, we find that

$$x = 4a \text{ and } y = 3b$$

$$\begin{aligned} (4a + 3b)^3 &= (4a)^3 + (3b)^3 + 3(4a)(3b)(4a + 3b) \\ &= 64a^3 + 27b^3 + 144a^2b + 108ab^2 \end{aligned}$$

(ii) Comparing the expression with $(x - y)^3$, we find that

$$x = 3x, \quad y = 5y$$

$$\begin{aligned} \therefore (3x - 5y)^3 &= (3x)^3 - (5y)^3 - 3(3x)(5y)(3x - 5y) \\ &= 27x^3 - 125y^3 - 135x^2y + 225xy^2 \end{aligned}$$

Example 21. Using the suitable identity evaluate each of the following :

$$(i) (102)^3 \qquad (ii) (998)^3$$

Solution : (i) $(102)^3 = (100 + 2)^3$

$$\begin{aligned} &= (100)^3 + (2)^3 + 3(100)(2)(100 + 2) \text{ (Using the Identity VI)} \\ &= 1000000 + 8 + 60000 + 1200 \\ &= 1061208 \end{aligned}$$

(ii) $(998)^3 = (1000 - 2)^3$

$$\begin{aligned} &= (1000)^3 - (2)^3 - 3(1000)(2)(1000 - 2) \\ &= 1000000000 - 8 - 6000000 + 12000 \\ &= 994011992 \end{aligned}$$

Example 22. Factorise : $8x^3 + 27y^3 + 36x^2y + 54xy^2$

Solution : $8x^3 + 27y^3 + 36x^2y + 54xy^2$

$$\begin{aligned} &= (2x)^3 + (3y)^3 + 3(2x)^2(3y) + 3(2x)(3y)^2 \\ &= (2x + 3y)^3 \text{ (Using Identity VI)} \\ &= (2x + 3y)(2x + 3y)(2x + 3y) \end{aligned}$$

Let us find out an important identity :

On expanding $(x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx)$, we get

$$\begin{aligned}
&= x(x^2 + y^2 + z^2 - xy - yz - zx) + y(x^2 + y^2 + z^2 - xy - yz - zx) + \\
&\quad z(x^2 + y^2 + z^2 - xy - yz - zx) \\
&= x^3 + xy^2 + xz^2 - x^2y - xyz - x^2z + x^2y + y^3 + yz^2 - xy^2 - y^2z - xyz \\
&\quad + x^2z + y^2z + z^3 - xyz - yz^2 - xz^2 \\
&= x^3 + y^3 + z^3 - 3xyz \text{ (By simplification)}
\end{aligned}$$

So we obtain the following identity.

$$\text{Identity VIII: } x^3 + y^3 + z^3 - 3xyz = (x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx)$$

Example 23. Factorise : $27x^3 + y^3 + z^3 - 9xyz$

$$\begin{aligned}
\text{Solution : } 27x^3 + y^3 + z^3 - 9xyz &= (3x)^3 + (y)^3 + (z)^3 - 3(3x)(y)(z) \\
&= (3x + y + z)\left[(3x)^2 + y^2 + z^2 - 3x \cdot y - y \cdot z - z \cdot 3x\right] \\
&\quad \text{(Using the identity (VIII))} \\
&= (3x + y + z)(9x^2 + y^2 + z^2 - 3xy - yz - 3zx)
\end{aligned}$$

Exercise 3.5

1. Use the suitable identities to find the product of :
 - (i) $(x + 3)(x + 7)$
 - (ii) $(x - 5)(x + 8)$
 - (iii) $(2x + 7)(3x - 5)$
 - (iv) $(5 - 3x)(3 + 2x)$
 - (v) $\left(x^2 + \frac{3}{5}\right)\left(x^2 - \frac{3}{5}\right)$
 - (vi) $(x + 2)(x - 5)$
2. Using the algebraic identities, find the product of following :
 - (i) 104×109
 - (ii) 94×97
 - (iii) 103×97
3. Using the suitable identities, factorise the following.
 - (i) $x^2 + 6xy + 9y^2$
 - (ii) $x^2 - 4x + 4$
 - (iii) $\frac{x^2}{100} - y^2$
4. Expand the following with the help of suitable identities :
 - (i) $(2a - 3b - c)^2$
 - (ii) $(2 + x - 2y)^2$
 - (iii) $(a + 2b + 4c)^2$
 - (iv) $(m + 2n - 5p)^2$
 - (v) $(3a - 7b - c^2)^2$
 - (vi) $\left(\frac{x}{y} + \frac{y}{z} + \frac{z}{x}\right)^2$
5. Factorise :
 - (i) $9x^2 + 4y^2 + 16z^2 - 12xy - 16yz + 24xz$

$$(ii) x^2 + 2y^2 + 8z^2 + 2\sqrt{2}xy - 8yz - 4\sqrt{2}xz$$

6. Expand the following cubes :

$$(i) (3a - 2b)^3 \quad (ii) (1 + 2x)^3 \quad (iii) \left(\frac{2}{5}x + 3\right)^3 \quad (iv) \left(x - \frac{2}{3}y\right)^3$$

7. Evaluate the following using suitable identities :

$$(i) (98)^3 \quad (ii) (103)^3 \quad (iii) (999)^3$$

8. Factorise :

$$(i) x^3 + 8y^3 + 6x^2y + 12xy^2 \quad (ii) 27a^3 - 8b^3 - 54a^2b + 36ab^2$$
$$(iii) 27 - 125x^3 - 135x + 225x^2 \quad (iv) 125x^3 - 64y^3 - 300x^2y + 240xy^2$$

9. Factorise :

$$(i) 64a^3 + 27b^3 \quad (ii) 125x^3 - 8y^3$$

10. Verify that :

$$(i) x^3 + y^3 + z^3 - 3xyz = \frac{1}{2}(x + y + z)[(x - y)^2 + (y - z)^2 + (z - x)^2]$$

$$(ii) 27a^3 + b^3 + c^3 = (3a + b + c)[9a^2 + b^2 + c^2 - 3ab - bc - 3ac]$$

11. If $x + y + z = 0$, then verify that $x^3 + y^3 + z^3 = 3xyz$

12. Using the suitable identities compute :

$$(i) (30)^3 + (20)^3 + (-50)^3 \quad (ii) (-15)^3 + (28)^3 + (-13)^3$$

[Hint : Use identity if $x + y + z = 0$, then $x^3 + y^3 + z^3 = 3xyz$]

Important Points

1. A polynomial $p(x)$ in one variable x is an algebraic expression in x of the form

$$p(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_2 x^2 + a_1 x + a_0$$

Where a_0, a_1, a_2, \dots are constant and $a_n \neq 0$

$a_0, a_1, a_2, \dots, a_n$ are respectively the coefficients of x^0, x, x^2, \dots and n is called the degree of the polynomial.

$a_n x^n, a_{n-1} x^{n-1}, \dots, a_0$ where $a_n \neq 0$ is called a term of the polynomial $p(x)$

2. A polynomial of one term is called a monomial.
3. A polynomial of two terms is called a binomial.
4. A polynomial of three terms is called a trinomial.
5. A polynomial of degree one is called a linear polynomial.
6. A polynomial of degree two is called a quadratic polynomial.
7. A polynomial of degree three is called a cubic polynomial.
8. A real number ' a ' is a zero of the polynomial $p(x)$, if $p(a) = 0$.
9. Every linear polynomial in one variable has a unique zero, a non-zero constant polynomial has no zero, and every real number is a zero of the zero polynomial.
10. Remainder Theorem : If $p(x)$ is any polynomial of degree greater than or equal to 1 and a is a real number. If $p(x)$ is divided by the linear polynomial $(x - a)$, then the remainder is $p(a)$
11. Factor theorem : $(x - a)$ is a factor of the polynomial $p(x)$, if $p(a) = 0$. Also if $x - a$ is a factor of $p(x)$, then $p(a) = 0$
12. $(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2xz$
13. $(x + y)^3 = x^3 + y^3 + 3xy(x + y)$
14. $(x - y)^3 = x^3 - y^3 - 3xy(x - y)$
15. $x^3 + y^3 + z^3 - 3xyz = (x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx)$

Answer

Exercise 3.1

1. (i) Polynomial, one variable.
(ii) Polynomial, one variable
(iii) and (v) are not polynomial, because each exponent is not a whole number.
(iv) Constant polynomial
(vi) Polynomial, three variables.
2. (i) 1 (ii) 0 (iii) 0 (iv) $\frac{\pi}{2}$
3. $5x^{45} + 7$ (other polynomials may be possible)
4. $3x^{120}$ (other polynomials may be possible)
5. $2x^8 + 3x^4 + 5x$ (other polynomials are also possible)
6. Possible, write yourself.
7. (i) 3 (ii) 1 (iii) 0 (iv) 2

Exercise 3.2

1. (i) 10 (ii) -210 (iii) 12 (iv) -20
2. (i) 3, 1, 1 (ii) 3, 0, -1 (iii) 8, 1, 0 (iv) 0, 3, 2
4. (i) 4 (ii) 0 (iii) 0 (iv) -3
(v) $\frac{1}{2}$ (vi) $-\frac{7}{3}$ (vii) $-\frac{d}{c}$

Exercise 3.3

1. (i) 1 (ii) $\frac{31}{16}$ (iii) $\pi^4 - \pi^3 - 3\pi^2 - 3\pi + 1$
(iv) $-\frac{137}{16}$ (v) 1
2. 6a
3. Yes, because remainder is zero.
4. $a = -5$

Exercise 3.4

- (i) and (iii) has a factor $(x-1)$;
(ii) and (iv) do not have a factor $(x-1)$
- (i) Yes (ii) Yes (iii) No (iv) Yes
- (i) $k = -8$
- $k = -(2 + \sqrt{2})$
- $a = -2, b = 2$
- (i) $(x+2)(3x+1)$; (ii) $(x-1)(4x+3)$;
(iii) $(3x-1)(4x-1)$; (iv) $(2x+3)(3x-2)$
- (i) $-1, -2, -3$; (ii) $-2, -1, 1$; (iii) $-2, -1, 2, 3$;
(iv) $-1, 1, 2$; (v) $-1, 5$; (vi) $1, 10, 12$

Exercise 3.5

- (i) $x^2 + 10x + 21$ (ii) $x^2 + 3x - 40$ (iii) $6x^2 + 11x - 35$
(iv) $15 + x - 6x^2$ (v) $x^4 - \frac{9}{25}$ (vi) $x^2 - 3x - 10$
- (i) 11336 (ii) 9118 (iii) 9991
- (i) $(x+3y)(x+3y)$ (ii) $(x-2)(x-2)$ (iii) $\left(\frac{x}{10} + y\right)\left(\frac{x}{10} - y\right)$
- (i) $4a^2 \times 9b^2 + c^2 - 12ab + 6bc - 4ac$
(ii) $4 + x^2 + 4y^2 + 4x - 4xy - 8y$
(iii) $a^2 + 4b^2 + 16c^2 + 4ab + 16bc + 8ac$
(iv) $m^2 + 4n^2 + 25p^2 + 4mn - 20np - 10pm$
(v) $9a^2 + 49b^2 + c^2 - 42ab + 14bc - 6ac$
(vi) $\frac{x^2}{y^2} + \frac{y^2}{z^2} + \frac{z^2}{x^2} + \frac{2x}{z} + \frac{2y}{x} + \frac{2z}{y}$
- (i) $(3x - 2y + 4z)^2$ (ii) $(x + \sqrt{2}y - 2\sqrt{2}z)^2$

6. (i) $27a^3 - 8b^3 - 54a^2b + 36ab^2$ (ii) $1 + 8x^3 + 6x + 12x^2$
 (iii) $\frac{8}{125}x^3 + 27 + \frac{36}{25}x^2 + \frac{54}{5}x$ (iv) $x^3 - \frac{8}{27}y^3 - 2x^2y + \frac{4}{3}xy^2$
7. (i) 941192; (ii) 1092727; (iii) 997002999
8. (i) $(x+2y)^3$; (ii) $(3a-2b)^3$; (iii) $(3-5x)^3$; (iv) $(5x-4y)^3$
9. (i) $(4a+3b)(16a^2-12ab+9b^2)$; (ii) $(5x-2y)(25x^2+10xy+4y^2)$
12. (i) -90000; (ii) 16380

Linear Equations in Two Variables

4.01. Introduction

In previous classes you have studied about linear equations in one variable. The equations in which degree of variables is one are called linear equation. Some of the examples of the linear equations are

(i) $x + 3 = 8$

(ii) $2y + 10 = 28$

(iii) $4x - 7 = 2x + 3$

(iv) $5m = 40$

The equations satisfied by a value of the variable, which when substituted in place of variable in the equation i.e. the left hand side and right hand side of the equation is equal is called the solution of the equation.

We know the fact about the equations that the solution of a linear equation is not affected when :

- (i) the same number is added or subtracted from both sides of an equation.
- (ii) the same number is multiplied or same non zero number divide both the sides of the equation.

In general, a linear equation in one variable can be expressed in the form of $ax + b = 0$, where a and b are real number. Here $a \neq 0$ and x is the variable. The solution of equation $ax + b = 0$ is $x = \frac{-b}{a}$. The equation in one variable has a unique (one and only one) solution, that is called the root of equation .

Linear Equations in Two Variables

The equation in which there are two variables of degree one is called linear equation in two variables.

Let us understand the co-ordinate system before study of the equations in two variables.

4.02. Rectangular Co-ordinate System

We shall clear the concept of rectangular co-ordinate system before solving a linear equation graphically.

(a) Rectangular Co-ordinate System :

We have already learnt that how to represent a real number on the number line. There are many situations, in which to find a point we are need to describe its position with reference to more than one line. Sometimes a point does not fall on the number line instead it is located somewhere in the plane. So we expand the principle related to number line.

So a dot in the plane can be represented by two perpendicular lines one of them is horizontal and other one is vertical. The horizontal line is called x -axis as well as vertical line is called y -axis and they are represented as XOX' and YOY' respectively.

The intersecting point of both lines known as origin and is denoted by symbol O . Positive integers lie to the right side of origin on x -axis (towards OX) and negative integers lie to the left side (towards OX'). Similarly positive integers lie above the origin of y -axis (towards OY) and negative integers lie below the origin (towards OY')

(b) Quadrants

Two axes XX' and YY' divide the plane into four parts which are called quadrants (means one fourth part). The expansion of these quadrants is infinite. XOY , YOX' , $X'OY'$ and $Y'OX$ are the first, second, third and fourth quadrants respectively.

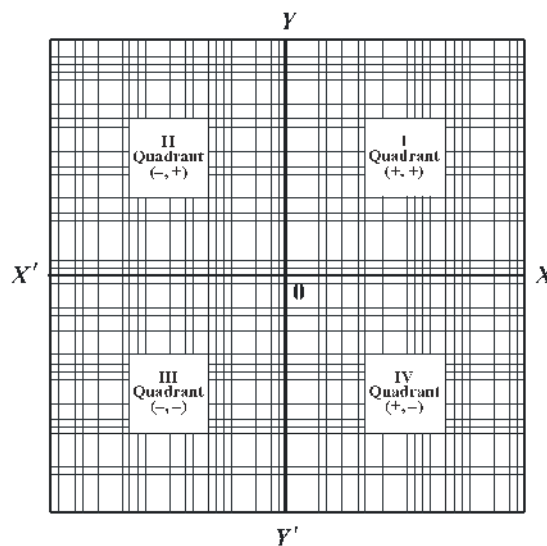


Fig. 4.01

(c) Plotting of Points

Let P be a point in the first quadrant. To reach this point we have to move 3 units towards OX and then 4 units towards OY , then this point can be expressed as $P(3, 4)$. 3 is the x -coordinate of P on x -axis and 4 is the y -coordinate of P on y -axis. x -coordinate

is called **abscissa** and y -coordinate is called **ordinate**. In this way, there is an abscissa x and an ordinate y for every point in the plane. These are represented by a ordered pair (x,y) . Ordered pair (x,y) is called the coordinate of that point.

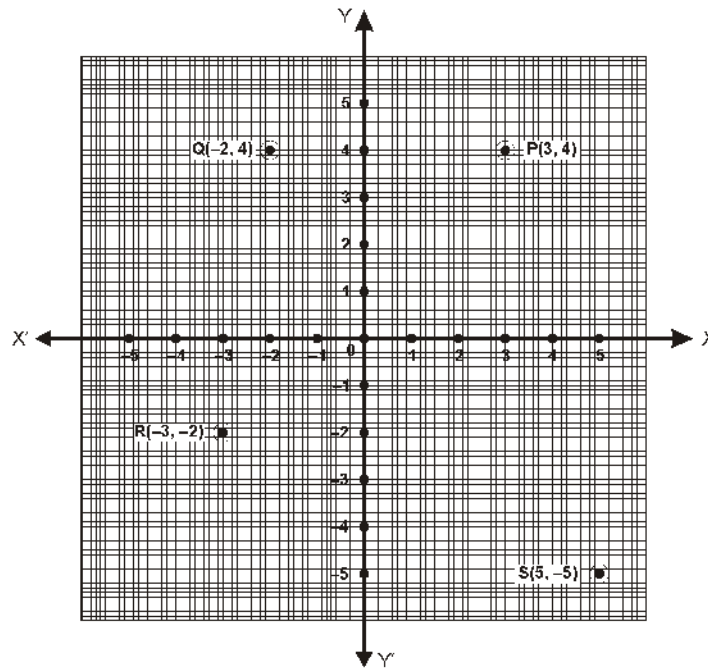


Fig. 4.02

Now consider the point $P(3,4)$. We observe that this point is above the x -axis and right to the y -axis, so its both abscissa and ordinate are positive. Therefore, the point $P(3,4)$ lie in the first quadrant.

Similarily we see that the coordinate in ordered pair form of points Q, R and S are resectively $(-2, 4)$, $(-3, -2)$ and $(5, -5)$ as shown in the figure 4.02.

Again if we plote the point $P(3, 4)$ then we move 3 unit in the direction towards OX right to O and then we move upward 4 unit parallel to OY from that point. This is the actual position of point $P(3, 4)$ in the plane.

So, to locate the point $Q(-2, 4)$ we move 2 units in the direction of OX' form O and then we move upward 4 units parallel to OY. The point Q lie in the IInd quadrant.

In the same way we can plot the points $R(-3, -2)$ and $S(5, -5)$ in the plane.

Note :

1. The ordinate of each point on x - axis is zero.
2. The abscissa of each point on y - axis is zero.
3. Coordinates of origin are $(0,0)$.

4.03. Graph of Linear equation in two variables

Take an example $x + y = 9$

The solutions of the equation are the values of variables x and y which satisfy the given equation. Let us see what values of x and y satisfy the above equation. See the following table.

x	0	1	2	3	4	5	6	7
y	9	8	7	6	5	4	3	2

These are some solutions that satisfy the equation but we can say that infinitely many values of variables x and y satisfy the solution.

We should plot the value of x variable on the X-axis and the value of variable y on Y-axis, and coordinates of x and y are written as (x,y) . When all these points are joined, we get a straight line that is called the graph of the equation. Solutions of the equation are pointed on the obtained line and according to the graph every point on the line is the solution of the equation.

Construction of a line is a series of infinitely many points. So we can say that an equation in two variables has infinitely many solutions.

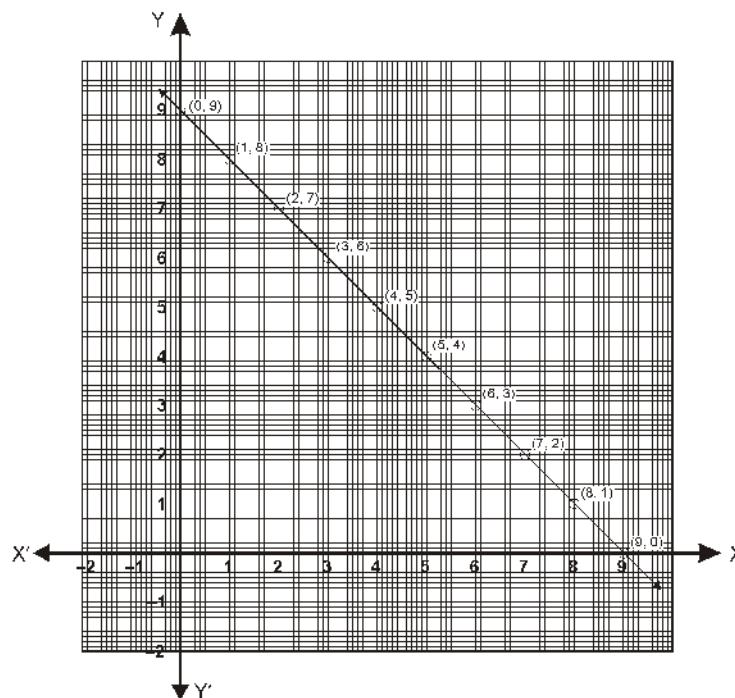


Fig. 4.03

Note :

1. The graphical representation of an equation in two variables is always a straight line.

2. Every point on the graph line gives the solution of the equation.
3. Any point, which does not lie on graph line, is not a solution of the equation. To obtain the graph of a linear equation in two variables, it is enough to plot two points corresponding to two solutions and join them by a line. However, it is advisable to plot more than two such points so that we can immediately check the corresponding to two solutions and join them by a line. However, it is advisable to plot more than two such points so that we can immediately check the correction of the graph.

Example 1. Draw the graph of equation $3x + y = 2$

Solution : Given equation is

$$3x + y = 2 \quad \Rightarrow \quad y = 2 - 3x$$

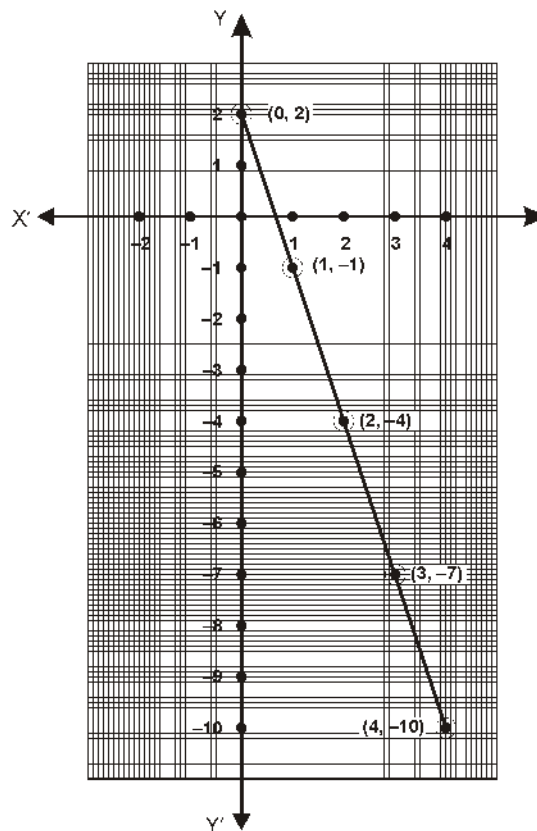


Fig. 4.04

We prepare a table as follows by writing the values of y below the corresponding values of x

x	0	1	2	3	4
y	2	-1	-4	-7	-10

From the graph of the equation, it is clear that every point on the graph line is a solution of the equation. If more than one equations in two variables are to be graphed on the same graph paper, then following conditions may be obtained :

- (i) The graph line of two equations may intersect each other on a point.
- (ii) Graph lines of two equations may be parallel and they never intersects each other.
- (iii) Both lines may be coincident. In the first condition, the intersecting point of two lines shows the solution of both the lines. So, the coordinates of that point satisfy the two equations.

To obtain the unique solution of a linear equation in two variables, two linear equations will be required. Such linear equations in two variables are called the simultaneous equations.

The solution of the pair of linear equation by graph.

Illustrative Examples

Example 2. Solve the following equations by graphical method.

$$x + y = 3 ; 3x - 2y = 4$$

Solution : By the given equations we make the separate tables from their possible solutions.

Given, $x + y = 3$... (i)

$\Rightarrow x = 3 - y$

x	1	2	3
y	2	1	0

Similarly, $3x - 2y = 4$ or $x = \frac{4 + 2y}{3}$... (ii)

x	2	4	6
y	1	4	7

Draw the graph by plotting the above points given in the tables and then by joining the line.

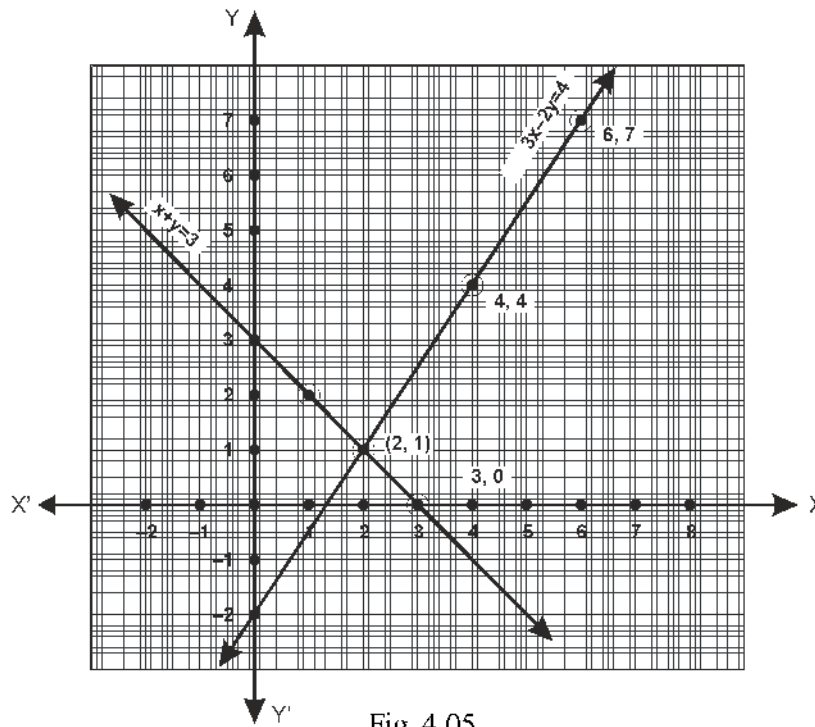


Fig. 4.05

Both the lines intersect at a point. The coordinates of point are (2, 1). Thus, the solution of the given equations is $x = 2$ and $y = 1$.

Example 3. Solve the following pair of equations for x and y by graphical

Method : $2x + 3y = 13$; $5x - 2y = 4$

Solution : Given system of equations is :

$$2x + 3y = 13 \quad \dots(1)$$

$$5x - 2y = 4 \quad \dots(2)$$

Graph of the equation $2x + 3y = 13$ or $y = \frac{13 - 2x}{3}$

We have the following table for some possible solutions of the equation :

x	-1	2	5
y	5	3	1

Similarly equation $5x - 2y = 4$ or $y = \frac{5x - 4}{2}$.

x	-2	4	2
y	-7	8	3

Plotting the points $(-1, 5)$, $(2, 3)$ and $(5, 1)$ and drawing a line joining them, we get the graph of the equation $2x + 3y = 13$ as shown in Fig. 4.06. We have the following table for values of (x, y) .

Plotting the points $(-2, -7)$, $(4, 8)$ on the same graph paper and drawing a line joining them, we obtain the graph of the equation $5x - 2y = 4$.

Clearly, the two lines intersect at point $P(2, 3)$. Thus, $x = 2$, $y = 3$ is the solution of the given system.

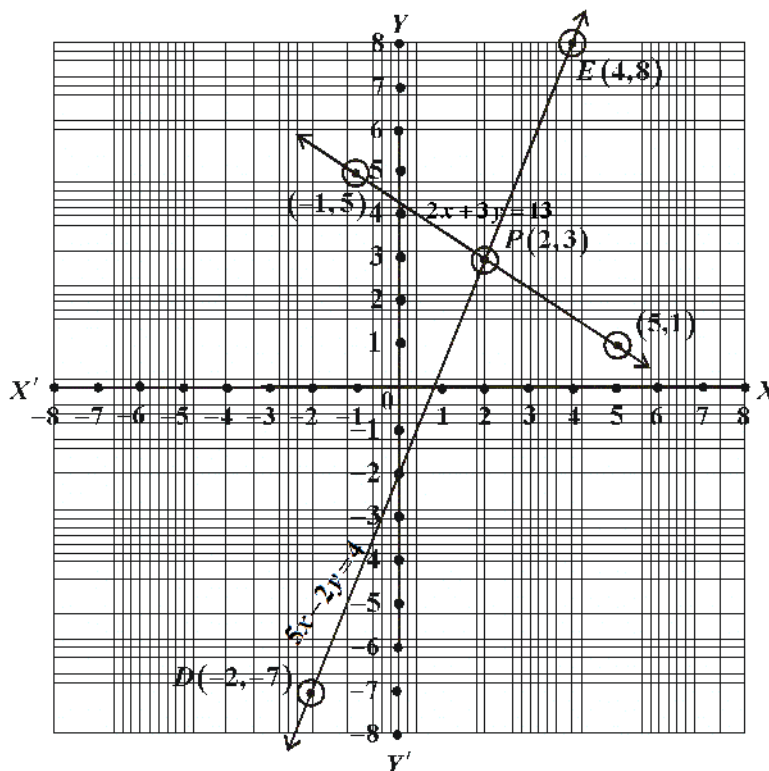


Fig. 4.06

Example 4. Solve the following equations by graphical method.

$$2x - 6y + 10 = 0; 3x - 9y + 15 = 0$$

Solution : Given system of equation is :

$$2x - 6y + 10 = 0 \quad \dots (1)$$

$$3x - 9y + 15 = 0 \quad \dots (2)$$

Table for equation

$$2x - 6y + 10 = 0$$

x	1	-5	7
y	2	0	4

Table for equation

$$3x - 9y + 15 = 0$$

x	4	-2	-8
y	3	1	-1

Now, we plot the points $(1, 2)$, $(-5, 0)$ and $(7, 4)$ on a graph and join them. We get a straight line AB in the form of the graph of equation $2x - 6y + 10 = 0$. Again, we plot the points $(4, 3)$, $(-2, -1)$ and $(-8, -1)$. We see that all three points lie on the line AB . So, both the lines are coincident. So, the equations have infinitely many solutions. And the solution of the equation $2x - 6y + 10 = 0$, will be the solution of the system.

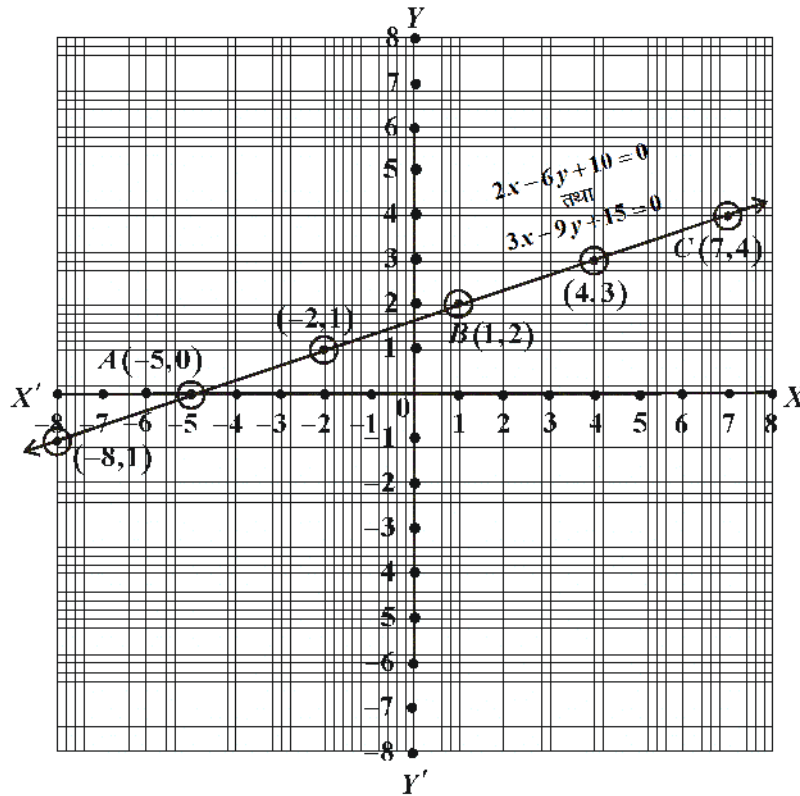


Fig. 4.07

Example 5. Solve the system of equations given below graphically. Also find the nature of system $2x + 3y = 12$; $2x + 3y = 6$.

Solution : We have two linear equations.

$$2x + 3y = 12 \quad \dots (1)$$

$$2x + 3y = 6 \quad \dots (2)$$

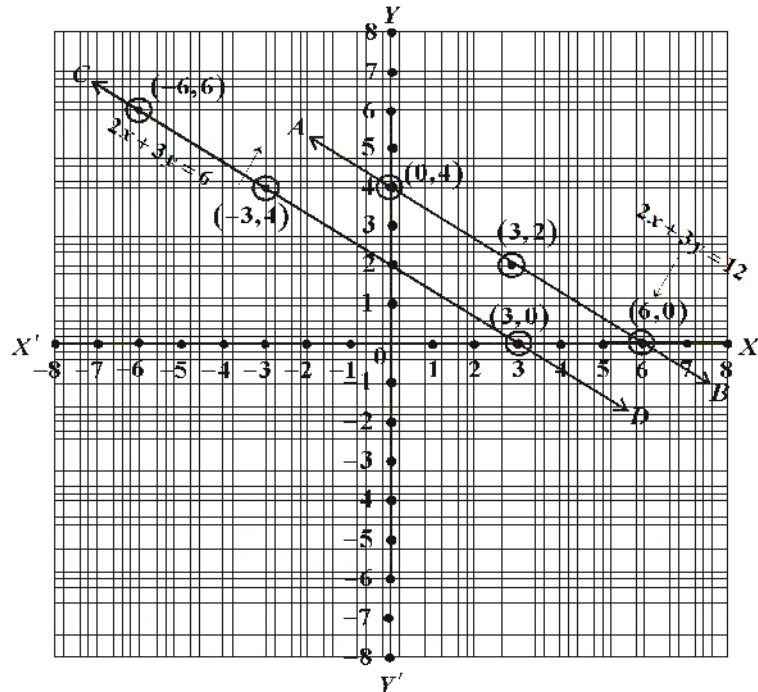


Fig. 4.08

Now on plotting the points $(6, 0), (3, 2), (0, 4)$. When we join these three points, we obtain a straight line AB . Which is a graph of equation $2x + 3y - 12 = 0$.

Again on plotting the points $(3, 0), (-3, 4)$ and $(-6, 6)$. We get the graph of equation $2x + 3y = 6$ is a straight line CD by joining these points.

Now we see that the lines AB and CD which we obtained graphically are parallel it means the given system of linear equations is inconsistent. Hence, there is no solution of the given equations.

Exercise 4.1

Solve the following equations graphically.

- | | |
|-----------------------|------------------|
| 1. $x + 3y = 6$ | 2. $2x + y = 6$ |
| $2x - 3y = 12$ | $2x - y + 2 = 0$ |
| 3. $x - 2y = 6$ | 4. $x + y = 4$ |
| $3x - 6y = 0$ | $2x - 3y = 3$ |
| 5. $2x - 3y + 13 = 0$ | |
| $3x - 2y + 12 = 0$ | |

6. $3x - 4y = 1; -2x + \frac{8}{3}y = 5$
7. $2x + \frac{y}{2} - 5 = 0; \frac{x}{2} + y = -4$
8. $0 \cdot 3x + 0 \cdot 4y = 3 \cdot 2; 0 \cdot 6x + 0 \cdot 8y = 2 \cdot 4$
9. $2x + 3y = 8; 4x - \frac{3}{2}y = 1$
10. $3x - y = 2; 6x - 2y = 4$
11. $3x + 2y = 0; 2x + y = -1$

4.04. Algebraic Methods of Solving Simultaneous Linear Equations

Simultaneous equations is the pair of two linear equations in two variables. The values of both variables that satisfy both the equations are solutions of the simultaneous equations.

The solution of the given system of linear equations can be obtained by using following algebraic method.

- (i) Method of elimination (by substitution)
- (ii) Method of elimination (by equating the co-efficient)
- (iii) Method of cross multiplication (General method)

(i) Method of Elimination (by Substitution) :

In this method, the value of a variable in an equation of simultaneous linear equation is expressed in the form of other variable of the equation. Now the value of the variable so obtained substituted in another equation of simultaneous linear equation. Consequently the second equation is equation in one variable. Solving this equation in one variable, we can easily find the value of the variable used in the equation. Then substituting this value in any of equation we obtain the value of other variable. The example given below will be helpful to understand this method.

Example 6. Solve the following system of equations by substituting method.

$$x + 3y = 11$$

$$4x - y = 5$$

Solution : The given system of equations is :

$$x + 3y = 11 \quad \dots (1)$$

$$4x - y = 5 \quad \dots (2)$$

From equation (1), we get

$$x = 11 - 3y \quad \dots (3)$$

Substituting this value of x in (2), we get.

$$4(11 - 3y) - y = 5$$

$$\begin{aligned} \text{or} & \quad 44 - 12y - y = 5 \\ \text{or} & \quad 44 - 13y = 5 \\ \text{or} & \quad 13y = 39 \\ \text{or} & \quad y = \frac{39}{13} \\ \therefore & \quad y = 3 \end{aligned}$$

Putting $y = 3$ in equation (1), we get :

$$\begin{aligned} & \quad x = 11 - 3(3) \\ \text{or} & \quad x = 11 - 9 \\ \text{or} & \quad x = 2 \end{aligned}$$

Hence, the solution of the given system of equations is $x = 2, y = 3$

(ii) Method of Elimination (by Equating the Coefficient) :

In this method, one or both equations of the simultaneous equations are multiplied by such number so that the co-efficients of one variable in two equations may be equal. Now, according to the situation both the equations are added or subtracted, so that we can get an equation in one variable easily. Now this value is substituted in any of the two equations. In this way, the value of other variable is found.

Example 7. Solve the pair of equations using the method of elimination by equating the co-efficient.

$$4x + 5y = 31$$

$$7x - 2y = 22$$

Solution : The given system of equations is :

$$4x + 5y = 31 \quad \dots (1)$$

$$7x - 2y = 22 \quad \dots (2)$$

Let us eliminate y from the given equations. The coefficients of y in the given equations are 5 and 2 respectively. The LCM of 5 and 2 is 10. So, to make the coefficients of y equal, we multiply equation (1) by 2 and equation (2) by 5, we get

$$8x + 10y = 62 \quad \dots (3)$$

$$35x - 10y = 110 \quad \dots (4)$$

Adding the equations (3) and (4) we get :

$$43x = 172 \quad \text{or} \quad x = \frac{172}{43}$$

$$\therefore x = 4$$

Substituting $x = 4$ in equation (1), we get

$$4(4) + 5y = 31 \quad \text{or} \quad 16 + 5y = 31$$

$$\text{or} \quad 5y = 31 - 16 \quad \Rightarrow \quad y = \frac{15}{5}$$

$$\therefore y = 3$$

Hence, the solution of the given system of equations is $x = 4, y = 3$.

Using this method, we can solve such equations which are made of reciprocals of variables.

The method is so clear by the example given below.

Example 8. Solve the following equations :

$$\frac{20}{x} + \frac{2}{y} = 6, \quad \frac{10}{x} - \frac{1}{y} = 2$$

Solution : Given, system of equations is :

$$\frac{20}{x} + \frac{2}{y} = 6 \quad \dots (1)$$

$$\frac{10}{x} - \frac{1}{y} = 2 \quad \dots (2)$$

Multiply equation (2) by 2, we get

$$\frac{20}{x} - \frac{2}{y} = 4 \quad \dots (3)$$

Adding equation (1) and (3), we get

$$\frac{40}{x} = 10 \quad \Rightarrow \quad x = \frac{40}{10}$$

$$\text{or} \quad x = 4$$

Substituting $x = 4$ in equation (1), we get

$$\frac{20}{4} + \frac{2}{y} = 6 \quad \text{or} \quad 5 + \frac{2}{y} = 6$$

$$\text{or} \quad \frac{2}{y} = 6 - 5 \quad \text{or} \quad \frac{2}{y} = 1$$

$$\text{or} \quad y = 2$$

Thus, the solution of the given system of equations is, $x = 4$, $y = 2$

Example 9. Find the solution of following system of equations :

$$5x + 6y = 3xy, \quad 10x + 9y = 5xy$$

Solution : The given system of equations :

$$5x + 6y = 3xy \quad \dots (1)$$

$$10x + 9y = 5xy \quad \dots (2)$$

Dividing the equation (1) and (2) both by xy , we have

$$\frac{5}{y} + \frac{6}{x} = 3 \quad \dots (3)$$

$$\frac{10}{y} + \frac{9}{x} = 5 \quad \dots (4)$$

Taking $\frac{1}{x} = m$ and $\frac{1}{y} = n$. The given system of equations become

$$5n + 6m = 3 \quad \dots (5)$$

$$10n + 9m = 5 \quad \dots (6)$$

Multiply equation (5) by 2, we get

$$10n + 12m = 6 \quad \dots (7)$$

Subtracting equation (6) from equation (7), we get

$$3m = 1 \Rightarrow m = \frac{1}{3}$$

Putting $m = \frac{1}{3}$ in equation (6), we get

$$10n + 9\left(\frac{1}{3}\right) = 5$$

$$\text{or } 10n + 3 = 5 \quad \Rightarrow \quad 10n = 5 - 3$$

$$\text{or } 10n = 2 \quad \Rightarrow \quad n = \frac{2}{10}$$

$$\text{or } n = \frac{1}{5}$$

$$\text{Now } m = \frac{1}{3} \Rightarrow \frac{1}{x} = \frac{1}{3} \Rightarrow x = 3$$

and $n = \frac{1}{5} \Rightarrow \frac{1}{y} = \frac{1}{5} \Rightarrow y = 5$

Thus, the solution of the equation is $x = 3, y = 5$

Exercise 4.2

Solve the following equations by the method of elimination (by substitution):

1. $2x + 3y = 9$

$3x + 4y = 5$

3. $3x + 2y = 11$

$2x + 3y = 4$

5. $4x - 5y = 39$

$2x - 7y = 51$

2. $x + 2y = -1$

$2x - 3y = 12$

4. $8x + 5y = 9$

$3x + 2y = 4$

6. $5x - 2y = 19$

$3x + y = 18$

Solve the following equations by method of elimination by equating the coefficients:

7. $2x + y = 13$

$5x - 3y = 16$

9. $\frac{x}{7} + \frac{y}{3} = 5$

$\frac{x}{2} - \frac{y}{9} = 6$

11. $3x - 7y + 10 = 0$

$y - 2x = 3$

8. $0.4x + 0.3y = 1.7$

$0.7x - 0.2y = 0.8$

10. $11x + 15y = -23$

$7x - 2y = 20$

12. $x + 2y = \frac{3}{2}$

$2x + y = \frac{3}{2}$

Solve the following equations:

13. $8v - 3u = 5uv$

$6v - 5u = -2uv$

14. $\frac{1}{2x} - \frac{1}{y} = -1$

$\frac{1}{x} + \frac{1}{2y} = 8$

15. $\frac{5}{(x+y)} - \frac{2}{(x-y)} = -1$

$\frac{15}{(x+y)} + \frac{7}{(x-y)} = 10$

Cross- Multiplication Method

Cross-multiplication is a general method to solve the simultaneous equations. Here we are going to clear this method.

Let the given system of equations be

$$a_1x + b_1y + c_1 = 0 \quad \dots (1)$$

$$a_2x + b_2y + c_2 = 0 \quad \dots (2)$$

Multiplying equation (1) by b_2 and (2) by b_1 respectively, we get

$$a_1b_2x + b_1b_2y + b_2c_1 = 0 \quad \dots (3)$$

$$a_2b_1x + b_1b_2y + b_1c_2 = 0 \quad \dots (4)$$

Subtracting equation (4) from equation (3), we get

$$(a_1b_2 - a_2b_1)x + b_2c_1 - b_1c_2 = 0$$

or $(a_1b_2 - a_2b_1)x = b_1c_2 - b_2c_1$

$$\Rightarrow x = \frac{b_1c_2 - b_2c_1}{a_1b_2 - a_2b_1} \quad \dots (5)$$

Similarly multiplying equation (1) by a_2 and equation (2) by a_1 , we get

$$a_1b_2x + a_2b_1y + c_1a_2 = 0 \quad \dots (6)$$

$$a_1a_2x + a_1b_2y + c_2a_1 = 0 \quad \dots (7)$$

Subtracting equation (7) from equation (6)

$$(a_2b_1 - a_1b_2)y + c_1a_2 - c_2a_1 = 0$$

or $(a_2b_1 - a_1b_2)y = -c_1a_2 + c_2a_1$

or $y = \frac{c_1a_2 - c_2a_1}{a_1b_2 - a_2b_1} \quad \dots (8)$

Hence, $x = \frac{b_1c_2 - b_2c_1}{a_1b_2 - a_2b_1}$ and $y = \frac{c_1a_2 - c_2a_1}{a_1b_2 - a_2b_1}$

The above solution of equation can be written as :

$$\frac{x}{b_1c_2 - b_2c_1} = \frac{y}{c_1a_2 - c_2a_1} = \frac{1}{a_1b_2 - a_2b_1}$$

This result is shown by the following diagram, so that we can remembering the solution of given equation easily.

$$\frac{x}{\begin{array}{l} b_1 \nearrow c_1 \\ b_2 \searrow c_2 \end{array}} = \frac{y}{\begin{array}{l} c_1 \nearrow a_1 \\ c_2 \searrow a_2 \end{array}} = \frac{1}{\begin{array}{l} a_1 \nearrow b_1 \\ a_2 \searrow b_2 \end{array}}$$

In the above diagram, the direction of arrows shows the multiplication of the related numbers. First of all we multiply downwards and then subtract the multiplication upward from it.

In this method, the numbers are multiplied across, so it is called the cross multiplication method. In this system all the terms are taken to the left hand side and then right hand side become zero.

4.05. Condition for Solvability

In a pair of equations $a_1x + b_1y + c_1 = 0$, $a_2x + b_2y + c_2 = 0$, the solution of the equation depends on the ratio of corresponding coefficients.

1. First condition : if $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$, then the pair of linear equations is consistent with a unique solution.

2. Second condition : if $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$, then the pair of equations is inconsistent, *i.e.*, it has no solution.

3. Third condition : if $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$, then the pair of equations has infinitely many solutions.

Example 10. Solve the equations given below by cross multiplication method.

$$2x + 3y - 17 = 0 ; 3x - 2y - 6 = 0$$

Solution : By cross multiplication method, we get

$$\frac{x}{\begin{array}{l} 3 \nearrow -17 \\ -2 \searrow -6 \end{array}} = \frac{y}{\begin{array}{l} -17 \nearrow 2 \\ -6 \searrow 3 \end{array}} = \frac{1}{\begin{array}{l} 2 \nearrow 3 \\ 3 \searrow -2 \end{array}}$$

$$\Rightarrow \frac{x}{(3)(-6) - (-2)(-17)} = \frac{y}{(-17)(3) - (-6)(2)} = \frac{1}{(2)(-2) - (3)(3)}$$

$$\Rightarrow \frac{x}{-18-34} = \frac{y}{-51+12} = \frac{1}{-4-9}$$

$$\Rightarrow \frac{x}{-52} = \frac{y}{-39} = \frac{1}{-13}$$

$$\Rightarrow x = \frac{-52}{-13} \text{ and } y = \frac{-39}{-13}$$

$$\Rightarrow x = 4 \text{ and } y = 3$$

Hence, the solution of the equations is $x = 4, y = 3$

Example 11. Check the consistency of the equations given below. If they are consistent, solve them.

$$2x + 3y = 7$$

$$6x + 9y = 15$$

Solution : The equations are

$$2x + 3y = 7$$

$$6x + 9y = 15$$

Taking all terms to the left side, we get

$$2x + 3y - 7 = 0$$

and $6x + 9y - 15 = 0$

here $\frac{a_1}{a_2} = \frac{2}{6} = \frac{1}{3}$

and $\frac{b_1}{b_2} = \frac{3}{9} = \frac{1}{3}$

also $\frac{c_1}{c_2} = \frac{-7}{-15} = \frac{7}{15}$

Here, we see that $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

Hence, the pair of equations is inconsistent and there is no solution.

Exercise 4.3

Check the equations given below, if they have a unique solution, no solution or infinitely many solutions. In case there is a unique solution, find it;

1. $2x + y = 35$

2. $2x - y = 6$

- $$3x + 4y = 65$$
- $$x - y = 2$$
3. $3x + 2y + 25 = 0$ 4. $x + 2y + 1 = 0$
 $2x + y + 10 = 0$ $2x - 3y - 12 = 0$
5. If the following system of equation has no solution, find the value of k .
(i) $2x + ky = 1$, $3x - 5y = 7$
(ii) $kx + 2y = 5$, $3x + y = 1$
6. Find the solution of system of equations $mx - ny = m^2 + n^2$, $x + y = 2m$
7. Find the value of λ if the system of equations $3x + \lambda y + 1 = 0$, $2x + y - 9 = 0$
(i) has a unique solution (ii) has no solution.

4.06 Application of linear equations in two variables

With the help of a pair of simultaneous linear equations we can solve the practical problems related to our daily life. In solving such problems, we may use the following steps:

- (i) We use the variable for unknown quantities involved in the given problem.
- (ii) Convert the conditions which are given in the verbal form of problem in equation form by using the variables.
- (iii) On solving these equation by appropriate method. We get the value of variables.

Example 12. 10 students of a class participated in a essay competition. If the number of boys is 4 more than the number of girls, find the number of boys and the girls separately participated in the competition.

Solution : Let the number of boys participating in the competition be x and number of girls be y

Given that the total number of students participating in the competition is 10. So
number of boys + number of girls = 10

$$\Rightarrow \quad \quad \quad x + y = 10$$

It is also given that the number of boys is 4 more than the number of girls

Number of boys – number of girls = 4

$$x - y = 4$$

According to the situation we get a pair of equations as :

$$x + y = 10 \quad \quad \quad \dots (1)$$

$$x - y = 4 \quad \quad \quad \dots (2)$$

Adding equations (1) and (2), we get

$$2x = 14$$

$$\text{or } x = 7$$

Putting the value of x in equation (1), we get

$$7 + y = 10$$

$$\text{or } y = 10 - 7$$

$$\text{or } y = 3$$

Hence, the number of boys are 7 and the number of girls are 3.

Example 13. The ratio of the salaries of two persons is 9 : 7 and the ratio of their expenditure is 4 : 3. If each of the two persons saves ₹ 2000 per month, find out their salaries.

Solution : Let the salary of first person be ₹ x and the salary of second person by ₹ y .

According to the problem $x : y = 9 : 7$

$$\text{or } \frac{x}{y} = \frac{9}{7} \quad \text{or } 7x = 9y$$

$$\text{or } 7x - 9y = 0 \quad \dots (1)$$

The two persons save ₹ 2000 per month separately.

Their monthly expenditure are ₹ $(x - 2000)$ and ₹ $(y - 2000)$ respectively.

The ratios of their expenditure is 4 : 3

$$\therefore (x - 2000) : (y - 2000) = 4 : 3$$

$$\frac{(x - 2000)}{(y - 2000)} = \frac{4}{3}$$

$$\text{or } 3(x - 2000) = 4(y - 2000)$$

$$\text{or } 3x - 6000 = 4y - 8000$$

$$\text{or } 3x - 4y + 2000 = 0 \quad \dots (2)$$

From equation (1), we get $x = \frac{9y}{7}$

Substituting $x = \frac{9y}{7}$ in equation (2), we get

$$3\left(\frac{9y}{7}\right) - 4y + 2000 = 0$$

$$\text{or } \frac{27y}{7} - 4y + 2000 = 0$$

$$\text{or } 27y - 28y + 14000 = 0 \quad \text{or} \quad -y = -14000$$

$$\text{or } y = 14000$$

Putting the value of y in equation (1), we get

$$7x - 9(14000) = 0$$

$$\text{or } 7x - 126000 = 0$$

$$\text{or } 7x = 126000 \quad \text{or} \quad x = \frac{126000}{7}$$

$$\text{or } x = 18000$$

Hence the salaries of two persons are ₹ 18000 and ₹ 14000 respectively.

Example 14. The sum of the digits of a two digit number is 12. If 18 is subtracted from the number, the place of two digits is interchanged. Find digits the number.

Solution : Let the one's digit and ten's digits of the number be x and y respectively.

So the number is $x + 10y$

According to the problem, the sum of the digits of this number is 12. So

$$x + y = 12 \quad \dots (1)$$

and $x + 10y - 18 = 10x + y$ or $10y - y + x - 10x = 18$

$$\text{or } 9y - 9x = 18 \quad \text{or } 9x - 9y = -18$$

$$\text{or } x - y = -2 \quad \dots (2)$$

Adding equation (1) and (2), we have

$$2x = 10 \quad \text{or} \quad x = 5$$

Putting the value of x in equation (1), we get

$$5 + y = 12$$

$$\text{or } y = 12 - 5$$

$$\text{or } y = 7$$

Thus, the number $10y + x = 10 \times 7 + 5 = 75$

Exercise 4.4

Solve the problems given below :

1. In a two-digit number, the one's digit is 3 times the ten's digit. If 10 is added to the 2 times of the number, its digits interchange their places in the new number. Find the number.
2. The perimeter of a rectangle is 56 cm. The ratio of length and its breadth is 4 : 3. Find the length and breadth of the rectangle.
3. The ratio of two numbers is 3 : 4. If 5 is subtracted from each of the number, the ratio becomes 5 : 7. Find the numbers.
4. The age of a father is 5 years more than the age of 6 times of his son's age. After 7 years the age of father will be 3 more than the age of 3 times of his son's age. Find their present ages.
5. Ram said to Shyam, "If you give me ₹ 100, my money will be doubled to your money." Then Shyam said to Ram, "If you give me ₹ 10, my money will be 6 times to yours. Find how many rupees does each of them have?"
6. The cost of 4 chairs and 3 tables is ₹ 2100 and the cost of 5 chairs and 2 tables is ₹ 1750. Find the cost of one chair and one table separately.
7. If 3 times of a larger number is divided by a smaller number the quotient is 4 and remainder is 3 and when 7 times of the smaller number is divided by the larger number, then quotient is 5 and remainder is 1. Find the two numbers.
8. A two-digit number is 4 times the sum of its digits and 2 times the product of its digits. Find the number.
9. If one is added to the numerator and the denominator separately of a fraction, then the fraction becomes $\frac{4}{5}$ and if 5 is subtracted from both numerator and denominator, then the fraction becomes $\frac{1}{2}$. Find the fraction.
10. 5 years ago Geeta's age was 3 times of Kamla's age. After 10 years Geeta's age will be 2 times of Kamla's age. Find their present ages.
11. A man travels 370 km partly by train and partly by car. If he covers 250 km by train and the rest by car, he takes 4 hours. But, if he travels 130 km by train and the rest by car, he takes 18 minutes longer, Find the speed of the train and car.

Important Points

1. A linear equation in two variables is in the form of $ax + by + c = 0$, where a, b , and c are real numbers and $a \neq 0, b \neq 0$.
2. A linear equation in two variables has infinitely many solutions.
3. The nature of a pair of two linear equations $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ is given as:
 - (a) If $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$, the pair of equations are consistent and has a unique solution.
 - (b) If $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$, the pair of equations is consistent with infinitely many solutions.
 - (c) If $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$, the pair of equation is inconsistent and have no solution.
4. If x and y are digits at one's and ten's places respectively in a number, then the number will be $10y + x$.

Miscellaneous Exercise-4

Choose the correct answer (Questions 1 to 10)

1. Find the value of x , if $y = 2x - 3$ and $y = 5$.
(a) 1 (b) 2 (c) 3 (d) 4
2. The pair of solution that satisfy the equation $2x + y = 6$ is:
(a) (1, 2) (b) (2, 1) (c) (2, 2) (d) (1, 1)
3. If $\frac{4}{x} + 5y = 7$ and $x = -\frac{4}{3}$, then the value of y is:
(a) $\frac{37}{15}$ (b) 2 (c) $\frac{1}{2}$ (d) $\frac{1}{3}$
4. If $\frac{3}{x} + 4y = 5$ and $y = 1$, then the value of x is:
(a) 3 (b) $\frac{1}{3}$ (c) -3 (d) $-\frac{1}{3}$
5. If $x = 1$, then find the value of y in the equation $\frac{4}{x} + \frac{3}{y} = 5$:
(a) 1 (b) $\frac{1}{3}$ (c) 3 (d) -3
6. If one's digit and ten's digit of a number are x and y respectively, then the number will be:
(a) $10x + y$ (b) $10y + x$ (c) $x + y$ (d) xy
7. The son's age is one third of his mother's age. If the present age of the mother is x years, then after 12 years son's age will be:
(a) $\frac{x}{3} + 12$ (b) $\frac{x + 12}{3}$ (c) $x + 4$ (d) $\frac{x}{3} - 12$

8. The coordinate of a point at the x -axis is:
 (a) (2,3) (b) (2,0) (c) (0,2) (d) (2,2)
9. The coordinates of origin are:
 (a) (0,0) (b) (0,1) (c) (1,0) (d) (1,1)
10. In which quadrant does the point (3, -4) lie?
 (a) First (b) Second (c) Third (d) Fourth
11. In equation $5y - 3x - 10 = 0$, express y in terms of x , Find the point on the graph. Where the line represented by $5y - 3x - 10 = 0$ cuts y -axis.
12. Taking the values of x from -2 to 2 prepare a table of the equation $y = 2x + 1$. Also draw the graph of the equation.
13. Solve the simultaneous equations given below:
 $0.5x + 0.6y = 2.3$; $0.2x + 0.7y = 2.3$
14. Solve the system of the equations:
 $2x + 3y = 9$; $3x + 4y = 5$
15. Solve the simultaneous equations
 $\frac{1}{2x} - \frac{1}{y} = -1$; $\frac{1}{x} + \frac{1}{2y} = 8$; where $x \neq 0, y \neq 0$
16. There are two numbers such that if 7 is added to the smaller number then sum becomes double to the larger number and if 4 is added to the larger number it becomes three times of the smaller one. Find the two numbers.
17. Numerator of a fraction is 4 less than its denominator. If 2 is subtracted from the numerator and 1 is added to the denominator, then the denominator becomes 8 times of the numerator. Find the fraction.
18. The cost of 5 books and 7 pens is ₹ 79 and the cost of 7 books and 5 pens is ₹ 77. Find the cost of 1 book and 2 pens.
19. When a two digit number is multiplied by 9, it becomes 2 times of the number that is obtained by interchanging the digits. If the difference of the digits is 7, find the number.
20. In a triangle ABC , $\angle A = x^\circ$, $\angle B = 3x^\circ$ and $\angle C = y^\circ$. Now, if $5x^\circ - 3y^\circ + 30^\circ = 0$, prove that given triangle is a right angled triangle.
21. Solve the following equations graphically:
 (a) $x + y = 4$; $x = y$ (b) $x + y = 3$; $2x + 5y = 12$
 (c) $2x - 3y - 6 = 0$; $2x + y + 10 = 0$ (d) $2x + y - 3 = 0$; $2x - 3y - 7 = 0$
22. Solve the system of equations $2x - y = 1$; $x + 2y = 8$ graphically and find the coordinates of the points where corresponding lines intersect y -axis.

Answer

Exercise 4.1

- (1) $x = 6, y = 0$ (2) $x = 1, y = 4$ (3) No solution (4) $x = 3, y = 1$
(5) $x = -2, y = 3$ (6) inconsistent, no solution (7) $(4, -6)$
(8) inconsistent, no solution (9) $(1, 2)$
(10) coincide, infinitely many solutions (11) $(-2, 3)$

Exercise 4.2

- (1) $x = -21, y = 17$ (2) $x = 3, y = -2$ (3) $x = 5, y = -2$ (4) $x = -2, y = 5$
(5) $x = 1, y = -7$ (6) $x = 5, y = 3$ (7) $x = 5, y = 3$ (8) $x = 2, y = 3$
(9) $x = 14, y = 9$ (10) $x = 2, y = -3$ (11) $x = -1, y = 1$ (12) $x = \frac{1}{2}, y = \frac{1}{2}$
(13) $u = \frac{22}{31}, v = \frac{11}{23}$ (14) $x = \frac{1}{6}, y = \frac{1}{4}$ (15) $x = 3, y = 2$

Exercise 4.3

- (1) $x = 15, y = 5$ (2) $x = 4, y = 2$ (3) $x = 5, y = -20$ (4) $x = 3, y = -2$
(5) (i) $k = -\frac{10}{3}$; (ii) $k = 6$ (6) $x = (m+n), y = m-n$
(7) (i) For unique solution $\lambda \neq \frac{3}{2}$ (ii) For no solution $\lambda = \frac{3}{2}$

Exercise 4.4

- (1) 26 (2) length 16 cm, width 12 cm (3) 30 and 40
(4) age of father is 29 years and age of his son is 4 years.
(5) Ram's money is ₹ 40 and Shyam's money is ₹ 170.
(6) Cost of chair ₹ 150 and table is ₹ 500
(7) The larger number is 25 and smaller number is 18. (8) 36 (9) $7/9$
(10) the age of Geeta is 50 years, age of Kamla is 20 years.
(11) Train 100 km/h and car 80 km/h.

Miscellaneous Exercise 4

1. (D) 2. (C) 3. (B) 4. (A) 5. (C) 6. (A) 7. (A)
8. (B) 9. (A) 10. (D) 11. $y = \frac{3x+10}{5}, (0, 2)$
12.

x	-2	-1	0	1	2
y	-3	-1	1	3	5

 13. $x = 1, y = 3$ 14. $x = -21, y = 17$
15. $x = \frac{1}{6}, y = \frac{1}{4}$ 16. 5, 3 17. $\frac{3}{7}$ 18. ₹ 20 19. 18
21. (a) $x = 2, y = 2$ (b) $x = 1, y = 2$ (c) $x = -3, y = -4$ (d) $x = 2, y = -1$
22. $x = 2, y = 3$ first line meets the y-axis at point $(0, -1)$ and second line meets the y-axis at point $(0, 4)$.

Introduction to Plane Geometry, Line and Angle

5.01 Historical Introduction

Harappa and Mohan-jo-dro (now in Pakistan), Kalibanga (Rajasthan) and Lothal (Gujrat) testify clearly that a rich civilization flourished in a large track of land during the period extending from 2500 B.C. to 1750 B.C. in ancient India. The relics of this civilization prove that the people of this period had special knowledge of Geometry and Geometrical formations. On the basis of this knowledge they constructed buildings, roads, circles, arcs where mensuration is of great importance. Babylonian has also formulated formulae for finding out the area of various linear shapes which are available in Rhind Papyrus (1650 B.C.)

5.02 History of Indian Geometry

India has been the birth place of Geometry too. The foundation of Geometry had been laid in the shulav age or in the age of Vedang and Astronomy. During this period it was known by various names, such as Shulav mathematics, Shulav Science, Rajju Mathematics, Rajju-Figures. Rajju being synonym of shulav it began to be called Rajju Mathematics which later on changed to Geometry.

Similarly such terms as Rajju Mathematics, Kshetra Samas, Kshetra behaviour, Mensuration, Geometry and Boomiti were used for the work of measuring fields. Since ancient times altars for the performance of yagya used to be built. Geometry was the basis of their construction.

It is said in astronomy that :-

“Veda hi yajanartham bhi pravarithah”

That is, the Vedas also have been used in the work of performing yagya. According to the need of various yajanans (yagya). Different types of altars were required to be built. For this, various shape such as rectangular arcs, rectangular, triangular etc. were developed while constructing an altar care was taken that the areas of all they altars must be equal to the area of standard altar. Hence for this knowledge of geometrical formation such as forming a square on straight line, converting it into circle equal to the area of the square drawing a circle around

the square, drawing a circle within the square and doubling the area of the circle etc. was very essential.

If we think a bit seriously, we come to know that two words are very important in Geometry. They are Rajju (rope) and measurement. Hence the science or mathematics which was developed with the help of Shulav, began to be called Shulav science and Shulav mathematics. Indian mathematicians contributed a lot in this field. They formulated formulae for forming various shapes which were known by their names such as Baudhayan Shulav Formula, Apastamb Shulav formula, Katyayan Shulav formula, Manav Shulav formula, Mestrayan Shulav formula, Varah Shulav formula, Baudhav Shulv formula etc.

Among the main achievements of Shulav period “Sumkon Tribhuz ka Prameya” (theorem of right angle triangle), that is a square formed on hypotenuse is equal to the sum of the squares formed on remaining two arms. This theorem was widely used in India many centuries before Pythagoras (580 B.C.).

Baudhayan Theorem (800 B.C.)

**“Deergh chatur srisya khasnya Rajjuh Parshavmani
Tiryankmani yatprithambhute kurutastadubhayam karoti”**

The sum of the areas of squares formed on the perpendicular line and the base line of a rectangle is equal to the area of the square formed on the diagonal. It is worth noting that Pythagoras (580 B.C.) established this theorem about 300 years after 'Baudhayan'. Hence it is proper to call this theorem as Baudhayan theorem.

Among Indian Geometricians are Bhraham Gupta (598 A.D.) who found out the area of cyclic quadrilateral in terms of its perimeter and radius, Arya Bhat (476 A.D.) who found out the area of equilateral triangle, volume of pyramid and value of π and Bhaskar-II (1114 A.D.) who proved the Baudhayan theorem, by split method.

Later on the Greek mathematicians (300 B.C.) systematized this knowledge by providing its facts through inductive reasoning and published it in the book titled 'Elements'. These days we study Geometry in this way.

5.03 Basic Concepts

Geometry is studied by taking some basic concepts as basis. These basic concepts are understood by examples and experiences. For these, no proofs are given. There are three basic concepts in the study of Geometry which are very important. These are (1) Point (2) Line and (3) Surface. Now we shall study these with the help of some examples.

(1) Point : A minute sign made by a fine pointed pencil, the corner of the black board are the examples of a point. If the point of the pencil is fine, it will make a fine point. Generally the points are shown by the capital letter of English alphabet that is A, B, C, D etc.

(2) Line : If we fold a piece of paper, a line is formed on the surface of the piece of paper. Line is also regarded as a concept in the same way as a point is regarded. In the figure 5.01, the line AB is shown by AB . The line can also be shown by small letters of English alphabet such as l, m, n .

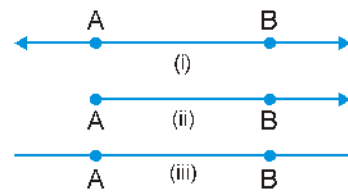


Fig. 5.01

(3) Plane : Floor, roof, wall, table are similar examples of plane. Although there are some dimensions of these examples, yet the geometrical planes extend endlessly in all the directions. A plane is extended in length and width, it has no thickness.

(4) Postulates : In Geometry there are some concepts which are regarded as true without any proof and on the basis of which other geometrical facts are proved. Such facts are called 'Postulates'. Some of the main postulates are-

- (i) There are infinite points on a line.
- (ii) A line can be extended as much as we desire.
- (iii) Infinite number of lines can be drawn from a point.
- (iv) One and only one straight line can be drawn through two points.
- (v) One and only one line can be drawn from a point parallel to a line.
- (vi) All the right angles are equal.
- (vii) Like supplementary and complementary angles are equal to each other.
- (viii) A line segment can be bisected at one point only.
- (ix) An angle can be bisected by one line only.

5.04 Inductive and Deductive Reasoning

Various rules which are established by various examples. Empirical findings are called inductive reasoning. Such findings are always true.

A special method of reasoning wherein the rules are proved with the help of evidences is known as deductive reasoning.

Theorem (Prameya) and Construction (Nirmeya) :

(1) Theorem : The rules which are verified by the inductive reasoning are called theorem. In geometry following steps are adopted in proving a theorem.

Corollary : On proving the theorem some results are drawn, which are understood easily. Such results are called "Corollary".

Constructions : The geometrical forms formed by using geometric rules are called 'Construction'.

5.05 Geometric Symbols

The terms used in geometry are written in the form of symbols. Symbols of some words are given in the following table :

S.No.	Words	Symbol	S.No.	Words	Symbol
1.	Because	\therefore	8.	Right angle	\perp
2.	Therefore	\therefore	9.	Perpendicular	\perp
3.	Greaterthan	$>$	10.	Triangle	\triangle
4.	Lessthan	$<$	11.	Parallel	\parallel
5.	Congruent	\cong	12.	Circle	\bigcirc
6.	Similar	\sim	13.	Arc	\frown
7.	Angle	\angle	14.	Not equal to	\neq

5.06 Angle and its Measurements

Angle : Any two rays whose starting point is same make an angle. In fig 5.02, starting from O and two rays \overrightarrow{OA} and \overrightarrow{OB} are starting. In this figure angle at point O is known as angle. According to figure, one ray is at point A and second ray is at point B, then this angle can be expressed as $\angle AOB$ or $\angle BOA$, OA and OB are sides of or Common point O is known as vertex of angle. Sometimes for convenience an angle is denoted by numbers or words as given is fig. 5.03 and 5.04.

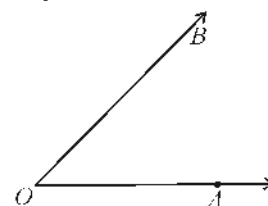


Fig. 5.02

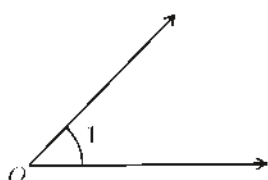


Fig. 5.03

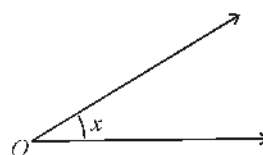


Fig. 5.04

Measurement of Angle : Suppose a revolving line with respect to point O is in OA position reaches in OB position as shown in fig. 5.05, then this revolving position is known as $\angle AOB$ and quantity of this angle is known as its measurement.

If a line OA by completing a round around O and comes in its original position, then measurement of such angle is distributed in 360 equal parts and it is denoted by 360° (degree) so similarly 1 Part = 1 degree = 1° and 360 part = 360° .

In rotation form 1 degree; 1 minute and 1 seconds represents 1° , 1' and 1'' respectively.

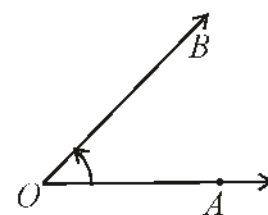


Fig. 5.05

If every degree is divided into 60 equal parts, then such each part is known as one kala (1 minute), if one minute is divided into 60 equal parts, then each part is known as vikala (second).

$$1^\circ = 60 \text{ (minutes)} = 60 \text{ kala}$$

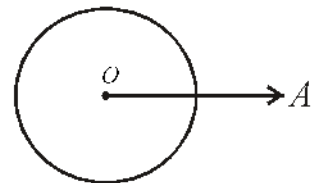
$$1 \text{ kala (minuts)} = 60 \text{ vikala}$$

In symbolic form of one degree, one minute and one second can be expressed 1° , $1'$ and $1''$. For measurement of angle use of a protractor, which have 0° to 180° scale.

Vertically Opposite Angles

Two angles are called a pair of vertically opposite angles, if their arms form two pairs of opposite rays. In Fig. 5.07 lines AB and CD intersect each other at point O and makes angle

$\angle AOC$ and $\angle BOD$, $\angle AOD$, $\angle BOC$ which are vertically opposite angle.



(1 round = 360°)

Fig. 5.06

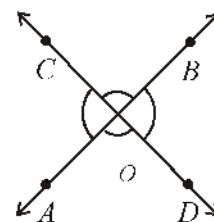


Fig. 5.07

Angle around a point

The sides of vertically angles are opposite rays.

Angle on around of a point : if various rays are starting from a point and angles obtained in this manner are know as angles around a point. As we studied that in this measurement an angle made around a point by revolving ray is 360° .

Here, sum of angle around point O is 360° .

$$\text{or, } \angle 1 + \angle 2 + \angle 3 + \angle 4 + \angle 5 + \angle 6 = 360^\circ$$

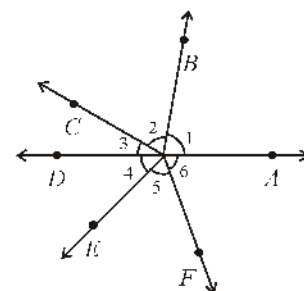


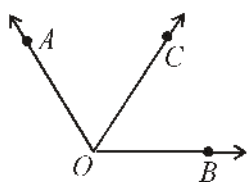
Fig. 5.08

5.07 A linear Pair of Angles

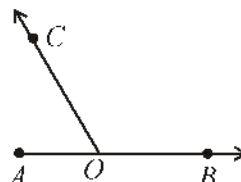
In Fig. 5.09, carefully observe the pair of angles.

Every pair of angles ($\angle AOC$ and $\angle BOC$) are adjacent angles.

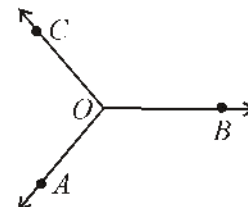
In Fig. 5.09 (ii), a pair of angle is such that their total is 180° such angle is known as linear angle combination. It is clear that linear angle combination of adjacent angles are supplementary.



(i)



(ii)



(iii)

Fig. 5.09

Defination :

If the non-common arms of two adjacent angles form a line, then these angles are called linear pair of angles.

Linear Pair Axiom

Theorem 5.1 : If two lines intersect each other, then the vertically opposite angles are equal to each other.

Given : Lines AB and CD intersect each other at point O.

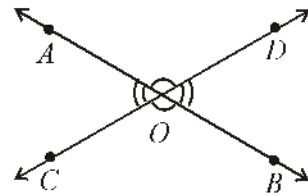


Fig. 5.10

To Prove : Vertically opposite angles

$$\angle AOC = \angle DOB \text{ and } \angle AOD = \angle BOC$$

Proof:

\therefore Ray OD stands on line AB.

$$\therefore \angle AOD + \angle DOB = 180^\circ \quad (\text{By linear pair Axiom}) \quad \dots\dots(i)$$

Similiary, $\angle AOC + \angle AOD = 180^\circ \quad \dots\dots(ii)$

From equation (i) and (ii), we get

$$\angle AOD + \angle DOB = \angle AOC + \angle AOD$$

$$\Rightarrow \angle AOC = \angle DOB$$

Similarly, we can prove that :

$$\angle AOD = \angle BOC$$

Hence proved.

Cordallary 1 : If two or more than two lines intersect each other on a point, then the sum of the angles on intersecting point is 360° .

Cordallary 2 : Bisector of vertically opposite angles are in a line.

Illustrative Examples

Example 1. In Fig. 5.11, $\angle 1$ and $\angle 2$ are linear pair $\angle 2 - \angle 1 = 18^\circ$. If then find out the value of $\angle 1$ and $\angle 2$.

Sol. : $\angle 2 + \angle 1 = 180^\circ$ (Linear pair axiom) $\dots\dots(1)$

Given, $\angle 2 - \angle 1 = 18^\circ \quad \dots\dots(2)$

On adding equation (1) and (2), we get

$$\angle 2 = \frac{198}{2} = 99^\circ \quad \dots\dots(3)$$

Substitution $\angle 2 = 99^\circ$ in equation(1), we get

$$99^\circ + \angle 1 = 180^\circ$$

$$\angle 1 = 180^\circ - 99^\circ = 81^\circ$$

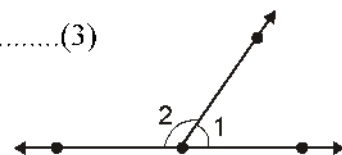


Fig. 5.11

Thus, $\angle 1 = 81^\circ$ and $\angle 2 = 99^\circ$

Example 2. In Fig. 5.12, find the value of $\angle AOB$, $\angle BOC$, $\angle COD$ and $\angle DOE$ where $\angle AOE = 100^\circ$.

Sol : We know that the sum of the angles at a point is 360°

$$\begin{aligned} \therefore y + 2y + 4y + 6y + 100^\circ &= 360^\circ \\ \Rightarrow 13y &= 260^\circ \\ \Rightarrow y &= \frac{260}{13} = 20^\circ \\ \angle AOB &= y = 20^\circ, \\ \angle BOC &= 2y = 40^\circ, \\ \angle COD &= 4y = 80^\circ \\ \angle DOE &= 6y = 120^\circ \end{aligned}$$

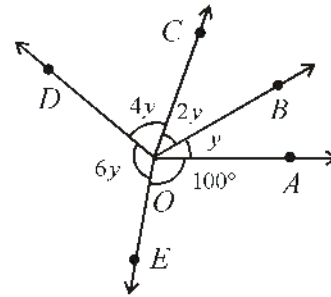


Fig. 5.12

Example 3. In Fig. 5.13, find the value of $\angle x$, $\angle y$ and $\angle z$.

Sol : Here, $\angle y = 140^\circ$ (Vertically opposite angles)
and $\angle x + 140^\circ = 180^\circ$ (Linear pair)

$$\begin{aligned} \Rightarrow \angle x &= 40^\circ \\ \text{Also } \angle x &= \angle z \text{ (Vertically opposite angle)} \\ \Rightarrow \angle z &= 40^\circ \end{aligned}$$

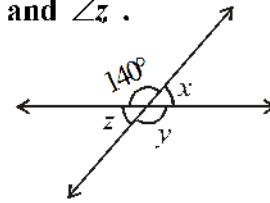


Fig. 5.13

Example 3. In Fig. 5.14, line AB and line OP meet at point O. line OD and OE are the bisectors of angle $\angle BOP$ and $\angle POA$, then find the measure of $\angle EOD$.

Sol : Let $\angle BOP = x$, and $\angle POA = y$

From figure,

$$\angle x + \angle y = 180^\circ \text{ (Linear pair) } \dots (i)$$

From figure

$$\angle x = 2\angle 1, \angle y = 2\angle 2 \dots (ii)$$

\therefore From equation (i) and (ii), we get

$$2\angle 1 + 2\angle 2 = 180^\circ \text{ (Linear pair)}$$

$$\Rightarrow 2(\angle 1 + \angle 2) = 180^\circ$$

$$\Rightarrow \angle 1 + \angle 2 = 90^\circ$$

Thus $\angle EOD = 90^\circ$

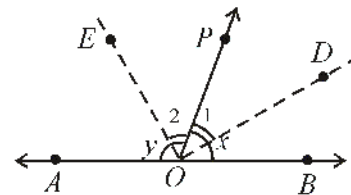


Fig. 5.14

Example 5. An angle is half of its supplementary angle, then find out the value of each angle.

Sol : Let one of the angle is x , then the value of its supplement is angle = $\frac{(180 - x)}{2}$

We know that sum of supplementary angles is 180° .

$$\text{So } x = \frac{180 - x}{2}$$

$$2x = 180 - x$$

$$3x = 180^\circ$$

$$\therefore x = 60^\circ$$

Hence, the angles are 120° and 60° .

Exercise 5.1

- If angles $(2x + 4)$ and $(x - 1)$ form a linear pair, then find out the measure of the angles.
- In Fig. 5.15 :
 - Find the measure of $\angle BOD$
 - Find the measure of $\angle AOD$

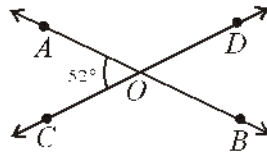


Fig. 5.15

- Which pair of angles have vertically opposite angles?
 - Find the adjacent supplementary angles of $\angle AOC$.
- In the given figure 5.16. if $\angle PQR = \angle PRQ$, then prove $\angle PQS = \angle PRT$.

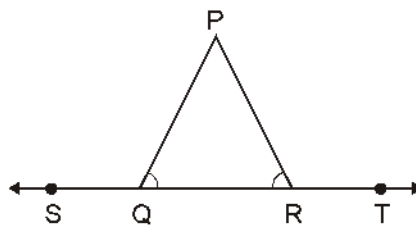


Fig. 5.16

- In fig. 5.17, OP, OQ, OR and OS are four rays, then prove that :
 $\angle POQ + \angle QOR + \angle ROS + \angle SOP = 360^\circ$

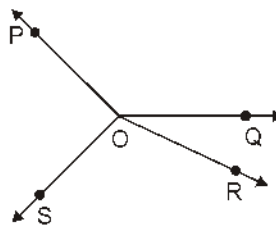


Fig. 5.17

5. In Fig. 5.18 $\angle x + \angle y = \angle p + \angle q$, then prove that AOB is a straight line.

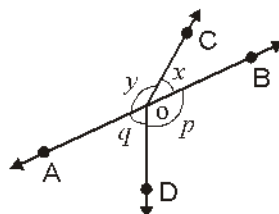


Fig. 5.18

5.08 Intersecting lines and Parallel Lines

If you are asked to draw pairs of two straight lines, then lines will be definitely shown as in Fig. 5.19.

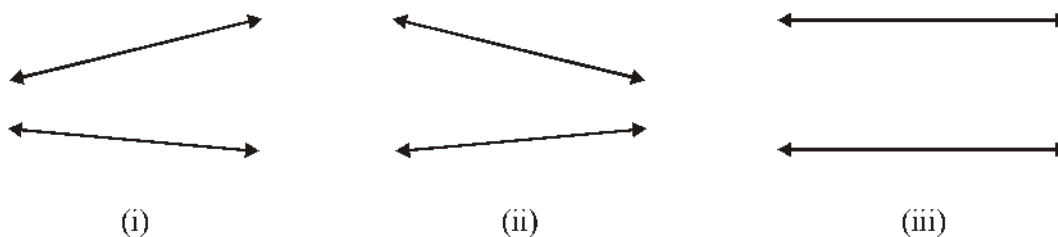


Fig. 5.19

Now, measure the distance between the lines in two different places. What do you observe?

No doubt you will see that in Fig. 5.19 (i) and (ii), the distance between the pair of lines is not equal at every point, i.e. if we extend these lines in forward or backward direction, then these lines will intersect each other. Hence, these lines are intersecting lines. In Fig. 5.19 (iii), the distance between the lines is equal at every point, if we extend the lines in forward or backward direction up to infinite, then they will not intersect each other, Hence, there are parallel lines.

Transversal : If the group of two or more lines are intersected by a line at different points, then it is known as transversal as shown in Fig. 5.20, line l , intersect lines m and n at points P and Q respectively. So line l is a transversal for lines m and n . Do you observe that four angles at points P and Q . Yes, at point P , four angles $\angle 1, \angle 2, \angle 3, \angle 4$ and at point Q four angles $\angle 5, \angle 6, \angle 7, \angle 8$ are formed.

Here $\angle 1, \angle 4, \angle 6$ and $\angle 7$ are exterior angles and $\angle 2, \angle 3, \angle 5$ and $\angle 8$ are interior angles.

Remember you have studied the numbering of angles made by a transversal. Let us remind them again.

- (a) Corresponding angle :
 (i) $\angle 1$ and $\angle 5$ (ii) $\angle 2$ and $\angle 6$
 (iii) $\angle 3$ and $\angle 7$ (iv) $\angle 4$ and $\angle 8$
- (b) Alternate interior angles :
 (i) $\angle 2$ and $\angle 8$ (ii) $\angle 3$ and $\angle 5$
- (c) Alternate exterior angles:
 (i) $\angle 1$ and $\angle 7$ (ii) $\angle 4$ and $\angle 6$
- (d) Interior angles on the same side of the transversal:
 (i) $\angle 2$ and $\angle 5$ (ii) $\angle 3$ and $\angle 8$

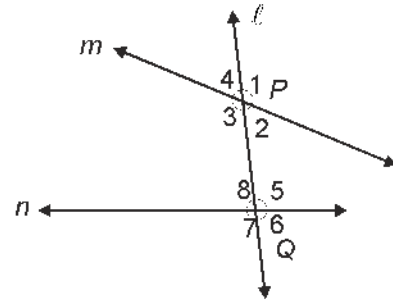


Fig. 5.20

When a transversal line intersect two or more parallel lines, then :

- (i) Corresponding angles are equal
 (ii) Alternate angles are equal
 (iii) Interior angles on the same side of a transversal line are supplementary and Opposite all of the statements are also true. If a transversal line cuts two lines and corresponding angles are same then the lines are parallel.

Theorem 5.2. *If a transversal intersect two or more parallel lines, then alternate interior angles are equal to each other.*

Given : l and m are two parallel lines and a transversal intersect them at points G and H . ($\angle 2, \angle 1$) and ($\angle 3, \angle 4$) are pair of alternate angles.

To Prove : $\angle 1 = \angle 2$ and $\angle 3 = \angle 4$

Proof

$$\angle 2 = \angle 6 \text{ (Vertically opposite angles)}$$

... (i)

$$\angle 1 = \angle 6 \text{ (Corresponding angles)}$$

... (ii)

From equation (1) and (2), we get

$$\Rightarrow \angle 1 = \angle 2$$

$$\text{Similarly } \angle 4 = \angle 5 \text{ (Corresponding angles) } \dots \text{ (iii)}$$

$$\text{and } \angle 3 = \angle 5 \text{ (Vertically opposite angles) } \dots \text{ (iv)}$$

From equation (iii) and (iv), we get

$$\Rightarrow \angle 3 = \angle 4$$

Hence proved

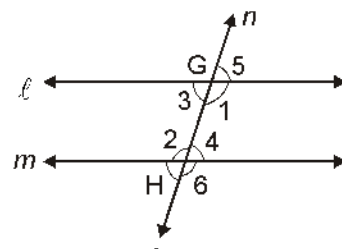


Fig. 5.21

Theorem 5.3 (Converse of Theorem 5.2)

If a transversal intersects two lines and alternate interior angles are equal, then the lines are parallel to each other.

Given : l and m are two lines which is intersected by a transversal n at points G and H , then alternate angles are $\angle 2 = \angle 8$ and $\angle 3 = \angle 5$.

To Prove : $l \parallel m$

Proof :

$$\angle 1 = \angle 3 \text{ (Alternate interior angles) } \dots\dots(1)$$

$$\angle 3 = \angle 5 \text{ (Given) } \dots\dots(2)$$

From equation (1) and (2), we get $\angle 1 = \angle 5$ (But these are corresponding angles)

So, $l \parallel m$

Hence Proved.

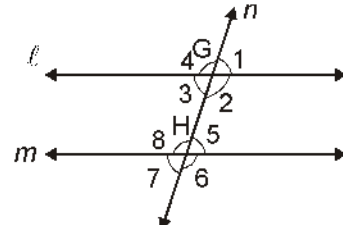


Fig. 5.22

Theorem 5.4. *If a transversal intersect two parallel lines, then the sum interior angles is equal to two right angles or 180° .*

Given : l and m two parallel lines which are intersected by a transversal n at point G and H and angles at G and H are $\angle 1, \angle 2, \angle 3, \angle 4$ and $\angle 5, \angle 6, \angle 7, \angle 8$ are formed.

To Prove : $\angle 2 + \angle 5 = 180^\circ$ and $\angle 3 + \angle 8 = 180^\circ$

Proof $\angle 1 + \angle 2 = 180^\circ$ (Linear pair) ... (i)

$$\angle 1 = \angle 5 \text{ (Corresponding angles) } \dots \text{ (ii)}$$

From equation (i) and (ii), we get

$$\angle 2 + \angle 5 = 180^\circ$$

Similarly, $\angle 3 + \angle 4 = 180^\circ$ (linear pair) ... (iii)

and $\angle 4 = \angle 8$ (Corresponding angles) ... (iv)

From equations (iii) and (iv), we get

$$\angle 3 + \angle 8 = 180^\circ$$

Hence proved

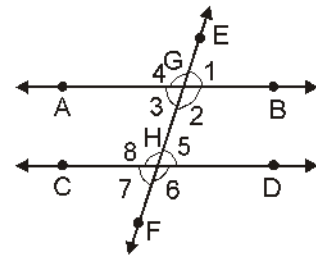


Fig. 5.23

Theorem 5.5 (Converse of Theorem 5.4)

If a transversal intersect two lines, and the sum of angles of the same side of the transversal is two right angle, then lines are parallel.

Given : l and m are two lines which is intersected by a transversal n at points G and H , and formed angles $\angle 1, \angle 2, \angle 3, \angle 4$ and $\angle 5, \angle 6, \angle 7, \angle 8$ then alternate angles are such that : $\angle 2 + \angle 5 = 180^\circ$ and $\angle 3 + \angle 8 = 180^\circ$

To prove : $l \parallel m$

Proof :

$$\angle 1 + \angle 2 = 180^\circ \quad \text{(Linear pair axiom)} \quad \dots(1)$$

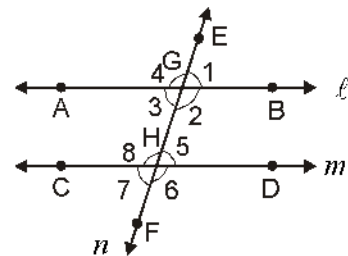


Fig. 5.24

$$\angle 2 + \angle 5 = 180^\circ \quad (\text{Given}) \quad \dots(2)$$

From equation (1) and (2), we get

$$\angle 1 = \angle 5$$

But these are corresponding angles.

Thus, $\ell \parallel m$

Theorem 5.6 : *If two lines are parallel to third line, then all three lines will be parallel to each other, i.e., if $\ell \parallel n$ and $m \parallel n$, then $\ell \parallel m$*

To prove : $\ell \parallel m$

Proof : Since, $\ell \parallel n$

$$\Rightarrow \angle 1 = \angle 9 \quad (\text{Corresponding angles}) \quad \dots(1)$$

$m \parallel n$ and PQ is transversal

$$\angle 5 = \angle 9 \quad (\text{Corresponding angles}) \quad \dots(2)$$

From equations (1) and (2)

$$\angle 1 = \angle 5 \quad \dots(3)$$

Since, $\angle 1$ and $\angle 5$ are corresponding angles, therefore by corresponding angles property $\ell \parallel m$.

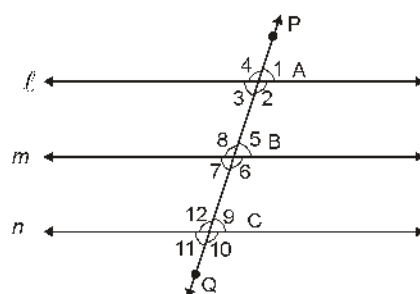


Fig. 5.25

Hence Proved

Example 6. In Fig. 5.26 lines $\ell \parallel m$ and a transversal n intersects them. If $\angle 1 = 55^\circ$, then find out the remaining angles.

Sol : Since, $\angle 3 = \angle 1$ (Vertically opposite angles)

$$\angle 1 = 55^\circ \quad (\text{Given})$$

Therefore,

$$\angle 3 = 55^\circ$$

$$\angle 1 + \angle 4 = 180^\circ \quad (\text{Linear Pair axiom})$$

$$\Rightarrow 55^\circ + \angle 4 = 180^\circ \Rightarrow \angle 4 = 125^\circ$$

$$\text{Since, } \angle 2 = \angle 4 \quad (\text{Vertically opposite angles})$$

$$\Rightarrow \angle 2 = 125^\circ$$

$$\text{Since, } \angle 1 = \angle 6 \quad (\text{Corresponding angles})$$

$$\angle 1 = 55^\circ$$

$$\text{Since, } \angle 4 = \angle 8 \quad (\text{Corresponding angles})$$

$$\angle 4 = 125^\circ$$

$$\text{Since, } \angle 7 = \angle 6 \quad (\text{Vertically opposite angles})$$

$$\Rightarrow \angle 6 = \angle 7 = 55^\circ$$

$$\text{Since, } \angle 5 = \angle 8 \quad (\text{Corresponding angles})$$

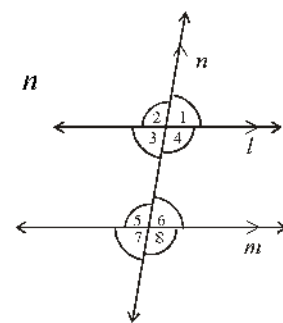


Fig. 5.26

$$\begin{aligned} \Rightarrow \quad & \angle 6 = 55^\circ \\ & \angle 5 = \angle 8 \\ \text{and} \quad & \angle 8 = 125^\circ \quad \therefore \quad \angle 5 = 125^\circ \end{aligned}$$

Example 7. In Fig. 5.27, $\ell \parallel m$, then find out the equal angles and also write the reason.

Sol: Here $\angle 1 = \angle 2$ (Alternate angle)
 $\angle 3 = \angle 4$ (Alternate angle)
 $\angle 5 = \angle 6$ (Vertically opposite angle)
 $\angle 3 = \angle 7$ (Vertically opposite angle)
 $\angle 4 = \angle 7$ (Corresponding angle)

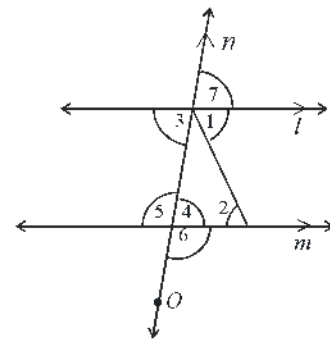


Fig. 5.27

Example 8. In Fig. 5.28, m and n two plane mirror which are perpendicular to each other. Show that incident ray CA is parallel to reflected ray BD .

Sol: Let us consider that perpendicular of A and B meet at point P because both the glasses are perpendicular.
 \therefore $BP \parallel OA$ and $AP \parallel OB$

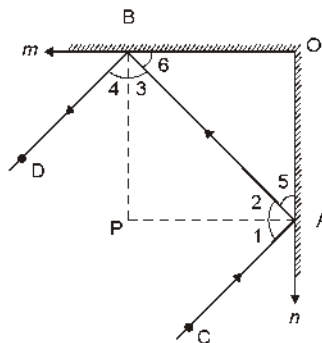


Fig. 5.28

\therefore $BP \parallel OA$ and BA is a transversal.
 \therefore $\angle 3 = \angle 5$ (Alternate angles) ... (1)

and $PA \perp OA \Rightarrow \angle PAO = 90^\circ$
 $\angle PAO = \angle 2 + \angle 5 = 90^\circ$... (2)

From equation (1) and (2), we get
 $\angle 2 + \angle 3 = 90^\circ$... (3)

\therefore $\angle 1 = \angle 2$ and $\angle 4 = \angle 3$ (angle of Incidence = angle Reflection) ... (4)

From equation (3) and (4), we get
 $\angle 1 + \angle 4 = 90^\circ$... (5)

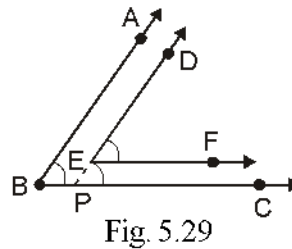
Adding equations (3) and (5), we get

$$\angle 1 + \angle 2 + \angle 3 + \angle 4 = 180^\circ$$

$\Rightarrow \angle CAB + \angle DBA = 180^\circ$ (Interior angles on the same side of the transversal)

Thus, $CA \parallel BD$

Example 9. In Fig. 5.29, $BA \parallel ED$ and $BC \parallel EF$. Show that : $\angle ABC = \angle DEF$



Sol : Extend DE in such away that it will meet BC at point P .

\therefore $BC \parallel EF$ (Given)

If we consider DP as transversal, then

$$\angle DEF = \angle DPC \quad (\text{Corresponding angles}) \quad \dots (1)$$

$$AB \parallel DE \quad (\text{Given})$$

then $AB \parallel DP$ (DE is extended up to P) and consider BC as a transversal.

$$\angle ABC = \angle DPC \quad (\text{Corresponding angles}) \quad \dots (2)$$

From equation (1) and (2), we get

$$\angle ABC = \angle DEF$$

Exercise 5.2

- In Fig. 5.30, lines AB, CD and EF are parallel to each other. Find the values of $\angle x, \angle y, \angle z$ and $\angle p$

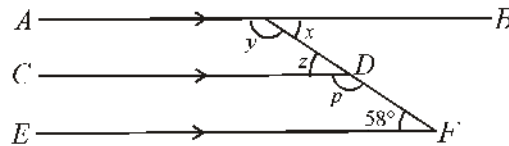


Fig. 5.30

- In Fig. 5.31, $AB \parallel EF$ find the values of $\angle x$ and $\angle y$.

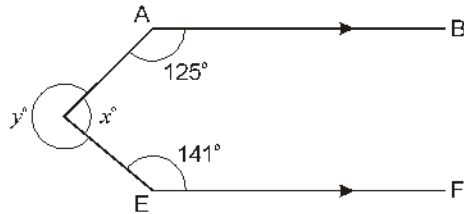


Fig. 5.31

3. In Fig. 5.32, if $\ell \parallel m$, then find the angles equivalent to $\angle 1$.

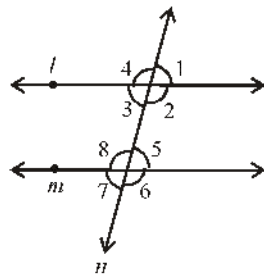


Fig. 5.32

4. In Fig. 5.33, $\angle 1 = 60^\circ$ and $\angle 6 = 120^\circ$, then show that m and n are parallel.

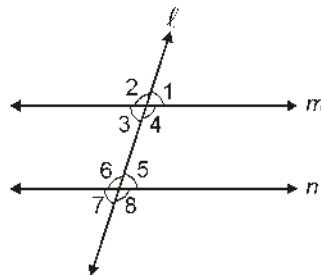


Fig. 5.33

5. AP and BQ are two bisectors of alternate angles of two parallel lines ℓ and m and its transversal n . Show that $AP \parallel BQ$.
6. In Fig. 5.34, $BA \parallel ED$, then show: $\angle ABC + \angle DEF = 180^\circ$.

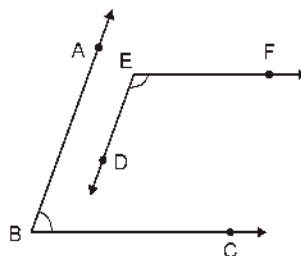


Fig. 5.34

7. In Fig. 5.35, $DE \parallel QR$ and AP and BP are bisectors of $\angle EAB$ and $\angle RBA$, then

find out the value of $\angle APB$.

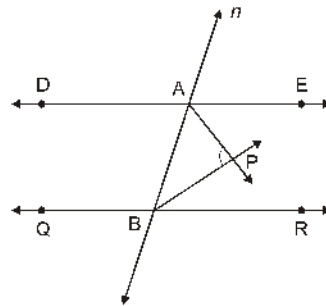


Fig. 5.35

8. If two straight lines are perpendicular to two parallel lines, then show that these straight lines are parallel to each other.
9. In Fig. 5.36, $AB \parallel CD$, $CD \parallel EF$ and $y : z = 3 : 7$, then find the value of x .

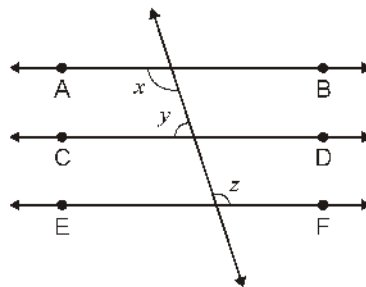


Fig. 5.36

10. In Fig. 5.37, PQ and RS are two mirrors placed parallel to each other. An incident ray AB strikes the mirror PQ to B , the reflected ray moves along the path BC and strikes the mirror RS at C and again reflects back along CD . Prove that $AB \parallel CD$.

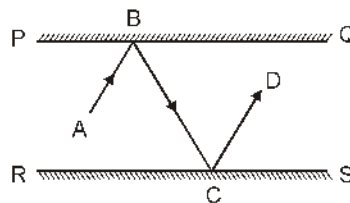


Fig. 5.37

11. In Fig. 5.38, if $PQ \parallel RS$, $\angle MXQ = 135^\circ$ and $\angle MYR = 40^\circ$, then find $\angle XMY$.

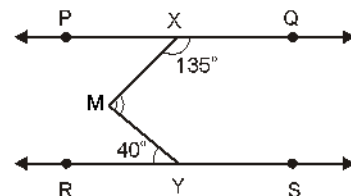


Fig. 5.38

5.9. Basic Constructions

While proving the theorem or solving the questions related to theorem which figures, shapes are made they have very less accuracy but geometric construction with the help of geometric tools like ruler, protector, set squares and compasses can be made more accurate. During drawing following points should be remember.

1. Whatever geometric is to be made, first we have to draw the rough diagram with the dark pencil.
2. Steps of drawing should be explained in words.

Few Simple Construction :

You have studied some geometric construction in previous classes such as to draw a line of desired length and to draw an angle of desired measure. You have also studied various geometric facts in pervious chapter in this book. On the basis of previous chapter's knowledge and facts we can draw few geometric construction.

Geometric construction 1

Bisection of a given line segment i.e., bisection of line segment AB.

Construction : Draw a line segment AB of desired length. Taking A and B as a centre and take arc of more than half radius on the both side of line segment AB which intersect each other at point P and Q. Now, join PQ where it will intersect the line AB marked that point O. Point O is the point of bisection of line AB as shown in Fig. 5.39.

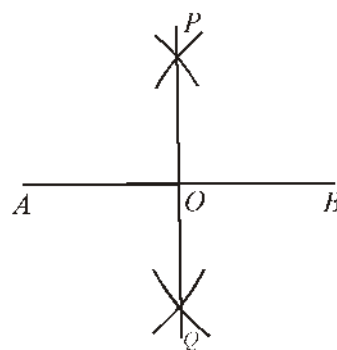


Fig. 5.39

Base of Construction : The point which have equal distance from the two given points is perpendicular bisector of the line joining them. PQ is the perpendicular bisector of line segment AB.

Geometric Construction 2

Bisection of any given angle say $\angle BAC$

Construction : Taking point A as centre of given angle $\angle BAC$ and with any radius draw an arc, which interest the side AB and AC at point P and Q. Now, taking P and Q as a centre and radius more than half of PQ, draw two arcs which intersect each other at point O. Now, join AO which bisect $\angle BAC$.

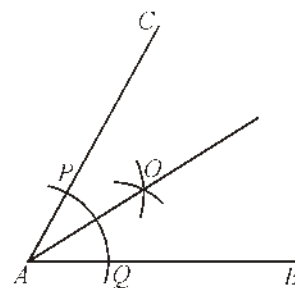


Fig. 5.40

Base of Construction : The points which have equal distance from lines AC and AB is bisector of $\angle CAB$. Now join AO. The line between the angles is known as bisector of $\angle BAC$.

Geometric Construction 3

Draw an angle of 60° with ruler and compass.

Draw an angle of 60° on line segment AB at point O .

Construction : Taking O as centre draw an arc of any radius on line AB which intersect AB at point P . Now taking P as centre draw an arc with same radius which will intersect the arc at point Q , join OQ . Thus, $\angle BOQ = 60^\circ$ is the required angle as shown in Fig. 5.41.

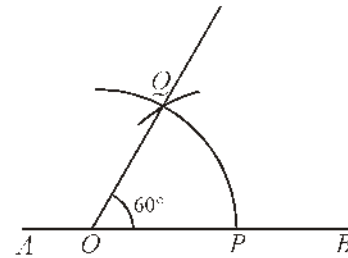


Fig. 5.41

Base of Construction : Equilateral triangle ΔPOQ which have every angle 60° .

Geometric Construction 4

Draw an angle of 120° with the help of ruler and compass.

Draw an angle of 120° on line AB at point O .

Construction : Taking O as a centre draw an arc of any radius which will intersect AB at point P . Now, taking P as centre draw an arc of same radius as two times (twice) which will intersect the arc on points Q and R respectively. Now, join OR . So, $\angle BOR = 120^\circ$, as shown in Fig. 5.42.

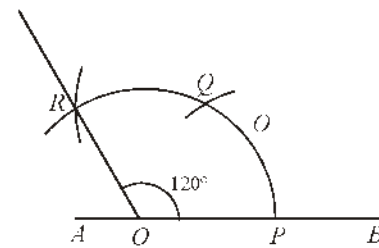


Fig. 5.42

Geometric Construction 5

Construction of various angles

(A) *Construction of angle 30° :*

$$\left[\because 30^\circ = \frac{60^\circ}{2} \right]$$

Construction : With the help of ruler and compass draw an angle BOQ of 60° according to compass and geometric construction no. 2. Draw the bisector of $\angle BOQ$. Hence, $\angle BOR = 30^\circ$ (Fig. 5.43).

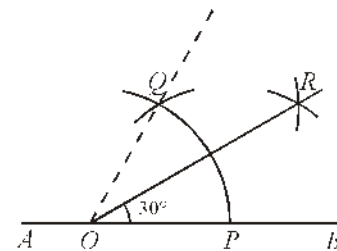


Fig. 5.43

(A) *Construction of 90° angle :*

First Method :

$$\left[\because 90^\circ = 60^\circ + \frac{60^\circ}{2} \right]$$

Construction : With the help of ruler and compass

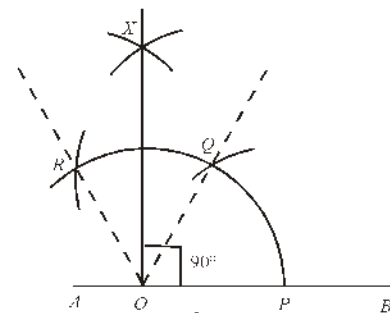


Fig. 5.44

draw 60° and 120° take two arcs and mark then Q and R points. On the basis of compass and geometric construction 2. Draw the bisector $\angle QOR$ and draw OX similar $\angle BOX = 90^\circ$ (fig. 5.44)

Second Method :

$$\left[\because 90^\circ = \frac{180^\circ}{2} \right]$$

Construction : Angle on straight line is $BOA = 180^\circ$ according to compass and geometric construction and bisector OR.

$\therefore \angle BOR = 90^\circ$ (Fig. 5.45)

(C) Construction of 45°

$$\left[\because 45^\circ = \frac{90^\circ}{2} \right]$$

Construction : Draw a 90° angle with the help of ruler and compass $\angle BOR = 90^\circ$. According to compass and geometric construction 2, bisector of $\angle BOR$ by OS (Fig. 5.46).

So, $\angle BOS = 45^\circ$

(D) Construction of 135° :

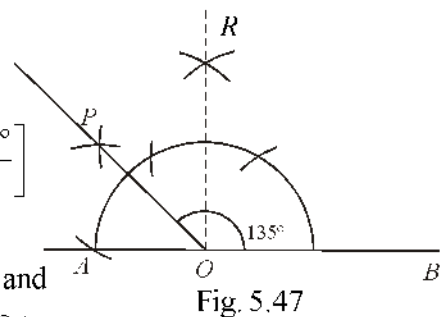
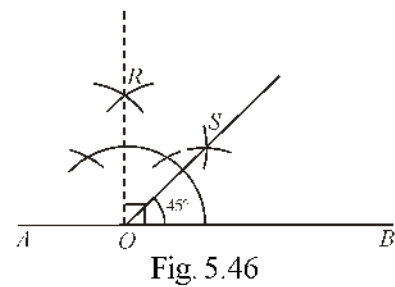
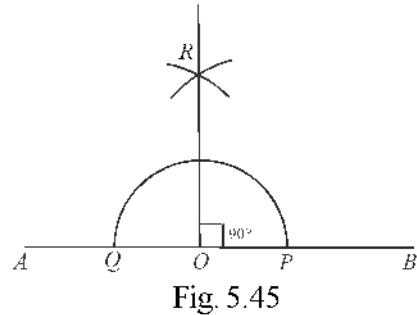
$$\left[\because 135^\circ = 90^\circ + \frac{90^\circ}{2} \text{ or } 180^\circ - \frac{90^\circ}{2} \right]$$

Construction : Draw a 90° angle with the help of ruler and compass $\angle BOR$ according to compass and geometric construction 2. Draw the bisector of $\angle ROA$ and its bisector by OP .

So, $\angle BOP = 135^\circ$.(Fig. 5.47)

Similarly, you can draw the following angles

1. $15^\circ = \frac{30^\circ}{2}$
2. $75^\circ = 60^\circ + 15^\circ$
3. $105^\circ = 90^\circ + 15^\circ$
4. $150^\circ = 120^\circ + 30^\circ$ or $150^\circ = 180^\circ - \frac{60^\circ}{2}$
 $= 180^\circ - 30^\circ$



Geometric Construction 6

Draw an equivalent angle to an angle drawn on a point of a line.

Construction : Draw an equivalent angle equal to $\angle D$ of line segment AB at a point O . Taking D as centre and taking appropriate radius. Draw an arc which will intersect the sides at P and Q . Now, taking O as centre, draw the same arc which will intersect the line segment AB at R .

Now, taking R as a centre, draw an arc of radius PQ which will intersect the previous arc at S . Now, join OS then $\angle BOS$ is desired angle.

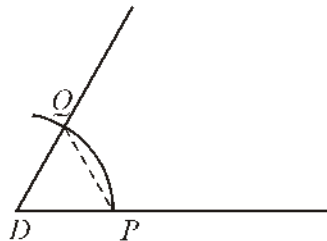


Fig. 5.48

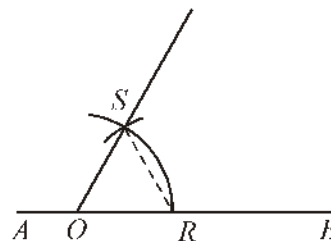


Fig. 5.49

Geometric Construction 7.

In previous classes we have learnt to draw the various angles with the help of angle measuring instrument. Let us draw the various angles without angle measurement instrument with the help of scale and protractor.

Step 1 : Draw a line segment AB of 6 cm as shown in Fig. 5.50.

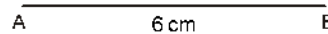


Fig. 5.50

Step 2 : Open the compass upto 6 cm and taking A as centre, draw a circle such that it passes through B . Now, taking 6 cm as radius draw equal arcs on the circle and mark the cuts as C, D, E, F and G .

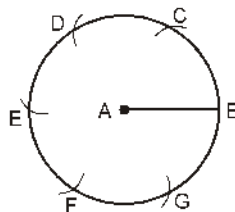


Fig. 5.51

Step 3 : Now, join AB and BC . Join in such away that it will start from where O is started and marked them 1-1 cm. Similarly, CD, DE, EF, FG and GB .

Here marked after one cm distance makes a 10° angle from each outer from point A .

if it divided into mm, then it will make 1° degree angle at distance of 1 mm from point A.

Suppose, we have to draw an angle of 40° , then we have to move up to 4th point where 40 is written. Now, join A to that point $\angle BAH = 40^\circ$.

Suppose, we have to draw 130° angle then we have to move up to point D and after that 130° , then $\angle BAK = 130^\circ$.

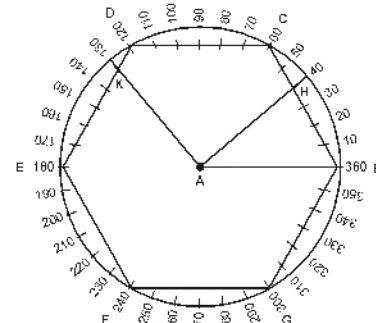


Fig. 5.52

Note: In this method, by using millimeter scale is used then we can draw angle 1° to 360° .

Remark: In the above method 6 cm straight line and another end where angle to be drawn. A 6 cm circle to be drawn and divide circle in 6 equal parts. (Hence, every part But not joining every point we have to join the desired line).

Geometric Construction 8

On any given line to draw a perpendicular from a point which is outside the line.

Method 1: AB is straight line there is a point P outside the line. Draw a perpendicular.

Composition: Taking P as a centre and radius more than the distance from P to AB draw two arcs which will intersect the line AB at points C and D. Taking C and D as a centre and radius more than half of CD draw two arcs below AB which will intersect each other at Q. Now join PQ. It will meet AB at point O. Thus, PO is the desired perpendicular.

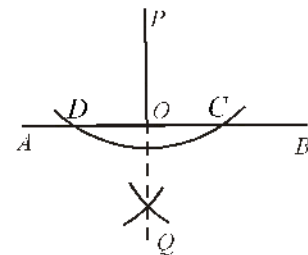


Fig. 5.53

Base of Construction: PQ is the perpendicular bisector of C and D which have equal distance.

Method 2: Keep the one side of set square on line AB. Keep second set square in such a way so that it can move on scale and it should be stable. Move set square so that it should move up to P. Draw line PQ to AB from point P. Hence, PQ is the required perpendicular (Fig. 5.54).

Geometric Construction 9

Draw a perpendicular at any point O on line AB.

Construction: Let a is any point on ; line AB Taking O as centre of given line AB draw an arc which will intersect the line AB at points C and

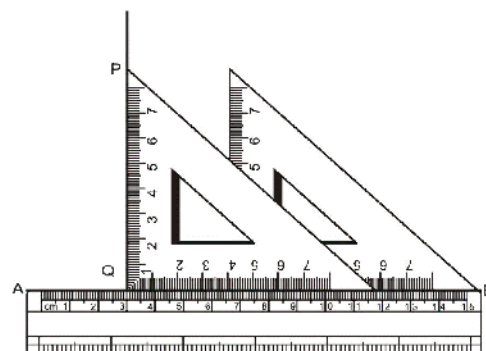


Fig. 5.54

D. Now taking C and D as centre taking more than half distance in compass draw an arc which will intersect each other at point P. Now join PO. Hence, PO is the required perpendicular Fig. (5.55).

Base of Construction : If we join PC and PD, then $\triangle POC$ and $\triangle POD$ are congruent. Result is $\angle POC = \angle POD = 90^\circ$.

5.10. Construction of Parallel Lines

Geometric Construction 10

From any outer point P draw a line parallel to a given line.

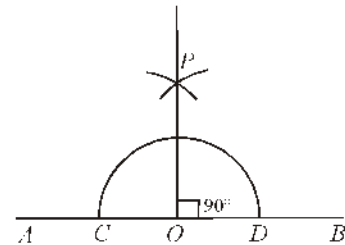


Fig. 5.55

When two lines are intersected by a transversal, corresponding and alternate angles are equal then lines are parallel to each other.

(A) Corresponding Angle Method :

Taking a point Q on line AB and join with P. Extend QP to R. Now, at point P on QR draw $\angle DPR$ which is equal to $\angle BQR$. Hence, CD is the required line parallel to AB as shown in (Fig. 5.56).

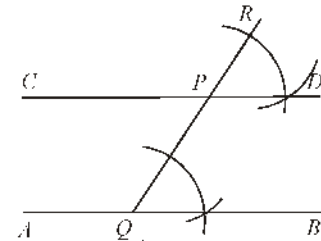


Fig. 5.56

(B) Alternate Angle Method :

Take a point Q on AB and join this with P of QP draw alternate angles $\angle CPQ = \angle BQP$. Hence, CD is the required line parallel to AB as shown in Fig. 5.57.

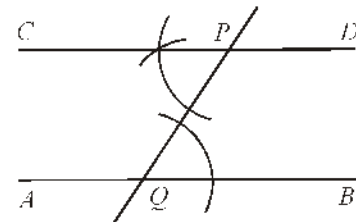


Fig. 5.57

(C) Construction of Parallel Lines with the help of Set Square :

Let AB be a straight line to which a parallel line has to be drawn from point P. Except the diagonal of set square keep other side on AB. Its another side set in such a way that its one side should be fix. Move the set square in such a way that it should be moved on P. Now, draw line PC. Hence, PC is the required parallel line (Fig. 5.58).

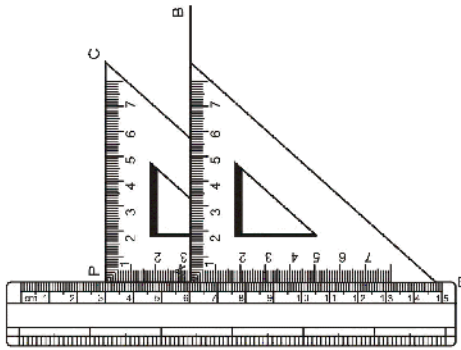


Fig. 5.58

Exercise 5.3

1. Draw a line segment $AB = 10$ cm. Draw its bisector and verify your answer.
2. Draw an angle of 120° . Draw bisector of this angle. Measure both the angles and verify your answer.
3. Draw an angle of 40° with the protractor also draw an angle equal to this angle equal to this angle with ruler and compass.
4. Draw a line of 6 cm. Draw a perpendicular to it from an outer point P .
5. Draw $\angle ABC = 120^\circ$. Draw a line parallel to BC from A .
6. Draw a line segment of 9 cm. Divide this in three equal parts with ruler and compass.
7. Draw a line segment of 10 cm. With the help of ruler and compass divide it into 3 : 2. Measure their length.
8. Draw a line segment of 6 cm. Divide it into 1 : 2 : 3 with the help of ruler and compass.
9. Draw the following angles with ruler and compass 45° , 75° , 105° , 150° .
10. Without using protractor draw the following angles :

(i) 12°	(ii) 20°	(iii) 80°	(iv) 100°
(v) 155°	(vi) 218°	(vii) 307°	(viii) 127°

Verify these angles with protractor.



Important Points

1. When two rays are originated from a point, then it makes a shape of angle. Common point of rays is known as vertex and rays are known as sides of angle.
2. If one line is standing on another line, then they make a right angle.
3. If a revolving ray takes a complete round, then it makes angles equal to four right angles, *i.e.*, 360° .
4. An acute angle has a magnitude between 0° and 90° .
5. An obtuse angle has a magnitude between 90° and 180° .
6. Measurement of straight angle on a plane is equal to two right angles.
7. A reflex angle has a magnitude of more than two right angles but less than four right angles.
8. If the sum of two angles is equal to 90° , then these angles are known as complementary angles.
9. If the sum of two angles is equal to 180° , then these angles are known as supplementary angles.
10. If in two vertex and one side is common and other side is opposite to common side then angles are known as adjacent angle.
11. If the measurement of two adjacent angles is 180° , then these angles are known as linear pair.
12. Two angles are called a pair of vertically opposite angles, if their arms form two pairs of opposite rays and they are equal.
13. If two parallel lines are intersected by a transversal, then :
 - (i) Corresponding angles are equal,
 - (ii) Alternate angles are equal,
 - (iii) Sum of interior angles of one side is equal to 180° .
14. If two lines are intersected by a transversal and the angles made by them are such that
 - (i) Corresponding angles are equal, or
 - (ii) Alternate angles are equal, or
 - (iii) Sum of interior angle on one side is equal to 180° , then the lines are parallel to each other.

Miscellaneous Exercise-5

1. In fig. 5.59 If $AB \parallel CD \parallel EF$, $PQ \parallel RS$, $\angle RQD = 25^\circ$ and $\angle CQP = 60^\circ$, then the value of $\angle QRS$ is equal to :

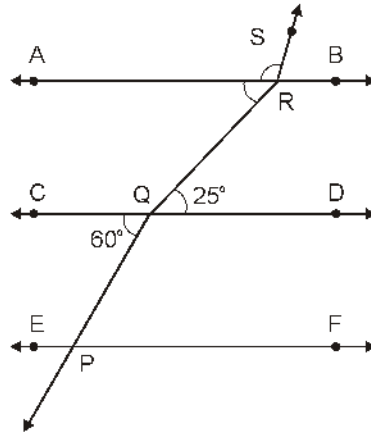


Fig. 5.59

- (A) 85° (B) 135° (C) 145° (D) 110°
2. In Fig. 5.60., POQ is a straight line. The value of x is :

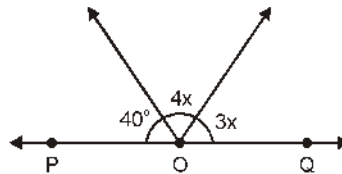


Fig. 5.60

- (A) 20° (B) 25° (C) 30° (D) 35°
3. Fig. 5.61, $OP \parallel RS$, $\angle OPQ = 110^\circ$ and $\angle QRS = 130^\circ$, then $\angle PQR$ is equal to :

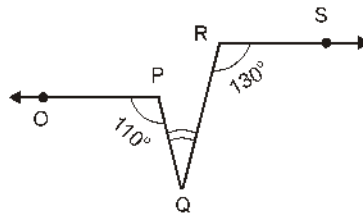


Fig. 5.61

- (A) 40° (B) 50° (C) 60° (D) 70°

4. In Fig. 5.62, reflex angle $\angle AOB$ is equal to :

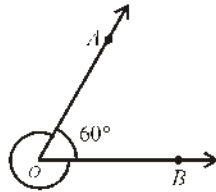


Fig. 5.62

- (A) 60° (B) 120° (C) 300° (D) 360°
5. In Fig. 5.63, two straight lines AB and CD are intersecting each other at point O . Angles so formed are marked Fig. 5.63.

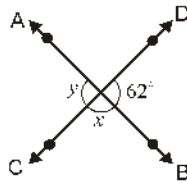


Fig. 5.63

Here, value of $\angle x - \angle y$ is:

- (A) 56° (B) 118° (C) 62° (D) 180°
6. In Fig. 5.64., which pair of angles is not a pair of corresponding angles :

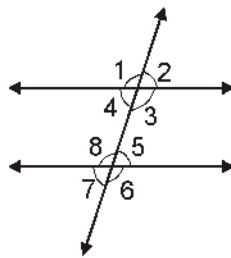


Fig. 5.64

- (A) $\angle 1, \angle 5$ (B) $\angle 2, \angle 6$ (C) $\angle 3, \angle 7$ (D) $\angle 3, \angle 5$
7. In fig. 5.65, if l and m are two parallel lines which are intersected by transversal line at points G and H and the angles so formed are shown.

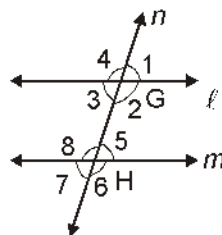


Fig. 5.65

If $\angle 1$ is an acute angle, then which statement is false :

- (A) $\angle 1 + \angle 2 = 180^\circ$ (B) $\angle 2 + \angle 5 = 180^\circ$
 (C) $\angle 3 + \angle 8 = 180^\circ$ (D) $\angle 2 + \angle 6 = 180^\circ$

8. Find the value of x in Fig. 5.66.

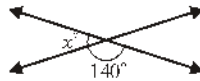


Fig. 5.66

9. In Fig. 5.67, $AB \parallel CD$. Find the value of $\angle x$ and $\angle y$.

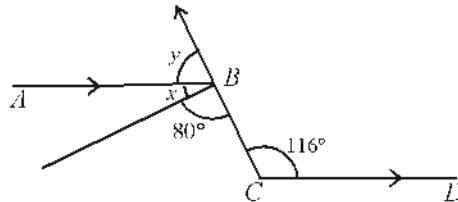


Fig. 5.67

10. In Fig. 5.68, lines l and m are parallel. Find the value of $\angle x$.

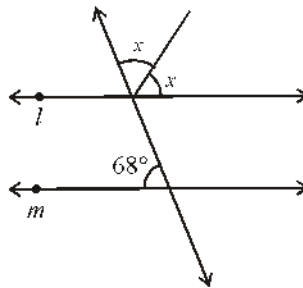


Fig. 5.68

11. In Fig. 5.69, which lines are parallel and why?

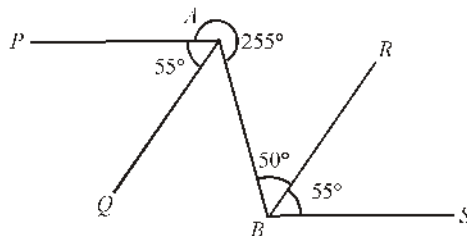


Fig. 5.69

12. In Fig. 5.70, $AC \parallel PQ$ and $AB \parallel RS$ then find the value of $\angle y$. Also, write the statements you have used.

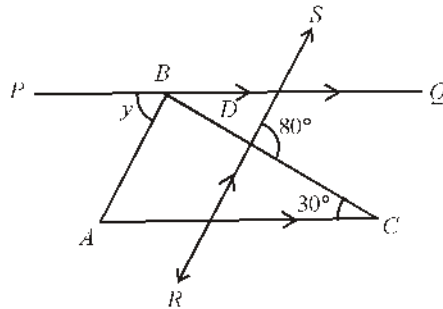


Fig. 5.70

13. In Fig. 5.71, $AB \parallel CD$ and $PQ \parallel EF$, then find the value of $\angle x$.

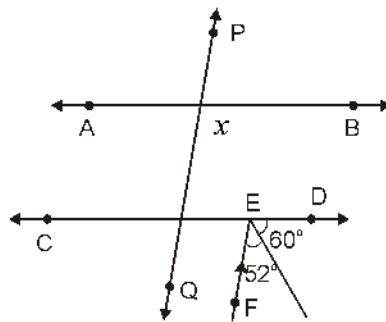


Fig. 5.71

14. In Fig. 5.72, out of lines l, m, n, p, q and r which lines are parallel and why.

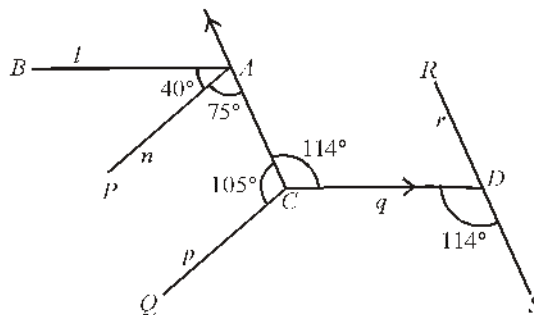


Fig. 5.72

15. In Fig. 5.73, two straight lines are intersecting each other. If $\angle 1 + \angle 2 + \angle 3 = 230^\circ$ then find the values of $\angle 1$ and $\angle 4$.

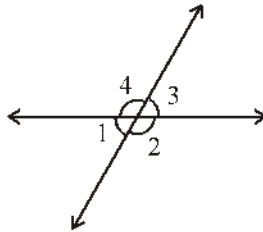


Fig. 5.73

16. In Fig. 5.74, PQ and QR are two mirrors which are joined at Q at an angle of 30° to each other. If incident ray AB is parallel to mirror RC , then find the values of $\angle BCQ$, $\angle CBQ$ and $\angle BDC$.

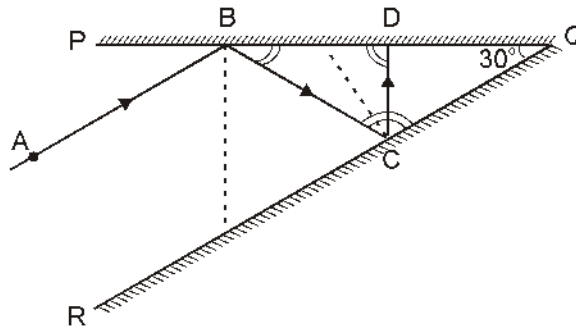


Fig. 5.74

Answer

Exercise 5.1

1. 122° and 58°
2. (i) 52° (ii) 128° (iii) $\angle AOC$, $\angle BOD$ and $\angle AOD$, $\angle BOC$
(iv) $\angle AOD$ and $\angle BOC$

Exercise 5.2

1. $\angle x = 58^\circ$, $\angle y = 122^\circ$, $\angle z = 58^\circ$, $\angle p = 122^\circ$
2. $\angle x = 94^\circ$, $\angle y = 266^\circ$
3. $\angle 3$, $\angle 5$, $\angle 7$
7. 90°
9. 126°
11. 85°

Miscellaneous Exercise - 5

1. (C) 2. (A) 3. (C) 4. (C)
5. (A) 6. (D) 7. (B)
8. 40°
9. $x = 36, y = 64$
10. $x = 56^\circ$
11. $AQ \parallel BR, AP \parallel BS$
12. $y = 110^\circ$
13. 112°
14. $m \parallel r, n \parallel p$
15. $\angle 1 = 50, \angle 4 = 113^\circ$
16. $\angle BCQ = 120^\circ, \angle CBQ = 30^\circ, \angle BDC = 90^\circ$

RECTILINEAR FIGURES

6.01 Triangle and its Angles :

We have studied about angles, made by straight lines at a point. In this chapter, we shall study about a plane figure formed by more than two lines. If we take three non-collinear points in a plane and join them by taking two points at one time, then we will get three line segments. Thus, the figure so formed bounded by three line segments is called a triangle.

"A plane figure bounded by three line segments by joining three non-collinear points in a plane is called a triangle."

Fig. 6.01 three non-collinear points A , B and C are joined. Figure ABC , so formed by joining line segments AB , BC and CA , is called triangle. Symbol ' Δ ' is used in place of the word 'triangle'. So, triangle ABC will be denoted by ΔABC . These three points which makes a triangle are called vertices of triangle. Three line segments of triangle are called its **sides**. Angles formed at the vertices of a triangle by three line segments are called **angles** of triangle.

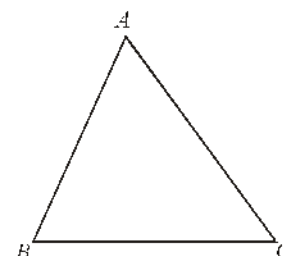


Fig. 6.01

From Fig. 6.02, it is clear that in ΔABC :

- (i) points A , B and C are its vertices.
- (ii) line segments AB , BC and CA are its sides.
- (iii) $\angle CAB$, (or $\angle A$), $\angle ABC$ (or $\angle B$) $\angle BCA$ (or $\angle C$)

are the angles of the triangle. If the sides of ΔABC are extended in order, then angle between extended side and adjacent side is called an **exterior angle** of triangle.

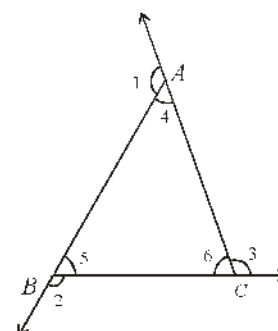


Fig. 6.02

In figure 6.2 $\angle 1, \angle 2, \angle 3$ are exterior angles of triangle ABC. $\angle 4, \angle 5, \angle 6$ are interior angles of triangle ABC.

Triangles can be classified on the basis of their sides or angles.

6.02 Classification of Triangles on the Basis of Sides

(i) Scalene Triangle : A triangle having all the three sides of different measure is called scalene triangle. In Fig. 6.03, $\triangle ABC$ is a scalene triangle.

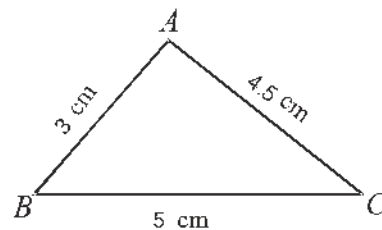


Fig. 6.03

(ii) Isosceles Triangle : If two sides of a triangle are of equal measure, then it is called an isosceles triangle. In Fig. 6.04, $\triangle PQR$ is an isosceles triangle in which $PQ = PR$.

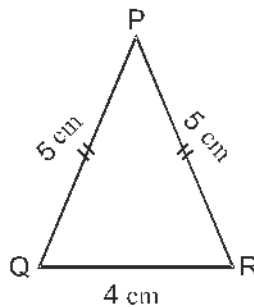


Fig. 6.04

(iii) Equilateral Triangle : A triangle, whose all sides are equal is called an equilateral triangle. In Fig. 6.05, $\triangle ABC$ is an equilateral triangle in which $AB = BC = CA$.

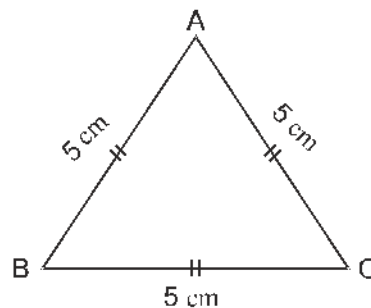


Fig. 6.05

6.03 Classification of Triangle on the basis of Angles :

(i) Acute-angled Triangle : A triangle, whose each angle is acute, is called an acute-angled triangle. In Fig. 6.06, ΔPQR is an acute-angled triangle, since $\angle P$, $\angle Q$ and $\angle R$ are acute angles.

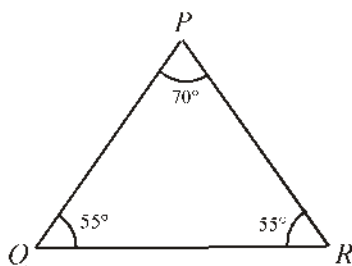


Fig. 6.06

(ii) Right-angled Triangle : A triangle with one angle a right angle is called a right-angled triangle. In Fig. 6.07, ΔABC is a right-angled triangle, since $\angle B = 90^\circ$.

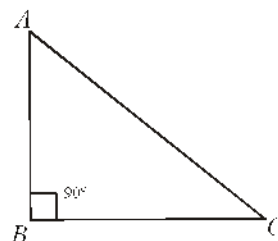


Fig. 6.07

(iii) Obtuse-angled Triangle : A triangle with one angle an obtuse angle, is known as an obtuse-angled triangle. In Fig. 6.08, ΔPQR is an obtuse-angled triangle, since here $\angle PQR = 120^\circ$.

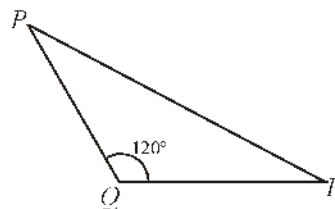


Fig. 6.08

The sum of three angles of a triangle is 180° proof of this geometrical fact is following.

Theorem 6.1. *The sum of the three angle of a triangle is equal to two right angles.*

Given : A triangle ABC and its angles namely $\angle 1$, $\angle 2$ and $\angle 3$.

To prove $\angle 1 + \angle 2 + \angle 3 = 180^\circ$

Construction : Through A , draw a line parallel to BC .

Proof : $\because BC \parallel \ell$

$$\angle 2 = \angle 5 \quad (\text{Alternate angles}) \dots(1)$$

and $\angle 3 = \angle 4 \quad (\text{Alternate angles}) \dots(2)$

On adding equations (1) and (2), we get

$$\angle 2 + \angle 3 = \angle 5 + \angle 4 \quad \dots(3)$$

Adding to both sides of equation (3), we get

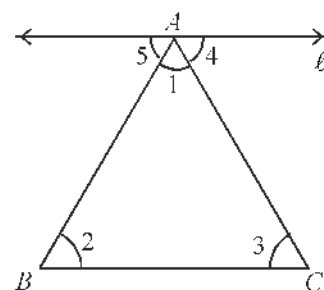


Fig. 6.09

$$\angle 1 + \angle 2 + \angle 3 = \angle 1 + \angle 5 + \angle 4 \quad \dots(4) \quad \dots (iv)$$

(Sum of angles at a point on a line is 180°)

$$\angle 1 + \angle 5 + \angle 4 = 180^\circ \quad \dots(5)$$

From (4) and (5),

Thus $\angle 1 + \angle 2 + \angle 3 = 180^\circ$ Hence Proved

Corollary 1. If a side of a triangle is produced the exterior angle so formed is equal to the sum of the two interior opposite angles.

Sum of the three interior angles of a triangle is 180°

In Fig. 6.10, $\angle 1 + \angle 2 + \angle 3 = 180^\circ$... (i)

By linear pair property

$$\angle 3 + \angle 4 = 180^\circ \quad \dots (ii)$$

From equations (i) and (ii), we get

$$\angle 1 + \angle 2 + \angle 3 = \angle 3 + \angle 4$$

Thus, $\angle 1 + \angle 2 = \angle 4$

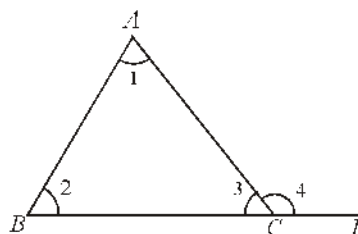


Fig. 6.10

Corollary 2. An exterior angle of a triangle is greater than either of the interior opposite angles.

In Fig. 6.10, $\angle 4 = \angle 1 + \angle 2$ (From Corollary 1)

$$\Rightarrow \angle 4 > \angle 1$$

$$\text{and } \angle 4 > \angle 2$$

Corollary 3. In a right-angled triangle, right angle is the greatest angle,

$$\therefore \text{Sum of the three angles of a triangle} = 180^\circ$$

$$\therefore \text{Sum of one right angle} + \text{Sum of two other angles} = 180^\circ$$

$$\Rightarrow 90^\circ + \text{Sum of other two angles} = 180^\circ$$

$$\therefore \text{Sum of other two angles} = 90^\circ$$

$$\Rightarrow \text{Remaining two angles are acute angles.}$$

Thus, Right angle is greater than remaining two acute angles.

Note: In each triangle, at least two angles are acute angles.

Corollary 4. Sum of the four angles of a quadrilateral is equal to four right angles.

In Fig. 6.11, ABCD is a quadrilateral having four angles $\angle A, \angle B, \angle C$ and $\angle D$. Line AC divides quadrilateral into two triangles.

$$\text{In } \triangle ABC \quad \angle 1 + \angle 2 + \angle 3 = 180^\circ \quad \dots(1)$$

$$\text{and in } \triangle ADC, \quad \angle 4 + \angle 5 + \angle 6 = 180^\circ \quad \dots(2)$$

From equations (i) and (ii), we get

$$\begin{aligned} & \angle 1 + \angle 2 + \angle 3 + \angle 4 + \angle 5 + \angle 6 = 360^\circ \\ \Rightarrow & (\angle 1 + \angle 4) + \angle 2 + (\angle 3 + \angle 5) + \angle 6 = 360^\circ \\ \text{Thus,} & \angle A + \angle B + \angle C + \angle D = 360^\circ \end{aligned}$$

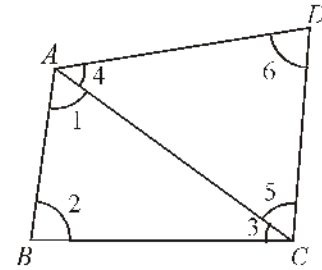


Fig. 6.11

Illustrative Example

Example 1 : In Fig. 6.12, one angle of $\triangle ABC$ is 40° . If the difference between remaining two angles is 30° then find them.

Solution : Let $\angle x$ and $\angle y$ are remaining two angles of $\triangle ABC$.

$$\begin{aligned} \therefore & \angle x + \angle y + 40^\circ = 180^\circ \\ \Rightarrow & \angle x + \angle y = 140^\circ \quad \dots\dots (i) \\ \Rightarrow & \angle x - \angle y = 30^\circ \quad (\text{Given}) \quad \dots\dots (ii) \end{aligned}$$

On adding (i) and (ii), we get

$$\begin{aligned} & \angle x + \angle y + \angle x - \angle y = 140^\circ + 30^\circ \\ \Rightarrow & 2\angle x = 170^\circ \\ \Rightarrow & \angle x = 85^\circ \end{aligned}$$

$$\begin{aligned} \text{and from (i)} \quad & \angle y = 140^\circ - \angle x \\ & = 140^\circ - 85^\circ = 55^\circ \end{aligned}$$

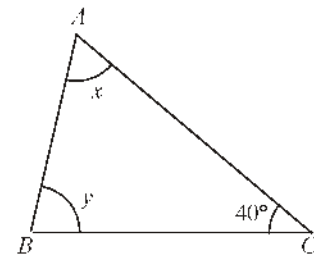


Fig. 6.12

Thus, the required angles are $\angle x = 85^\circ$ and $\angle y = 55^\circ$.

Example 2. From Fig. 6.13, find $\angle RPQ$, $\angle QRP$ and $\angle PQR$.

Solution : From Fig. 6.13

$$\begin{aligned} & \angle x + \angle x = 126^\circ \\ \Rightarrow & 2\angle x = 126^\circ \\ \Rightarrow & \angle x = 63^\circ \\ \text{and} & \angle RPQ = 63^\circ \\ \text{and} & \angle PQR = 63^\circ \\ \text{now} & \angle y + 126^\circ = 180^\circ \quad (\text{Linear pair}) \\ \therefore & \angle y = 180^\circ - 126^\circ = 54^\circ \\ \text{or} & \angle QRP = 54^\circ \end{aligned}$$

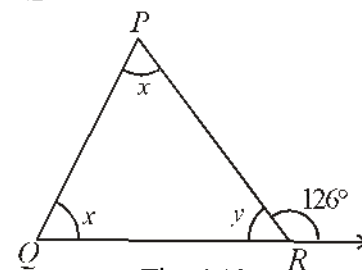


Fig. 6.13

Example 3. In Fig. 6.14, Find $\angle x$, $\angle y$ and $\angle ACD$. Here, $BA \parallel CE$.

Solution : Here, $\angle x = 42^\circ$ (Alternate angles)

Now $\angle ACD = \angle x + 66^\circ$
 $= 42^\circ + 66^\circ$
 $= 108^\circ$
 $\angle y + \angle ACD = 180^\circ$... (i)
or $\angle y + 108^\circ = 180^\circ$
 $\Rightarrow \angle y = 180^\circ - 108^\circ$
 $\Rightarrow \angle y = 72^\circ$

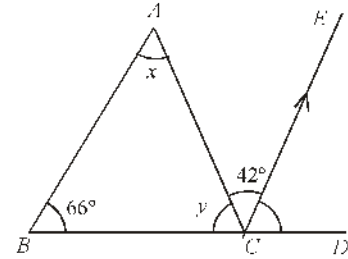


Fig. 6.14

Example 4. If in a ΔABC , bisectors of angle B and C intersect each other at point 'O', then show that $\angle BOC = 90^\circ + \frac{1}{2}\angle A$.

Solution : Draw ΔABC as shown in Fig. 6.15 and then draw BO and CO , the bisectors of $\angle B$ and $\angle C$.

$\angle A + \angle ABC + \angle ACB = 180^\circ$
(Sum of three angles of a Δ is 180°)
or $\frac{1}{2}\angle A + \frac{1}{2}\angle ABC + \frac{1}{2}\angle ACB = \frac{1}{2} \times 180^\circ$
or $\frac{1}{2}\angle A + \angle OBC + \angle OCB = 90^\circ$ (1)

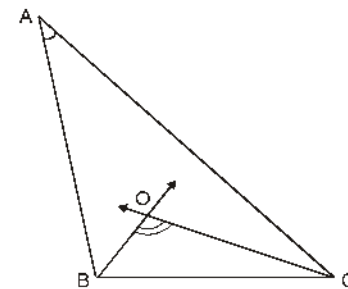


Fig. 6.15

Given BO and CO are the bisectors of $\angle B$ and $\angle C$ respectively.

$\therefore \angle BOC + \angle OBC + \angle OCB = 180^\circ$ (the dangles of ΔOBC)(2)
Subtracting (1) from (2), we get

$\angle BOC + \angle OBC + \angle OCB - \frac{1}{2}\angle A - \angle OBC - \angle OCB = 180^\circ - 90^\circ$
 $\Rightarrow \angle BOC - \frac{1}{2}\angle A = 90^\circ$
Thus, $\angle BOC = 90^\circ + \frac{1}{2}\angle A$.

Example 5. In Fig. 6.16, if $BE \perp AC$, $\angle EBC = 30^\circ$ and $\angle FAC = 20^\circ$, then find the values of $\angle x$ and $\angle y$.

Solution : In ΔBCE ,
 $90^\circ + 30^\circ + x = 180^\circ$

or $120^\circ + \angle x = 180^\circ$

or $\angle x = 180^\circ - 120^\circ$

or $\angle x = 60^\circ$

Now, $\angle y = \angle FAC + \angle x$ (Exterior angle = Sum of interior opposite angles)

$$\angle y = 20^\circ + 60^\circ = 80^\circ$$

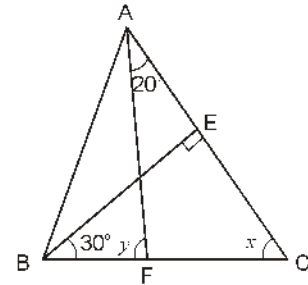


Fig. 6.16

Exercise 6.1

1. In Fig. 6.17, find all the angles of $\triangle ABC$

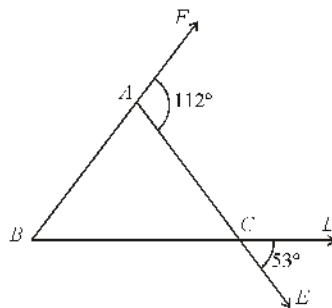


Fig. 6.17

2. In Fig. 6.18, $\triangle ABC$ is an equilateral triangle. Find the values of $\angle x$, $\angle y$ and $\angle z$ from the figure.

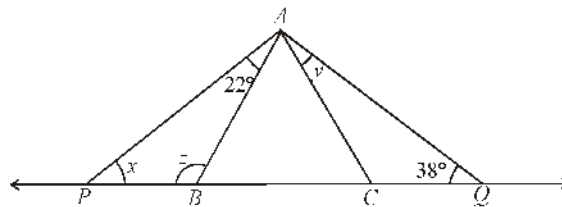


Fig. 6.18

3. In Fig. 6.19, sides AB and AC of $\triangle ABC$ are produced to E and D respectively. If angle bisectors BO and CO of $\angle CBE$ and $\angle BCD$ meet each other at point O , then prove that;

$$\angle BOC = 90^\circ - \frac{\angle x}{2}$$

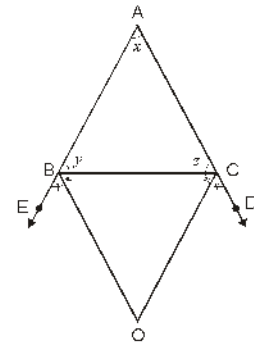


Fig. 6.19

4. In Fig. 6.20, $\angle P = 52^\circ$ and $\angle PQO = 64^\circ$, if QO and RO are the angle bisectors of $\angle PQR$ and $\angle PRQ$ respectively, then find the values of $\angle x$ and $\angle y$.

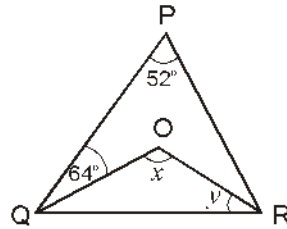


Fig. 6.20

5. In Fig. 6.21, if $AB \parallel DE$, $\angle BAC = 35^\circ$ and $\angle CDE = 53^\circ$, then find the value of $\angle DCE$.

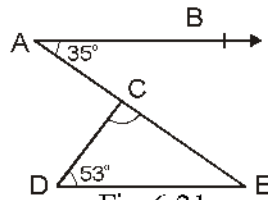


Fig. 6.21

6. In Fig. 6.22, if lines PQ and RS intersect each other at point T , such that $\angle PRT = 40^\circ$, $\angle RPT = 95^\circ$ and $\angle TSQ = 75^\circ$, then find $\angle SQT$.

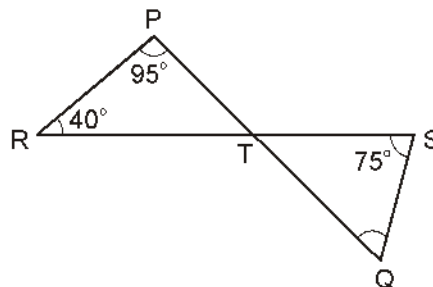


Fig. 6.22

7. In Fig. 6.23, sides QP and RQ of a triangle PQR are produced upto S and T respectively. If $\angle SPR = 135^\circ$ and $\angle PQT = 110^\circ$ then find $\angle PRQ$.

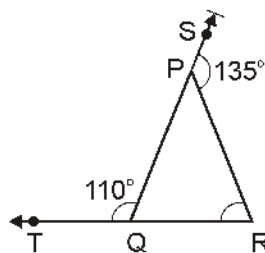


Fig. 6.23

8. In Fig. 6.24. If $PQ \perp PS$, $PQ \parallel SR$, $\angle SQR = 28^\circ$ and $\angle QRT = 65^\circ$, then find the values of x and y .

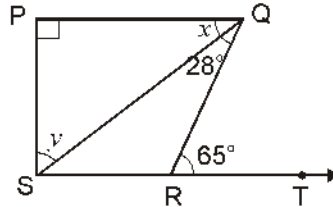


Fig. 6.24

9. In Fig. 6.25, side QR of $\triangle PQR$ is produced to point S . If the bisectors of $\angle PQR$ and $\angle PRS$ meet at point T , then prove that : $\angle QTR = \frac{1}{2} \angle QPR$

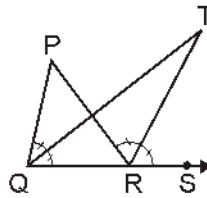


Fig. 6.25

10. In $\triangle ABC$, A is right angle. L is any point on BC such that $AL \perp BC$. Prove that : $\angle BAL = \angle ACB$
11. The angles of a triangle are in ratio 2:3:4. Find the all angles of the triangle.

6.04 Rectilinear Figures

A closed plane figure, bounded by at least three straight lines is called rectilinear figure. If we take n ($n \geq 3$) different points on the plane such that :

- (i) Line segments made by any two points out of n considered points, should not pass through any remaining point ($n - 2$ points) except its own end points.

- (ii) Two line segments from a point is not collinear.

Then join these points in an order and figure so obtained is called polygon of n sides. These points are called vertices of the polygon and line segments which make polygon are called **sides of polygon**. Angles subtended by line segments at the vertices are called **interior angles** of polygon.

It is clear that in a n sided polygon :

- (i) There are n vertices.
 (ii) There are n sides.
 (iii) There are n interior angles.

Polygons are classified on the basis of number of sides and the measure of angles.

(i) Triangle : When $n=3$, then rectilinear figure is called triangle.

(ii) Quadrilateral : When $n=4$, the rectilinear figure is called quadrilateral.

(iii) Pentagon : A polygon with 5 sides is called pentagon. In Fig. 6.26, ABCDE is a pentagon, which AB, BC, CD, DE and EA are its sides and $\angle EAB$, $\angle ABC$, $\angle BCD$, $\angle CDE$ and $\angle DEA$ are its interior angles.

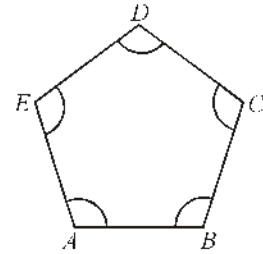


Fig. 6.26

(iv) Hexagon : A polygon with 6 sides is called hexagon. In Fig. 6.27 ABCDEF is a hexagon. Angles namely $\angle 1$, $\angle 2$, $\angle 3$, $\angle 4$, $\angle 5$ and $\angle 6$ are its interior angles and AB, BC, CD, DE, EF and FA are its sides.

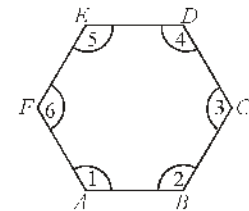


Fig. 6.27

(v) Heptagon : A polygon with 7 sides is called heptagon. In Fig. 6.28, ABCDEFG is a heptagon.

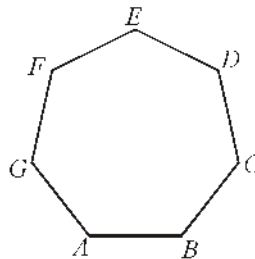


Fig. 6.28

(vi) Octagon : A polygon with 8 sides is called an octagon. In Fig. 6.29, ABCDEFGH is an octagon.

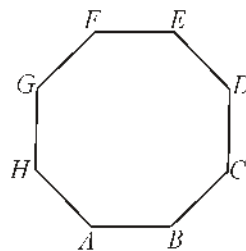


Fig. 6.29

Convex Polygon : If each angle of a polygon is less than two right angles, then the polygon is called a convex polygon. Unless otherwise stated, a polygon means a convex polygon.

Concave Polygon : If at least one angle of the polygon is greater than two right angles, it is called a concave polygon. In the given figure 6.30, ABCDEFGH is a concave polygon, since its interior angle $\angle FED$ is greater than two right angles.

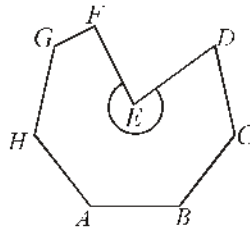


Fig. 6.30

Equilateral Polygon : A polygon with all sides equal is called an equilateral polygon. (Fig. 6.31)

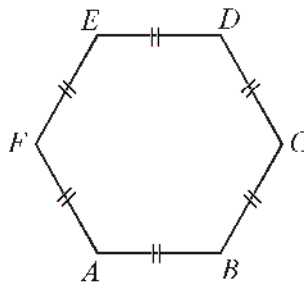


Fig. 6.31

Equiangular Polygon : A polygon with all interior angles equal is called an equiangular polygon. (Fig. 6.32)

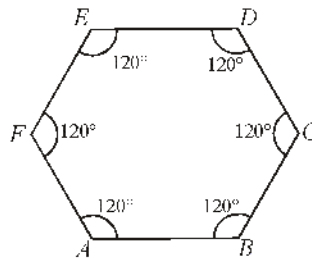


Fig. 6.32

Regular Polygon : A polygon which is equilateral and equiangular, is called regular polygon (Fig. 6.33).

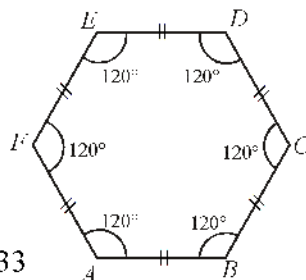


Fig. 6.33

Exterior angles of a polygon : if the sides of a polygon are produced in order (*i.e.* clockwise or anticlockwise direction) then the angles outside the polygon, which are supplementary angles of interior angles, and called exterior angles of the polygon.

In Fig. 6.34, the sides of polygon ABCDEF is produced in same order and we obtained exterior angles as

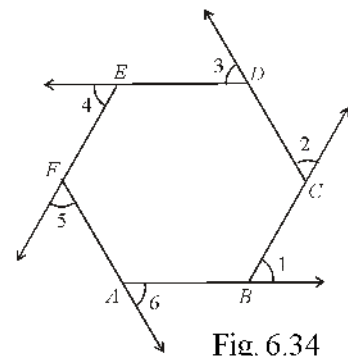


Fig. 6.34

$\angle 1, \angle 2, \angle 3, \angle 4, \angle 5$ and $\angle 6$.

Perimeter of a Polygon : The sum of all sides of a polygon is called its **perimeter**.

Diagonals of a Polygon : By joining the vertices of a polygon we get the straight lines (that are not the sides) are called diagonals of polygon. In Fig. 6.35, lines AC, AD and AE are diagonals from point A. Here, Total 9 diagonals

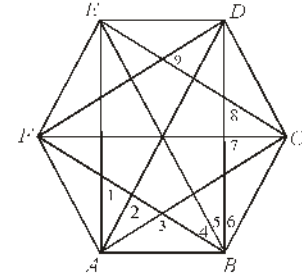


Fig. 6.35

Total number of diagonals in n-sided polygon = $\frac{n(n-1)}{2} - n$

e.g. if $n=6$, then number of diagonals

$$= \frac{6(6-1)}{2} - 6 = \frac{6 \times 5}{2} - 6 = 15 - 6 = 9$$

In a n -sided polygon, the diagonals drawn from vertices make $(n-2)$ triangles. In Fig. 6.36, diagonals AC and AD of pentagon ABCDE makes three triangles. On the basis of above facts we will establish the formula to find the sum of all the interior angles of any polygon, and on this basis we will find the sum of all exterior angles of polygon.

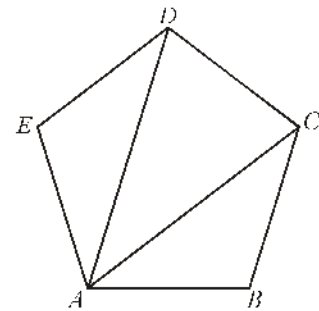


Fig. 6.36

Theorem 6.2

(i) *Sum of interior angles of an n-sided polygon is equal to $(2n-4)$ right angle.*

(ii) *Sum of exterior angles of an n-sided polygon is equal to 4 right angles.*

(iii) *Each interior angles of a regular polygon is equal to $\frac{1}{n}(2n-4)$ right angle.*

The sides AB, BC, CD, DE, EF, FA, ... of a n sided polygon are produced in same order. In This way $\angle 1, \angle 2, \angle 3, \angle 4, \angle 5, \angle 6 \dots$ are respectively exterior angles at the vertices A, B, C, D, E, F, ...

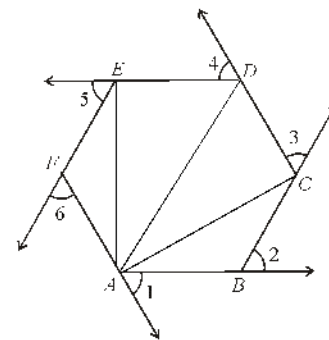


Fig. 6.37

To prove

(i) Sum of all interior angles = $(2n-4)$ right angles

(ii) Sum of all exterior angles = 4 right angles

(iii) Each interior angle of a regular polygon = $\frac{(2n-4)}{n}$ right angle

Construction : Draw diagonals AC, AD, AE, ... from the vertex A. In the way we get $(n-2)$ triangles in total.

Proof (i) : We know diagonals of polygon from its vertices divides it into $(n-2)$ triangles.

Sum of all interior angles of n sided polygon

= Sum of interior angles of $(n-2)$ triangles made by the diagonals from the vertices

= $(n-2) \times 2$ right angle

= $(2n-4)$ right angle

Hence Proved

Proof (ii) : We know that when the sides of polygon are produced, then sum of its pair of interior and exterior angles is equal to 2 right angles. Thus, there are n pairs of interior and exterior in a n -sided polygon and their sum = $n \times 2$ right angles = $2n$ right angles.

Sum of n interior angles = $(2n-4)$ right angle.

Sum of n exterior angles = $2n$ right angle - $(2n-4)$ right angle.

= $(2n-2n+4)$ right angle = 4 right angle.

Proof (iii) : If polygon is regular then its each interior angle = $\frac{1}{n}(2n-4)$ right angle

Proof : We know that the sum of all the interior angles of n -sided polygon is equal to $(2n-4)$ right angles. We know that, each angle of regular polygon is same. Let each angle of regular

polygon is x° , then $nx = (2n-4)$ right angles (sum of all n angles)

$$x = \frac{1}{n}(2n-4) \text{ right angles}$$

Hence Proved.

Illustrative Examples

Example 6. In a regular polygon, number of sides is 10. Find its each interior angle.

Solution : First Method : \therefore Each exterior angles or of n -sided regular polygon

$$= \frac{4}{n} \quad (\text{right angles})$$

$$= \frac{4}{10} \times 90^\circ \quad (\text{Here } n = 10)$$

$$= 36^\circ$$

$$\therefore \text{Each interior angle} = 180^\circ - 36^\circ \\ = 144^\circ$$

Second Method :

$$\text{Each interior angle of } n \text{ sided regular polygon} = \frac{(2n-4) \text{ right angle}}{n}$$

$$= \frac{(2 \times 10 - 4) \times 90^\circ}{10}$$

$$= \frac{16 \times 90^\circ}{10} = 144^\circ$$

Example 7. Find the sum of all interior angles of a heptagon.

Solution : Sum of interior angles of n sided regular polygon = $(2n - 4)$ right angle.

\therefore Sum of all the interior angles of 7 sided regular polygon.

$$= (2 \times 7 - 4) \times 90^\circ = 10 \times 90^\circ = 900^\circ$$

$$= 10 \times 90^\circ = 900^\circ$$

Alternative method :

Number of triangles in regular polygon made by all the diagonals from a vertex of a heptagon = $7 - 2 = 5$.

\therefore Sum of interior angles of a triangle = 180°

\therefore Sum of interior angles of 5 triangles = $5 \times 180^\circ = 900^\circ$

Example 8. If each interior angle of a regular polygon is 175° , then find number of sides.

Solution : \therefore 1 interior angle + 1 exterior angle = 180°

$$1 \text{ exterior angle} = 180^\circ - 1 \text{ interior angle}$$

$$= 180^\circ - 175^\circ = 5^\circ$$

\therefore Sum of n exterior angles = 360° (Let the number of sides are n .)

$$\therefore n \times 5^\circ = 360^\circ$$

Thus,
$$n = \frac{360^\circ}{5^\circ} = 72$$

Example 9. Can a regular polygon be possible in which each interior angle is 115° ?

Check it.

Solution : \therefore It is given that each interior angle of regular polygon = 115° (If it possible.)

$$\text{Each exterior angle} = 180^\circ - 115^\circ = 65^\circ$$

let the number of sides be n .

\therefore Sum of exterior angles of a polygon = 4 right angles

$$\Rightarrow n \times 65^\circ = 360^\circ$$

$$\Rightarrow n = \frac{360^\circ}{65^\circ} = \frac{72}{13} = 5 \frac{7}{13} \neq \text{a whole number}$$

Thus, a regular polygon cannot possible in which each interior angle is 115° .

Exercise 6.2

1. If a regular polygon has 8 sides, then:
 - (i) find the sum of its exterior angles.
 - (ii) find the measure of each exterior angle.
 - (iii) find the sum of its interior angle.
 - (iv) find the measure of each interior angle.
2. If the sum of interior angles of a regular polygon is 2160° , then what will be the number of sides of polygon? Find it.
3. Check, whether a regular polygon is possible with each interior angles 137° .
4. Find $\angle CED$ and $\angle BDE$ in the following Fig. 6.38.

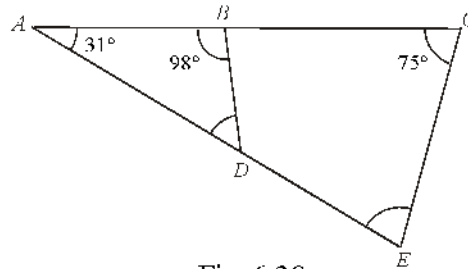


Fig. 6.38

Important Points

1. A plane figure bounded by n line segments is called n -sided polygon. In a n -sided polygon, there are n vertices, n interior angles and n sides.
2. Triangle, quadrilateral, pentagon, hexagon, heptagon and octagon are the name of the polygon according to $n=3,4,5,6,7,8$ respectively.
3. Sum of three interior angles of a triangle is 180° .
4. Sum of all interior angles of n -sided regular polygon is equal to $(2n-4)$ right angles.
5. A polygon with equal sides and equal angles is called regular polygon.
6. Each interior angles of n -sided regular polygon = $\frac{(2n-4)}{n}$ right angles.
7. Sum of all exterior angles of a polygon = 360°
8. Each exterior angle of n -sided regular polygon = $\frac{4}{n}$ right angles.

Miscellaneous Exercise - 6

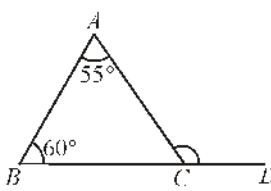
1. If two angles of a triangle are of measure 90° and 30° then the third angles is:
 (A) 90° (B) 30° (C) 60° (D) 120° []
 2. Three angles of triangle are in ratio 2:3:4, then the measure of its greatest angle is :
 (A) 80° (B) 60° (C) 40° (D) 180° []
 3. Measure of each angle of an equilateral triangle is :
 (A) 90° (B) 30° (C) 45° (D) 60° []
 4. Four angles of a quadrilateral are in the ratio 1 : 2 : 3 : 4 ,then the measure of its smallest angle is :
 (A) 120° (B) 36° (C) 18° (D) 10° []
 5. In Fig.6.39, side BC of $\triangle ABC$ is produced upto points D , If $\angle A = 55^\circ$ and $\angle B = 60^\circ$,then the measure of $\angle ACD$ is :
 (A) 120° (B) 110°
 (C) 115° (D) 125° []
- 

Fig.6.39
6. Sum of all interior angles of a hexagon is :
 (A) 720° (B) 360° (C) 540° (D) 1080° []
 7. The sum of exterior angles made by producing sides (in same order) of n -sided regular polygon :
 (A) n -right angles (B) $2n$ right angles
 (C) $(2n-4)$ right angles (D) 4 right angles []
 8. In a regular polygon, number of sides in n , then the measure of each interior angle is :
 (A) $\frac{360^\circ}{n}$ (B) $\left(\frac{2n-4}{n}\right)$ right angles
 (C) n right angles (D) $2n$ right angles []
 9. If one angles of a triangle is equal to the sum of other two angles, then the triangle is :
 (A) Isosceles triangle (B) Obtuse angled triangle
 (C) Equilateral triangle (D) Right angled triangle []
 10. The exterior angle of a triangle is 105° and the interior opposite angles are same. Each of the equal angle is :
 (A) $37\frac{1}{2}^\circ$ (B) $52\frac{1}{2}^\circ$ (C) $72\frac{1}{2}^\circ$ (D) 75° []

11. The angles of a triangle are in the ratio 5 : 3 : 7. This triangle is :
 (A) Acute angled triangle (B) Obtuse angled triangle
 (C) Right angled triangle (D) Isosceles triangle []
12. If one angle of a triangle is 130° , then angle between the angular bisector of two angles of a triangle may be :
 (A) 50° (B) 65° (C) 145° (D) 155° []
13. In Fig. 6.40, find $\angle A$.

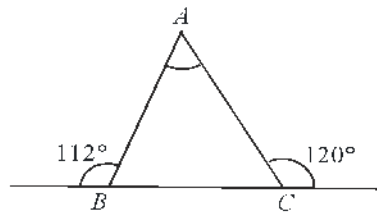


Fig. 6.40

14. In Fig. 6.41, $\angle B = 60^\circ$ and $\angle C = 40^\circ$. Find the measure of $\angle A$.

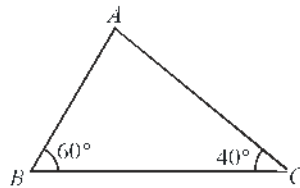


Fig. 6.41

15. In Fig. 6.42 $m \parallel QR$, then find $\angle QPR$

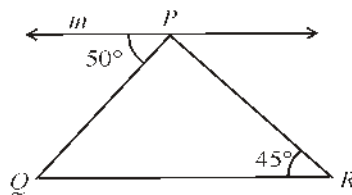


Fig. 6.42

16. In Fig. 6.43, find $\angle A$

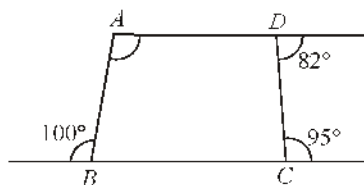


Fig. 6.43

17. Four interior angles of a pentagon are 40° , 75° , 125° and 135° , then find the measure of the fifth angle.
18. If each exterior angle of a regular polygon is 45° , then find the number of its sides.
19. The number of sides of a regular polygon is 12, then find the measure of each interior angle.
20. Sum of all interior angles of a polygon is 10 right angles. Find the number of sides.
21. Check, whether a polygon is possible if each of its interior angle is of measure 110° .
22. If in a $\triangle ABC$, $\angle A + \angle B = \angle C$, then find the greatest angle of $\triangle ABC$.
23. Find the sum of interior angles of an octagon.
24. Find each interior angle of regular decagon.
25. 110° , 130° and x° are the exterior angles obtained by producing the sides of a triangle in same order. Find the value of x° .
26. A hexagon has one interior angle of measure 165° and remaining each interior angle of measure x° , then find measure of remaining angle.
27. In Fig. 6.44, $AB \parallel DC$, Find $\angle x$, $\angle y$ and $\angle z$.

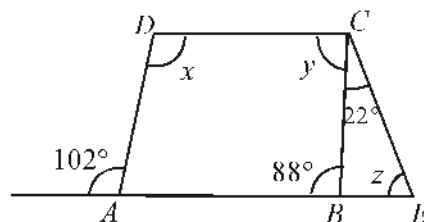


Fig. 6.44

28. In Fig. 6.45, find the value of $\angle x$ and $\angle y$, where $\angle x - \angle y = 10^\circ$.

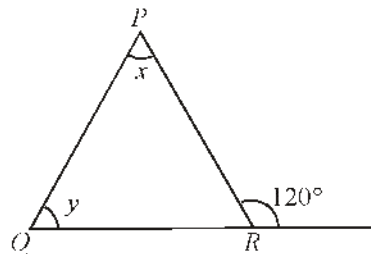


Fig. 6.45

29. In a polygon, two angles are of 90° each remaining each angle is of measure 150° . Find the number of sides of this polygon.

30. In Fig. 6.46, prove that : $\angle x + \angle y = \angle A + \angle C$.

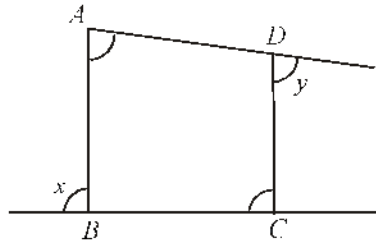


Fig. 6.46

31. In Fig. 6.47, lines BO and CO are the bisectors of $\angle B$ and $\angle C$, Find angle x

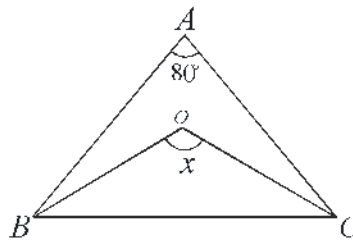


Fig. 6.47

32. In Fig 6.48, $\angle Q > \angle R$, PA is the bisector of QPR and $PM \perp QR$. Prove that :

$$\angle APM = \frac{1}{2}(\angle Q - \angle R)$$

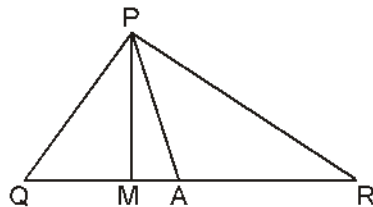


Fig. 6.48

Answers

Exercise 6.1

1. $\angle A = 68^\circ$, $\angle B = 59^\circ$, $\angle C = 53^\circ$
2. $\angle x = 38^\circ$, $\angle y = 22^\circ$, $\angle z = 120^\circ$
4. $\angle x = 116^\circ$, $\angle y = 32^\circ$
5. 92°
6. 60°
7. 65°
9. $\angle x = 37^\circ$ and $\angle y = 53^\circ$
11. $40^\circ, 60^\circ, 80^\circ$

Exercise 6.2

1. (i) 360° (ii) 45° (iii) 1080° (iv) 135°
2. 14 3. *No* 4. 74°

Miscellaneous Exercise 6

1. (C) 2. (A) 3. (D) 4. (B) 5. (C) 6. (A)
7. (D) 8. (B) 9. (D) 10. (B) 11. (A) 12. (D)
13. $\angle A = 52^\circ$ 14. 80° 15. 85° 16. 97 17. 165
18. 8 19. 150° 20. 17 21. *No* 22. $\angle C$ 23. 1080°
24. 144° 25. 120° 26. 111° 27. $x = 102^\circ$, $y = 92^\circ$, $z = 66^\circ$
28. $x = 65^\circ$, $y = 55^\circ$ 29. 6 30. (B) 31. 140°

□

Congruence and Inequalities of Triangles

7.1 Introduction

Earlier, we have studied about triangles and their properties. In this chapter, we will study about the congruence rules of triangles and some other properties of triangles and inequalities in triangles.

7.2 Congruence of triangles

You have ever made several copies of your photograph of same size from a photographer. Similarly, you have seen bangles of same size in the wrist of your mother and seen postal stamps with same photo. Such figures are identical. If you choose any two such figures out of these and placed upon each other, then they exactly coincide.

Do you know, in geometry these figures are known by which name? These are called congruent figures. Congruent mean identically equal, i.e., figures with same shape and same size.

Thus, two triangles are congruent if and only if one of them can be made to superpose on the other, so as to cover it exactly.

Axiom : (1)

If two sides and one included angle of a triangle are equal to the corresponding two sides and included angle of the other triangle, then the two triangles are congruent. (SAS rule of congruence)

Theorem 7.1. Angle-Side-Angle Rule (ASA Rule)

If two angles and the included side of one triangle are equal to the corresponding two angles and the included side of the other triangle, then the triangles are congruent.

Given : ABC and DEF are two triangles, in which $\angle ABC = \angle DEF$, $\angle ACB = \angle DFE$ and $BC = EF$

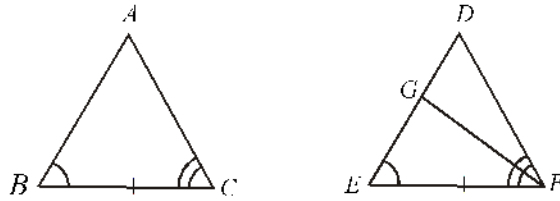


Fig. 7.01

To prove : $\triangle ABC \cong \triangle DEF$

Proof : Here, comparing the length of sides AB and DE of $\triangle ABC$ and $\triangle DEF$, following three conditions are possible :

(i) $AB = DE$ (ii) $AB < DE$ (iii) $AB > DE$

Condition (i) : If $AB = DE$, then in $\triangle ABC$ and $\triangle DEF$

$$AB = DE \quad (\text{Say})$$

$$\angle ABC = \angle DEF \quad (\text{Given})$$

$$BC = EF \quad (\text{Given})$$

Thus, $\triangle ABC$ and $\triangle DEF$ are congruent by Side-Angle Side rule.

$$\text{i.e.,} \quad \triangle ABC \cong \triangle DEF$$

Condition (ii) : When $AB < DE$, then take a point G on side DE such that $AB = GE$ and join GF (Fig. 7.01)

For $\triangle ABC$ and $\triangle GEF$

$$AB = GE \quad (\text{Say})$$

$$BC = EF \quad (\text{Given})$$

$$\angle ABC = \angle GEF \quad (\text{Given}) \quad [\because \angle GEF = \angle DEF]$$

i.e., by SAS rule $\triangle ABC \cong \triangle GEF$

$$\text{Thus,} \quad \angle ACB = \angle GFE \quad \dots (1)$$

$$\text{and} \quad \angle ACB = \angle DFE \quad (\text{Given}) \quad \dots (2)$$

From (1) and (2) $\angle GFE = \angle DFE$ is impossible, unless GF and DF do not coincide. It means points G and D coincide.

$$\therefore \quad AB = DE$$

Thus, by SAS rule,

$$\triangle ABC \cong \triangle DEF.$$

Condition (iii) : When $AB > DE$ then according to Fig. 7.02 take a point G on side AB of $\triangle ABC$ such that $BG = ED$

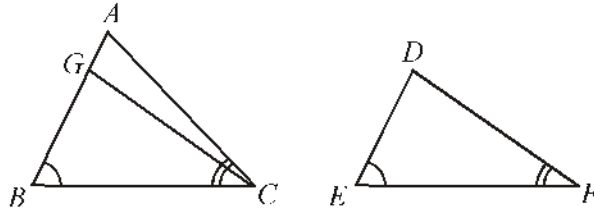


Fig. 7.02

Here, according to condition (ii), we can prove that point G will coincide point A i.e., $AB = DE$ and by Side-Angle-Side rule, $\triangle ABC \cong \triangle DEF$.

Thus, in all three condition, $\triangle ABC \cong \triangle DEF$. Hence proved.

Note : We know that the sum of three interior angles of a triangle is 180° . Therefore, when two angles of a triangle are equal to two angles of another triangle, then their third angles will automatically be same. We will prove the following corollary on the basis of this law.

Corollary : Angle-Angle-Side Rule (AAS Rule)

If two angles and one side of one triangle are equal to the corresponding two angles and one side of the other triangle, then the two triangles are congruent.

Given : In $\triangle ABC$ and $\triangle DEF$ $\angle B = \angle E$; $\angle A = \angle D$ and side $BC =$ side EF

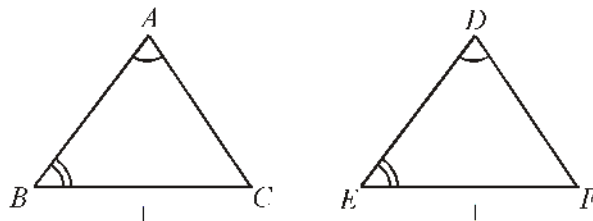


Fig. 7.03

To prove : $\triangle ABC \cong \triangle DEF$

Proof: We know that the sum of three interior angles of a triangle is 180° .

$$\angle A + \angle B + \angle C = 180^\circ \quad \dots (1)$$

$$\angle D + \angle E + \angle F = 180^\circ \quad \dots (2)$$

From (1) and (2) $\angle A + \angle B + \angle C = \angle D + \angle E + \angle F \quad \dots (3)$

Given that $\angle B = \angle E$ $\angle A = \angle D$

Thus, $\angle C = \angle F$ [From (3)] $\dots (4)$

Now, in $\triangle ABC$ and $\triangle DEF$

$$\angle B = \angle E \quad \text{(Given)}$$

$$BC = EF \quad \text{(given)}$$

$$\angle C = \angle F \quad \text{[From (4)]}$$

By Angle-Side-Angle rule $\triangle ABC \cong \triangle DEF$.

Hence proved

Illustrative Examples

Example 1. In Fig. 7.04, C is the mid-point of AB , $\angle BCD = \angle ACE$ and $\angle DAB = \angle EBA$ then show that :

(i) $\triangle DAC \cong \triangle EBC$

(ii) $DA = EB$.

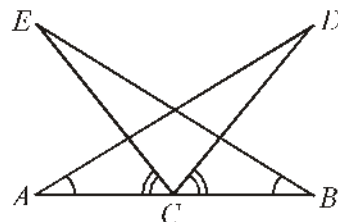


Fig. 7.04

Sol: Given : In Fig. 7.04,

$AC = BC$, $\angle DAB = \angle EBA$ and $\angle BCD = \angle ACE$

To prove : (i) $\triangle DAC \cong \triangle EBC$ (ii) $DA = EB$.

Proof: It is given that C , is the mid-point of side AB .

So, $AC = BC$... (1)

and $\angle BCD = \angle ACE$ (Given) ... (2)

Adding $\angle DCE$ both sides,

$$\angle BCD + \angle DCE = \angle ACE + \angle DCE$$

or $\angle ECB = \angle DCA$... (3)

Now, in $\triangle DAC$ and $\triangle EBC$

$\angle DAC = \angle EBC$ (Given that)

$AC = BC$ [From (1)]

$\angle DCA = \angle ECB$ [From (3)]

By Angle-Side-Angle rule,

$\triangle DAC \cong \triangle EBC$

By the property of congruence, corresponding sides of two triangles are same.

Thus, $DA = EB$ Hence proved.

Example 2. In fig. 7.05, in a quadrilateral $ABCD$ $BC = AD$ and $\angle ADC = \angle BCD$ then prove that :

(i) $AC = BD$ (ii) $\angle ACD = \angle CDB$.

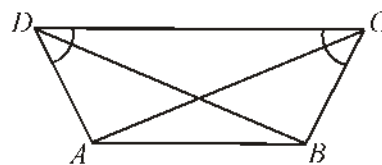


Fig. 7.05

Sol: According to Fig. 7.05, it is given that :

$BC = AD$ and $\angle ADC = \angle BCD$

So, in $\triangle ADC$ and $\triangle BCD$

$AD = BC$ (Given)

$CD = CD$ (Common side)

$\angle ADC = \angle BCD$ (Given)

So, by Side-Angle-Side rule, $\triangle ADC \cong \triangle BCD$

Since, corresponding sides and corresponding angles of congruent triangles are same.

Therefore, $AC = BD$ and $\angle ACD = \angle CDB$. or $\angle ACD = \angle CDB$. Hence proved.

Example 3. AB is a line segment and line ℓ is its perpendicular bisector. If P is any point on ℓ , then show that P is equidistant from points A and B

Sol. : AB is a line segment and line ℓ passes through the mid-point C of AB (see fig. 7.06). We have to show that $PA = PB$. For this, think about $\triangle PCA$ and $\triangle PCB$. It is given that

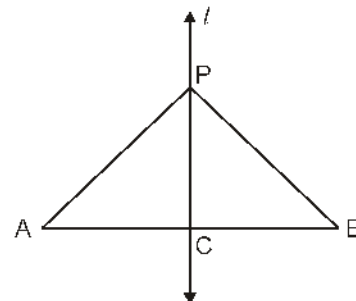


Fig. 7.06

$$AC = BC \quad (C \text{ is the mid-point of } AB)$$

$$\angle PCA = \angle PCB = 90^\circ \quad (\text{Given})$$

$$PC = PC \quad (\text{Common})$$

So, $\triangle PCA \cong \triangle PCB$ (SAS Rule)

Therefore, $PA = PB$ (Corresponding sides of congruent triangles) Hence proved

Example 4. In Fig. 7.07, $AE = EC$ and $DE = BE$ then show that :

(i) $\triangle AED \cong \triangle CEB$ (ii) $\angle A = \angle C$.

Sol. : According Fig. 7.07, it is given that

$$AE = EC$$

$$DE = BE \quad \dots (1)$$

Now, for $\triangle AED$ and $\triangle BEC$

$$AE = EC \quad (\text{given})$$

$$\angle AED = \angle CEB \quad (\text{vertically opposite angles})$$

$$DE = EB \quad (\text{given})$$

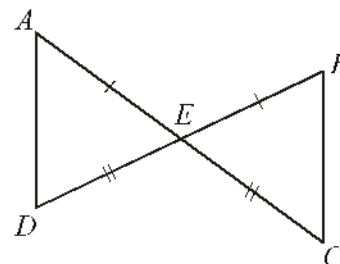


Fig. 7.07

Therefore, by Side-Angle-Side rule $\triangle AED \cong \triangle CEB$ and their corresponding angles $\angle A = \angle C$ $\angle D = \angle B$ Hence proved

Example 5. In Fig. 7.08, $AD = BC$ and $BD = CA$ then show that :

(i) $\angle ADB = \angle BCA$

(ii) $\angle DAB = \angle CBA$.

Sol. : In Fig. 7.08 $AD = BC$ and $BD = CA$

So, in $\triangle ABD$ and $\triangle ABC$

$$\left. \begin{array}{l} AD = BC \\ BD = CA \end{array} \right\} \quad (\text{Given})$$

$$AB = AB \quad (\text{Common})$$

So, by Side-Side-Side rule

$$\triangle ABD \cong \triangle ABC$$

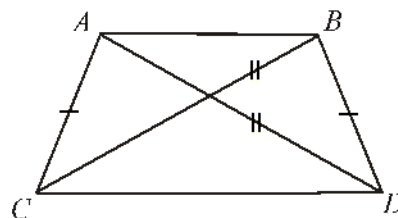


Fig. 7.08

So, corresponding angles

$$(i) \angle ADB = \angle BCA \quad (ii) \angle DAB = \angle CBA$$

Hence proved

Exercise 7.1

1. In $\triangle ABC$ and $\triangle PQR$ $\angle A = \angle Q$ and $\angle B = \angle R$. Which side of $\triangle PQR$ should be equal to the side AB of $\triangle ABC$, so that two triangles become congruent? Give reason to your answer.
2. In triangles ABC and PQR $\angle A = \angle Q$ and $\angle B = \angle R$. Which side of $\triangle PQR$ should be equal to side BC of $\triangle ABC$, so that two triangles are congruent? Give reason to your answer.
3. If two sides and one angle of a triangle is equal to the two sides and one angle of other triangle, then two triangles should be congruent. Is this statement true? Why?
4. If two angles and one side of a triangle is equal to the two angles and one side of other triangle then triangles sure should be congruent. Is this statement true? Why?
5. It is given that $\triangle ABC \cong \triangle RPQ$. Is $BC = QR$ true? Why?
6. If $\triangle PQR \cong \triangle EDF$, then is this true $PR = EF$? Give reason your answer.
7. In fig. 7.09, diagonal AC of quadrilateral $ABCD$, is the bisector of $\angle A$ and $\angle C$? Prove that: $AB = AD$ and $CB = CD$.

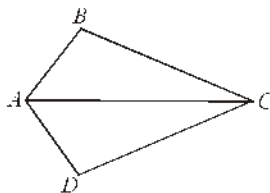


Fig. 7.09

8. In Fig. 7.10, in quadrilateral $ADBC$, $\angle ABC = \angle ABD$ and $BC = BD$, then prove that $\triangle ABC \cong \triangle ABD$.

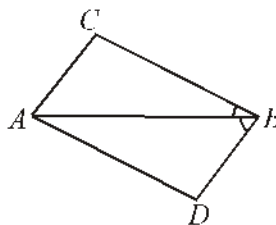


Fig. 7.10

9. According to Fig. 7.11 $AB \parallel DC$ and $AD \parallel BC$ then prove that: $\triangle ADB \cong \triangle CBD$.

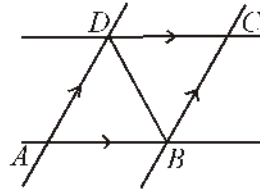


Fig. 7.11

10. In Fig. 7.12, if $AB \parallel DC$ and E is the mid-point of side AC , then prove that E is the mid-point of side BD

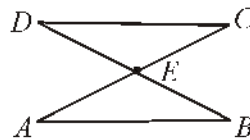


Fig. 7.12

7.3 Special Properties of Triangle

You have already studied about two conditions of congruence of triangles. Now, we will use their results to prove the theorems related to an isosceles triangle and remaining theorems of congruence of triangles.

7.4 Isosceles Triangle

A triangle with two equal sides is called an isosceles triangle.

Theorem 7.3 : If two sides of a triangle are equal, then their opposite angles are also equal.

or

In an isosceles triangle, angles opposite to equal sides are equal.

Given : $\triangle ABC$ is an isosceles triangle

Where, $AB = AC$

To prove : $\angle B = \angle C$

Construction : Draw bisector AD of $\angle A$, which meets BC at D .

Proof : In $\triangle ABD$ and $\triangle ACD$,

$$AB = AC \quad (\text{Given})$$

$$\angle BAD = \angle CAD \quad (\text{By construction})$$

$$AD = AD \quad (\text{Common side})$$

By Side Angle Side rule, $\triangle ABD \cong \triangle ACD$

Since, corresponding angles of congruent triangles are equal.

$$\angle B = \angle C$$

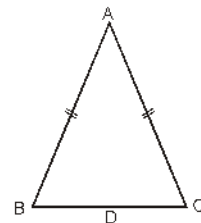


Fig. 7.13

Hence proved.

Theorem 7.4 : If two angles in a triangle are equal, then their opposite sides will be also equal.

Given : $\triangle ABC$ in which $\angle B = \angle C$

To prove : $AB = AC$

Construction : Draw AD , the bisector of $\angle BAC$

Proof : In $\triangle ABD$ and $\triangle ACD$

$$\angle B = \angle C \quad (\text{Given})$$

$$AD = AD \quad (\text{common side})$$

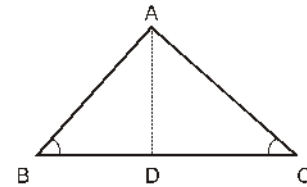
$$\angle BAD = \angle CAD \quad (\text{by construction})$$

By Angle Side Angle rule,

$$\triangle ABD \cong \triangle ACD$$

Thus, corresponding sides $AB = AC$

Fig. 7.14



Hence proved.

Illustrative Examples

Example 6. In $\triangle ABC$, the bisector AD of $\angle A$, is perpendicular to side BC . Show that $\triangle ABC$ is an isosceles triangle.

Sol : In $\triangle ABD$ and $\triangle ACD$

$$\angle BAD = \angle CAD \quad (\text{given that } AD \text{ is bisector of } \angle A)$$

$$AD = AD \quad (\text{Common side})$$

$$\angle ADB = \angle ADC = 90^\circ \quad (\text{Given})$$

$$\triangle ABD \cong \triangle ACD \quad (\text{By ASA rule})$$

Therefore, $AB = AC$

Thus, $\triangle ABC$ is an isosceles triangle.

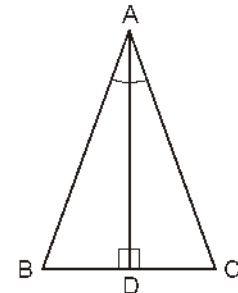


Fig. 7.15

Example 7. According to Fig. 7.16, $ABCD$ is a square and $\triangle CDE$ is an equilateral triangle, then prove that $AE = BE$.

Sol : Given, $ABCD$ is a square and $\triangle CDE$ is an equilateral triangle.

To prove : $AE = BE$

Proof : $\triangle CDE$ is an equilateral triangle

Thus, $CD = DE = CE \quad \dots (1)$

$$\angle DEC = \angle EDC = \angle DCE = 60^\circ \quad \dots (2)$$

and $ABCD$ is a square, so

$$\angle ADC = \angle BCD = 90^\circ$$

Adding $\angle EDC$ to both sides

$$\angle ADC + \angle EDC = \angle BCD + \angle EDC$$

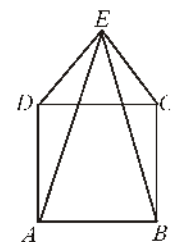


Fig. 7.16

$$\Rightarrow \angle EDA = \angle ECB \quad \dots (3)$$

Now, in $\triangle ADE$ and $\triangle BCE$

$$AD = BC \quad (\text{sides of square})$$

$$\angle EDA = \angle ECB \quad [\text{From (3)}]$$

$$DE = EC \quad [\text{From (1)}]$$

By Side-Angle-Side rule $\triangle ADE \cong \triangle BCE$.

Thus, corresponding sides $AE = BE$

Hence proved

Example 8. Prove that the medians, which bisect equal sides of an isosceles triangle, are equal.

Sol. Given : In an Isosceles $\triangle ABC$, D and E are mid points of equal sides AB and AC .

To prove : $BE = CD$

Proof : $\triangle ABC$ is an isosceles triangle whose sides AB and AC are equal.

$$AB = AC \quad \dots (1)$$

$$\text{and } \angle ABC = \angle ACB \quad \dots (2)$$

and D and E are the mid points of sides AB and AC .

$$\text{Thus, } DB = DA = EC = AE \quad \dots (3)$$

Now, in $\triangle BCD$ and $\triangle BCE$

$$BC = BC \quad (\text{Common side})$$

$$\angle DBC = \angle ECB \quad [\text{From (2)}]$$

$$BD = CE \quad [\text{From (3)}]$$

By Side-Angle-Side rule $\triangle BCD \cong \triangle BCE$

Corresponding sides will be equal i.e.,

$$CD = BE \text{ or } BE = CD$$

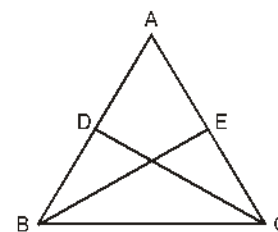


Fig. 7.17

Hence proved.

Example 9. In Fig. 7.18, $AB = AC$ and D is a point in $\triangle ABC$ such that $\angle DBC = \angle DCB$. Prove that AD bisects $\angle BAC$.

Sol. Given : In $\triangle ABC$, $AB = AC$ and $\angle DBC = \angle DCB$.

To prove : AD is the bisector of $\angle BAC$.

$$\text{i.e. } \angle BAD = \angle CAD$$

Proof : In $\triangle BDC$, $\angle DBC = \angle DCB$ so their opposite sides will be same.

$$CD = BD$$

Now, in $\triangle ABD$ and $\triangle ACD$

$$BD = CD \quad [\text{From (1)}]$$

$$AD = AD \quad [\text{Common side}]$$

$$AB = AC \quad (\text{Given})$$

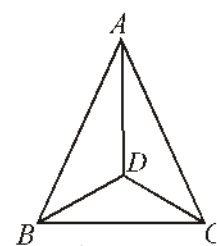


Fig. 7.18

... (1)

By Side-Side-Side rule, $\triangle ABD \cong \triangle ACD$.

So, corresponding angles will be same. i.e., $\angle BAD = \angle CAD$.

Thus, AD is the bisector of $\angle BAC$.

Hence proved.

Example 10. If perpendiculars drawn from mid-point of a side of a triangle to the other two sides are equal, then prove that triangle will be isosceles.

Sol. Given : D is the mid point of side BC of $\triangle ABC$. DE and DF are the perpendiculars on AC and AB respectively, and $DE = DF$.

To prove : $\triangle ABC$ is an isosceles triangle, i.e., $AB = AC$

Construction : Join AD

Proof : In $\triangle BDF$ and $\triangle CDE$

Hypotenuse $BD =$ Hypotenuse CD (Given)

$$\angle DFB = \angle DEC = 90^\circ$$

$$\text{and } DF = DE \quad (\text{Given})$$

By Right Angle Hypotenuse Side rule

$$\triangle BDF \cong \triangle CDE.$$

Thus, corresponding angles $\angle B = \angle C$ and opposite sides to two equal angles will be equal i.e., $AB = AC$. Hence proved

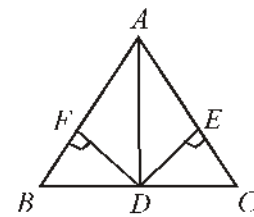


Fig. 7.19

Example 11. In an isosceles triangle ABC , $AB = AC$ and D, E, F are mid-points of sides BC, AC and AB , then prove that $DE = DF$.

Sol. : According to Fig. 7.20 in $\triangle ABC$

$$AB = AC \quad \dots (1)$$

And D, E, F are the mid-points of sides BC, AC and AB respectively

So, in $\triangle BDF$ and $\triangle CDE$

$$BD = CD \quad [D \text{ is the mid-point of side } BC]$$

$$CE = BF \quad [\text{Given } AB = AC]$$

and $\angle B = \angle C$ [Angles opposite to equal sides are equal]

By Side-Angle-Side-rule

$$\triangle BDF \cong \triangle CDE$$

Thus, or $DE = DF$

Hence Proved

Example 12. In Fig. 7.21, ABC is a right-angled triangle, in which $\angle B = 90^\circ$, such that $\angle BCA = 2\angle BAC$. Show that hypotenuse $AC = 2BC$.

Sol. Given : A right angled ABC such that $\angle B = 90^\circ$ and $\angle BCA = 2\angle BAC$

To prove : $AC = 2BC$

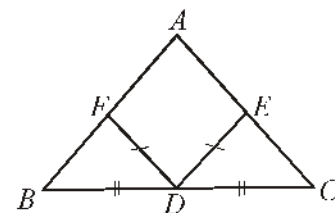


Fig. 7.20

Construction : Produce **CB** upto **D** such that **BC = BD** and join **AD**

Proof : Let $\angle BAC = x$, then $\angle BCA = 2 \angle BAC = 2x$.

In right triangle ABC

$$\angle BAC + \angle BCA + \angle ABC = 180^\circ$$

$$\Rightarrow x + 2x + 90^\circ = 180^\circ$$

$$\Rightarrow 3x = 180^\circ - 90^\circ$$

$$\Rightarrow x = 30^\circ$$

$$\therefore \angle BCA = 2 \times 30^\circ = 60^\circ \quad \text{and} \quad \angle BAC = 30^\circ \quad \dots(i)$$

In $\triangle ABC$ and $\triangle ABD$

$$BC = BD \quad (\text{By construction})$$

$$\angle ABC = \angle ABD \quad (\text{Each is } 90^\circ)$$

$$AB = AB \quad [\text{Common side}]$$

$$\therefore \triangle ABC \cong \triangle ABD \quad (\text{By SAS rule})$$

$$\Rightarrow \angle CAB = \angle DAB$$

$$\text{and} \quad AC = AD$$

$$\text{Now} \quad \angle BAC = \angle BAD = 30^\circ \quad [\text{Using (1)}]$$

$$\therefore \angle A = 2 \times 30^\circ = 60^\circ$$

$$\text{and} \quad AC = AD$$

$$\Rightarrow \angle C = \angle D \quad [\text{Angles opp. to equal sides are equal}]$$

$$\Rightarrow \angle C = \angle D = 60^\circ$$

In $\triangle ACD$,

$$\angle A = \angle C = \angle D = 60^\circ$$

$$\Rightarrow \triangle ACD \text{ is an equilateral triangle.}$$

$$\Rightarrow AC = CD = AD \quad \dots(2)$$

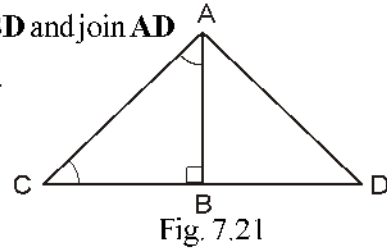
$$\text{But} \quad BC = BD$$

$$\therefore CD = BC + BD.$$

$$\Rightarrow CD = BC + BC$$

$$\Rightarrow CD = 2 BC$$

$$\Rightarrow AC = 2 BC. \quad [\text{Using (2)}] \quad \text{Hence Proved.}$$



Exercise 7.2

1. In Fig. 7.22 $AB = AC$ and $\angle B = 58^\circ$ then find the value of $\angle A$

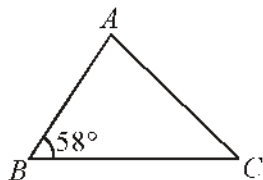


Fig. 7.22

2. In Fig. 7.23 $AD = BD$ and $\angle C = \angle E$, then prove that $BC = AE$.

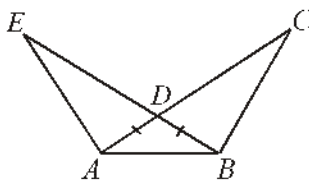


Fig. 7.23

3. If AD is the median of an isosceles triangle ABC and $\angle A = 120^\circ$ and $AB = AC$, then find the value of $\angle ADB$
4. If the bisector of any angle of a triangle also bisects the opposite side then prove that the triangle is an isosceles triangle.
5. In Fig. 7.24, $AB = AC$ and $BE = CD$, then prove that: $AD = AE$.

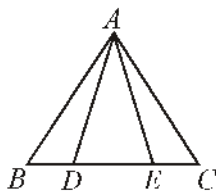


Fig. 7.24

6. E and F are two points on sides AD and BC respectively of square $ABCD$, such that $AF = BE$. Show that :
- (i) $\angle BAF = \angle ABE$ (ii) $BF = AE$
7. AD and BC are two equal perpendiculars on a line segment AB (see fig. 7.25). Show that CD bisects line segment AB .

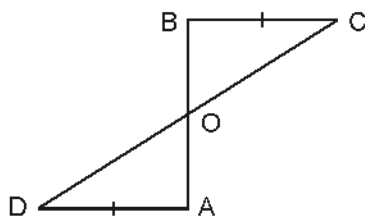


Fig. 7.25

8. The bisectors of angles B and C of an isosceles triangle with $AB = AC$, intersect each other at point O. Produce BO upto point M. Prove that : $\angle MOC = \angle ABC$
9. Line ℓ bisects angle A and B is any point on line ℓ . BP and BQ are the perpendiculars drawn on the sides of angle A from point B (See Fig. 7.26)

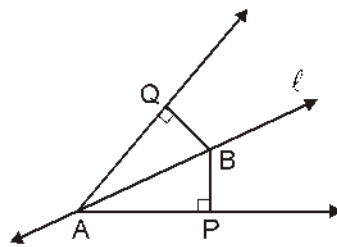


Fig. 7.26

- (i) $\triangle APB \cong \triangle AQB$
- (ii) $BP = BQ$ it means point B is equidistant from the sides of $\angle A$.
11. In Fig. 7.27 $AC = AE$, $AB = AD$ and $\angle BAD = \angle EAC$. Show that : $BC = DE$

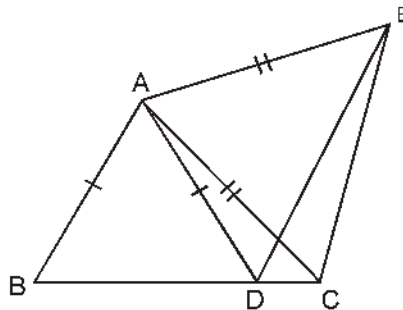


Fig. 7.27

12. In right triangle ABC, angle C is right angle M is the mid-point of hypotenuse AB Join C to M and produce it upto D such that $DM = CM$. Join point D and B (see fig. 7.28). Show that :
- (i) $\triangle AMC \cong \triangle BMD$
- (ii) $\angle DBC$ is a right angle
- (iii) $\triangle DBC \cong \triangle ACB$
- (iv) $CM = \frac{1}{2} AB$

7.5 Some other concepts for the Congruence of Triangles

If three angles of a triangle are equal to three angles of another triangle, then it is not necessary that these two triangles are congruent.

If three sides of triangle are equal to three sides of another triangle then according to you whether these triangles be congruent? Definitely, they will be congruent. Now, we prove this theorem by using obtained result.

Theorem 7.5 : Side, Side, Side-Rule (S S S Rule)

If three sides of a triangle are equal to the corresponding three sides of other triangle, then two triangles are congruent.

Given : Corresponding sides of $\triangle ABC$ and $\triangle DEF$ are same, i.e.,

$$AB = DE; BC = EF \text{ and } AC = DF$$

To prove : $\triangle ABC \cong \triangle DEF$

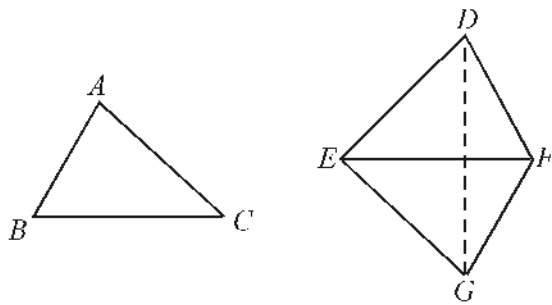


Fig. 7.29

Construction : Draw a line segment EG in the opposite side of $\triangle DEF$ such that $EG = AB$ and $\angle ABC = \angle FEG$. Join GE and DG .

Proof: In $\triangle ABC$ and $\triangle GEF$

$$AB = GE \quad (\text{By construction})$$

$$\angle ABC = \angle GEF \quad (\text{By construction})$$

$$BC = EF \quad (\text{Given})$$

By Side-Angle-Side rule, $\triangle ABC \cong \triangle GEF$

So, corresponding angles and corresponding sides of congruent triangles are equal.

$$\angle A = \angle G; AB = GF \quad \dots(1)$$

Now, $AB = EG$ (By construction, and $AB = DE$ (Given)

$$\text{So} \quad EG = DE \quad \dots(2)$$

Similarly, $AC = GF$ from equation (1) and $AC = DF$ (Given)

$$\therefore GF = DF \quad \dots(3)$$

\Rightarrow In $\triangle EDG$, angles opposite to equal sides EG and DE are same.

$$\angle EDG = \angle EGD \quad \dots(4)$$

Similarly, in $\triangle FDG$, angles opposite to equal sides GF and DF are same.

$$\angle GDF = \angle DGF \quad \dots(5)$$

Adding equations (4) and (5)

$$\begin{aligned} & \angle EDG + \angle GDF = \angle EGD + \angle DGF \\ \Rightarrow & \angle D = \angle G \quad \dots(6) \end{aligned}$$

But from equation (1)

$$\angle A = \angle G \quad \dots(7)$$

From equation (6) and (7)

$$\angle A = \angle D \quad \dots(8)$$

Thus, in $\triangle ABC$ and $\triangle DEF$

$$\begin{aligned} AB &= DE && \text{(Given)} \\ \angle A &= \angle D && \text{[From (8)]} \\ AC &= DF && \text{(Given)} \end{aligned}$$

By SAS rule.

$$\triangle ABC \cong \triangle DEF \quad \text{Hence Proved}$$

Now, we try to verify this theorem by the following activity :

Construct two triangles each of which have sides 4 cm, 3.5 cm and 3 cm.

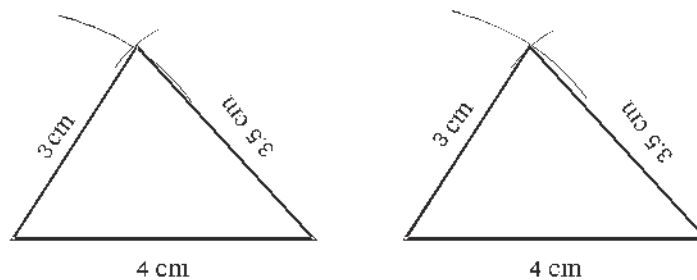


Fig. 7.30

Now cut them and place one of them on the other. What do you see? By keeping equal sides in mind when we placed one upon the other triangle they cover each other completely. This is possible only when two triangles are congruent.

It means two triangles are congruent.

You have already observed SAS rule of congruence. By SAS rule of congruence

pair of equal angles may be between pair of corresponding sides. If not so, then two triangles may not be congruent.

Let verify it by an activity.

Draw two right angled triangles in which each one of hypotenuse is 5 cm and a side is 4 cm (See Fig. 7.31).

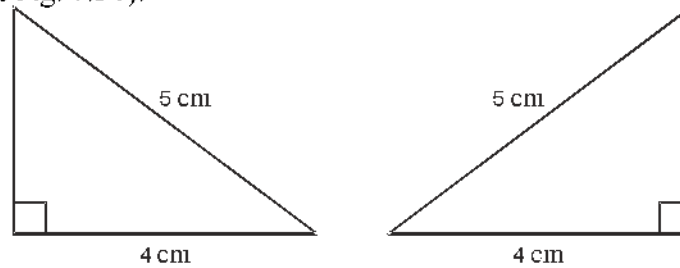


Fig. 7.31

Cut them and place one on the other such that equal sides coincide. If necessary, rotate them. What do you see? You see, on placing one on the other, they exactly cover each other so they are congruent. Repeat this activity by taking differe of right angled triangles. What do you see? You will see that if their hypotenuse and one pair of side are equal, then two right angled triangles will be congruent.

Note that, in this condition right angle is not the angle included in the hypotenuse and side.

In this way we conclude an important fact for right angled triangles which can be proved by theorem.

Theorem 7.6. Right Hypotenuse Side Rule (RHS congruence rule) :

Two right triangles are congruent if and only if the hypotenuse and a side of one triangle are equal to the hypotenuse and the corresponding side of the other triangle.

Given : In two right triangles, ABC and DEF

$$\angle B = \angle E = 90^\circ$$

Hypotenuse AC – hypotenuse DF
and side AB – side DE

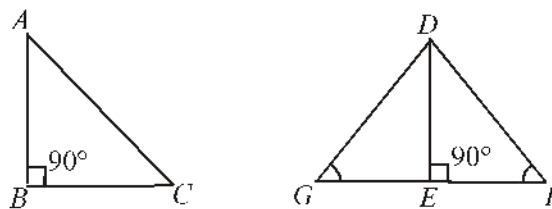


Fig. 7.32

To prove : $\triangle ABC \cong \triangle DEF$

Construction : In $\triangle DEF$, produce E upto G such that $GE = BC$ and join G to D .

Proof : Here, $\angle DEF = 90^\circ$

\therefore and $\angle DEG = 90^\circ$... (1)

Now, in $\triangle ABC$ and $\triangle DEG$

$$AB = DE \quad (\text{Given})$$

$$BC = GE \quad (\text{By construction})$$

$$\angle ABC = \angle DEG = 90^\circ \quad [\text{From (i)}]$$

By Side-Angle-Side rule, $\triangle ABC$ and $\triangle DEG$ are congruent. So, their corresponding sides and corresponding angles will be equal.

\therefore $AC = DG$ and $\angle C = \angle G$... (2)

But it is given that $AC = DF$... (3)

From equations (2) and (3),

$$DG = DF \quad \dots (4)$$

\therefore In $\triangle DGF$, angles opposite to equal sides ($DG = DF$) will be equal

$$\angle G = \angle F \quad \dots (5)$$

From equations (2) and (5), $\angle C = \angle F$... (6)

Now, in $\triangle ABC$ and $\triangle DEF$

$$AB = DE \quad (\text{Given})$$

$$\angle C = \angle F \quad [\text{from (6)}]$$

and $\angle ABC = \angle DEF = 90^\circ$ (Given)

By Angle-Angle-Side rule,

$$\triangle ABC \cong \triangle DEF \quad \text{Hence Proved}$$

Illustrative Examples

Example 13. AB is a line segment and points P and Q are situated on the opposite side of AB such that each of them is equidistant from A and B . (see fig. 7.33). Show that line segment PQ is the perpendicular bisector of the line segment AB .

Solution : Here, $PA = PB$ and $QA = QB$ is given. We have to show that $PQ \perp AB$ and PQ bisects AB . Let PQ intersects line segment AB at C .

You can see two congruent triangles in this figure? Let us take $\triangle PAQ$ and $\triangle PBQ$.

$$AP = BP \quad (\text{Given})$$

$$AQ = BQ \quad (\text{Given})$$

\therefore $PQ = PQ$ (Common side)
 $\Delta PQA \cong \Delta PBQ$ (BySSS rule)
 $\angle APQ = \angle BPQ$
 So, In ΔPAC and ΔPBC , we get
 $AP = BP$ (Given)
 $\angle APC = \angle BPC$ ($\angle APQ = \angle BPQ$ has been proved)
 $PC = PC$ (Common side)

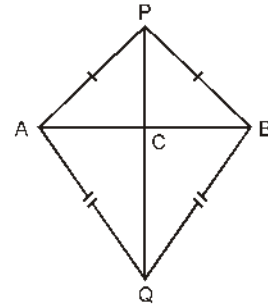


Fig. 7.33

$\therefore \Delta PAC \cong \Delta PBC$
 and $\angle ACP = \angle BCP$
 and $AC = CB$
 Now, $\angle ACP + \angle BCP = 180^\circ$ (Linear pair) ... (1)
 So, $2 \angle ACP = 180^\circ$
 or, $\angle ACP = 90^\circ$... (2)

From equations (1) and (2), we conclude that line PQ is perpendicular bisector of AB .

Note that without proving congruence of ΔPAQ and ΔPBQ , we cannot show $\Delta APQ = \Delta BPQ$, whereas $AP = BP$ (given) $PC = PC$ (common) and $\angle PAC = \angle PBC$.

(Opposite angles of equal sides in ΔPAB). We obtained this by SSA rule which is not always acceptable for the congruence of triangles and angle is not (included) between the equal sides. Let us take some other examples.

Example 14: P is a point equidistant from two lines l and m intersecting at point A (see fig 7.34). Show that line AP bisects the angle between them.

Solution : It is given that lines l and m intersect at A . Let $PB \perp l$ and $PC \perp m$.

It is given that $PB = PC$ ($\because P$ is equidistant from l and m .)

To Prove : $\angle PAB = \angle PAC$

Proof: Now, in ΔPAB and ΔPAC

$$PB = PC \quad (\text{Given})$$

$$\angle PBA = \angle PCA = 90^\circ \quad (PB \perp l \text{ and } PC \perp m)$$

$$PA = PA \quad (\text{Common hypotenuse})$$

$\therefore \Delta PAB \cong \Delta PAC$ (ByRHSrule)

Therefore $\angle PAB = \angle PAC$

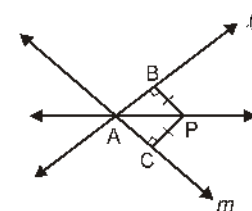


Fig. 7.34

Hence proved

Exercise 7.3

- $\triangle ABC$ and $\triangle DBC$ are two isosceles triangles on the same base BC such that vertices A and D are situated on the same side of BC (see Fig. 7.35). If AD is extended to intersect BC at P , then show that

 - $\triangle ABD \cong \triangle ACD$
 - $\triangle ABP \cong \triangle ACP$
 - AP bisects both $\angle A$ and $\angle D$
 - AP is the perpendicular bisector of line segment BC .

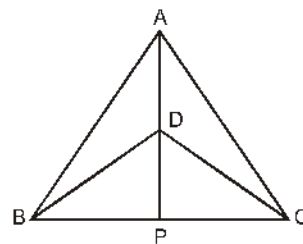


Fig. 7.35

- AD is an altitude of an isosceles triangle ABC in which $AB = AC$. Show that :

 - AD bisects line segment BC .
 - AD bisects $\angle A$
- Two sides AB and BC and median AM of a $\triangle ABC$ are respectively equal to the corresponding sides PQ and QR and median PN of other triangle (see Fig. 7.36). Show that:

 - $\triangle ABM \cong \triangle PQN$
 - $\triangle ABC \cong \triangle PQR$

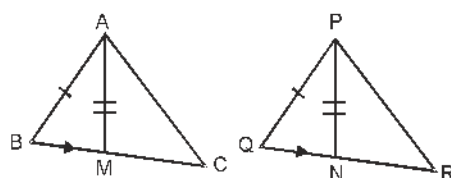


Fig. 7.36

- BE and CF are two equal altitudes of a triangle ABC . By using RHS rule of congruence, prove that $\triangle ABC$ is an isosceles triangle.
- ABC is an isosceles triangle in which $AB = AC$. By drawing $AP \perp BC$, show that $\angle B = \angle C$

7.8. Inequalities of a triangle :

In the previous chapter, you have studied about scalene triangles, isosceles triangle and equilateral triangle on the basis of sides of triangle and acute angled triangle, right-angled triangle and obtuse-angled triangle on the basis of angles.

Did you ever think that if measure of sides of triangle are change, then angles also change and if angles of a triangle are change then measure of sides are also change

Why?

Let us try to understand this by the following activity and theorems.

Theorem 7.7. *If two sides of a triangle are unequal, then angle opposite to longest side is greater than the angle opposite to smaller side.*

Given : In $\triangle ABC$, $AB > AC$

To Prove $\angle C > \angle B$

Construction : Draw line CD from vertex C such that $AC = AD$.

Proof : $AC = AD$ (By construction)

\therefore Their opposite angles will be same.

$\therefore \angle ACD = \angle ADC$... (1)

$\therefore \angle ADC$ is exterior angle of $\triangle BDC$

$\therefore \angle ADC > \angle B$... (2)

From equations (1) and (2) $\angle ACD > \angle B$... (3)

From figure $\angle ACB > \angle ACD$... (4)

From equations (3) and (4), $\angle ACB > \angle ACD > \angle B$

$\Rightarrow \angle ACB > \angle B$

Thus, $\angle C > \angle B$

Hence Proved

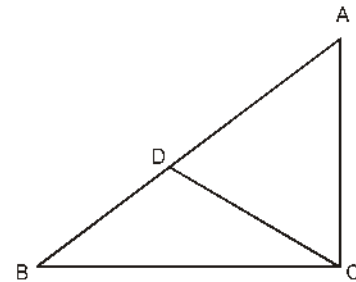


Fig. 7.37

Theorem 7.8. (Converse of Theorem 7.7)

In a triangle, side opposite to greatest angle is greater than the side opposite to smaller angle.

Given : In $\triangle ABC$, $\angle B > \angle C$

To Prove: $AC > AB$

Proof : Following are the three possibilities for sides AC and AB of $\triangle ABC$. Out of them only one is possible.

(i) $AC = AB$

(ii) $AC < AB$, and

(iii) $AC > AB$

Condition : (i) When $AC = AB$

If $AC = AB$, then in $\triangle ABC$, angles opposite to equal sides will be equal i.e., $\angle B = \angle C$ which is impossible because it is given that $\angle B > \angle C$

Thus, $AC \neq AB$

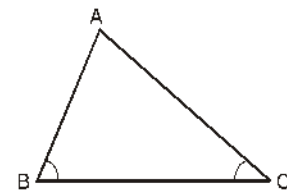


Fig. 7.38

Condition : (ii) When $AC < AB$

We know that angle opposite to longest side is greater.

$$\therefore AC < AB \Rightarrow AB > AC$$

$$\Rightarrow \angle C > \angle B$$

Which is contradiction of given statement

$$\therefore AC < AB$$

Condition : (iii) Since, AC is neither less than nor equal to AB

$$AC \text{ must } > AB$$

Hence, $AC > AB$ is true.

Hence Proved

Theorem 7.9 : Sum of any two sides of a triangle is greater than its third side.

Given : A triangle ABC

To Prove :

(i) $AB + BC > AC$

(ii) $BC + AC > AB$

(iii) $AC + AB > BC$

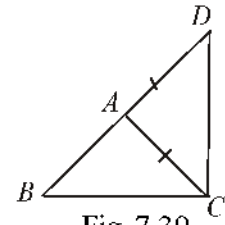


Fig. 7.39

Construction : Produce BA upto D such that $AD = AC$

Proof: In ADC

$$AD = AC \quad (\text{By construction})$$

\therefore Angle opposite to equal sides will be equal.

$$\angle ACD = \angle ADC \quad \dots(1)$$

and $\angle BCD > \angle ACD \quad \dots(2)$

From equations (1) and (2)

$$\angle BCD > \angle ADC - \angle BDC$$

$$\therefore BD > BC \quad [\because \text{Side opposite to greater angle is longer}]$$

Thus $BD > BC$

$$\Rightarrow BA + AD > BC \quad [\because BD = BA + AD]$$

$$\Rightarrow BA + AC > BC \quad [\because AD = AC, \text{ by construction}]$$

Similarly, we can prove that

$$AB + BC > AC$$

$$BC + AC > AB$$

Hence Proved

7.7. Lines and Perpendicular Distance from an External Point:

Distance between a line and its external only point is equal to the length of perpendicular drawn from that point on the line.

Theorem 7.10. *Out of the all line segments drawn from an external point to a straight line (line segment), then the perpendicular line segment is the shortest.*

AB is a line and C is an external point not lying on it. $CE \perp AB$ and D is any point on AB other than E .

To prove : $CE < CD$

Proof : In $\triangle CED$

$$\angle CED = 90^\circ \quad [\because CE \perp AB]$$

In $\triangle CED$, $\angle CED = 90^\circ$

$$\angle CDE + \angle CDE = 90^\circ$$

$\Rightarrow \angle CDE < \angle CED$

$\Rightarrow \angle CED > \angle CDE$

$\Rightarrow CD > CE$

[Side opposite to greater angle is longer]

And $\angle DCE < \angle CED$

$\Rightarrow \angle CED > \angle DCE$

$\Rightarrow CD > DE$

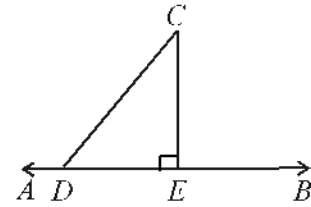


Fig. 7.40

It means out of all line segments drawn from an external point to a straight line, the perpendicular line segment is the shortest. **Hence proved**

Illustrative Examples

Example 15 : In Fig. 7.41, AD is the median of $\triangle ABC$, then prove that $AB + AC > 2AD$

Or

Prove that the sum of two sides of a triangle is more than twice the median draw on third side.

Sol : Given : AD is the median of $\triangle ABC$

To Prove : $AB + AC > 2AD$

Construction : According to figure, produce AD upto E

such that $DE = AD$. Join C to E .

Proof : In $\triangle ADB$ and $\triangle EDC$

$$AD = DE \text{ (By construction)}$$

$$BD = DC \text{ (Given)}$$

$$\angle ADB = \angle EDC \text{ (Vertically opposite angles)}$$

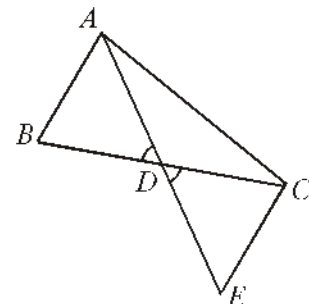


Fig. 7.41

⇒ By Side Angle Side rule, $\triangle ADB \cong \triangle EDC$

$$AB = CE$$

Now in $\triangle ACE$

$$AC - CE > AE$$

⇒ $AC - AB > AE$ [$\because CE = AB$]

⇒ $AC - AB > 2AD$ [$\because AE = 2AD$]

Hence Proved

Example 16. If $ABCD$ is a quadrilateral, then prove that

(i) $AB + BC + CD + DA > 2AC$

(ii) $AB + BC + CD + DA > AC + BD$

Solution : Given : According to figure 7.42, $ABCD$ is a quadrilateral.

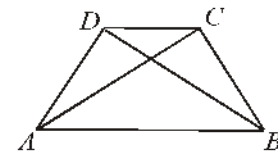


Fig. 7.42

To Prove : (i) $AB + BC + CD + DA > 2AC$

(ii) $AB + BC + CD + DA > AC + BD$

Construction : Join diagonals AC and BD .

Proof : We know that the sum of two sides of a triangle is more than the third side. So

In $\triangle ABC$, $AB + BC > AC$... (1)

In $\triangle ADC$, $AD + DC > AC$... (2)

In $\triangle ABD$, $AB + AD > BD$... (3)

In $\triangle BCD$, $BC + CD > BD$... (4)

Adding (1) and (2), we get

$$AB + BC + AD + CD > 2AC \quad \dots(i)$$

Again, adding (1), (2), (3) and (4), we get

$$2(AB + BC + AD + DC) > 2(AC + BD)$$

⇒ $AB + BC + AD + DC > AC + BD$... (ii)

Hence Proved

Example 17 : In Fig. 7.43, O is any point in the interior of $\triangle ABC$, then prove that

$$AB + AC > OB + OC$$

Solution : Given : O is an interior point in $\triangle ABC$.

To Prove : $AB + AC > OB + OC$

Construction : Produce BO , which meets AC at D .

Proof : We know that in a triangle, sum of two sides is more than the third side.

∴ In $\triangle ABD$, $AB + AD > BD$

⇒ $AB + AD > OB + OD$

... (1)

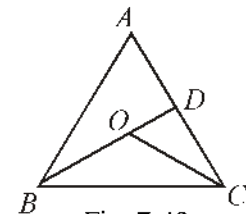


Fig. 7.43

Similarly, in $\triangle OCD$, $OD + DC > OC$... (2)

Adding (1) and (2), we get

$$\begin{aligned} & AB + AD + OD - DC > OB - OD + OC \\ \Rightarrow & AB + (AD + DC) > OB + OC \\ \Rightarrow & AB + AC > OB + OC \end{aligned}$$

Hence Proved

Important points

In this chapter, you have studied the following points.

1. Two figures are congruent, if they have same size and same measure.
2. Two circles of same radius are congruent.
3. Two squares of same sides are congruent.
4. If $\triangle ABC$ and $\triangle PQR$ are congruent under the correspondence $A \leftrightarrow P, B \leftrightarrow Q$ and $C \leftrightarrow R$, then in notation form, they are written as $\triangle ABC \cong \triangle PQR$.
5. If two sides and one included angle of one triangle are equal to the corresponding two sides and included angle of other triangle, then two triangles are congruent. (SAS rule of congruence)
6. If two angles and included side of a triangle are equal to the corresponding two angles and included side of other triangle, then the two triangles are congruent. (ASA rule of congruence)
7. If two angles and one side of a triangle are equal to the corresponding two angles and one side of other triangle, then two triangles are congruent. (AAS rule of congruence)
8. Angles opposite to equal sides of a triangle are equal.
9. Side opposite to equal angles of a triangle are equal.
10. Each angle of an equilateral triangle is 60° .
11. If the three sides of a triangle are equal to corresponding three sides of other triangle, then two triangles are congruent. (SSS rule of congruence)
12. 'If in two right triangles, hypotenuse and one side of a triangle are equal to hypotenuse and one side of other triangle, then two triangles are congruent. (RHS rule of congruence)
13. In a triangle, angle opposite to longer side is greater.
14. In a triangle, side opposite to greater angle is longer.
15. In a triangle, sum of two sides is greater than the third side.

- (c) CA (d) None of theite
13. Difference of two sides of a triangle to its third side is :
 (a) More (b) same (c) less (d) half
14. If two sides of a triangle are unequal, then angle opposite to longest side is :
 (a) Greater (b) smaller (c) equal (d) half
15. The perimeter of triangle is then sum of its medians.
 (a) More (b) Less (c) Equal (d) Half
16. The sum of three altitudes of triangles is its perimeter :
 (a) more (b) equal (c) half (d) less
17. If in $\triangle ABC$, $AB = AC$ and $\angle A < 60^\circ$ then write the relation between BC and AC .
18. In Fig 7.44, write the relation between AB and AC .

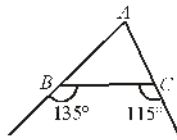


Fig. 7.44

19. In any triangle ABC , $\angle A > \angle B$ and $\angle B > \angle C$, then what will be the smallest side?
20. Find all the angles of an equilateral triangle.
21. P is any point lie on the bisector of $\angle ABC$. If through P , a line is drawn parallel to BA meets line BC at Q then prove that $\triangle BPQ$ is an isosceles triangle.
22. ABC is a right angled triangle in which $AB = AC$. Bisector of $\angle A$ meets BC at D . Prove that $BC = 2 AD$.
23. $\triangle ABC$ and $\triangle DEC$ are lie on the same base BC such that points A and D are on the opposite side of BC , and $AB = AC$ and $DB = DC$. Show that AD is the perpendicular bisector of BC .
24. ABC is an isosceles triangle, in which $AC = BC$. AD and BE are altitudes on BC and AC respectively. Prove that $AE = BD$.
25. Prove that sum of any two sides of a triangle is more than twice the corresponding median of third side.
26. In a $\triangle ABC$, D is the mid-point of side AC such that $BD = \frac{1}{2} AC$. Show that $\angle ABC$ is a right angle.
27. In a right triangle prove that line segment joining the mid point of hypotenuse to its opposite vertex is half of the hypotenuse.

28. In Fig. 7.45, if $AB = AC$, then write the relation between AB and AD .

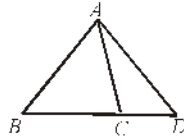


Fig. 7.45

29. AD is a median of $\triangle ABC$. Is it true that $AB + BC + CA > 2AD$. Give reason for your answer.
30. M is any point on side BC of $\triangle ABC$ such that AM is the bisector of $\angle BAC$. Is it true that perimeter of triangle is more than $2AM$? Give reason for your answer.
31. Q is any point situated on side SR of $\triangle PSR$ such that $PQ < PR$. Prove that : $PS > PQ$.
32. S is any point situated on side QR of $\triangle PQR$. Show that $PQ + QR + RP > 2PS$
33. D is any point situated on side AC of $\triangle ABC$, with $AB = AC$. Show that $CD < BD$.
34. In Fig. 7.46. $\angle B > \angle A$ and $\angle D > \angle E$, then prove that $AE > BD$.

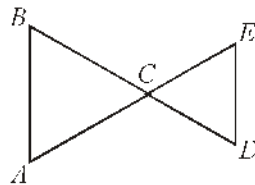


Fig. 7.46

35. In $\triangle ABC$, $AB > AC$ and D is any point on side BC prove that $AB > AD$.
36. Prove that sum of three sides of a triangle is more than the sum of its three medians. [Hint: use example 1]
37. In Fig. 7.47. O is the interior point in a triangle, then prove that :
 $(BC + AB + AC) < 2(OA + OB + OC)$

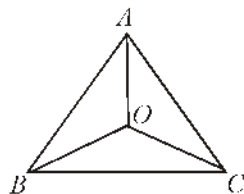


Fig. 7.47

38. Prove that sum of three altitudes of triangle is less than perimeter of triangle.
39. Prove that difference of any two sides of any triangle is less than its third side.
40. Bisectors of $\angle B$ and $\angle C$ of an isosceles triangle with $AB = AC$, intersect each other at point O . show that adjacent $\angle ABC$ is equal to exterior $\angle BOC$.
41. In Fig. 7.48 AD is the bisector of $\angle BAC$. Prove that $AB > BD$.

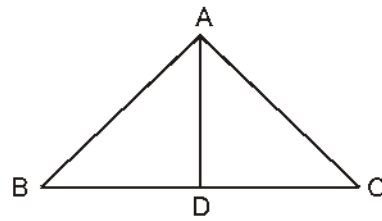


Fig. 7.48

Answer

Exercise 7.1

1. QR: These will be congruent by ASA
2. RP: These will be congruent by AAS
3. No, angle should be between the two sides.
4. No, sides should be corresponding.
5. No, BC should be equal to PQ
6. Yes, these are corresponding sides.

Exercise 7.2

1. 64°
3. 90°

Miscellaneous Exercise -7

- | | | | | | |
|-------|-------|-------|-------|-------|-------|
| 1. C | 2. B | 3. B | 4. C | 5. A | 6. B |
| 7. B | 8. D | 9. B | 10. A | 11. B | 12. A |
| 13. C | 14. A | 15. A | 16. D | | |
17. $BC < AC$
 18. $AB > AC$
 19. AB
 20. 60, 60, 60
 28. $AD > AB$
 29. Yes $AB + BD > AD$ and $AC + CD > AD$
 30. Yes $AB + BM > AM$ and $AC + CM > AM$

Construction of Triangles

8.1 Introduction

Every triangle has six components—three sides and three angles. We have already studied about the essential condition for congruency of triangles. To construct a triangle three components are required i.e. :

- (i) Three sides, or
- (ii) Two sides and angle between them, or
- (iii) Two angles and one side, or
- (iv) In right angled triangle hypotenuse and one side

Remark :

- (i) If all three angles are given, then construction of triangle is not possible,
- (ii) If two sides and their one opposite acute angle be given, then construction of triangle is in ambiguous situation.

In this chapter, we will learn to construct triangles in various conditions.

Construction 8.1 : Construction of triangle whose three sides are given

Construct a triangle ABC whose sides $a = 5$ cm, $b = 4$ cm and $c = 3.5$ cm

Here, ' a ' stands for side opposite to $\angle A$; ' b ' stands for side opposite to $\angle B$; ' c ' stands for side opposite to $\angle C$.

Before the construction of the triangle, we will draw a rough sketch of the triangle of given measurements. On the basis of that we will draw the required triangle.

Construction :

- (i) Draw a line segment $BC = a = 5$ cm
- (ii) Taking B as centre and 3.5 cm as radius draw an arc.

- (iii) Taking C as centre and 4 cm as radius draw another arc which cuts the previous arc at A .
- (iv) Join A to B and C

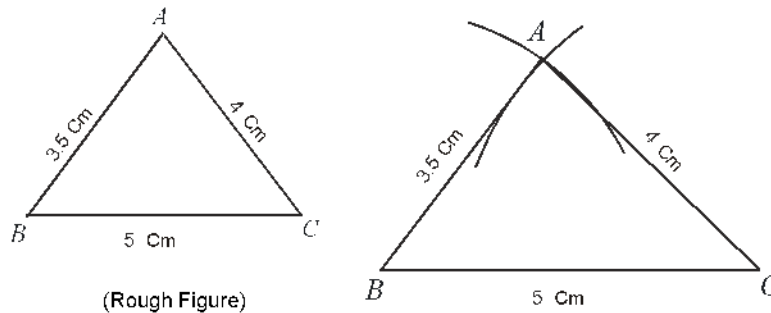


Fig. 8.01

Thus, ABC is the required triangle.

Exercise 8.1

1. Construct a triangle ABC in which $AB = 4\text{ cm}$, $BC = 5\text{ cm}$ and $CA = 6\text{ cm}$.
2. Two points A and B are at a distance 6.5 cm to each other. Find the position of point C which is at a distance of 7 cm and 6 cm from points A and B respectively.
3. Construct a triangle ABC in which sides $a = 6.5\text{ cm}$, $b = 7.2\text{ cm}$ and $c = 8\text{ cm}$. Draw the bisector of $\angle B$, which meets AC at point M .
4. Construct a ΔABC such that $a = 7\text{ cm}$, $b = 5\text{ cm}$ and $c = 4\text{ cm}$. Draw a perpendicular from A on BC .
5. Construct an equilateral triangle ABC whose each side is 5.5 cm .
6. Construct an isosceles triangle whose base is 3 cm and equal sides are 5 cm .

Construction 8.2

Construction of a triangle whose two sides and angle between them is given

Construct a triangle ABC whose sides $a = 4.5\text{ cm}$, $c = 3.5\text{ cm}$ and $\angle B = 45^\circ$.

Construction :

- (i) Draw a line segment $BC = a = 4.5\text{ cm}$
- (ii) At point B with the help of ruler and compass draw $\angle DBC = 45^\circ$ with BC
- (iii) Taking B as a centre and 3.5 cm as radius, draw an arc cutting BA from BD .
- (iv) Join A to C .

Thus, ΔABC is the required triangle.

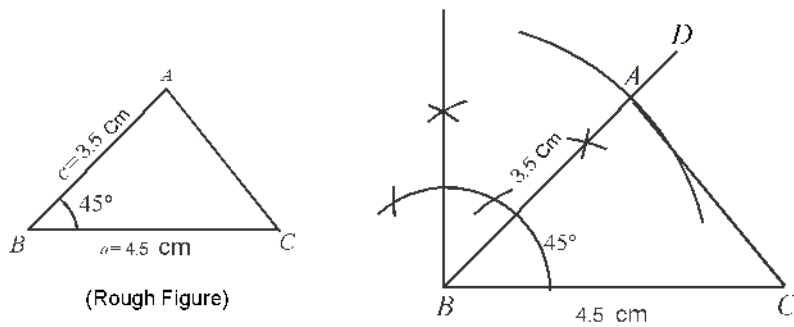


Fig. 8.02

Exercise 8.2

1. Construct a triangle ABC in which $a = 4$ cm, $b = 5$ cm and $\angle C = 60^\circ$.
2. Construct a triangle LMN in which $\angle L = 120^\circ$, $LM = 4$ cm, $LN = 5$ cm.
3. Construct a triangle ABC in which side $AB = AC = 8$ cm and $\angle A = 75^\circ$. Also draw the bisector of $\angle B$ which meets the opposite side.
4. Construct an isosceles triangle whose vertex angle is 120° and each equal sides is of length 5.5 cm.

Construction 8.3

Construction of a triangle whose one side and two angles are given :

Construct a ΔABC in which $\angle B = 60^\circ$, $\angle C = 75^\circ$ and side $BC = 4.6$ cm.

Construction :

- (i) Draw a line segment $BC = 4.6$ cm
- (ii) With the help of ruler and compass, at point B , draw $\angle EBC = 60^\circ$ and at point C , draw $\angle FCB = 75^\circ$ with BC .
- (iii) BE and CF intersect at point A . Thus, ΔABC is the required triangle.

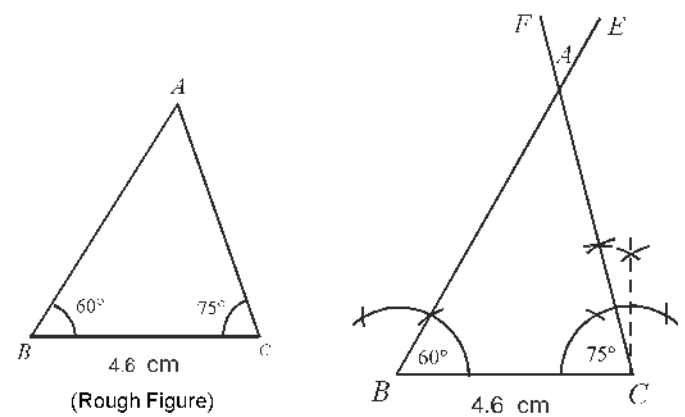


Fig. 8.03

Remark :

If one angle is given at the ends B and C on side BC and a vertex angle is given, then to find the base angle, we can subtract the sum of base and vertex angle from 180° and can get the other base angle, then construct the triangle by above geometric construction.

Exercise 8.3

1. Construct a triangle PQR in which $QR = 8$ cm, $\angle Q = 120^\circ$ and $\angle R = 30^\circ$.
2. Construct a triangle ABC in which $b = 7$ cm, $\angle A = 90^\circ$ and $\angle C = 60^\circ$.
3. Construct an isosceles triangle whose base is 4 cm and vertex angle is 30° . Draw perpendicular from the vertex on base.

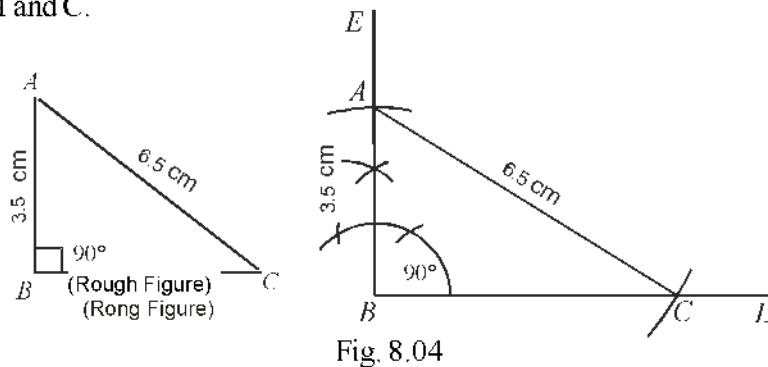
Construction 8.4

Construction of a right angled triangle in which hypotenuse and one other side be given.

Construct a right-angled triangle whose hypotenuse $AC = 6.5$ cm and side $AB = 3.5$ cm.

Construction :

- (i) Draw a line segment BD of suitable length and at B draw $\angle DBE = 90^\circ$.
- (ii) Taking B as centre and 3.5 cm as radius, draw an arc which cuts BE at A .
- (iii) Taking A as centre and 6.5 cm as radius, draw an arc which cuts BD at C .
- (iv) Join A and C .



Thus, ABC is the required triangle.

Exercise 8.4

1. Construct a right angled triangle whose hypotenuse is 5 cm and other side is 3 cm.
2. Construct a triangle ABC in which $\angle A = 90^\circ$, side $AC = 5.4$ cm and hypotenuse $BC = 10$ cm.
3. Construct a right angle triangle ABC in which $\angle A = 90^\circ$ and side $a = 10$ cm and side $b = 6$ cm. Draw a perpendicular on hypotenuse from vertex A .

Construction 8.5

Construct of a triangle whose two sides and an angle opposite to one of the given side, is given.

Construct a triangle $\triangle ABC$ in which $AB = 5$ cm, $AC = 3$ cm, and $\angle B = 30^\circ$.

Construction :

1. Draw a line segment BI' . Draw an angle $\angle EBF = 30^\circ$ at point B
2. With B as centre and 5 cm as radius, draw an arc which will intersect BE at point A .
3. With A as centre and 3 cm as radius, draw an arc which intersect BI' at C and C' .
Now join C and C' to A

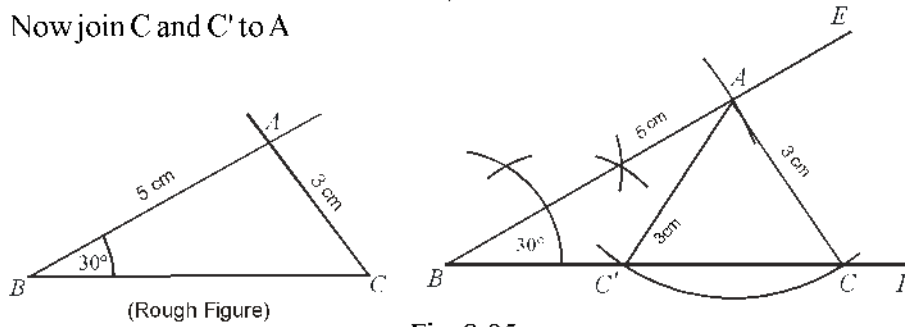


Fig. 8.05

4. Hence, $\triangle ABC$ and $\triangle ABC'$ are the required triangles.

Various conditions related to above construction

Condition 1 : When length of side b is less than the length of perpendicular p drawn from A to BF . Then taking A as centre and radius as b , drawn arc does not cut BF . In this condition construction of triangle is not possible (Fig 8.06)

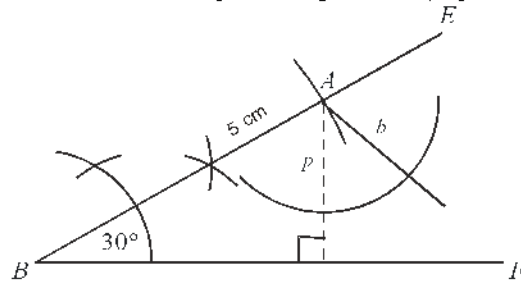


Fig. 8.06

Condition 2 : When length of side b is equal to the length of perpendicular p , then taking A as centre and b as radius, draw an arc will touch BI' . In this condition only one triangle can be constructed which will be right-angled triangle (Fig. 8.07).

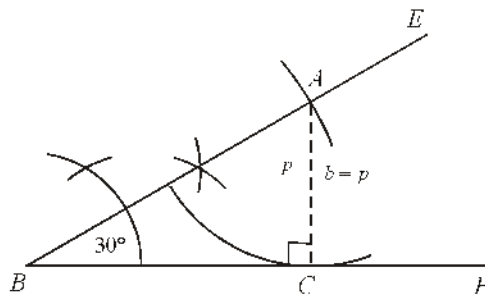


Fig. 8.07

Condition 3 : If side b is greater than perpendicular p and equal to c , then an isosceles triangle can be made in which side AB and AC are equal (Fig. 8.08).

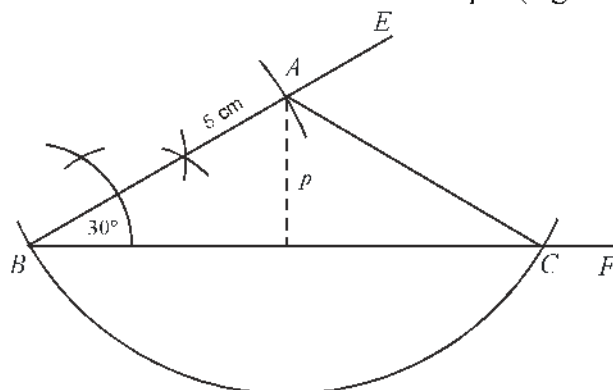


Fig. 8.08

Condition 4 : If side b , is greater than both the perpendicular p and side c , then arc will intersects line BF towards the right side of point B , at point C . In this condition, only one triangle ABC can be constructed. In this condition when arc CB is extended then it will intersect at C' . But $\Delta ABC'$ is not a triangle because in which $\angle ABC' \neq 30^\circ$ (Fig. 8.09).

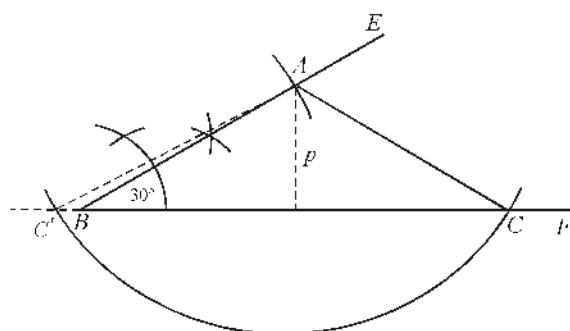


Fig. 8.09

Condition 5 : If side b is greater than perpendicular p and smaller than side c , then arc will intersect BF at different points, and so ΔABC and $\Delta ABC'$ are required triangles shown as in (Fig. 8.05)

Remark:

From above conditions, it is clear that, if $\angle B$ is not acute angle, then there is only one triangle is possible because in this condition side b is greater than side c . If $\angle B$ is an acute angle, then above any condition may be possible.

Exercise 8.5

1. Construct a $\angle XYZ$ in which $\angle XYZ = 60^\circ$, $XY = 5$ cm and $XZ = 4.5$ cm. How many triangles of such type can be drawn?
2. Construct a triangle PQR in which $\angle PQR = 45^\circ$, side $PQ = 6$ cm and $PR = 5$ cm.

3. Construct a triangle ABC in which $a = 5.4$ cm, $b = 6.8$ and $\angle A = 45^\circ$. Also find the measurement of side AB.

8.02 Difficult Construction of Triangles

Illustrative Example

Example 1. Construct a triangle ABC, when base $BC = 6$ cm, $\angle ABC = 60^\circ$ and $AB + AC = 7$ cm.

Solution : Draw a rough sketch of given measurements. Extend BA to point D such that $AC = AD$ so $BD = BA + AC$. Join D to C. Draw perpendicular bisector of DC. Then find the ΔABC .

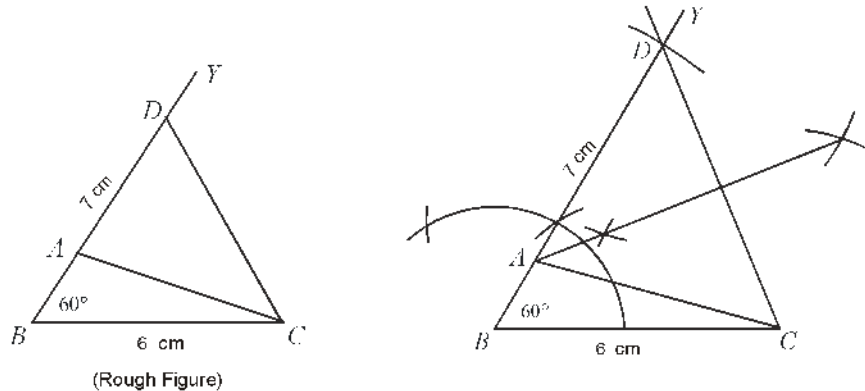


Fig. 8.10

Construction :

- (i) Draw the $BC = 6$ cm and at point B, construct $\angle YBC = 60^\circ$.
- (ii) Cut a line segment $BD = 7$ cm equal to $(BA + AC)$ from BY.
- (iii) Join D to C
- (iv) Draw perpendicular bisector of DC which intersects BD at point A.
- (v) Join A and C.

Thus, ABC is required triangle in which $BA + AC = 7$ cm

Note : A lies on the perpendicular bisector of CD, so $AD = DC$.

Example 2. Construct a triangle in which $AB + BC + CA = 8$ cm, $\angle B = 60^\circ$ and $\angle C = 80^\circ$

Solution : Draw the rough sketch with the given measurements

Extend BC in both side such that $BP = AB$ and $CQ = AC$. Join AP and AQ, obtain ΔABP and ΔACQ .

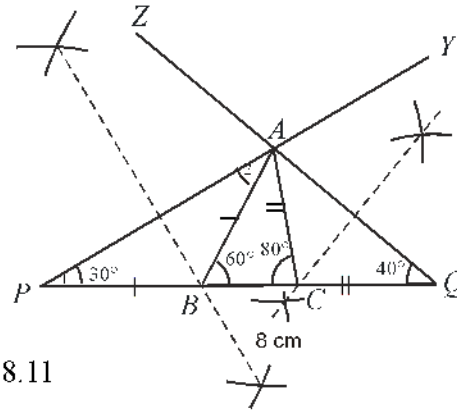
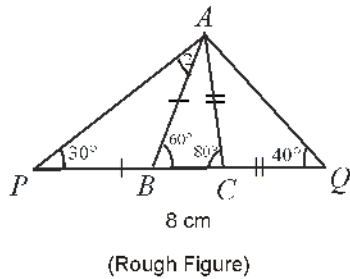


Fig. 8.11

Construction :

- (i) Draw a line segment $PQ = 8$ cm.
- (ii) At point P and Q, construct $\angle YPQ = 30^\circ$ and $\angle ZQP = 40^\circ$ respectively.
- (iii) PY and QZ intersects at point A
- (iv) Draw perpendicular bisectors of AP and AQ which intersects PQ at point B and C respectively.
- (v) Join AB and AC (Fig. 8.11)

Thus, ABC is the required triangle.

Example 3. Construct a triangle ABC in which $BC = 7$ cm, $b - c = 3.5$ cm and $\angle C = 30^\circ$

Solution : Draw a rough sketch figure with the given dimensions.

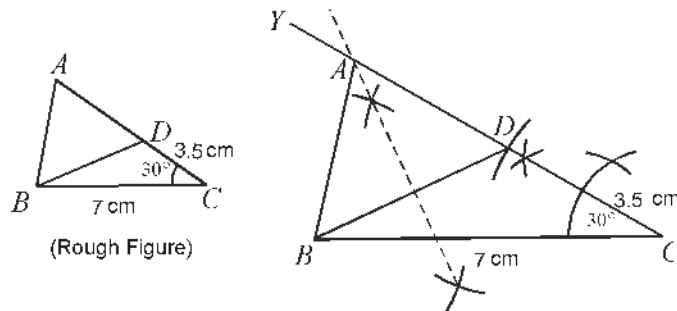


Fig. 8.12

In the figure, AC is bigger than AB. To get required triangle cut line segment AB from AC.

Steps of construction :

- (i) Draw a line segment $BC = 7$ cm and at point C, Construct $\angle YCB = 30^\circ$.
- (ii) Cut the line segment CD equal to $b - c = 3.5$ cm from CY.
- (iii) Join BD and draw the perpendicular bisector of BD.
- (iv) Let it intersect CY at point A. Join AB.

Thus, ABC is the required triangle.

Example 4. Construct a triangle ABC in which $BC = 6$ cm, and medians AD and CF are 9 cm and 7.5 cm respectively.

Solution : Draw the rough sketch with the given measurements.

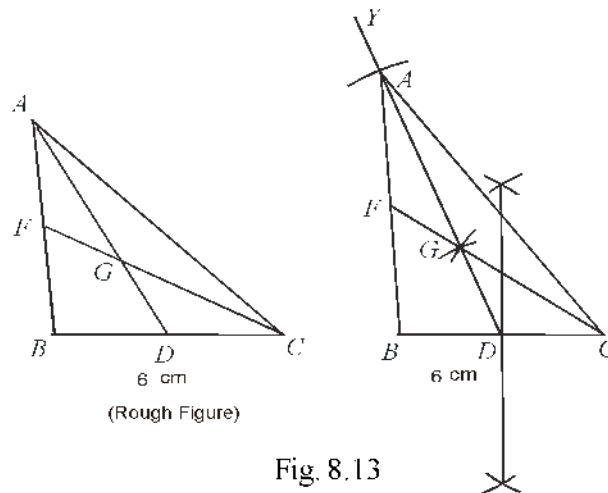


Fig. 8.13

$$DC = \frac{1}{2}BC = \frac{1}{2} \times 6 = 3 \text{ cm}$$

$$GC = \frac{2}{3}CF = \frac{2}{3} \times 7.5 = 5 \text{ cm}$$

$$GD = \frac{1}{3} \times 9 = 3 \text{ cm}$$

Construction :

Draw $BC = 6$ cm. Bisect BC at D . Taking D as a centre and radius $\left[GD = \frac{1}{3}AD\right]$

$= 3$ cm, draw an arc. Similarly, taking C as centre and radius $\left[GC = \frac{2}{3}CF\right] = 5$ cm, draw another arc, which will intersect the first arc at point G

Extend DG such that $DA = 9$ cm. Join AB and AC .

Thus, ABC is the required triangle.

Example 5. Construct a triangle ABC whose medians are 3.6 cm, 2.7 cm and 4.2 cm respectively.

Solution : Draw a rough sketch with given measurement

$$\text{From figure, } OB = \frac{2}{3} \times BE = \frac{2}{3} \times 2.7 = 1.8 \text{ cm}$$

$$OC = \frac{2}{3} \times CF = \frac{2}{3} \times 4.2 = 2.8 \text{ cm}$$

$$OA = \frac{2}{3} \times AD = \frac{2}{3} \times 3.6 = 2.4 \text{ cm}$$

Complete the figure by extending AD in such a way that $KC = OB$. Find the mid-point of OK say D and complete the triangle ABC .

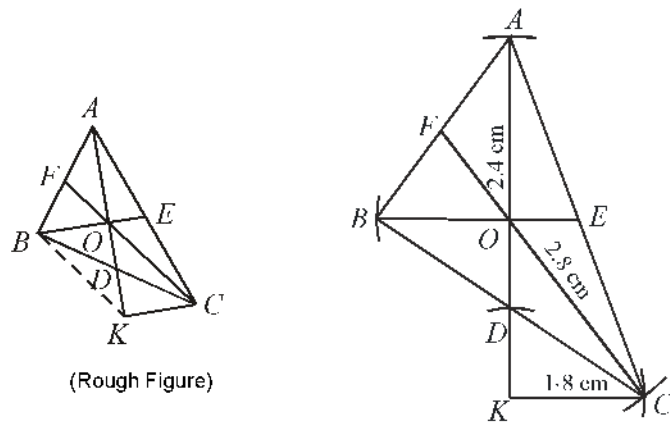


Fig. 8.14

Construction :

Construct triangle OKC in such a way that $OK = AO = 2.4$ cm, $OC = 2.8$ cm and $KC = OB = 1.8$ cm. Find the mid-point of OK say D . Extend KD forward such that $AD = 3.6$ cm. Extend CD backward such that $CD = BD$. Join AB and AC .

Thus, $\triangle ABC$ is required triangle whose median $AD = 3.6$ cm, $BE = 2.7$ cm and $CF = 4.2$ cm.

Exercise 8.6

1. Construct a triangle ABC in which $BC = 7$ cm, $\angle C = 50^\circ$ and $AC = AB = 8$ cm.
2. Construct a triangle PQR in which $PQ = 6$ cm, $\angle Q = 60^\circ$ and $PQ = PR = 8$ cm.
3. Construct a triangle PQR in which $QR = 5$ cm, $\angle R = 40^\circ$ and $PR = PQ = 1$ cm.
4. Construct a triangle ABC whose perimeter is 12 cm and base angles are 50° and 70° .
5. Construct a triangle ABC in which $BC = 6$ cm, medians AD and CF are 6 cm and 7.5 cm respectively.
6. Construct a triangle ABC whose, three medians are 3 cm, 2.7 cm and 6 cm respectively.

Important Points

To construct a triangle, following components are necessarily known:

(i) All three sides (side-side-side).

or

(ii) Two angles and one side (angle-side-angle).

or

(iii) Two sides and angle between them (side-angle-side).

or

(iv) In right angled triangle, hypotenuse and one side. (hypotenuse- side)

2. In the following conditions, construction of triangle is not possible,
- If all three angles are given.
 - If two sides and one of the angles opposite to them be acute angle given (In this condition, two triangles can be made but the final triangle cannot be decided) therefore, condition is not clear.

Miscellaneous Exercise-8

- Construct a triangle whose perimeter is 12 cm and ratio of sides is 1 : 2 : 3.
- Construct a triangle ABC in which $\angle B = 90^\circ$, $\angle C = 60^\circ$ and $c = 5$ cm.
- Construct a right angled triangle ABC in which hypotenuse BC is 8.2 cm and one side is 4.2 cm
- Construct a triangle ABC in which $\angle B = 45^\circ$, $\angle C = 60^\circ$ and perpendicular from A on BC is AD and its length is 4 cm.
- Construct a triangle ABC in which $a = 5.6$ cm, $b = c = 10.2$ cm and $\angle B = \angle C = 30^\circ$
(Hint: Vertex angle B is $90^\circ + \frac{1}{2}(\angle B - \angle C) = 105^\circ$)
- Construct a triangle whose all three medians are 4.2 cm, 4.8 cm and 5.4 cm.
- Construct an isosceles triangle whose height is 6 cm and equal sides are 7 cm each. Find the measurement of base.

Answers

Exercise 8.5

- 2.8 cm, 6.8 cm

Miscellaneous Exercise-8

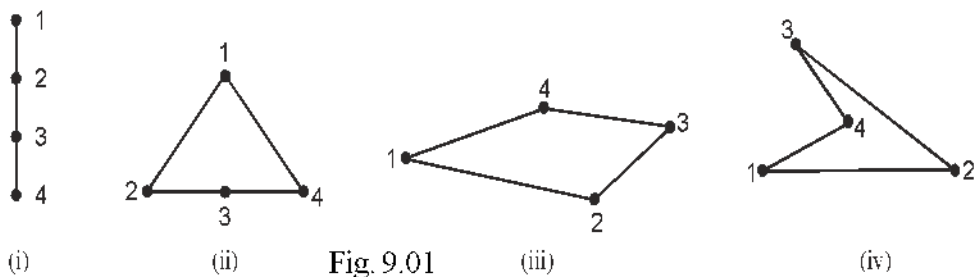
- 7 cm

Quadrilateral

9.01 Introduction

In chapters 5 and 6 you have studied, about many properties of triangles. You know that a triangle is formed by joining three non-collinear points.

Now, we mark the groups of 4-4 points on paper and join them one by one in some order and see that how many possible figures can be found?



Possible figures like 9.01 (i), (ii), (iii) and (iv) can be formed. In this chapter, we will study figures like 9.01 (iii), which we call quadrilateral.

9.02 Quadrilateral

A figure enclosed by four line segments is called a quadrilateral. A quadrilateral has four sides, four angles and four vertices. Like in Fig. 9.02, $PQRS$ is a quadrilateral where PQ, QR, RS and SP are four sides, P, Q, R and S are four vertices and $\angle P, \angle Q, \angle R$ and $\angle S$ are four angles.

Opposite Sides and Opposite Angles : In Fig. 9.02 RS is the opposite side of PQ and QR is the opposite side of PS . $\angle P$ is opposite of $\angle R$ and $\angle Q$

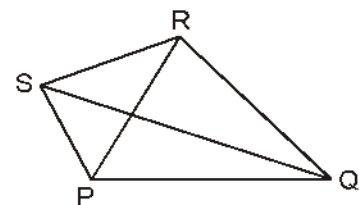


Fig. 9.02

is opposite of $\angle S$.

In Fig. 9.02, pair of **adjacent sides** are PQ, QR and PS, SR . Similarly, SP, PQ and SR, RQ are also pair of adjacent sides.

Diagonal : Line joining opposite vertices is called diagonal. In Fig 9.02, PR and QS are the diagonals of quadrilateral $PQRS$.

9.03 Sum of the Angles of Quadrilateral

Sum of all four angles of a quadrilateral is 4 right angles (360°). We have learnt this property of quadrilateral in chapter 5 through Corollary 4.

9.04 Types of Quadrilateral

- **Kite** : In Fig 9.03 $WXYZ$ is a quadrilateral whose two pairs of adjacent sides *i. e.*, WX, XY and WZ, YZ are equal. It is called kite. Such quadrilateral whose two pair of adjacent sides are equal, is called kite.
- **Trapezium** : In Fig 9.04, $ABCD$ is a quadrilateral whose one pair of opposite sides AB and DC are parallel. This quadrilateral is known as trapezium.

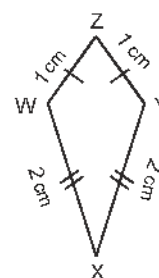


Fig. 9.03
Kite

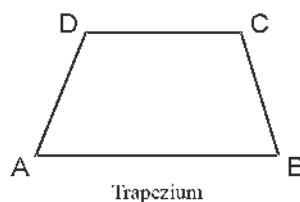


Fig. 9.04

- **Parallelogram** : In Fig. 9.05, $PQRS$ is a parallelogram whose two pair of opposite sides PQ, RS and PS, QR are parallel

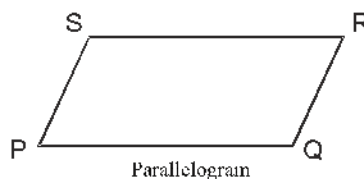


Fig. 9.05

A parallelogram is a trapezium but a trapezium is not a parallelogram.

- **Rectangle** : In Fig 9.06, $EFGH$ is a special parallelogram called rectangle whose each angle is 90° .

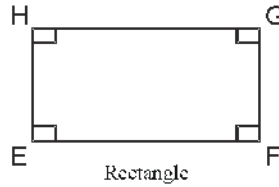


Fig. 9.06

A rectangle is a parallelogram but a parallelogram is not necessarily a rectangle.

A rectangle is a trapezium but a trapezium is not rectangle.

- **Rhombus** : In fig. 9.07, $TUVW$ is a special parallelogram called rhombus, whose each side is equal in measure.

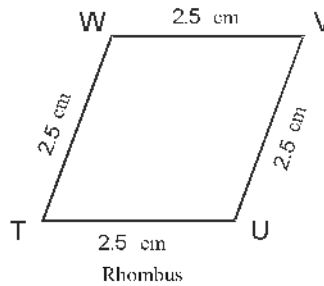


Fig. 9.07

Such parallelogram whose each side is equal, is known as rhombus.

A rhombus is a parallelogram but a parallelogram is not necessarily a rhombus.

A rhombus is a parallelogram, but a parallelogram is not a rhombus.

- **Square** In Fig. 9.08, $KLMN$ is a special rectangle called square whose all sides are equal or a special parallelogram whose each side is equal and each angle is 90° .

A square is a trapezium but a trapezium is not a square. A square is a parallelogram but a parallelogram is not necessarily a square. A square is a rectangle but a rectangle is not necessarily a square. A square is a rhombus but a rhombus is not necessarily a square.

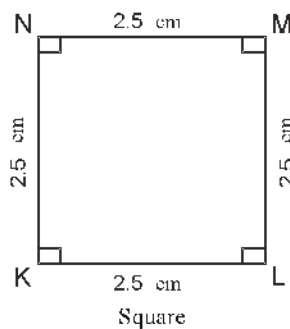


Fig. 9.08

9.05 Properties of Parallelogram

Theorem 9.1. *The diagonal of a parallelogram divides it into two congruent triangles.*

Given : $ABCD$ is a parallelogram and BD its diagonal.

To Prove : $\triangle ABD \cong \triangle BCD$

Proof: In Fig. 9.09, $ABCD$ is a parallelogram.

$AB \parallel CD$ and BD is a transversal

$\therefore \angle ABD = \angle CDB$ (Alternate interior angles) ... (i)

$AD \parallel BC$ and BD is a transversal

$\therefore \angle ADB = \angle CBD$ (Alternate interior angles) ... (ii)

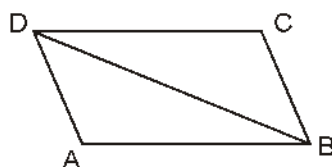


Fig. 9.09

Now, in $\triangle ABD$ and $\triangle CDB$

$\angle ABD = \angle CDB$

[From equ. (i)]

$BD = BD$

[Common]

$\angle ADB = \angle CBD$

[From equ (ii)]

$\therefore \triangle ABD \cong \triangle CDB$

(By ASA congruence rule)

Hence proved

Theorem 9.2. *Opposite sides of a parallelogram are equal.*

Given : In Fig. 9.09, $ABCD$ is a parallelogram.

To Prove : $AB = CD$ and $AD = BC$

Construction : Draw a diagonal BD .

Proof : From theorem 9.1, $\triangle ABD \cong \triangle CDB$

Since, corresponding parts of a congruent triangle are equal.

$AB = CD$ and $AD = BC$

Hence Proved

Theorem 9.3. (Converse of Theorem 9.2)

If each pair of opposite sides of a quadrilateral be equal, then it is a parallelogram.

Given : $ABCD$ is a quadrilateral whose opposite sides $AB = CD$ and $BC = AD$.

To prove : $ABCD$ is a parallelogram.

Construction : Join A to C .

Proof: In $\triangle ABC$ and $\triangle CDA$,

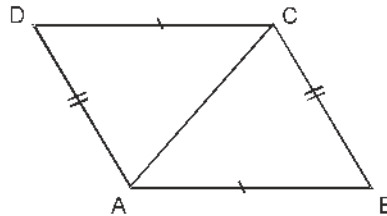


Fig. 9.10

$$AB = DC \quad \text{(Given)}$$

$$BC = AD \quad \text{(Given)}$$

$$AC = AC \quad \text{(Common)}$$

$$\therefore \Delta ABC \cong \Delta CDA \quad \text{[By SSS congruence rule]}$$

$$\text{or} \quad \angle CAB = \angle ACD \quad \text{(CPCT)}$$

Transversal line AC intersects two lines AB and CD such that alternate interior angles $\angle CAB$ and $\angle ACD$ are equal

$$\therefore AB \parallel DC \quad \dots(i)$$

$$\text{and} \quad \angle ACB = \angle CAD \quad \text{(CPCT)}$$

Transversal line AC intersects two lines BC and AD such that interior angles $\angle ACB$ and $\angle CAD$ are equal.

$$\therefore BC \parallel AD \quad \dots(ii)$$

Thus, from (i) and (ii), quadrilateral $ABCD$ is a parallelogram. **Hence Proved**

Theorem 9.4. Opposite angles of a parallelogram are equal.

Given : $ABCD$ is a parallelogram.

To Prove : $\angle A = \angle C$ and $\angle B = \angle D$

Proof: $ABCD$ is a parallelogram.

$$AB \parallel DC \text{ and } AD \parallel CB$$

Since, sum of the interior angles on the same side of a transversal is 180° .

$$\therefore \angle A + \angle D = 180^\circ \quad \dots(1)$$

$$\text{and} \quad \angle C + \angle D = 180^\circ \quad \dots(2)$$

From (1) and (2)

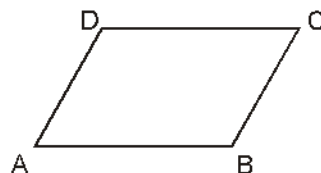


Fig. 9.11

$$\angle A + \angle D = \angle C + \angle D$$

$$\therefore \angle A = \angle C$$

Similarly, we can prove $\angle B = \angle D$

Hence Proved.

Now, converse of this theorem is also true? Let us prove it.

Theorem 9.5. *If opposite angles of a quadrilateral are equal, then it is a parallelogram.*

Given : A quadrilateral ABCD in which

$$\angle A = \angle C \quad \text{and} \quad \angle B = \angle D$$

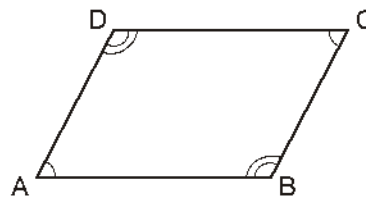


Fig. 9.12

To Prove : ABCD is a parallelogram.

Proof : In a quadrilateral ABCD

$$\angle A = \angle C \quad \dots(i)$$

and

$$\angle B = \angle D \quad \dots(ii)$$

on adding (1) and (2), we get

$$\angle A + \angle B = \angle C + \angle D \quad \dots(1)$$

But

$$\angle A + \angle B + \angle C + \angle D = 360^\circ \quad \dots(2)$$

From (1) and (2), we get

$$\angle A + \angle B = \angle C + \angle D = 180^\circ$$

$$\angle A + \angle B = 180^\circ$$

A transversal line AB intersects two lines AD and BC such that sum of consecutive interior angles is 180°

$$\therefore AD \parallel BC$$

$$\angle C + \angle D = 180^\circ \Rightarrow \angle A + \angle D = 180^\circ \quad [\because \angle C = \angle A]$$

A transversal line AD intersects two lines AB and CD such that sum of consecutive interior angles is 180° .

$$AB \parallel DC \quad \dots(6)$$

From (5) and (6), we get

Thus, ABCD is a parallelogram.

Hence Proved

Theorem 9.6. *Diagonals of a parallelogram bisect each other.*

Given : A parallelogram ABCD whose diagonals AC and BD intersect each other at O.

To Prove : OA = OC and OB = OD

Proof: $\because ABCD$ is a parallelogram.

$\therefore AD \parallel BC$ and transversal BD intersects them.

$$\angle ADB = \angle DBC \quad (\text{Alternate interior angles})$$

$$\Rightarrow \angle ADO = \angle OBC \quad \dots(1)$$

Again $AD \parallel BC$ and transversal AC intersects them

$$\angle DAC = \angle BCA \quad (\text{Alternate interior angle})$$

$$\Rightarrow \angle DAO = \angle BCO \quad \dots(2)$$

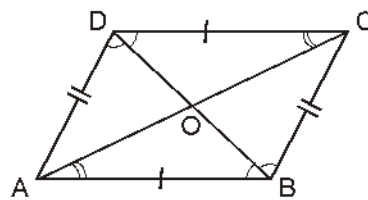


Fig. 9.13

In $\triangle AOD$ and $\triangle COB$

$$\angle ADO = \angle CBO \quad [\text{From equ (1)}]$$

$$AD = BC \quad [\text{Opposite side of a parallelogram}]$$

$$\angle ADO = \angle BCO \quad [\text{From equ (2)}]$$

$$\triangle AOD \cong \triangle COB \quad [\text{By ASA congruency rule}]$$

Since, corresponding parts of congruent triangles are equal,

$$\text{so, } OD = OB$$

$$\text{and } OA = OC$$

Hence Proved

Theorem 9.7. (Converse of Theorem 9.6)

If diagonals of a quadrilateral bisect each other, then it is a parallelogram.

Given: A quadrilateral $ABCD$ whose diagonals AC and BD bisect each other at O , i.e., $OA = OC$ and $OB = OD$

To Prove: $ABCD$ is a parallelogram.

Proof: In $\triangle AOB$ and $\triangle COD$,

$$OA = OC \quad (\text{Given})$$

$$\angle AOB = \angle COD \quad (\text{Vertically opposite angles})$$

$$\text{and } OB = OD \quad (\text{Given})$$

$$\triangle AOB \cong \triangle COD \quad (\text{By SAS congruency rule})$$

Since, corresponding parts of congruent triangles are equal so

$$\angle OAB = \angle OCD \Rightarrow \angle CAB = \angle ACD$$

Transversal AC intersects two lines AB and DC such that alternate interior angles CAB and ACD are equal.

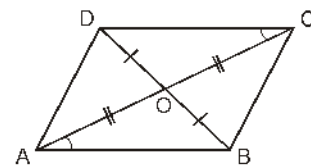


Fig. 9.14

$$AB \parallel CD$$

Similarly, we can prove $AD \parallel BC$

$ABCD$ is a parallelogram.

Hence Proved

Theorem 9.8. A quadrilateral is a parallelogram if a pair of opposite sides is equal and parallel.

Given : A quadrilateral $ABCD$, in which $AB \parallel DC$ and $AB = DC$

To Prove : $ABCD$ is a parallelogram.

Construction : Join A to C .

Prove : $AB \parallel DC$ and AC is a transversal

$$\angle BAC = \angle DCA \quad (\text{Alternate interior angles}) \quad \dots(1)$$

Now, in $\triangle ABC$ and $\triangle CDA$

$AB = DC$	(Given)
$\angle BAC = \angle DCA$	[From (1)]
$AC = AC$	(Common)
$\triangle ABC \cong \triangle CDA$	(By SAS congruency)

Since, corresponding parts of congruent triangles are equal,

so $\angle ACB = \angle CAD$

Now, AD and BC both intersect by transversal AC such that alternate angles

$$\angle ACB \text{ and are equal } \angle CAD$$

$\therefore AD \parallel BC \quad \dots(2)$

and $AB \parallel CD \quad (\text{Given})$

Thus, $ABCD$ is a parallelogram.

Hence Proved

Illustrative Examples

Example 1. Two line segments AC and BD bisect each other at point P .

Prove that $ABCD$ is a parallelogram.

Solution : **Given** AC and BD intersect each other at point P .

To Prove : $ABCD$ is a parallelogram.

Construction : Join AB , BC , CD and DA respectively.

Proof : In $\triangle APB$ and $\triangle CPD$

From figure $ABCD$ is a quadrilateral in which AC and BD are diagonals.

Since, $AP = PC$ and $BP = PD \quad (\text{Given})$

So, AC and BD bisect each other.

\therefore From theorem 9.7, $ABCD$ is a parallelogram.

Hence Proved

Example 2. In a parallelogram $ABCD$, it is not defined that bisectors of $\angle A$ and $\angle B$ intersect at point P . Prove that $\angle APB = 90^\circ$.

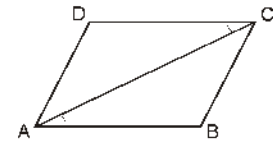


Fig. 9.15

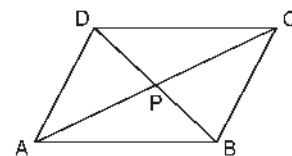


Fig. 9.16

Solution : Given : In Fig. 9.17, bisectors of adjacent angles $\angle A$ and $\angle B$, intersect at P .

To Prove : $\angle APB = 90^\circ$

Proof : We know that the sum of the adjacent angles of a parallelogram is 180°

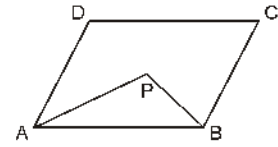


Fig. 9.17

$$\therefore \angle A + \angle B = 180^\circ \quad \dots(1)$$

$$\angle PAB = \frac{1}{2} \angle A \quad [AP \text{ is the bisector of } \angle A]$$

and $\angle PBA = \frac{1}{2} \angle B \quad [BP \text{ is the bisector of } \angle B]$

$$\therefore \angle PAB + \angle PBA = \frac{1}{2} (\angle A + \angle B) \quad \dots(2)$$

From (1) and (2), we have

$$\angle PAB + \angle PBA = 90^\circ \quad \dots(3)$$

In $\triangle PAB$, sum of the angles of a triangle is 180° .

$$\therefore \angle PAB + \angle PBA + \angle APB = 180^\circ$$

From (3) and (4)

$$\Rightarrow \angle APB = 90^\circ$$

Hence Proved

Example 3. Two points P and Q are situated on diagonal BD of a parallelogram $ABCD$ such that $DQ = BP$. Prove $APCQ$ is a parallelogram.

Solution :

Given : Two points P and Q are situated on diagonal BD of a parallelogram $ABCD$ such that $BP = QD$.

To Prove : $APCQ$ is a parallelogram.

Proof : In $\triangle AQD$ and $\triangle CPB$.

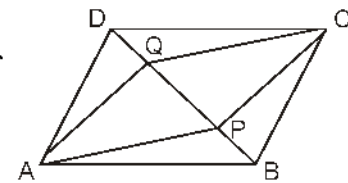


Fig. 9.18

$$AD = BC \quad [\text{Opposite sides of a parallelogram}]$$

$$\angle ADQ = \angle CBP \quad (\text{Alternate interior angles})$$

$$QD = BP \quad (\text{Given})$$

$$\triangle AQD \cong \triangle CPB \quad (\text{By SAS congruency})$$

Since, corresponding parts of congruent triangles are equal

$$AQ = CP \quad \dots(1)$$

Similarly, $\triangle CQD \cong \triangle APB$

and $CQ = AP \quad \dots(2)$

From (1) and (2), $APCQ$ is a parallelogram.

Hence Proved

Example 4. Diagonals of a quadrilateral $ABCD$ intersect each other at point O such that $OA : OC = 3 : 2$. Is $ABCD$ a parallelogram? Clarify with reason.

Solution : Given, $OA : OC = 3 : 2$

$$\Rightarrow OA \neq OC$$

Thus, $ABCD$ is not a parallelogram because diagonals of a quadrilateral $ABCD$ do not bisect each other.

Hence Proved

Example 5. The angles of a quadrilateral are in ratio 3:4:4:7. Find all the angles of the quadrilateral.

Solution : Let the angles of a quadrilateral be $3x, 4x, 4x$ and $7x$.

Since, the sum of the angles of a quadrilateral is 360° ,

$$\therefore 3x + 4x + 4x + 7x = 360^\circ$$

$$\Rightarrow 18x = 360^\circ,$$

$$\Rightarrow x = 20^\circ$$

$$\therefore \text{Required angles are } 3x = 3 \times 20^\circ = 60^\circ$$

$$4x = 4 \times 20^\circ = 80^\circ$$

$$4x = 4 \times 20^\circ = 80^\circ$$

$$7x = 7 \times 20^\circ = 140^\circ$$

Thus, the angles are $60^\circ, 80^\circ, 80^\circ$ and 140° .

Example 6. A diagonal of a parallelogram bisects its one angle. Prove that this diagonal will also bisect its opposite angle.

Solution : Let us draw a Fig 9.19 as per given conditions. In which diagonal AC bisects $\angle BAD$ of a parallelogram $ABCD$.

Given: $\angle BAC = \angle DAC$

To prove : $\angle BCA = \angle DCA$

Proof : Given, $AB \parallel CD$ and AC is a transversal.

$$\therefore \angle BAC = \angle DCA \quad (\text{Alternate interior angles}) \quad \dots(1)$$

$$\text{Similarly, } \angle DAC = \angle BCA \quad (\text{Alternate interior angles, as } AD \parallel BC) \dots(2)$$

$$\text{But } \angle BCA = \angle DAC \quad (\text{Given}) \quad \dots(3)$$

From (1), (2) and (3), we get

$$\angle BCA = \angle DCA$$

Hence Proved

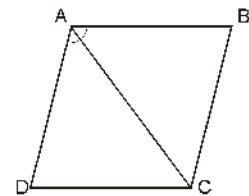


Fig. 9.19

Example 7. PQ and RS are two equal and parallel line segment. Point M which is not on side PQ or RS , is joined with Q and S respectively. A line parallel to QM and passing through P is drawn, and another line parallel to SM and passing through R is drawn. These lines meet at N . Prove that the line segments MN and PQ are equal and parallel to each other.

Solution : Draw a diagram according to the (Fig. 9.20) given conditions

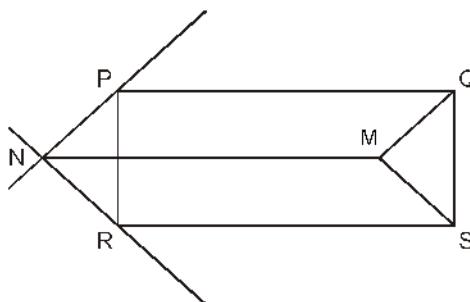


Fig. 9.20

Given that $PQ = RS$ and $PQ \parallel RS$

$\therefore PQSR$ is a parallelogram.

$\therefore PR = QS$ and $PR \parallel QS$... (1)

Now $PR \parallel QS$

$\therefore \angle RPQ + \angle PQS = 180^\circ$ (Interior angles of the same side of a transversal)

$\Rightarrow \angle RPQ + \angle PQM + \angle MQS = 180^\circ$... (2)

Also $PN \parallel QM$ (By construction)

$\therefore \angle NPR + \angle RPQ + \angle PQM = 180^\circ$ [From (2) and (3)] ... (3)

$\Rightarrow \angle NPR - \angle RPQ + \angle PQM = \angle RPQ + \angle PQM + \angle MQS$

$\therefore \angle NPR = \angle MQS$... (4)

Similarly, $\angle NRP = \angle MSQ$... (5)

In $\triangle PNR$ and $\triangle QMS$

$PR = QS$ [From (1)]

$\angle NPR = \angle MQS$ [From (4)]

$\angle NRP = \angle MSQ$ [From (5)]

$\therefore \triangle PNR \cong \triangle QMS$ [By ASA, (1), (4) and (5)]

So, $PN = QM$ and $NR = MS$

Since, $PN \parallel QM$, so $PQMN$ is a parallelogram.

Thus, $MN = PQ$ and $MN \parallel PQ$.

Hence Proved

Exercise 9.1

1. The angles of a quadrilateral are in the ratio 3:5:9:13. Find all the angles of the quadrilateral.
2. Diagonals AC and BD of a parallelogram $ABCD$ intersect each other at point O , where $OA = 3$ cm and $OD = 2$ cm. Find the lengths of AC and BD .

3. The diagonals of a parallelogram are perpendicular to each other. Is this statement true? Give reason in support of your answer.
4. State whether the angles 110° , 80° , 70° and 95° , are angles of a quadrilateral? Why and why not?
5. Are all the angles of a quadrilateral, be obtuse angles? Give reason for your answer.
6. One angle of a quadrilateral is 108° and other three angles are equal. Find the measure of each of the three equal angles.
7. $ABCD$ is a trapezium in which $AB \parallel DC$ and $\angle A - \angle B = 45^\circ$. Find $\angle C$ and $\angle D$ of this trapezium.
8. The angle between two altitudes, drawn from the vertex of an obtuse angle to the opposite sides of a parallelogram is 60° . Find all the angles of this parallelogram.
9. Points E and F lie on diagonal AC of a parallelogram $ABCD$ such that $AE = CF$. Show that $BFDE$ is a parallelogram.
10. In Fig. 9.21, $ABCD$ is a parallelogram. AQ and CP are the bisectors of $\angle A$ and $\angle C$ respectively. Prove that $APCQ$ is a parallelogram.

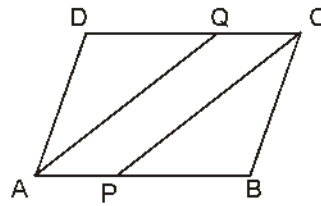


Fig. 9.21

11. In Fig. 9.22, $ABCD$ and $A FEB$ are parallelograms. Prove that $CDFE$ is a parallelogram.

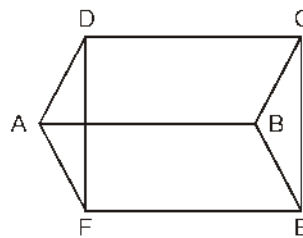


Fig. 9.22

12. The perpendiculars AP and CQ are drawn from points A and C respectively on diagonal BD of a parallelogram $ABCD$. Prove that $AP = CQ$.
13. In Fig. 9.23, $ABCD$ is a quadrilateral in which $AB \parallel DC$ and $AD = BC$. Prove that $\angle A = \angle B$.

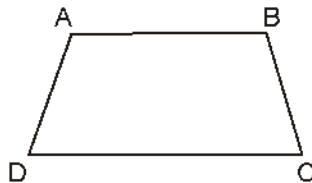


Fig. 9.23

14. In Fig. 9.24, $ABCD$ is a parallelogram. P and Q are the mid points of opposite sides AB and CD . Prove that $PRQS$ is a parallelogram.

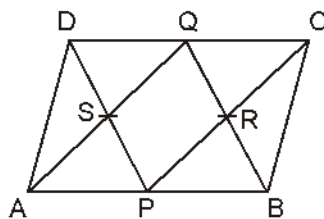


Fig. 9.24

9.06 Special Parallelogram and their Properties

You have studied different types of parallelogram in this chapter and you have also verified some properties.

Let us try to understand some existing properties of those specific parallelograms with the help of some theorems.

Theorem 9.9. *If diagonals of a parallelogram are equal, then it is a rectangle.*

Given : $ABCD$ is a parallelogram in which $AC = BD$.

To Prove : $ABCD$ is a rectangle.

Proof : In $\triangle ABC$ and $\triangle BAD$

$$BC = AD \quad (\text{Opp. sides of parallelogram})$$

$$AB = AB \quad (\text{Common side})$$

$$AC = BD \quad (\text{Given})$$

By SSS property of congruency

$$\triangle ABC \cong \triangle BAD$$

Since, corresponding angles of congruent triangles are equal,

$$\therefore \angle DAB + \angle CBA = 180^\circ$$

$$\Rightarrow \angle DAB + \angle DAB = 180^\circ$$

$$\therefore \angle DAB = \angle CBA = 90^\circ$$

Thus, $ABCD$ is a rectangle.

Hence Proved

Converse : *Diagonals of a rectangular are equal to each other.*

Theorem 9.10. *If the diagonals of a parallelogram are perpendicular to each other, then it is a rhombus.*

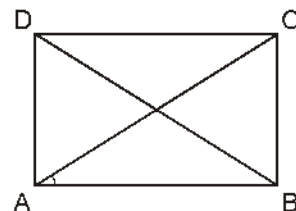


Fig. 9.25

Given : Diagonals AC and BD of parallelogram $ABCD$ are perpendicular to each other.

To prove : $ABCD$ is a rhombus

Proof : In $\triangle AOB$ and $\triangle COB$

$$OB = OB \text{ (Common)}$$

$$\angle AOB = \angle BOC = 90^\circ \text{ (Given)}$$

$$AO = CO \text{ (Diagonals of parallelogram bisect each other)}$$

$$\triangle AOB \cong \triangle COB \quad \text{(By SAS congruency)}$$

Since, corresponding parts of congruent triangles are equal,

$$AB = BC$$

Thus, $ABCD$ is a rhombus.

Hence Proved

Converse : *Diagonals of a rhombus are perpendicular to each other.*

Theorem 9.11. *If the diagonals of a parallelogram are equal and perpendicular to each other, then it is a square.*

Given : $ABCD$ is a parallelogram where $AC = BD$ and

$$AC \perp BD$$

To Prove : $ABCD$ is a square.

Proof : \therefore Diagonals of a parallelogram bisect each other

$$BO = OD \quad \dots(1)$$

In $\triangle ABO$ and $\triangle ADO$

$$BO = OD \quad \text{[From (1)]}$$

$$\therefore \angle AOB = \angle AOD = 90^\circ \text{ (Given)}$$

$$AO = AO \quad \text{(Common)}$$

$$\triangle ABO \cong \triangle ADO \quad \text{(By SAS congruency)}$$

Since, corresponding parts of congruent triangles are equal,

$$\therefore AB = AD \quad \dots(2)$$

Now, in $\triangle ABD$ and $\triangle BAC$

$$BD = AC \quad \text{(Given)}$$

$$AB = AB \quad \text{(Common)}$$

$$AD = BC \quad \text{(Opposite sides of parallelogram)}$$

$$\triangle ABD \cong \triangle BAC \quad \text{(By SSS congruency)}$$

Since, corresponding parts of congruent triangles are equal,

$$\therefore \angle DAB = \angle CBA \quad \dots(3)$$

We know that the sum of two adjacent angles of a parallelogram is 180° ,

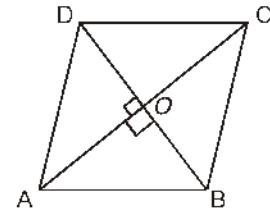


Fig. 9.26

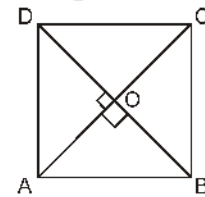


Fig. 9.27

$$\therefore \angle DAB + \angle CBA = 180^\circ \quad \dots(4)$$

From (3) and (4), we get

$$\angle DAB - \angle CBA = 90^\circ \quad \dots(5)$$

And from (2) and (5), we get

ABCD is a square.

Hence Proved

Converse : *Diagonals of a square are equal and perpendicular to each other.*

9.07 Mid-Point Theorem

You have studied about some properties of triangles and quadrilaterals. Now let us study another result which is related to the mid-point theorem.

Theorem 9.12. *A line segment joining the mid points of any two sides of a triangle, is half and parallel to its third side.*

Given : In $\triangle ABC$, points D and E are mid-points of sides AB and AC respectively.

To Prove : (i) $DE \parallel BC$, and (ii) $DE = \frac{1}{2}BC$

Construction : Extend DE to F where $EF = DE$. Join C to F .

Proof : In $\triangle ADE$ and $\triangle CFE$

$$\begin{aligned} AE &= CE && \text{(Given)} \\ \angle AED &= \angle CEF && \text{(Vertically opposite angles)} \\ DE &= EF && \text{(By construction)} \\ \triangle ADE &\cong \triangle CFE && \text{(By SAS congruency rule)} \end{aligned}$$

Since, corresponding parts of congruent triangles are equal.

$$\therefore AD = CF \quad \dots(1)$$

$$\text{and} \quad \angle EAD = \angle ECF$$

Transversal AC intersects lines AB and CF such that alternate angles EAD and ECF are equal.

$$\therefore AD \parallel CF \text{ and } BD \parallel CF$$

$$\text{But } AD = BD \quad \text{(Given)} \quad \dots(3)$$

From (1) and (3), we get

$$BD = CF \text{ and } BD \parallel CF$$

$\therefore BCFD$ is a parallelogram.

$$\therefore DF = BC \text{ and } DF \parallel BC$$

$$\Rightarrow \frac{1}{2}DF = \frac{1}{2}BC$$

$$\Rightarrow DE = EF = \frac{1}{2}DF. \text{ (By construction)}$$

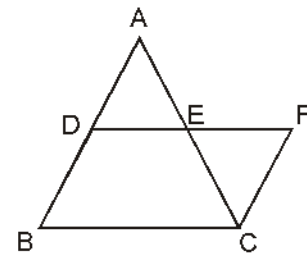


Fig. 9.28

$$\Rightarrow \quad DE = \frac{1}{2} BC$$

Thus, $DE = \frac{1}{2} BC$ and $DE \parallel BC$ **Hence Proved**

Theorem 9.14. (Converse of Theorem 9.12)

The line drawn through the mid-point of one side of a triangle, parallel to another side bisects the third side.

Given : In $\triangle ABC$, D is the mid-point of side AB in Fig. 9.29 $DE \parallel BC$ and it intersects AC at point E .

To Prove : $AE = EC$

Construction : Draw $CF \parallel BD$ which intersects DE (extended) at point F .

Proof : \therefore $BC \parallel DF$ (Given)
 and $BD \parallel CF$ (By construction)
 \therefore $BCFD$ is a parallelogram.
 \therefore $BD = CF$
 But $BD = AD$ (Given)
 \therefore $AD = CF$... (1)

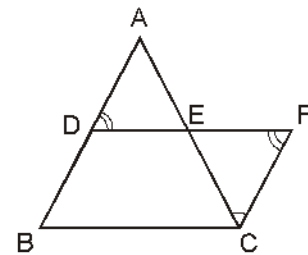


Fig. 9.29

Now, in $\triangle ADE$ and $\triangle CFE$

$\angle ADE = \angle CFE$ (Alternate interior angles)
 $AD = CF$ [From (1)]
 $\angle DAE = \angle ECF$ [Alternate interior angles]
 $\triangle ADE \cong \triangle CFE$ (By ASA congruency rule)
 $AE = CE$ **Hence Proved**

9.08. Intercept

If there are two lines l_1 and l_2 in a plane and line l_3 intersects these lines on different points A and B , then the line segment AB is called intercept made by lines l_1 and l_2 on line l_3 .

Here, will prove the following theorem for three parallel lines. This theorem can be extended for more than three lines.

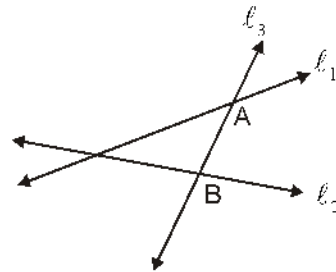


Fig. 9.30

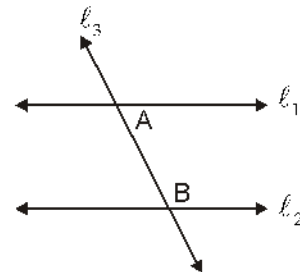


Fig. 9.31

Theorem 9.15. *If there are three or more than three parallel lines and intercepts made by them on a transversal line are equal, then the corresponding intercepts on the other transversal will be also equal.*

Given : In Fig 9.32, l, m, n are three parallel lines and two transversals lines l_1 and l_2 intersect them on points A, B, C and D, E, F respectively and $AB = BC$.

To Prove : $DE = EF$

Construction : Draw $GH \parallel l_1$ which passes through E .

Proof : $\therefore l$ and m are parallel lines

$$\therefore AG \parallel BE$$

and $AB \parallel GE$ (By construction)

$\therefore ABEG$ is a parallelogram.

$$\therefore AB = GE \quad \dots(1)$$

Similarly, $BCHE$ is a parallelogram.

$$\therefore BC = EH \quad \dots(2)$$

$$\text{Given} \quad AB = BC$$

$$\therefore GE = EH \quad \dots(3)$$

Again $l \parallel n$ and transversal l_2 intersects them.

$$\angle GDE = \angle EFH \quad (\text{Alternate interior angles}) \dots(4)$$

Now, in $\triangle GDE$ and $\triangle HFE$

$$\angle GDE = \angle EFH \quad [\text{From (4)}]$$

$$GE = EH \quad [\text{From (3)}]$$

$$\angle GED = \angle HFE \quad [\text{Vertically opposite angles}]$$

$$\triangle GDE \cong \triangle HFE \quad (\text{By ASA congruency rule})$$

Since, corresponding parts of congruent triangles are equal,

$$\text{Thus,} \quad DE = EF$$

Hence Proved

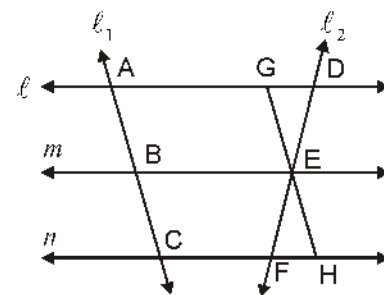


Fig. 9.32

Illustrative Examples

Example 8. D, E and F are respectively mid-points of sides BC, CA and AB of an equilateral triangle ABC . Prove that $\triangle DEF$ is an equilateral triangle.

Solution :

Given : $\triangle ABC$ in which D, E, F are the mid-points of sides BC, CA and AB respectively.

To Prove : $\triangle DEF$ is an equilateral triangle.

Proof : In $\triangle ABC$, D, E and F are the mid - points of sides BC, CA and AB respectively.

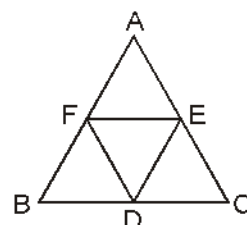


Fig. 9.33

$$\therefore DE = \frac{1}{2} AB \quad \dots(1)$$

$$EF = \frac{1}{2} BC \quad \dots(2)$$

$$FD = \frac{1}{2} AC \quad \dots(3)$$

But $\triangle ABC$ is an equilateral triangle.

$$AB = BC = CA \quad \dots(4)$$

From (1), (2), (3) and (4)

$$DE = EF = FD$$

Thus, $\triangle DEF$ is an equilateral triangle.

Hence Proved

Example 9. Prove that a line, joining the mid-points of the diagonals of a trapezium, will be parallel to its parallel sides and half of their difference.

Solution : Given : A trapezium $ABCD$ in which $AB \parallel DC$, F and G are mid-points of diagonals AC and BD respectively.

To Prove : (i) $FG \parallel AB$

$$(ii) FG = \frac{1}{2}(AB - CD)$$

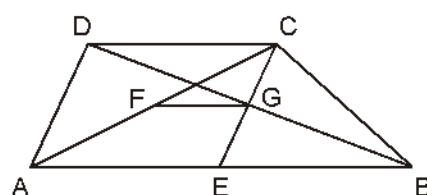


Fig. 9.34

Construction : Join CG and extend it such that it will meet AB at E .

Proof : In $\triangle CDG$ and $\triangle EBG$

$$\angle CDG = \angle EBG \quad \text{(Alternate interior angles)}$$

$$DG = GB \quad \text{(Given)}$$

$$\angle DCG = \angle BEG \quad \text{(Alternate interior angles)}$$

$$\triangle CDG \cong \triangle EBG \quad \text{(By ASA congruency rule)}$$

Since, corresponding parts of two congruent triangles are equal.

$$\therefore CG = EG \quad \dots(1)$$

$$\text{and } CD = EB \quad \dots(2)$$

Now in $\triangle ACE$, F and G are the mid points of sides AC and CE respectively.

$$FG \parallel AE \text{ and } FG = \frac{1}{2} AE \quad \dots(3)$$

$$\begin{aligned} \text{But } AE &= AB + EB \\ AE &= AB + CD \quad [\text{From (2)}] \end{aligned} \quad \dots(4)$$

From (3) and (4), we get

$$FG = \frac{1}{2} AE = \frac{1}{2} (AB + CD)$$

$$\text{and } FG \parallel AE$$

$$\text{Thus, } FG \parallel AB \text{ and } FG = \frac{1}{2} (AB + CD) \quad \text{Hence Proved}$$

Example 10. Prove that the quadrilateral obtained by joining the mid points of consecutive sides of a quadrilateral is a parallelogram.

Solution :

Given : In Fig 9.35 $ABCD$ is a quadrilateral where P, Q, R and S are the mid-points of its consecutive sides respectively.

To Prove : $PQRS$ is a parallelogram.

Construction : Join AC .

Proof : In $\triangle ABC$, P and Q are the mid-points of sides AB and BC respectively.

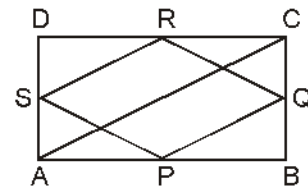


Fig. 9.35

$$PQ \parallel AC \text{ and } PQ = \frac{1}{2} AC \quad \dots(1)$$

In $\triangle ADC$, S and R are mid points of sides AD and DC respectively.

$$\therefore SR \parallel AC \text{ and } SR = \frac{1}{2} AC \quad \dots(2)$$

From (1) and (2), we get

$$PQ \parallel SR \text{ and } PQ = SR$$

Thus, in quadrilateral $PQRS$ one pair of opposite sides are equal and parallel.

Thus, $PQRS$ is a parallelogram

Example 11. In Fig 9.36, X and Y are respectively mid points of opposite sides AD and BC of a parallelogram $ABCD$. Also, BX and DY intersect line AC at points P and Q respectively. Show that $AP = PQ = QC$.

Solution : In fig. 9.36, X and Y are the mid-points of sides AD and BC respectively of a parallelogram $ABCD$.

$$\therefore DX = \frac{1}{2}AD \text{ and } BY = \frac{1}{2}CB$$

But $ABCD$ is a parallelogram.

$$\therefore AD = BC \text{ and } AD \parallel BC \quad [\text{Opposite sides of a parallelogram}]$$

$$\Rightarrow \frac{1}{2}AD = \frac{1}{2}BC \text{ and } AD \parallel BC$$

$$\Rightarrow DX = BY \text{ and } DX \parallel YB$$

\therefore One pair of opposite sides of quadrilateral XYD are equal and parallel

$\therefore XYD$ is a parallelogram

$$\Rightarrow PX \parallel QD$$

We know that the segment drawn from the mid-point of one side of a triangle and parallel to the other side bisects the third side.

In $\triangle CBP$, Y is the mid-point of BC and $YQ \parallel BP$

$\therefore Q$ is the mid-point of CP

$\Rightarrow P$ is the mid-point of AQ

$$\Rightarrow AP = PQ \quad \dots(i)$$

$$\text{Similarly } CQ = PQ \quad \dots(ii)$$

$$AP = PQ = QC \quad [\text{From (i) and (ii)}]$$

Example 12. In Fig. 9.37 AY and CX are respectively the bisectors of opposite angles A and C of parallelogram $ABCD$. Show that

$$AY \parallel CX$$

Solution : $\angle A = \angle C$ (opposite angles of parallelogram)

$$\Rightarrow \frac{1}{2}\angle A = \frac{1}{2}\angle C$$

[$\therefore AY$ and CX are the bisectors of $\angle A$ and $\angle C$ respectively]

$\therefore AB \parallel CD$ (opposite sides of parallelogram)

Also $AX \parallel YC$ and transversal CX intersects them

$$\angle YAX = \angle YCX \quad \dots(i)$$

A transversal line CX intersects two parallel lines AB and CD .

$$\text{So, } \angle CXB = \angle YCX \quad (\because \text{alternative angles}) \quad \dots(ii)$$

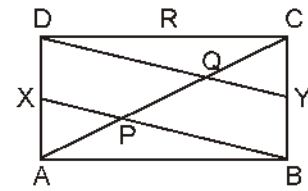


Fig. 9.36

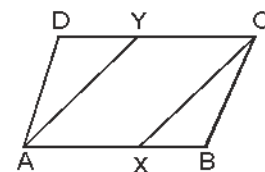


Fig. 9.37

From (i) and (ii) $\angle YAX = \angle CXB$

because the corresponding angles are equal.

$\therefore AX \parallel YC$ (because sum of interior angles on one side of a transversal line is 180°)

$\therefore AY \parallel CX$ **Hence Proved**

Example 13. Show that a quadrilateral, formed by joining the mid-points of the sides of a rhombus, in the same order, is a rectangle.

Solution : Let ABCD be a rhombus and P, Q, R, S are mid-points of sides AB, BC, CD and DA respectively. (Fig. 9.38). Join AC and BD.

\therefore From $\triangle ABD$, we get

$$SP = \frac{1}{2}BD \text{ and } SP \parallel DB$$

(S and P are the mid points of sides AB and AD respectively)

Similarly,

$$RQ = \frac{1}{2}BD \text{ and } RQ \parallel BD$$

$\therefore SP = RQ \text{ and } SP \parallel RQ$

\therefore PQRS is a parallelogram.

Also $AC \perp BD$ (Diagonals of a rhombus are perpendicular to each other)

$PQ \parallel AC$ (In $\triangle BAC$, P and Q are mid points of sides AB and BC respectively)

$$SP \parallel BD \quad \dots(1)$$

In $\triangle ABC$, P and Q are the mid point of AB and BC respectively

$\therefore PQ \parallel AC \quad \dots(2)$

From (1) and (2) $SP \perp PQ$ [$\because AC \perp BD$]

$\therefore \angle SPQ = 90^\circ$

\therefore PQRS is a rectangle. **Hence Proved**

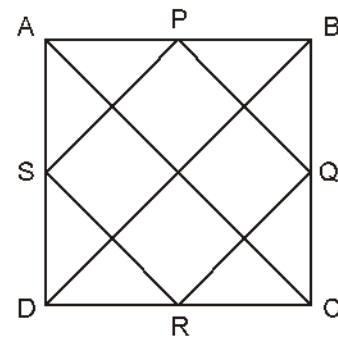


Fig. 9.38

Exercise 9.2

- In Fig. 9.39 ABCD and AEFG are two parallelograms. If $\angle C = 55^\circ$, then determine $\angle F$.

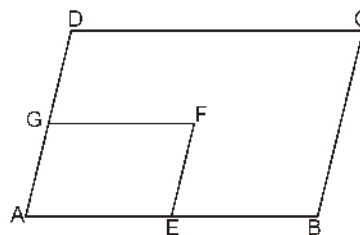


Fig. 9.39

2. Can all the angles of a quadrilateral be acute angles? Give reason for your answer.
3. Can all the angles of a quadrilateral be right angles? Give reason of your answer.
4. The diagonals of a quadrilateral ABCD bisect each other. If $\angle A = 35^\circ$, then determine $\angle B$.
5. Opposite angles of a quadrilateral ABCD are equal. If $AB = 4$ cm, then determine CD.
6. ABCD is a rhombus in which altitude from D on AB, bisects AB. Find the angles of the rhombus.
7. In a triangle ABC, lines RQ, PR and QP are respectively drawn parallel to lines BC, CA and AB passing through points A, B and C, as shown in fig. 9.40 Show that $BC = \frac{1}{2} = QR$.

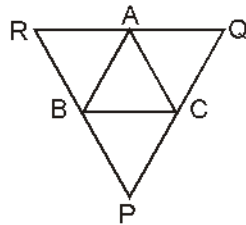


Fig. 9.40

8. D, E and F are respectively the mid points of sides BC, CA and AB respectively of an equilateral triangle ABC. Show that $\triangle DEF$ is also an equilateral triangle.
9. Points P and Q are respectively taken on opposite sides AB and CD of a parallelogram ABCD such that $AP = CQ$ (Fig 9.41). Show that AC and PQ bisect each other.

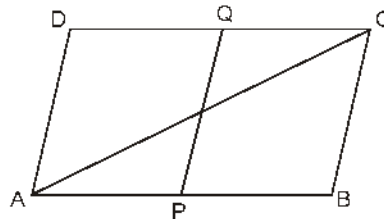


Fig. 9.41

10. E is the mid-point of side AD of a trapezium ABCD in which $AB \parallel DC$. Through point E a line is drawn parallel to AB, intersects BC at F. Show that F is the mid-point of BC. [Hint: Join AC].
11. In $\triangle ABC$, $AB = 5$ cm, $BC = 8$ cm and $CA = 7$ cm. If D and E are mid-points of AB and BC respectively, then find the length of DE.
12. In Fig 9.42, it is given that BDEF and FDCE are parallelograms. Can you say that $BD = CD$? Why and why not?

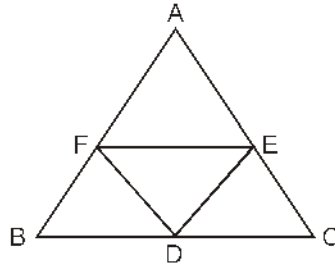


Fig. 9.42

13. In Fig. 9.43, D , E and F are mid points of sides BC , CA and AB respectively. If $AB = 4.3$ cm, $BC = 5.6$ cm and $AC = 3.9$ cm, then find the perimeter of the following :
 (i) $\triangle DEF$ (ii) quadrilateral $BDEF$

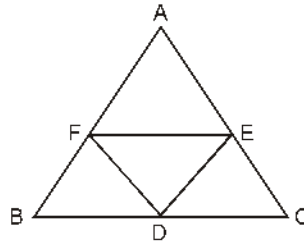


Fig. 9.43

14. Prove that a quadrilateral obtained by joining the mid points of consecutive sides of a square, is also a square.
15. The diagonals of a quadrilateral are perpendicular to each other. Prove that a quadrilateral obtained by joining the mid-points of its sides is a rectangle.
16. Prove that in a right-angled triangle, the bisecting median of hypotenuse is half of the hypotenuse.
17. Prove that a rhombus is obtained by joining the mid-points of the pairs of opposite sides of a rectangle.

Constructions of Quadrilaterals

9.09. Quadrilateral

A plane figure enclosed by four line segment, is called a quadrilateral. The line joining its opposite vertices is called its diagonal.

In Fig. 9.44 AB , BC , CD and DA are its four sides. A , B , C and D are its vertices and AC and BD are the diagonals of quadrilateral $ABCD$.

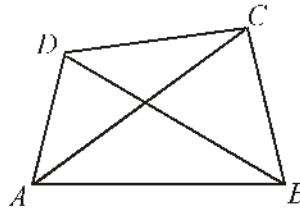


Fig. 9.44

9.10. Construction of Quadrilateral

When we have to construct a quadrilateral, a rough sketch should be drawn and mark the given facts. Generally, there is specific importance of diagonal in construction of a quadrilateral. So it should be considered by drawing a diagonal in rough sketch definitely. Is any triangle can be construct by it? It should be seen by drawing quadrilateral after formation of triangle. Construction of a quadrilateral can be completed by drawing triangle. It is not necessary to draw a diagonal in each condition. Some times without drawing any diagonal, quadrilateral can be constructed.

It can be understand clearly by the constructions given in this chapter.

Construction 9.16 : Construction of a quadrilateral when four sides and a diagonal are given.

Construct a quadrilateral $ABCD$ in which $AB = 3$ cm, $BC = 4.5$ cm, $CD = 6$ cm, $DA = 4$ cm and $AC = 4.8$ cm.

Construction : First draw a rough sketch on the basis of given measures and mark them on it.

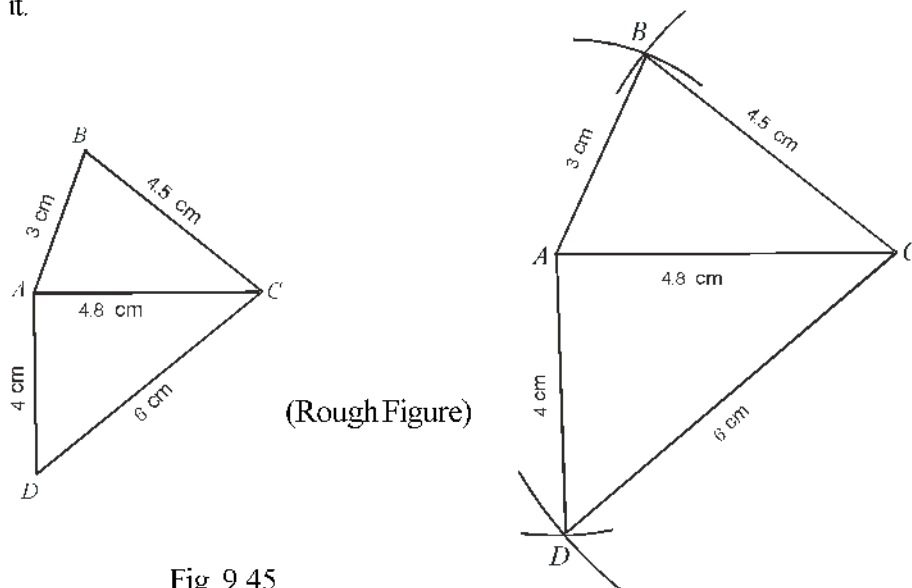


Fig. 9.45

According to the rough sketch, draw $AC = 4.8$ cm. Construct a triangle ABC by

drawing an arc of radius 3 cm from A and arc of radius 4.5 cm from C . Similarly, complete $\triangle ACD$ by drawing arc of length equal to AD and CD .

Thus, $ABCD$ is the required quadrilateral.

Construction 9.17. Construction of a Quadrilateral in which four sides and one angle are given.

Construct a quadrilateral $ABCD$ in which $AB = 4.8$ cm, $BC = 3.5$ cm, $CD = 4.5$ cm, $DA = 4$ cm and $\angle A = 60^\circ$

Construction : Draw a rough sketch according to the given measures and mark them.

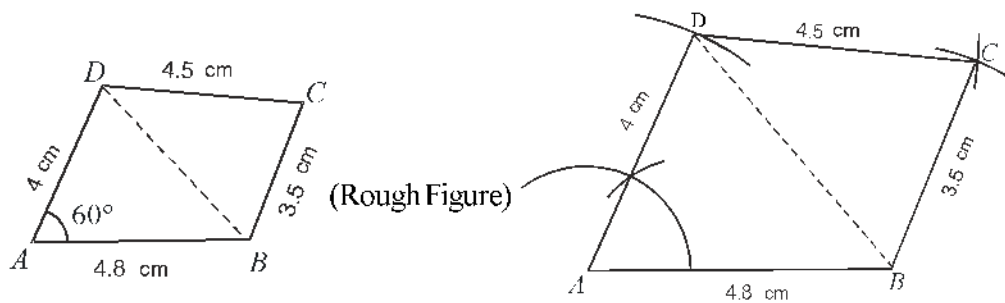


Fig. 9.46

Draw line segment $AB = 4.8$ cm. Draw $\angle DAB = 60^\circ$ at point A and cut $AD = 4$ cm from A . Draw two arcs from point D and B of radii 4.5 cm and 3.5 cm respectively they intersect at point C . Join DC and BC . Thus, $ABCD$ is a required quadrilateral.

Construction 9.18. Construction of a quadrilateral when three sides and two diagonals are given.

Construct a quadrilateral $ABCD$ in which $AB = 5.5$ cm, $BC = 3.3$ cm, $AD = 4.6$ cm and diagonals $AC = 5.7$ cm and $BD = 6$ cm.

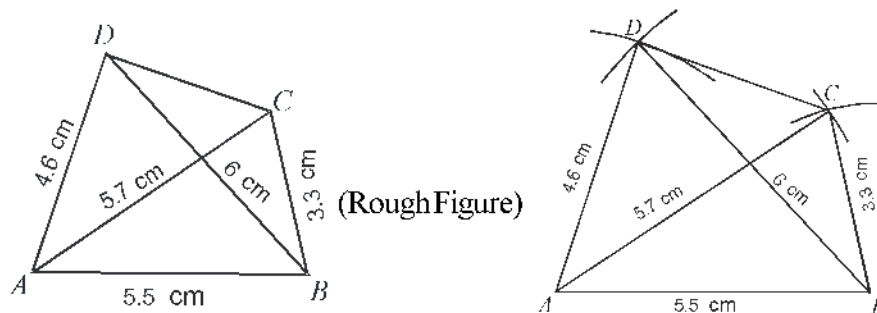


Fig. 9.47

Construction : Draw a rough sketch according to the given measures and mark them.

According to rough sketch draw line segment $AB = 5.5$ cm. Construct a $\triangle ABD$ by drawing the arcs of radii 4.6 cm and 6 cm from points A and B respectively. Similarly

again, construct $\triangle ABC$ by drawing arcs of radii 5.7 cm and 3.3 cm from points A and B . Join C and D .

Thus, $ABCD$ is the required quadrilateral.

Construction 9.19. Construction of a quadrilateral when three sides and two angles between them are given.

Construct a quadrilateral $ABCD$ in which $AB = 3.5$ cm, $BC = 5$ cm, $CD = 5.5$ cm, $\angle B = 120^\circ$ and $\angle C = 60^\circ$.

Construction : Draw a rough sketch according to given measures and mark them.

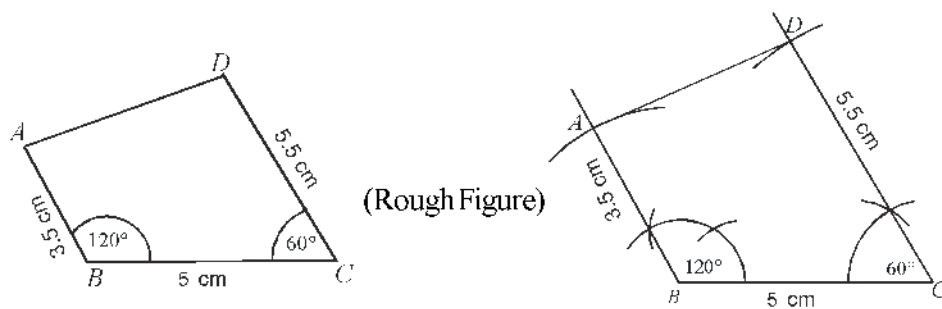


Fig. 9.48

Draw line segment $BC = 5$ cm. Draw $\angle B = 120^\circ$ and $\angle C = 60^\circ$ at points B and C respectively with BC and cut the given lengths $AB = 3.5$ cm and $CD = 5.5$ cm from B and C respectively. Join A and D to obtain the quadrilateral.

Thus $ABCD$ is required quadrilateral.

Construction 9.20. Construction of a quadrilateral when two consecutive sides and angle between them and other two angles are given.

Construct a quadrilateral $ABCD$ in which $AB = 5$ cm, $AD = 5.3$ cm, $\angle A = 60^\circ$, $\angle C = 105^\circ$ and $\angle D = 90^\circ$.

Construction : $\angle A + \angle B + \angle C + \angle D = 360^\circ$

$$\Rightarrow 60^\circ + 105^\circ + \angle B + 90^\circ = 360^\circ \Rightarrow 255^\circ + \angle B = 360^\circ$$

Draw a rough sketch according to given measures and mark them.

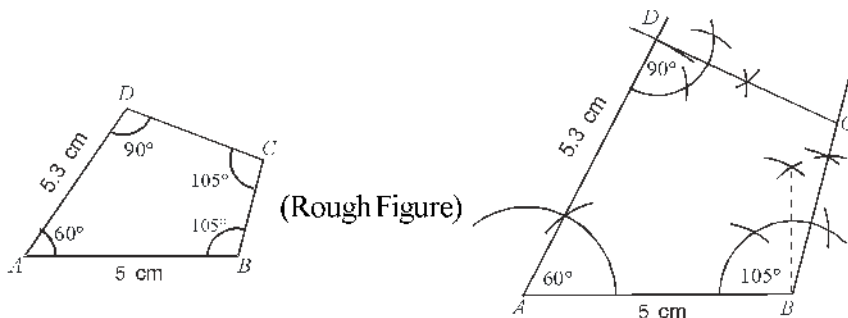


Fig. 9.49

Draw line segment $AB = 5$ cm. At points A and B , draw $\angle A = 60^\circ$ and $\angle B = 105^\circ$ with AB . From A cut, $AD = 5.3$ cm and at D , draw $\angle D = 90^\circ$ with AD . Thus, $ABCD$ is the required quadrilateral.

Exercise 9.3

Construct quadrilaterals for the following given facts with description of steps of construction.

1. In a quadrilateral $ABCD$, $AB = 3.5$ cm, $BC = 4.8$ cm, $CD = 5.1$ cm, $AD = 4.4$ cm and a diagonal $AC = 5.9$ cm.
2. In a quadrilateral $PQRS$, $PQ = 4$ cm, $QR = 3$ cm, $QS = 4.8$ cm, $PS = 3.5$ cm and $PR = 5$ cm.
3. In, quadrilateral $ABCD$, $AB = 4$ cm, $BC = 4.5$ cm, $CD = 3.5$ cm, $AD = 3$ cm and $\angle A = 60^\circ$
4. In, quadrilateral $ABCD$, $AB = 3.5$ cm, $BC = 3$ cm, $AD = 2.5$ cm, $AC = 4.5$ cm and $BD = 4$ cm.
5. In, quadrilateral $PQRS$, $PQ = 3$ cm, $QR = 4$ cm, $PS = 4.5$ cm, $PR = 6$ cm and $QS = 5.5$ cm.
6. In, quadrilateral $ABCD$, $AB = BC = 3.0$ cm, $AD = 5$ cm, $\angle A = 90^\circ$ and $\angle B = 120^\circ$.
7. In, quadrilateral $ABCD$, $AB = 3.8$ cm, $BC = 2.5$ cm, $CD = 4.5$ cm and $\angle B = 30^\circ$ and $\angle C = 150^\circ$.
8. In, quadrilateral $PQRS$, $PQ = 3$ cm, $QR = 3.5$ cm, $\angle Q = 90^\circ$ and $\angle P = 105^\circ$ and $\angle R = 120^\circ$.
9. In, quadrilateral $PQRS$, $PQ = 2.5$ cm, $QR = 3.7$ cm, $\angle Q = 120^\circ$, $\angle S = 60^\circ$ and $\angle R = 90^\circ$.

9.11 Constructions of Parallelograms and Rectangles

Before the construction of parallelogram, rectangle, square and rhombus, it is necessary to know the following facts.

1. In a parallelogram : (i) Opposite sides are equal,
(ii) opposite angles are equal,
(iii) Diagonals bisect each other,
(iv) Each diagonal bisects a parallelogram into two congruent triangles.
2. In a rectangle : (i) Each angle is right angle,
(ii) Opposite sides are equal.
(iii) Diagonals are equal.
(iv) Diagonals bisect each other.
3. In a square : (i) All four sides are equal.
(ii) Each angle is right angle.
(iii) Diagonals are equal.

- (iv) Diagonals bisect each other at right angle,
 - (v) Each diagonal makes an angle of 45° with sides.
4. In a rhombus :
- (i) All four sides are equal.
 - (ii) Opposite angles are equal.
 - (iii) Diagonals bisect each other at right angle.
 - (iv) Diagonals are bisectors of the vertex angles.

Construction 9.21 . Construction of a parallelogram when two sides and one diagonal are given.

Construct a parallelogram $ABCD$ if $AB = 5$ cm, $BC = 4$ cm and $BD = 7.7$ cm.

Construction : Draw rough sketch according to given measures and mark them.

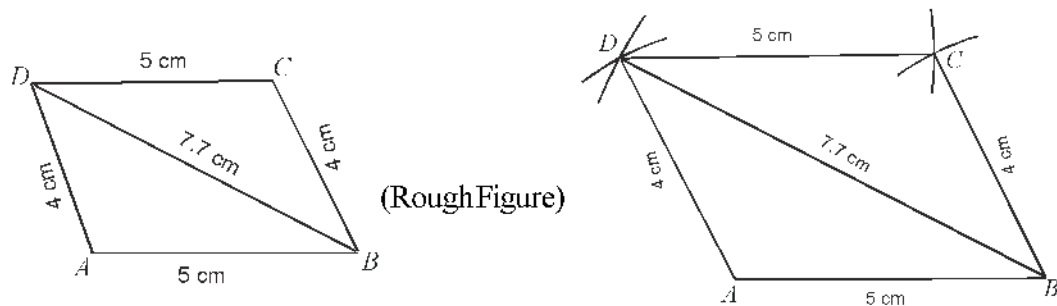


Fig. 9.50

Draw a line segment $AB = 5$ cm. Construct $\triangle ABD$ by drawing arcs of radii 4 cm and 7.7 cm from points A and B respectively. Similarly, construct $\triangle BCD$ by drawing arcs of radii 4 cm and 5 cm from points B and D respectively.

Thus, $ABCD$ is the required parallelogram.

Construction 9.22. Construction of a parallelogram when a side and two diagonals are given.

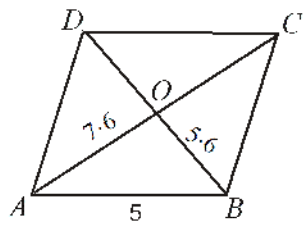
Construct a parallelogram $ABCD$ in which $AB = 5$ cm, diagonal $AC = 7.6$ cm and diagonal $BD = 5.6$ cm.

Hint: In a parallelogram, diagonal bisect each other

$$\therefore AO = OC = \frac{1}{2} AC \text{ and } BO = OD = \frac{1}{2} BD.$$

$$AO = \frac{1}{2} \times 7.6 = 3.8 \text{ cm and } BO = \frac{1}{2} \times 5.6 = 2.8 \text{ cm.}$$

Construction: Draw a rough sketch according to given measures and mark them.



(Rough Figure)

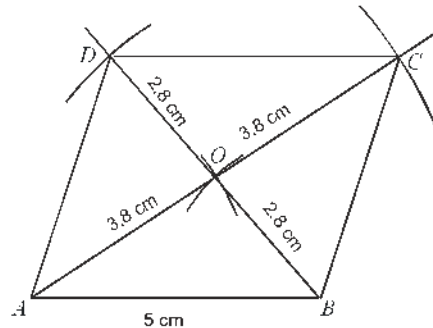


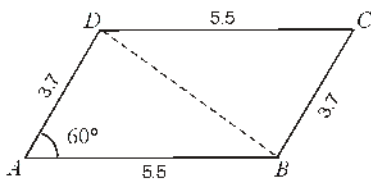
Fig. 9.51

Draw a line segment $AB = 5$ cm. Construct a $\triangle AOB$ by drawing arcs of radii 3.8 cm or half of diagonal AC and 2.8 cm or, half of diagonal BD . Extend AO and BO so that $AC = 7.6$ cm and $BD = 5.6$ cm. Join BC , CD and AD . Thus, $ABCD$ is the required parallelogram.

Construction 9.23. Construction of a parallelogram in which two adjacent sides and angle between them are given.

Construct a parallelogram $ABCD$ where $AB = 5.5$ cm, $BC = 3.7$ cm and $\angle A = 60^\circ$.

Construction : Draw a rough sketch according to given measures and mark them.



(Rough Figure)

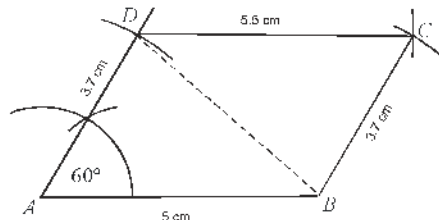


Fig. 9.52

Draw a line segment $AB = 5.5$ cm. Construct $\angle BAM = 60^\circ$ at point A and make a $\triangle ABD$ by taking $AD = 3.7$ cm. Similarly, by taking $BC = 3.7$ cm and $DC = 5.5$ cm construct $\triangle BDC$.

Thus, $ABCD$ is a required parallelogram.

Construction 9.24. Construction of a rectangle whose diagonal and one side are given.

Construct a rectangle in which diagonal $BD = 5.8$ cm and one side $AB = 5$ cm.

Construction : Draw a rough sketch according to given measures and mark them.

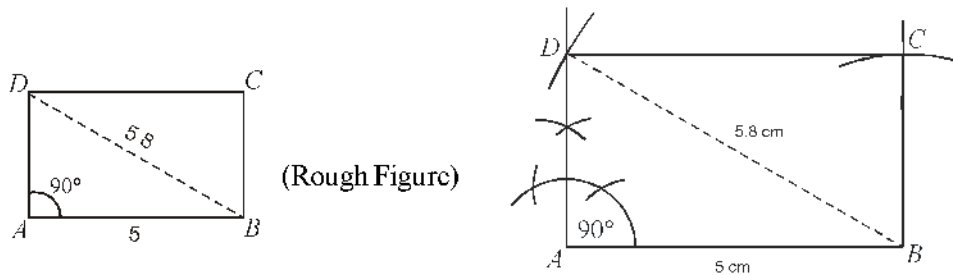


Fig. 9.53

Draw a line segment $AB = 5$ cm. At point A , draw $\angle A = 90^\circ$. Taking B as centre, draw an arc of radius $BD = 5.8$ cm which intersect at D . Now taking D and B as centres draw two arcs of radii AB and AD respectively which intersect, at C . Join CD and BC .

Thus, $ABCD$ is a required rectangle.

Construction 9.25: Construction of rhombus when two diagonals are given.

Construct a rhombus whose diagonals are 4 cm and 6 cm respectively.

Construction : Draw a rough sketch of given measures and mark them.

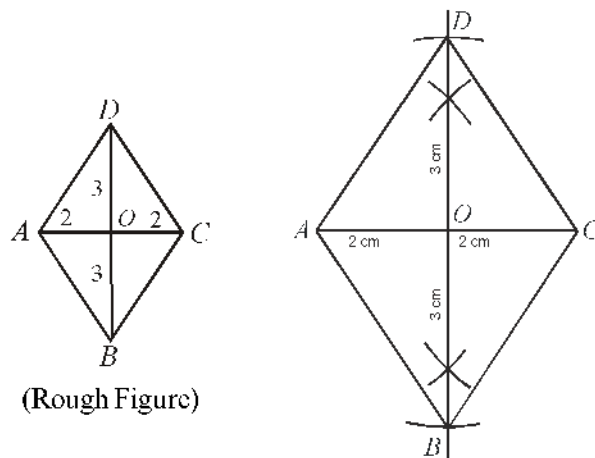


Fig. 9.54

Draw diagonal $AC = 4$ cm. Draw perpendicular bisector of AC which meets AC at O . Take O as centre and draw two arcs, of radius equal to half of the length of other

diagonal $BD = \left(\frac{1}{2} \times 6 = 3\text{ cm}\right)$ both sides of AC . These arcs intersect perpendicular bisector at B and D . Join AB , BC , CD and AD .

Thus $ABCD$ is a required rhombus.

Construction 9.26 : Construction of a square whose diagonals are given.

Construct a square whose diagonal is 5 cm.

Construction : Draw a rough sketch of given measures and mark them.

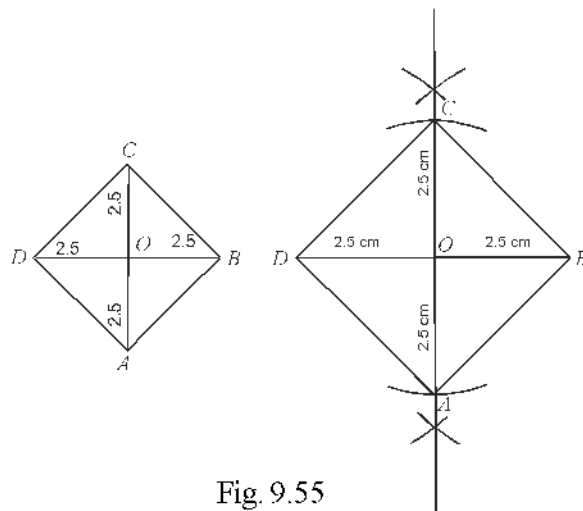


Fig. 9.55

Draw a diagonal $BD = 5$ cm. Draw its perpendicular bisector which meets BD at O . Take O as centre and cut $OC = OA = 2.5$ cm. Join AB, BC, CD and DA .

Thus, $ABCD$ is the required square.

Exercise 9.4

1. Construct a parallelogram $ABCD$ in which $AB = 4.7$ cm, $BC = 3.5$ cm and $AC = 7$ cm.
2. Construct a parallelogram $PQRS$ in which $PQ = 5$ cm, diagonal $PR = 7.6$ cm and diagonal $QS = 5.6$ cm.
3. Construct a parallelogram $ABCD$ whose two sides are 4.6 cm and 3 cm respectively and angle between them is 60° .
4. Construct a rectangle $ABCD$ in which $AB = 6$ cm and diagonal $AC = 10$ cm.
5. Construct a rhombus $ABCD$ whose diagonals $AC = 7$ cm and $BD = 5$ cm.
6. Construct a square $ABCD$ whose diagonal is 6 cm.

Construction 9.27 : Construction of a Trapezium

(a) When four sides of a trapezium are given and in which two sides are parallel.

Construct a trapezium $ABCD$ in which $AB = 7$ cm, $BC = 6$ cm, $CD = 4$ cm, $DA = 5$ cm and $AB \parallel CD$.

Construction : Draw a rough sketch based on given measures and mark all the lengths.

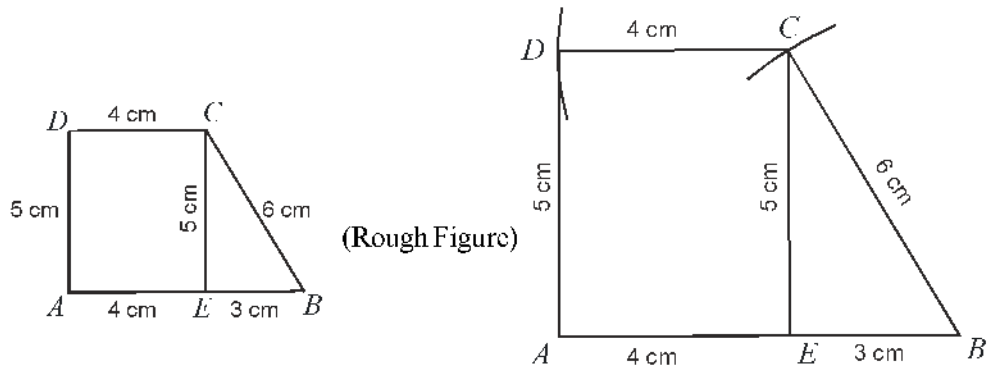


Fig. 9.56

Mark a point E on line AB such that $AE = DC = 4$ cm. Draw a line segment $AB = 7$ cm and mark point E such that $AE = 4$ cm. Take E and B as centres draw two arcs radii 5 cm ($= AD$) and 6 cm ($= BC$) respectively which intersect each other at point C . Again taking A and C as centres, draw two arcs of radii 5 cm and 4 cm such that they intersect each other at D . Join B to C , C to D and A to D to construct a complete quadrilateral.

Thus, $ABCD$ is the required trapezium.

(b) Construction of Trapezium if three sides and one angle are given and it is also given that which sides are parallel.

Construct a trapezium $ABCD$ in which $AB \parallel CD$, $\angle B = 90^\circ$, $AB = 4$ cm, $BC = 2.8$ cm, $AD = 3.5$ cm.

Construction : Draw a rough sketch of trapezium and mark all the given measures.

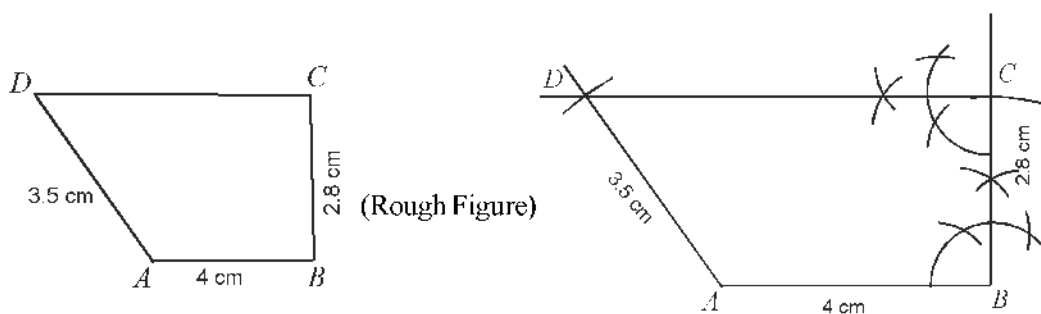


Fig. 9.57

For fair construction, take $AB = 4$ cm. Construct $\angle B = 90^\circ$ at B . Cut a point at a distance of 2.8 cm from line making right angle and mark it as C . Again construct $\angle C = 90^\circ$ at point C . ($\because AB \parallel CD$ and $\angle B = 90^\circ$, $\angle C = 90^\circ$)

Taking A as a centre, cut a point on a line drawn perpendicular at point C and mark it

as D . Join A and D to construct a quadrilateral $ABCD$.

Thus, $ABCD$ is a required trapezium.

Exercise 9.5

1. Construct a trapezium $ABCD$ in which $AB \parallel CD$, $AB = 4$ cm, $BC = 2.3$ cm, $CD = 2.8$ cm and $DA = 1.9$ cm.
2. Construct a trapezium $PQRS$ in which $PQ \parallel SR$, $PQ = 6$ cm, $RS = 3$ cm, $PS = 3$ cm and $QR = 5$ cm.
3. Construct a trapezium $ABCD$ in which $AB \parallel CD$, $AB = 8$ cm, $BC = 6$ cm, $CD = 4$ cm and $\angle B = 75^\circ$.
4. Construct a trapezium $ABCD$ in which $AB \parallel CD$, $AB = 4$ cm, $BC = 4$ cm, $AD = 5$ cm and $\angle B = 90^\circ$.

IMPORTANT POINTS

1. Sum of all the angles of a quadrilateral is 360° .
2. A diagonal of a parallelogram divides it into two congruent triangles.
3. In a parallelogram :
 - (i) opposite angles are equal.
 - (ii) opposite sides are equal.
 - (iii) diagonals bisect each other.
4. Any quadrilateral is a parallelogram, if :
 - (i) its opposite angles are equal,
 - (ii) its opposite sides are equal.
 - (iii) its diagonals bisect each other.
 - (iv) a pair of opposite sides is equal and parallel.
5. Diagonals of a rectangle bisect each other and are equal and vice-versa.
6. Diagonals of a rhombus, bisect each other at right angles and vice-versa.
7. Diagonals of a square bisect each other at right angles and are equal and vice-versa.
8. A line segment joining mid-points of two sides of a triangle, is parallel to third side and half of it.
9. A line passing through the mid-point of one side of a triangle side and parallel to another side, bisects the third side.
10. A quadrilateral, obtained by joining the mid-points of the sides of a quadrilateral, in a order, is a parallelogram.
11. It is necessary for the construction of a quadrilateral:
 - (i) four sides and a diagonal are given.
 - (ii) four sides and an angle are given.
 - (iii) three sides and two diagonals are given,

- (iv) three sides and angle between them are given.
 - (v) two adjacent sides and angle between them and other two angles are given.
12. It is necessary for the construction of a parallelogram :
 - (i) two adjacent sides and a diagonal are given.
 - (ii) one side and two diagonals are given.
 - (iii) two adjacent sides and two angle between them are given.
 13. It is necessary for the construction of a rectangle :
 - (i) two adjacent sides are given.
 - (ii) a side and a diagonal are given.
 14. For construction of square, it is necessary that:
 - (i) a side is given.
 - (ii) diagonal is given.
 15. For the construction of rhombus, it is necessary that :
 - (i) measure of a side and an angle between two adjacent sides,
 - (ii) diagonals are given.
 16. For the construction of a trapezium, it is necessary that :
 - (i) four sides are given and parallel sides are known.
 - (ii) three sides and an angle are given and parallel sides are known.
 17. Opposite sides of a parallelogram are equal and parallel and opposite angles are equal.
 18. Four sides of a square are equal and each angle is right angle. Diagonals of a square are equal and bisect each other at right angles.
 19. Opposite sides of a rectangle are equal and each angle is right angle.
 20. In a rhombus, four sides are equal and opposite angles are equal. Diagonals bisect each other at right angles.
 21. In a trapezium, only one pair of opposite sides be parallel.

Miscellaneous Exercise-9

Write the correct answers of each of the following questions (From question 1 to 15)

1. Three angles of a quadrilateral are 75° , 90° and 75° . Its fourth angle is :
 - (a) 90°
 - (b) 95°
 - (c) 105°
 - (d) 120°
2. A diagonal of rectangle is inclined at an angle of 25° with a side. The acute angle between its diagonals is :
 - (a) 55°
 - (b) 50°
 - (c) 40°
 - (d) 25°
3. $ABCD$ is a rhombus in which $\angle ACB = 40^\circ$. Then $\angle ADB$ is :
 - (a) 40°
 - (b) 45°
 - (c) 50°
 - (d) 60°

4. A quadrilateral, formed by joining the mid-points of the sides of a quadrilateral $PQRS$, in a order, is a rectangle if:
 - (a) $PQRS$ is a rectangle
 - (b) $PQRS$ is a parallelogram
 - (c) Diagonals of $PQRS$ are perpendicular to each other.
 - (d) Diagonals of $PQRS$ are equal.
5. A quadrilateral, formed by joining the mid-points of the sides of a quadrilateral $PQRS$, in a order, is a rhombus if:
 - (a) $PQRS$ is a rhombus
 - (b) $PQRS$ is a parallelogram
 - (c) Diagonals of $PQRS$ are perpendicular to each other
 - (d) Diagonals of $PQRS$ are equal.
6. If the ratio of angles A, B, C and D of a quadrilateral $ABCD$, taking in this order is $3:7:6:4$. Then $ABCD$ is a :
 - (a) rhombus (b) parallelogram (c) trapezium (d) kite
7. In a quadrilateral $ABCD$, bisectors of $\angle A$ and $\angle B$, intersect each other at P , bisectors of $\angle B$ and $\angle C$ at Q , bisectors of $\angle C$ and $\angle D$ at R , and bisectors of $\angle D$ and $\angle A$ intersect each other at S . Then $PQRS$ is a :
 - (a) rectangle (b) rhombus (c) parallelogram
 - (d) quadrilateral whose opposite angles are supplementary.
8. If AP and CQ are two parallel lines, then bisectors of $\angle APQ, \angle BPQ, \angle CQP$ and $\angle PQD$ make:
 - (a) a square (b) a rhombus (c) a rectangle (d) any other parallelogram
9. Joining the mid-points of the sides of rhombus, in a order, obtained a figure is :
 - (a) a rhombus (b) a rectangle (c) a square (d) any parallelogram
10. D and E are mid points of side AB and AC respectively of a ΔABC and O is any point on side BC . O is joined with A . If P and Q are mid points of OB and OC respectively, then $DEQP$ is a :
 - (a) square (b) rectangle (c) rhombus (d) parallelogram
11. The figure obtained by joining the mid-points of the sides of a quadrilateral $ABCD$ in order, is only a square if:
 - (a) $ABCD$ is a rhombus
 - (b) Diagonals of $ABCD$ are equal
 - (c) Diagonals of $ABCD$ are equal and perpendicular to each other
 - (d) Diagonals of $ABCD$ are perpendicular to each other

12. Diagonals AC and BD of a parallelogram $ABCD$ intersect each other at point O . If $\angle DAC = 32^\circ$ and $\angle AOB = 70^\circ$, then $\angle DBC$ is :
 (a) 24° (b) 86° (c) 38° (d) 32°
13. Which of the following statements is not true for a parallelogram :
 (a) Opposite sides are equal
 (b) Opposite angles are equal
 (c) Opposite angles are bisected by diagonals
 (d) Diagonals bisect each other.
14. D and E are mid points of sides AB and AC respectively of a ΔABC . DE is extended upto F . To prove that CF is equal to the line segment DA and parallel, we are required an other information, which is :
 (a) $\angle DAE = \angle EFC$ (b) $AE = EF$
 (c) $DE = EF$ (d) $\angle ADE = \angle ECF$
15. Diagonals of a parallelogram $ABCD$ intersect at point O . If $\angle BOC = 90^\circ$ and $\angle BDC = 50^\circ$, then $\angle OAB$ is :
 (a) 90° (b) 50° (c) 40° (d) 10°
16. $ABCD$ is a parallelogram. If its diagonals are equal, then find the value of $\angle ABC$.
17. Diagonals of a rhombus are equal and perpendicular to each other. Is this statement true? Give reason for your answer.
18. Three angles of a quadrilateral $ABCD$ are equal. Is this a parallelogram?
19. In quadrilateral $ABCD$, $\angle A + \angle D = 180^\circ$. What specific name can be given to this quadrilateral?
20. All angles of a quadrilateral are equal. What specific name is given to this quadrilateral?
21. Diagonals of rectangle are equal and perpendicular to each other. Is this statement true? Give reason for your answer.
22. Any square, inside an isosceles right-angled triangle is such that one angle is common in both square and triangle. Show that the vertex of the square, which is opposite to the vertex of the common angle, bisects the hypotenuse.
23. In a parallelogram $ABCD$, $AB = 10$ cm and $AD = 6$ cm. Bisector of $\angle A$ meets DC at E and producing AE and BC meet at F . Find the length of CF .
24. P, Q, R and S are the mid-points of sides AB, BC, CD and DA respectively in which $AC = BD$ and $AC \perp BD$. Prove that $PQRS$ is a square.
25. A diagonal of a parallelogram, bisects one of its angles. Prove that this parallelogram is a rhombus.

26. $ABCD$ is a quadrilateral in which $AB \parallel DC$ and $AD = BC$. Prove that $\angle A = \angle B$ and $\angle C = \angle D$.
27. E is the mid point of median AD of $\triangle ABC$. BE is extended to meet AC at F . Show that $AF = \frac{1}{3} AC$.
28. Show that a quadrilateral formed by joining the mid points of the consecutive side of a square is also a square.
29. Prove that a quadrilateral formed by the bisectors of the angles of a parallelogram is a rectangle.
30. P and Q are two points on opposite sides AD and BC of parallelogram $ABCD$ such that diagonal PQ passes through O , the point of intersection of diagonals AC and BD . Prove that PQ is bisected at point O .
31. $ABCD$ is a rectangle whose diagonal BD bisects $\angle B$. Show that $ABCD$ is a square.
32. D , E and F are respectively the mid-points of sides AB , BC and CA of a $\triangle ABC$. Prove that by joining the points D , E and F triangle ABC divided into four congruent triangles.
33. Prove that the line joining the mid-points of diagonals of a trapezium, is parallel to the parallel sides of that trapezium.
34. P is the mid point of side CD of a parallelogram $ABCD$. A line drawn passing through C and parallel to PA meets AB at Q and extended DA at R . Prove that $DA = AR$ and $CQ = QR$.
35. Construct a quadrilateral $ABCD$ in which $AB = 3.7$ cm, $BC = 3$ cm, $CD = 5$ cm, $AD = 4$ cm and $\angle A = 90^\circ$.
36. Construct a quadrilateral $ABCD$ in which $AB = AD = 3.2$ cm, $BC = 2.5$ cm, $AC = 4$ cm and $BD = 5$ cm.
37. Construct a quadrilateral $PQRS$ in which $PQ = 3.5$ cm, $QR = 3.5$ cm, $\angle P = 60^\circ$, $\angle Q = 105^\circ$ and $\angle S = 75^\circ$.
38. Construct a rhombus whose one side is 3.6 cm and one angle is 60° .
39. Construct a square in which $AB + BC + CD + DA = 12.8$ cm.
40. Construct a trapezium in which $AB \parallel CD$, $AB = 5$ cm, $BC = 3$ cm, $AD = 3.3$ cm and distance between parallel sides is 2.5 cm.
41. Construct a rhombus $ABCD$ in which $AB = 6$ cm and $\angle A = 120^\circ$.
42. Construct a trapezium where $AB = 2.3$ cm, $BC = 3.4$ cm, $CD = 5.4$ cm, $DA = 3.7$ cm and $AB \parallel CD$.
43. Construct a rhombus $ABCD$ whose diagonals are 5.6 cm and 7.2 cm.
44. Construct a rectangle $ABCD$ in which $AB = 4.5$ cm and $BD = 6$ cm.

Answers

EXERCISE 9.1

1. $36^\circ, 60^\circ, 108^\circ, 156^\circ$
2. $AC = 6 \text{ cm}, BD = 4 \text{ cm}$
3. No, diagonals of a parallelogram bisect each other.
4. No, sum of the angles of a quadrilateral should be 360° .
5. No, sum of angles will be greater than 360° which is not possible for a quadrilateral.
6. 84°
7. Each angle is 135°
8. $120^\circ, 60^\circ, 120^\circ, 60^\circ$

EXERCISE 9.2

1. 55°
2. No, sum of all angles will be less than 360° .
3. Yes, that will be rectangle or square.
4. 145°
5. 4 cm
6. 60, 120, 60, 120
13. (i) 6.9 cm (ii) 9.9 cm

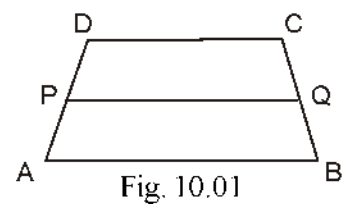
Miscellaneous Exercise 9

- | | | | |
|-------|-------|-------|-------|
| 1. D | 2. B | 3. C | 4. D |
| 5. C | 6. B | 7. D | 8. C |
| 9. B | 10. D | 11. C | 12. C |
| 13. C | 14. C | 15. C | |
16. 90° , this quadrilateral is a rectangle.
 17. This statement is not true because diagonals of a rhombus are perpendicular to each other but they are not equal.
 18. It is not necessary to be parallelogram.
 19. Parallelogram.
 20. Rectangle or square
 21. No, diagonals of rectangle are equal but not perpendicular to each other.
 23. 4 cm

Area of Triangles and Quadrilaterals

10.01 Introduction

We know that the study of Geometry, originated with the measurement of land in the process of recasting boundaries and distribution of the fields. For example, Kartik distributes his trapezium shaped field by joining the mid points of non-parallel sides between his two daughters. (See Fig 10.01). Is this distribution equal in area? To get answer to this type of problems, there is a need to have a look at area of plane figures.



10.02 Area

The part of plane enclosed by a simple closed figure, is called the plane region of that figure and magnitude or measure of this plane region is called the area of that field. This magnitude or measure is always expressed with the help of some units, for example 6 square cm (cm^2), 9 square metre (m^2), 12 hectare etc.

In chapter 7, we have studied about congruent figures. If two plane figures are same in shape and measure, they are called congruent. If they are cut and put on any plane, then both the figures have an equal plane region on that plane that means, two congruent figures are equal in areas. Is its converse also true? Let us try to understand the following activities.

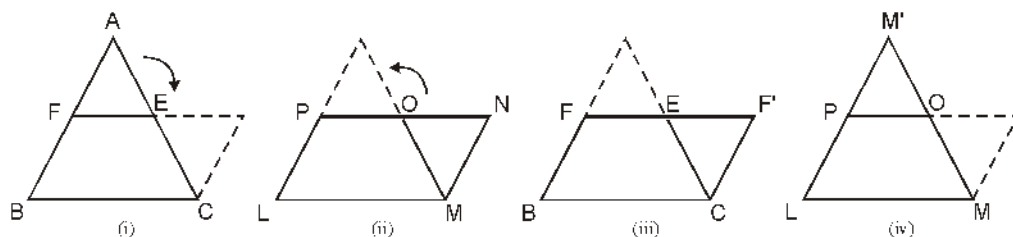


Fig. 10.02

Step-1

Prepare a carbon copy of Fig 10.02 (i), (iii) by leaving the dotted part with the help of pencil and scale on a white paper, by using carbon paper putting under this page of book. From prepared carbon copies, cut $\triangle AEF$ of fig 10.02 (i) and according to 10.02 (ii), keeping E on the same point, move $\triangle AEF$ so that A reach to point C and paste. Now we will get a quadrilateral $BCF'E$. Similarly, from Fig. 10.02 (iii), cut a $\triangle MNO$ and at O still, move such that N comes to P and paste as in Fig. 10.02 (iv). In this way a $\triangle IMM'$ will be obtained.

Step-2

Now put quadrilateral $BCF'E$ on Fig. 10.02 (iii) and $\triangle IMM'$ on Fig. 10.02 (i). Are they cover each other completely? Yes, they are covering. It means that quadrilateral $BCF'E \cong$ quadrilateral $LMNP$ and $\triangle IMM' \cong \triangle BCA$. Are these congruent figures equal in area? Definitely they are equal. But converse of it that $\triangle ABC$ and quadrilateral $BCF'E$ and quadrilateral $LMNP$ and $\triangle IMM'$ which are equal in area. Area $\triangle ABC$, quadrilateral $BCF'E$ and quadrilateral $LMNP$, $\triangle IMM'$ are congruent? Undoubtedly no.

So, we can say that ***congruent figures are equal in area but the figures equal in area need not necessarily congruent.***

If quadrilateral $BCF'E \cong$ quadrilateral $LMNP$ and $\triangle IMM' \cong \triangle BCA$, then we write it as :

$$ar(BCF'E) = ar(LMNP) \text{ and } ar(IMM') = ar(BCA)$$

Let us see the Fig 10.03. You may observe that plane region figure T is made by adding two figures X and Y in plane region. Now, you can easily see that
Area of figure T = Area of figure X + Area of figure Y

OR $ar(T) = ar(X) + ar(Y)$

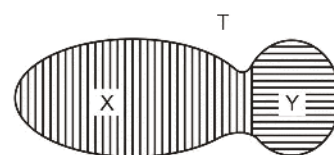


Fig. 10.03

In the previous classes, you have studied the formulae for finding the areas of different figures such as rectangle, square, triangle, etc. In this chapter, you will learn about the relation between area of geometrical figures under the condition when they lie on the same base and between the same pair of parallel lines. By studying this we will try to understand the deep knowledge of these formulae.

10.03 Figures Made on Same Base and Between Pair of Same Lines

Look at the following figures 10.04.

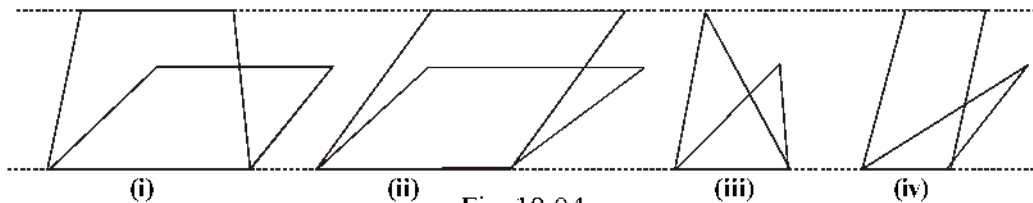


Fig. 10.04

In Fig. 10.04 (i), (ii), (iii) and (iv) in each case two figures are made on the same base but they are not made between same dotted wall of pair of parallel lines.

Now see the figures 10.05 given below.

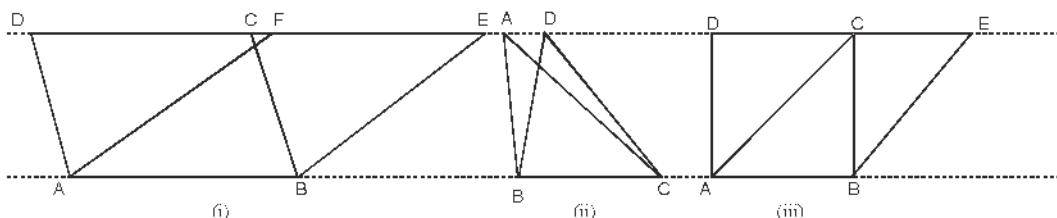


Fig. 10.05

In Fig. 10.05 in each case two figures are between two parallel lines. Here, in Fig. 10.05 (i) parallelogram $ABCD$ and $ABEF$ are on same base AB and between same parallel lines AB and DE . In Fig. 10.05 (ii), $\triangle ABC$ and $\triangle BCD$ are on same base BC and between same parallel lines BC and AD . Similarly, in Fig. 10.05 (iii) a square $ABCD$ and a parallelogram $ABEC$ are on same base AB and between same parallel lines AB and DE .

In Fig. 10.05 (i), (ii), (iii) are such figures that are said to be made on same base and between same parallel lines. In all the figures the bases are common in the two figures and the opposite-vertex of common base, is on the line drawn parallel to the base in each figure.

Now in article 10.06, keeping in mind the knowledge gained till now, which group of figures in fig. (i), (ii), (iii) and (iv) is made on the same base and between same parallel lines?

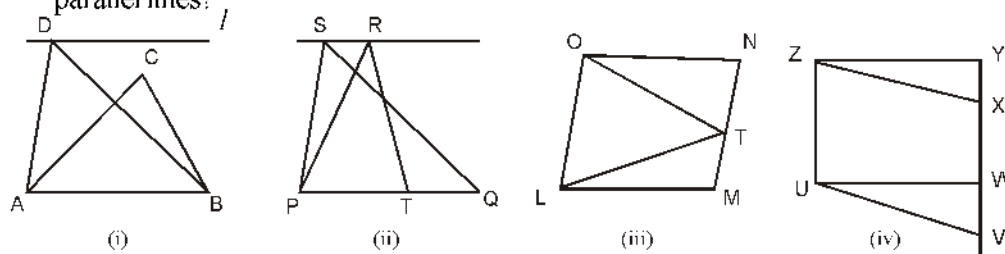


Fig. 10.06

Let us discuss

In Fig. 10.06 (i), $\triangle ABC$ and $\triangle ABD$ have common base AB , but the vertex C of $\triangle ABC$ does not lie on the line l which is parallel to base AB .

In Fig. 10.06 (ii) PT is the base of ΔPTR and PQ is base of ΔPQS means there is no common base of both the triangles but both the triangles are made in between two parallel lines PQ and SR .

In Fig. 10.06 (iii), parallelogram $LMNO$ and ΔLTO are on the same base LO and between the parallel lines LO and MN . Similarly, in Fig. 10.06 (iv), parallelogram $UVXZ$ and rectangle $UWYZ$ are made on same base UZ and between a pair of parallel lines UZ and VY . In this way Fig. 10.06 (i), (ii) are not in category of the figures, made on same base and between same parallel lines while Fig. 10.06 (iii) and (iv) are said to be in this category.

EXERCISE 10.1

- Which of the following figures are lying on the same base and between the same parallel lines? Write common base and pair of parallel lines in such a case.

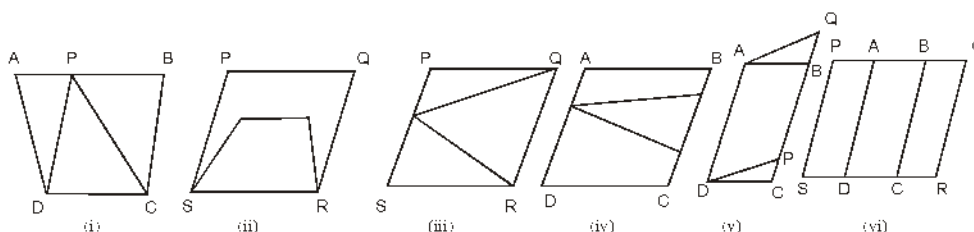


Fig. 10.07

- Draw the following figures, on the same base and between the same parallel lines-
 - An obtuse angled triangle and a trapezium.
 - A parallelogram and an isosceles triangle.
 - A square and a parallelogram.
 - A rectangle and a rhombus.
 - A rhombus and a parallelogram.

Activity 10.2

Step-1

Make two carbon copies of a parallelogram by keeping two carbon papers between three white papers and label their vertices by A, B, C, D . Mark a point P on side CD by pressing such that it also appears on carbon copies.

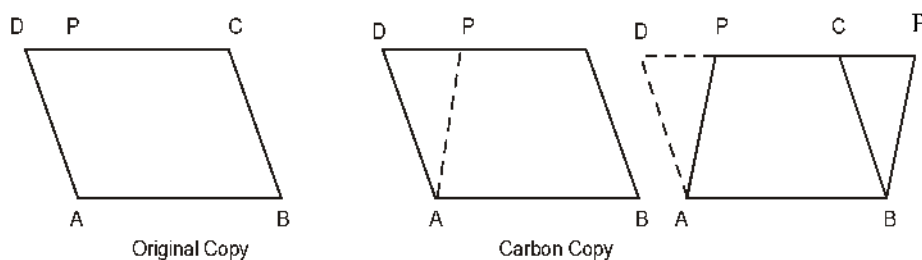


Fig. 10.08

Step-2

- (i) Cut original copy and paste it on a page of your exercise note book.
 - (ii) Cut ΔAPD made by joining P to A on carbon copy. Paste ΔAPD on otherside of carbon copy, in such a way that after cutting side AD should coincide with the side BC of trapezium $ABCP$. Keep in mind that A should be on B and D on C .
- Thus we are getting two new parallelograms $ABCD$ and $ABP'P$. Paste one of these two quadrilaterals on your exercise note book, on same page as fig. 10.08 (iii).

Step-3

Paste another new parallelogram $ABP'P$ on original copy such that side AB of both parallelograms should coincide. (See Fig 10.09).

In a new figure two parallelograms $ABCD$ and $ABP'P$ are made on same base and between a pair of same parallel lines.

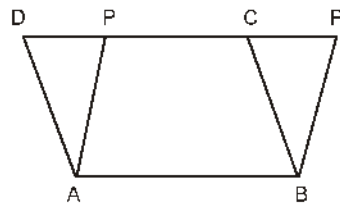


Fig. 10.09

Can you say that parallelogram $ABCD$ and $ABP'P$ are equal in area? Let us see.

$$\therefore \Delta APD \cong \Delta BP'C$$

(ΔAPD is pasted after cutting)

$$\therefore ar(\Delta APD) = ar(\Delta BP'C)$$

Adding $ar(ABCP)$ on both sides, we get

$$ar(\Delta APD) + ar(ABCP) = ar(\Delta BP'C) + ar(ABCP)$$

or $ar(ABCD) = ar(ABP'P)$

\Rightarrow Both parallelograms which are made on same base AB and between the parallel lines ($AB \parallel DP'$), are equal in area.

Let us try to prove this result by any other method.

Theorem 10.1. *Two parallelograms, made on same base and between the same parallel lines, are equal in areas.*

Given : Two parallelograms $ABCD$ and $ABFE$ whose base is AB and are between two parallel lines AB and DF .

To Prove : Area of parallelogram $ABCD$ = Area of parallelogram $ABFE$

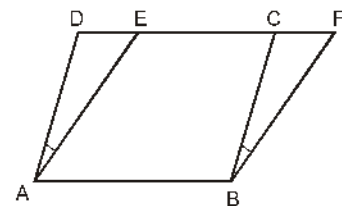


Fig. 10.10

Proof : In $\triangle ADE$ and $\triangle BCF$

$AE = BF$ (Opposite sides of parallelogram $ABFE$)
 $\angle DAE = \angle CBF$ (Corresponding angles)
 $AD = BC$ (Opposite sides of parallelogram $ABCD$)
 $\triangle ADE \cong \triangle BCF$ (By SAS congruency rule)

$\therefore \text{ar}(\triangle ADE) = \text{ar}(\triangle BCF)$

Adding $\text{ar}(ABCE)$ on both sides, we get

$\text{ar}(\triangle ADE) + \text{ar}(ABCE) = \text{ar}(\triangle BCF) + \text{ar}(ABCE)$

$\therefore \text{ar}(ABCD) = \text{ar}(ABFE)$

Hence Proved

Corollary 1.

A parallelogram and a rectangle are lying on same base and between two parallel lines, then their areas are equal and area of parallelogram is equal to the product of its base and distance between two parallel lines.

Given : In Fig. 10.11, $ABCD$ is a parallelogram and $EFCD$ is a rectangle.

Also $AL \perp DC$

To Prove : (i) $\text{ar}(ABCD) = \text{ar}(EFCD)$

(ii) $\text{ar}(ABCD) = DC \times AL$

Proof : (i) As rectangle is also a parallelogram,

$\therefore \text{ar}(ABCD) = \text{ar}(EFCD) \dots (i)$

(ii) \therefore Area of rectangle = length \times breadth

$\therefore \text{ar}(EFCD) = DC \times FC$

$\therefore \text{ar}(ABCD) = DC \times FE$

$\therefore AL \perp DC$ (given)

So, $ALCL$ is also a rectangle.

$\therefore AL = FC \dots (iii)$

Thus, $\text{ar}(ABCD) = DC \times AL$

[From (ii) and (iii)]

Hence Proved

Corollary 2.

If a triangle and a parallelogram are made on the same base and between pair of same parallel lines, then area of the triangle is half of the area of parallelogram.

Given : $\triangle ABP$ and parallelogram $ABCD$ are made on same base AB and between same parallel lines AB and PC .

To Prove : $\text{ar}(PAB) = \frac{1}{2} \text{ar}(ABCD)$

Construction : Draw $BQ \parallel AP$.

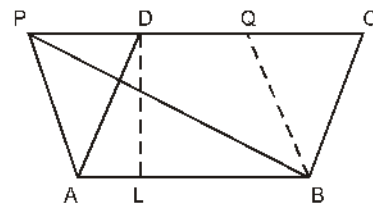


Fig. 10.11

[From (i)] \dots (ii)

Fig. 10.12

Proof :

$\therefore AB \parallel CD$ (given)
 $\therefore AB \parallel PQ$
 and $AP \parallel BQ$ (By construction)
 $\therefore ABQP$ is a parallelogram.
 $\therefore \text{ar}(ABCD) = \text{ar}(ABQP)$ (By theorem 10.1)
 And $\triangle ABP \cong \triangle QPB$ (A diagonal B divides a parallelogram in two congruent triangles)
 $\therefore \text{ar}(ABP) = \text{ar}(QPB)$
 $= \frac{1}{2} \text{ar}(ABQP)$
 $\Rightarrow \text{ar}(ABP) = \frac{1}{2} \text{ar}(ABCD)$ **Hence Proved**

Corollary 3.

$$\text{Area of triangle} = \frac{1}{2} \times \text{base} \times \text{height}$$

From Fig. 10.12, if $DL \perp AB$,

$\therefore \text{ar}(ABCD) = AB \times DL$ then by corollary

But $\text{ar}(PAB) = \frac{1}{2} \text{ar}(ABCD)$ (From corollary 2)

$\therefore \text{ar}(PAB) = \frac{1}{2} AB \times DL$

or **Area of triangle** $= \frac{1}{2} \times \text{base} \times \text{height}$ **Proved**

EXERCISE 10.2

- In Fig 10.13, $ABCD$ is a parallelogram, in which $AE \perp DC$ and $CF \perp AD$. If $AB = 16$ cm, $AE = 8$ cm and $CF = 10$ cm, then find the value of AD .

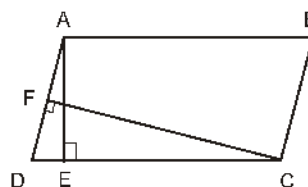


Fig. 10.13

2. If E, F, G and H are respectively the mid-points of sides of a parallelogram. Show that $\text{ar}(\text{EFGH}) = \frac{1}{2} \text{ar}(\text{ABCD})$
3. P and Q are respectively points lying on the side DC and AD respectively of a parallelogram $ABCD$. Show that $\text{ar}(\text{APB}) = \text{ar}(\text{BQC})$.
4. In Fig. 10. 14, P is any point in interior of a parallelogram $ABCD$. Show that :
 - (i) $\text{ar}(\text{APB}) + \text{ar}(\text{PCD}) = \frac{1}{2} \text{ar}(\text{ABCD})$
 - (ii) $\text{ar}(\text{APD}) + \text{ar}(\text{PBC}) = \text{ar}(\text{APB}) + \text{ar}(\text{PCD})$

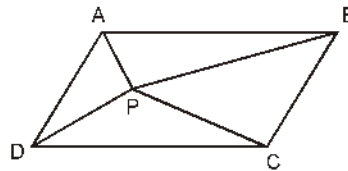


Fig. 10.14

5. In Fig. 10. 15, $PQRS$ and $ABRS$ are parallelograms and X is any point on side TR . Show that :
 - (i) $\text{ar}(\text{PQRS}) = \text{ar}(\text{ABRS})$
 - (ii) $\text{ar}(\text{AXS}) = \frac{1}{2} \text{ar}(\text{PQRS})$

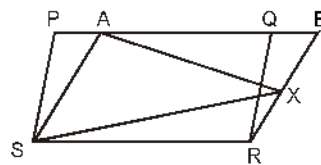


Fig. 10.15

6. A farmer had a field in the form of a parallelogram $PQRS$. He took any point A situated on RS and joined it to points P and Q . In how many parts the field is divided? What are the shapes of these parts? The farmer wants to sow wheat and pulses in equal portion of the field separately. How should he do it?

10.4. Triangles on the same base and between same parallel lines :

Theorem 10.2. *Area of triangles, on same base and between same parallel lines, are equal.*

Given : ΔABC and ΔDBC are on base BC and between parallel lines BC and AF .

To Prove : $ar(\Delta ABC) = ar(\Delta DBC)$

Construction : From point C draw two lines CE and CF parallel to AB and BD respectively.

Proof : $ABCE$ and $DBCF$ are between same parallel lines BC and AF .

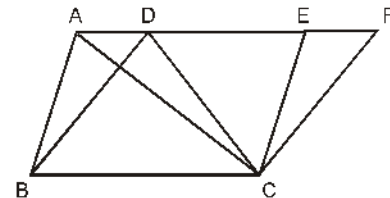


Fig. 10.16

$$\therefore ar(ABCE) = ar(DBCF) \quad \dots(1)$$

[From theorem 10.1]

$\therefore AC$ is a diagonal of parallelogram $ABCE$,

$$\therefore ar(\Delta ABC) = \frac{1}{2} ar(ABCE) \quad \dots(2)$$

Similarly, DC is a diagonal of parallelogram $DBCF$,

$$ar(DBC) = \frac{1}{2} ar(DBCF) \quad \dots(3)$$

From (1), (2) and (3), we get

$$ar(\Delta ABC) = ar(\Delta DBC)$$

Proved

Theorem 10.3. *If area of two triangles are equal and one side of a triangle is equal to one side of other triangle, then their corresponding altitudes are equal.*

Given : In ΔABC and ΔDEF

(i) $ar(\Delta ABC) = ar(\Delta DEF)$

(ii) $BC = EF$

To Prove : Altitude $AN =$ Altitude DM

Proof : In ΔABC , AN is the altitude to the corresponding side BC .

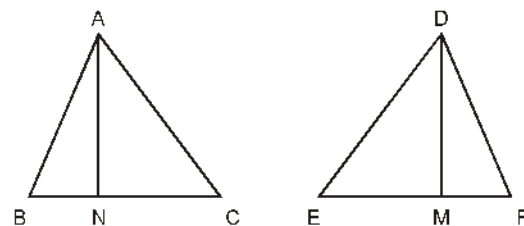


Fig. 10.17

$$\therefore ar(\Delta ABC) = \frac{1}{2} \times BC \times AN \quad \text{[By corollary 3]} \quad \dots(1)$$

$$\text{Similarly } ar(\Delta DEF) = \frac{1}{2} \times EF \times DM \quad \text{[By corollary 3]} \quad \dots(2)$$

But, given $ar(\Delta ABC) = ar(\Delta DEF)$

$$\therefore \frac{1}{2} \times BC \times AN = \frac{1}{2} \times EF \times DM$$

But $BC = EF$ (Given)

$\therefore AN = DM$

Hence Proved

10.5. Baudhayan Theorem

Baudhayan gave us a very important result on a right angled triangle which is known as Baudhayan Theorem. This theorem is also famous by the name of Pythagorus Theorem. Now we will prove it.

Theorem 10.4. *In a right angled triangle, square made on hypotenuse, is equal to the sum of the squares made on other two sides.*

Given : In $\triangle ABC$, $\angle C = 90^\circ$, and squares on sides AB , BC and CA are $ADEB$, $CBFG$ and $ACHK$ respectively.

To Prove : Square $ADEB =$ Square $ACHK +$ Square $CBFG$

Construction : From point C draw $CM \perp BE$ which intersects AB at L . Join BK and CD .

Proof : $\angle BAD = \angle CAK = 90^\circ$

Adding $\angle CAB$ both sides

$$\Rightarrow \begin{aligned} \angle BAD + \angle CAB &= \angle CAK + \angle CAB \\ \angle CAD &= \angle BAK \end{aligned} \quad \dots(1)$$

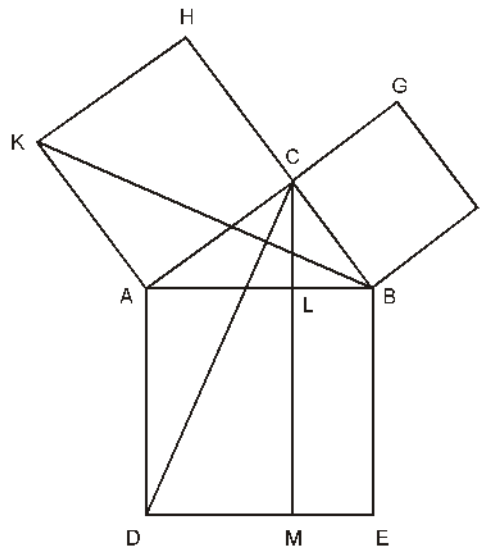


Fig. 10.18

In $\triangle BAK$ and $\triangle DAC$, we get

$$AB = AD$$

(Sides of a square $ABED$)

$$\angle BAK = \angle DAC$$

[From (1)]

$$AK = AC$$

[Sides of square $ACHK$]

$$\triangle BAK \cong \triangle CAD$$

(By SAS congruency rule) ... (2)

But $\angle BCA = \angle ACH = 90^\circ$
 $\angle BCA + \angle ACH = 180^\circ$

$\Rightarrow BCH$ is a straight line.

$CH \parallel AK$ (Opposite sides of square $ACHK$)

$\triangle BAK$ and square $ACHK$ are on same base AK and between same parallel lines AK and BH .

$$\therefore \text{ar}(\triangle BAK) = \frac{1}{2} \text{ar}(\text{square } ACHK) \quad \dots(3)$$

Similarly, $\triangle ADC$ and rectangle $ADML$ are on same base AD and between same parallel lines AD and CM .

$$\therefore \text{ar}(\triangle CAD) = \frac{1}{2} \text{ar}(\text{rectangle } ADML) \quad \dots(4)$$

\therefore From (2), (3) and (4), we get

$$\text{ar}(\triangle CAD) - \text{ar}(\triangle BAK) - \frac{1}{2} \text{ar}(\text{square } ACHK) = \frac{1}{2} \text{ar}(\text{rectangle } ADML)$$

$$\therefore \text{ar}(\text{square } ACHK) - \text{ar}(\text{rectangle } ADML) \quad \dots(5)$$

$$\text{Similarly, ar}(\text{square } CDFG) - \text{ar}(\text{rectangle } IMFE) \quad \dots(6)$$

Adding (5) and (6), we get

$$\text{ar}(\text{square } ACHK) + \text{ar}(\text{square } CDFG) = \text{ar}(\text{rectangle } ADML) + \text{ar}(\text{rectangle } IMFE)$$

$$\therefore \text{ar}(\text{square } ADEB) = \text{ar}(\text{square } ACHK) + \text{ar}(\text{square } CDFG) \quad \text{Proved}$$

Theorem 10.5. (Converse of Baudhayan Theorem)

In a triangle, if square of a side is equal to the sum of the squares of other two sides, angle opposite to this side, is a right angle.

Given : In $\triangle ABC$, $AB^2 + BC^2 = AC^2$

To Prove : $\angle B = 90^\circ$

Construction : Construct a $\triangle PQR$ such that

$\angle Q = 90^\circ$, $PQ = AB$ and $QR = BC$

Proof : In $\triangle PQR$ by Baudhayan theorem

$$PR^2 = PQ^2 + QR^2$$

But $PQ = AB$ and $QR = BC$.

$$\therefore PR^2 = AB^2 + BC^2 \quad \dots(1)$$

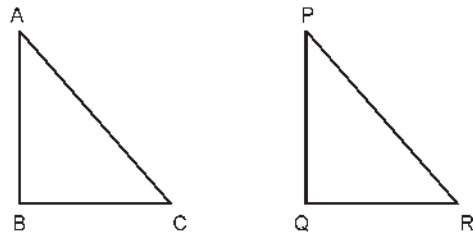


Fig. 10.19

But given that

$$AC^2 = AB^2 + BC^2 \quad \dots(2)$$

From (1) and (2)

$$PR^2 = AC^2 \Rightarrow PR = AC \quad \dots(3)$$

Now in ΔABC and ΔPQR , we get

$$PQ = AB \quad \text{(By construction)}$$

$$QR = BC \quad \text{(By construction)}$$

$$PR = AC \quad \text{[From (3)]}$$

$$\Delta ABC \cong \Delta PQR \quad \text{(By SSS congruency rule)}$$

$$\therefore \angle B = \angle Q = 90^\circ$$

$$\text{But } \angle Q = 90^\circ$$

$$\therefore \angle B = 90^\circ$$

Hence Proved

Illustrative Examples

Example 1. $PQRS$ is a square. T and U are the mid-points of PS and QR respectively (Fig. 10.20). Find the area of ΔOTS . If $PQ = 8$ cm and O is the point of intersection of TU and QS .

Solution : $PS = PQ = 8$ cm and $TU \parallel PQ$

$$\therefore ST = \frac{1}{2}PS = \frac{1}{2} \times 8 = 4 \text{ cm}$$

$$\text{also } PQ = TU = 8 \text{ cm and } PQ \parallel TU$$

In ΔPQS , T is the mid point of PS and $TU \parallel PQ$ then

$$OT = \frac{1}{2}PQ.$$

$$\therefore OT = \frac{1}{2}TU = \frac{1}{2} \times 8 = 4 \text{ cm}$$

$$\therefore \text{ar}(\Delta OTS) = \frac{1}{2} \times OT \times TS$$

[ΔOTS is a right angle triangle]

$$= \frac{1}{2} \times 4 \times 4 \text{ cm}^2 = 8 \text{ cm}^2$$

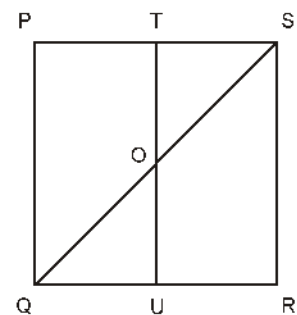


Fig. 10.20

Exercise 2. $ABCD$ is a parallelogram and BC is produced upto Q such that $AD = CQ$ (Fig 10.21). If AQ intersects side DC at P . Then show that :

$$\text{ar}(\Delta BPC) = \text{ar}(\Delta DPQ)$$

Solution : $\text{ar}(\text{ACP}) = \text{ar}(\text{BCP}) \quad \dots(1)$

[Triangles made on same base PC and between same parallel lines PC and AB]

also $\text{ar}(\text{ADC}) - \text{ar}(\text{ADQ}) \quad \dots(2)$

[Triangles made on same base AD and between same parallel lines AD and BQ]

$\text{ar}(\text{ADC}) - \text{ar}(\text{ADP}) = \text{ar}(\text{ADQ}) - \text{ar}(\text{ADP})$

$\text{ar}(\text{APC}) = \text{ar}(\text{DPQ}) \quad \dots(3)$

From (1) and (3)

$\therefore \text{ar}(\text{BCP}) = \text{ar}(\text{DPQ})$

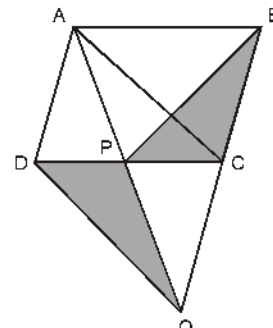


Fig. 10.21

Hence Proved

Example 3. In Fig. 10.22 ABCD is a parallelogram. Point P and Q divide side BC in three equal parts. Prove that $\text{ar}(\text{APQ}) = \text{ar}(\text{DPQ}) = \frac{1}{6} \text{ar}(\text{ABCD})$.

Solution : Draw PR and QS parallel to AB from points P and Q respectively (Fig. 10.22).

Now, PQRS is a parallelogram

whose base $PQ = \frac{1}{3} BC$.

Now $\text{ar}(\text{APD}) = \frac{1}{2} \text{ar}(\text{ABCD})$

[triangle and parallelogram ABCD lie on same base AD and between parallel lines AD and BC] ... (1)

also $\text{ar}(\text{AQD}) = \frac{1}{2} \text{ar}(\text{ABCD}) \quad \dots(2)$

From (1) and (2), we get

$\text{ar}(\text{APD}) = \text{ar}(\text{AQD}) \quad \dots(3)$

Subtracting $\text{ar}(\text{AOD})$ from both sides, we get

$\text{ar}(\text{APD}) - \text{ar}(\text{AOD}) = \text{ar}(\text{AQD}) - \text{ar}(\text{AOD})$

$\therefore \text{ar}(\text{APO}) = \text{ar}(\text{OQD}) \quad \dots(4)$

Adding $\text{ar}(\text{OPQ})$ to both sides, we get

$\text{ar}(\text{APO}) + \text{ar}(\text{OPQ}) = \text{ar}(\text{OQD}) + \text{ar}(\text{OPQ})$

$\text{ar}(\text{APQ}) = \text{ar}(\text{DPQ})$

$\therefore \text{ar}(\text{APQ}) = \frac{1}{2} \text{ar}(\text{PQRS})$

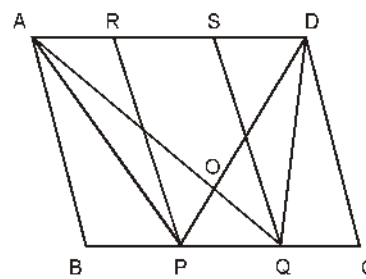


Fig. 10.22

and
$$ar(DPQ) = \frac{1}{2} ar(PQRS)$$

now
$$ar(PQRS) = \frac{1}{3} ar(ABCD)$$

$$\therefore ar(APQ) = ar(DPQ) = \frac{1}{2} ar(PQRS) = \frac{1}{2} \times \frac{1}{3} ar(ABCD)$$

$$= \frac{1}{6} ar(ABCD)$$
 Hence Proved

Example 4. In Fig. 10.23 l, m and n are lines such that $l \parallel m$ and line n intersects line l at P and line m at Q . $ABCD$ is a quadrilateral such that vertex A is situated on line l , vertices C and D are situated on line m and $AD \parallel n$. Show that:

$$ar(ABCQ) = ar(ABCDP)$$

Solution : $ar(ADP) - ar(ADQ) \dots(1)$

[On same base AD and between same parallel lines AD and PQ]

Adding $ar(ABCD)$ to both sides of (1), we get

$ar(ADP) + ar(ABCD) - ar(ADQ) + ar(ABCD)$

$ar(ABCDP) - ar(ABCQ)$

Hence Proved

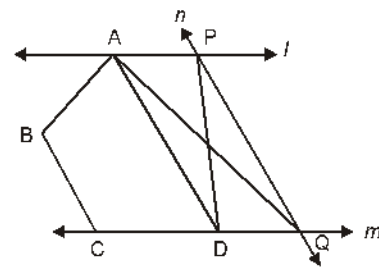


Fig. 10.23

Example 5. In Fig 10.24, $BD \parallel CA$, E is the mid-point of CA and $BD = \frac{1}{2} CA$.

Prove that $ar(ABC) = 2 ar(DBC)$.

Solution : Join DE . Here $BCED$ is a parallelogram.

$\therefore BD = CE \text{ and } BD \parallel CE$

$\therefore ar(DBC) = ar(EBC) \dots(1)$

(On same base BC and between same parallels BC and DE)

BE is median in ΔABC .

$\therefore ar(EBC) = \frac{1}{2} ar(ABC)$

Now $ar(ABC) = ar(EBC) + ar(ABE)$

$\therefore ar(ABC) = 2 ar(EBC)$

$\therefore ar(ABC) = 2 ar(DBC)$

[using (1)] **Hence Proved**

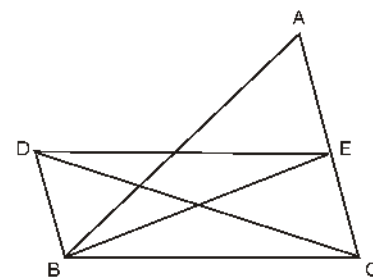


Fig. 10.24

Example 6. In an acute angled ΔABC , $\angle B$ is an acute angle. Therefore, all angles will be less than 90° . AD is perpendicular on BC . Prove that :

$$AB^2 = AC^2 + BC^2 - 2 BC \times DC$$

Solution : Given : ΔABC , $AD \perp BC$

To Prove :

$$AB^2 = AC^2 + BC^2 - 2 BC \times DC$$

Proof : In ΔABD , $\angle D = 90^\circ$

$$\therefore AB^2 = AD^2 + BD^2 \quad (\text{By Budhayan Theorem})$$

$$\Rightarrow AB^2 - AD^2 = (BC - DC)^2$$

$$= AD^2 + BC^2 + DC^2 - 2 BC \times DC$$

$$= (AD^2 - DC^2) - BC^2 - 2 BC \times DC \quad \dots(1)$$

Also in ΔADC , $\angle D = 90^\circ$

$$\therefore AC^2 = AD^2 + DC^2 \quad \dots(2)$$

From (1) and (2), we get

$$AB^2 - AC^2 = BC^2 - 2 BC \times DC$$

Hence Proved

Example 7. Prove that the sum of the squares of the sides of a rhombus, is equal to the sum of the squares of its diagonals.

Solution :

Given : Diagonals AC and BD of a rhombus $ABCD$ intersect at point O .

To Prove : $AB^2 + BC^2 + CD^2 + DA^2 = AC^2 + BD^2$

Proof : We know that diagonals of a rhombus intersect each other at right angles.

$$\text{Therefore, in } \Delta AOB, OA^2 + OB^2 = AB^2 \quad \dots(1)$$

$$\text{Similarly, in } \Delta BOC, OB^2 + OC^2 = BC^2 \quad \dots(2)$$

$$\text{In } \Delta COD, OC^2 + OD^2 = CD^2 \quad \dots(3)$$

$$\text{And in } \Delta AOD, OA^2 + OD^2 = AD^2 \quad \dots(4)$$

Adding (1), (2), (3) and (4), we get

$$2(OA^2 + OB^2 + OC^2 + OD^2) = AB^2 + BC^2 + CD^2 + AD^2$$

$$\therefore OA + OC = \frac{1}{2} AC$$

$$\text{and } OB + OD = \frac{1}{2} BD$$

$$AB^2 + BC^2 + CD^2 + AD^2 = 2 \left[\frac{AC^2}{4} + \frac{AC^2}{4} + \frac{BD^2}{4} + \frac{BD^2}{4} \right]$$

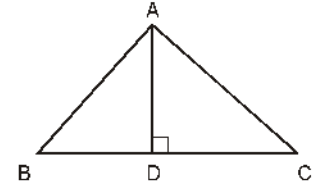


Fig. 10.25

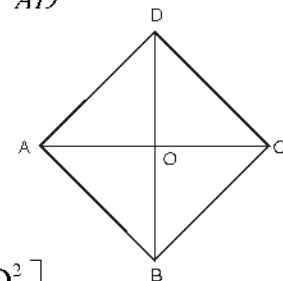


Fig. 10.26

$$\therefore AB^2 + BC^2 + CD^2 + AD^2 - 2 \left[\frac{AC^2}{2} + \frac{BD^2}{2} \right] = AC^2 + BD^2$$

Hence Proved

EXERCISE 10.3

Write true or false and give reason to your answer :

1. $ABCD$ is a parallelogram and X is the mid-point of AB . If $ar(AXCD) = 24\text{cm}^2$, then $ar(ABC) = 24\text{cm}^2$.
2. $PQRS$ is a rectangle which is inside a quadrant of a circle of radius 13 cm. A is any point on side PQ . If $PS = 5$ cm, then $ar(RAS) = 30\text{cm}^2$.
3. $PQRS$ is a parallelogram whose area is 180cm^2 and A is any point on diagonal QS . Then area of ΔASR is 90cm^2 .
4. ABC and BDE are two equilateral triangles such that D is the mid-point of side BC .

$$\text{Then } ar(BDE) = \frac{1}{4} ar(ABC).$$

5. In Fig 10.27, $ABCD$ and $EFGD$ are two parallelograms and G is the mid-point of side CD . Show that :

$$ar(DPC) = \frac{1}{2} ar(EFGD)$$

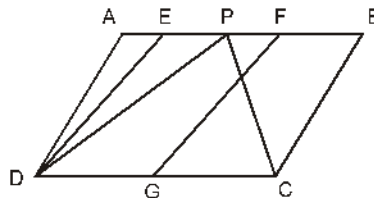


Fig. 10.27

6. In a trapezium $ABCD$, $AB \parallel CD$ and L is the mid-point of side BC . A line $PQ \parallel AD$ is drawn through L which meets AB on P and extended DC at Q (Fig. 10.28). Prove that : $ar(ABCD) = ar(APQD)$.

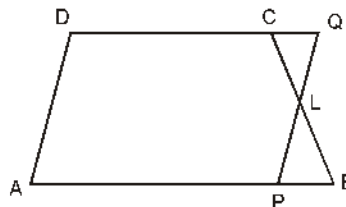


Fig. 10.28

7. If the mid-points of any quadrilateral are joined in a order, then prove that area of

such obtained parallelogram is half of the area of the given quadrilateral (Fig. 10.29).
 [Hint: Join BD and draw perpendicular from A on BD .]

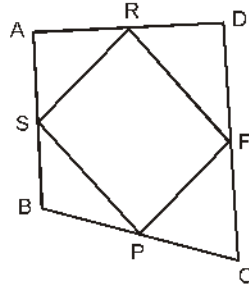


Fig. 10.29

8. A man walks 10 m in east and then 30 m in the north side. Find his distance from initial point.
9. A ladder is placed with a wall such that its lower end at a distance from wall is 7 m. If its other end at reached to the window height of 24 m. Find the length of the ladder.
10. Two poles of height 7 m and 12 m are standing on a plane ground. If distance between their feet is 12 m. Find the distance between upper ends of poles.
11. Find the length of altitude and area of an equilateral triangle whose length of side is a .
12. Find the length of diagonal of a square whose each side is 4 m.
13. If an equilateral triangle ABC , AD is perpendicular on BC then prove that $3AB^2 - 4AD^2$
14. O is any point inside a rectangle $ABCD$. Prove that $OB^2 + OD^2 = OA^2 + OC^2$
15. In an obtuse triangle ABC , $\angle C$ is an obtuse angle. $AD \perp BC$ meets BC at D on extending forward. Prove that :

$$AB^2 - AC^2 + BC^2 + 2BC \times CD$$

Important Points

1. If $\triangle ABC \cong \triangle PQR$, then $\text{ar}(\triangle ABC) = \text{ar}(\triangle PQR)$. Total area R of plane figure $ABCD$ is equal to the sum of area of triangular fields R_1 and R_2 or $\text{ar}(R) = \text{ar}(R_1) + \text{ar}(R_2)$ [Figure 10.30]

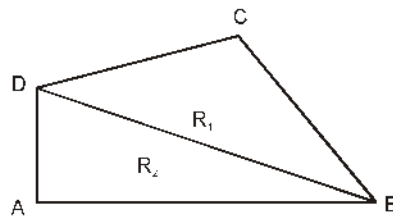


Fig. 10.30

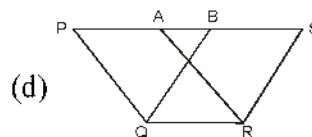
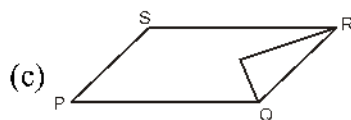
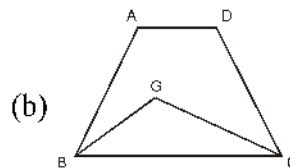
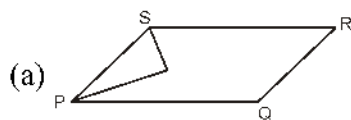
2. Area of two congruent figures are equal but converse of it is not true always.
3. A diagonal of a parallelogram, divides it into two triangles of equal area.
 - (i) Areas of parallelograms made on same base and between same parallel lines, are equal.
 - (ii) A parallelogram and a rectangle made on same base and between same parallel lines are equal in areas.
4. Parallelograms made on same base and between same parallel lines, are equal in areas.
5. Triangles made on same base and between same parallel lines are equal in areas.
6. Corresponding altitudes of triangles having equal bases and equal areas, are equal.
7. If a triangle and a parallelogram are made on same base and between same parallel lines, then area of triangle is half of the area of parallelogram.

Miscellaneous Exercise 10

Write correct answer in each of the following :

1. Median of a triangle divides it into two :

(a) triangles of equal areas	(b) congruent triangles
(c) right angled triangles	(d) isosceles triangles
2. In which of following figures, you find two polygons made on same base and between same parallel lines :



3. The figure, made by joining mid-points of adjacent sides 8 cm and 6 cm of a rectangle is :

(a) a rectangle of area 24 cm^2	(b) a square of area 25 cm^2
(c) a trapezium of area 24 cm^2	(d) a rhombus of area 24 cm^2

4. In Fig. 10.31, area of parallelogram ABCD is :

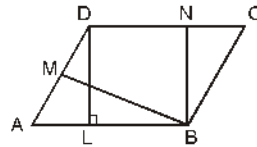


Fig. 10.31

- (a) $AB \times BM$ (b) $BC \times BN$
 (c) $DC \times DL$ (d) $AD \times DL$
5. In fig. 10.32, if parallelogram ABCD and rectangle ABEM are of equal areas, then :

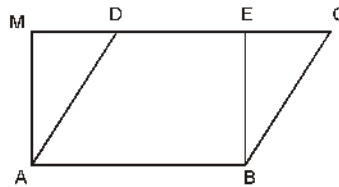


Fig. 10.32

- (a) perimeter of ABCD = Perimeter of ABEM
 (b) perimeter of ABCD < perimeter of ABEM
 (c) perimeter of ABCD > perimeter of ABEM
 (d) perimeter of ABCD = $\frac{1}{2}$ (Perimeter of ABEM)
6. Mid points of the sides of a triangle make a simple quadrilateral by taking with any vertex as fourth point, whose area is equal to :
- (a) $\frac{1}{2}$ ar (ABC) (b) $\frac{1}{3}$ ar (ABC)
 (c) $\frac{1}{4}$ ar (ABC) (d) ar (ABC)
7. Two parallelogram are on same base and between same parallel lines. Ratio of their areas is :
- (a) 1 : 2 (b) 1 : 1
 (c) 2 : 1 (d) 3 : 1
8. ABCD is a quadrilateral whose diagonal AC divides it into two parts of equal areas, then ABCD :
- (a) is a rectangle (b) is always a rhombus
 (c) is a parallelogram (d) is all of these

9. A triangle and a parallelogram are on same base and between same parallel lines, then ratio of areas of triangle with area of parallelogram is :
- (a) 1 : 3 (b) 1 : 2
(c) 3 : 1 (d) 1 : 4
10. ABCD is a trapezium whose sides AB = a cm and DC = b cm (Fig. 10.33) E and F are mid-points of non-parallel sides. Ratio of ar (ABFE) and ar (EFCD) is :

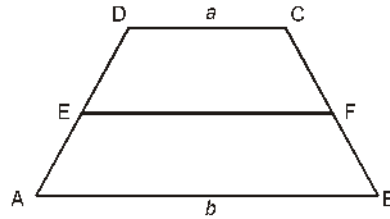


Fig. 10.33

- (a) a : b (b) $(3a + b) : (a + 3b)$
(c) $(a + 3b) : (3a + b)$ (d) $(2a + b) : (3a + b)$
11. If P is any point on median AD of $\triangle ABC$, then $\text{ar}(\text{ABP}) \neq \text{ar}(\text{ACP})$.
12. If in fig. 10.34, PQRS and EFRS are two parallelogram, then $\text{ar}(\text{MFR}) = \frac{1}{2} \text{ar}(\text{PQRS})$.

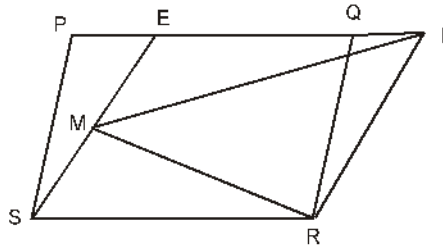


Fig. 10.34

13. In Fig. 10.35 PSDA is a parallelogram. Points Q and R on PS are taken such that $PQ = QR = RS$ and $PA \parallel QB \parallel RC$. Prove that $\text{ar}(\text{PQE}) = \text{ar}(\text{CFD})$

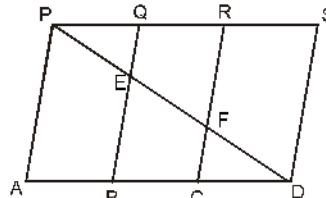


Fig. 10.35

14. X and Y are two points on side LN of $\triangle LMN$ such that $LX = XY = YN$. Through X a line is drawn parallel to LM, which meets MN at Z. (see Fig. 10.36). Prove that

: ar (LZY) = ar (MZYX)

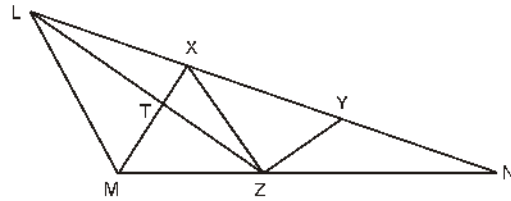


Fig. 10.36

15. Area of parallelogram ABCD is 90 cm^2 [Fig. 10.37]. Find the area of

(i) ar (ABEF) (ii) ar (ABD) (iii) ar (BEF)

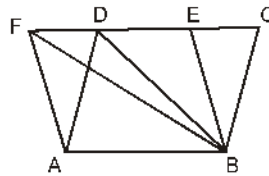


Fig. 10.37

16. In $\triangle ABC$, D is the mid-point of side AB and P is any point on side BC . If line segment $CQ \parallel PD$ meets side AB at Q (Fig. 10.38), then prove that :

$$\text{ar}(\triangle BPQ) = \frac{1}{2} \text{ar}(\triangle ABC)$$

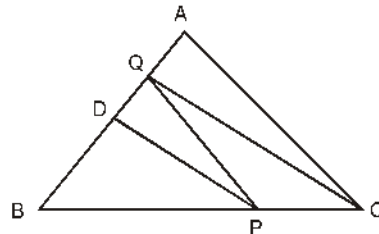


Fig. 10.38

17. ABCD is a square. E and F are respectively mid-points of sides BC and CD. If R is the mid-point of line segment EF (Fig. 10.39), then prove that :
 $\text{ar}(\triangle AER) = \text{ar}(\triangle AFR)$

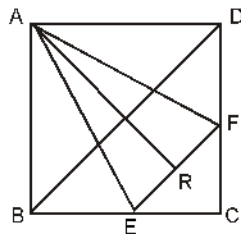


Fig. 10.39

18. O is any point on diagonal PR of a parallelogram PQRS (Fig. 10.40). Prove $\text{ar}(\text{PSO}) = \text{ar}(\text{PQO})$

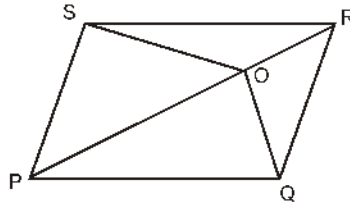


Fig. 10.40

19. ABCD is a parallelogram in which side BC is extended upto point E such that $CE = BC$ (Fig. 10.41). AE intersects side CD at F. If $\text{ar}(\text{DFB})$ is 3 cm^2 , then find the area of parallelogram ABCD.

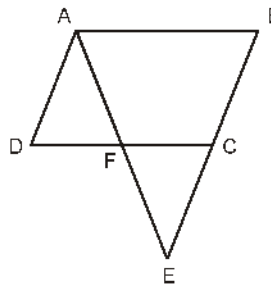


Fig. 10.41

20. Point E is taken on side BC of a parallelogram ABCD. AE and DC are extended so that they meet at F. Prove that $\text{ar}(\text{ADF}) = \text{ar}(\text{ABFC})$.
21. Diagonals of a parallelogram ABCD intersect at O. A line is drawn from O which meets AD at P and BC at Q. Show that PQ divides this parallelogram into two parts of equal areas.
22. Medians BE and CF of a $\triangle ABC$ intersect each other at point G. Prove that area of $\triangle GBC$ is equal to the area of quadrilateral AFGE.
23. In Fig. 10.42 $CD \parallel AE$ and $CY \parallel BA$. Prove that : $\text{ar}(\text{CBX}) = \text{ar}(\text{AXY})$

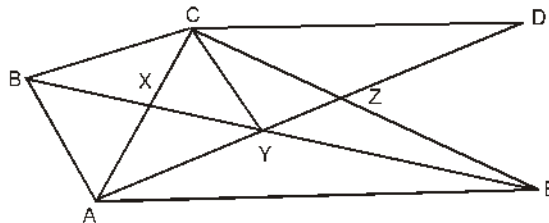


Fig. 10.42

24. ABCD is a trapezium in which $AB \parallel CD$, $CD = 30 \text{ cm}$ and $AB = 50 \text{ cm}$. If X and Y are mid-points of AD and BC respectively, then prove that :

$$\text{ar}(\text{DCYX}) = \frac{7}{9} \text{ar}(\text{XYBA}).$$

25. In $\triangle ABC$, L and M are point on sides AB and AC respectively such that $LM \parallel BC$. Prove that : $\text{ar}(\text{LOB}) = \text{ar}(\text{MOC})$ is LC and BM intersect at O.
26. In Fig. 10.43 ABCDE is a pentagon. BP drawn parallel to AC, meets extended DC at P and EQ drawn parallel to AD meets extended CD at Q. Prove that :
 $\text{ar}(\text{ABCDE}) = \text{ar}(\text{APQ}).$

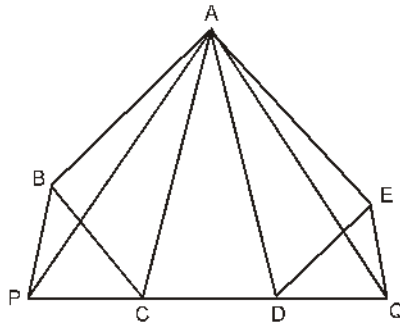


Fig. 10.43

27. If medians of a triangle ABC meet at point G, then prove that :

$$\text{ar}(\text{AGB}) = \text{ar}(\text{AGC}) = \text{ar}(\text{BGC}) = \frac{1}{3} \text{ar}(\text{ABC}).$$

28. In figure 10.44 X and Y are respectively the mid points of sides AC and AB. $QP \parallel BC$ and CYQ and BXP are straight lines. Show that : $\text{ar}(\text{ABP}) = \text{ar}(\text{ACQ})$

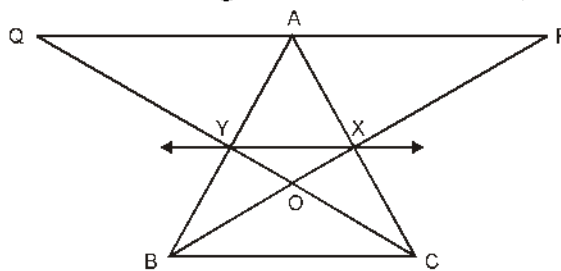


Fig. 10.44

29. In Fig. 10.45, ABCD and AEFD are two parallelogram. Prove that : $\text{ar}(\text{PEA}) = \text{ar}(\text{QFD}).$
 [Hint : Join PD]

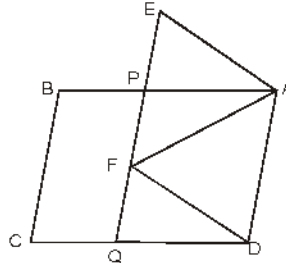


Fig. 10.44

Answers

EXERCISE 10.1

1. (i) DC and DC \parallel AB ; (iii) QR, QR \parallel PS ; (v) AD, AD \parallel QC

EXERCISE 10.2

1. 12.8 cm

EXERCISE 10.3

1. False $ar(A \times CD) = ar(ABCD) - ar(BC \times) = 48 - 12 = 36 \text{ cm}^2$

2. True $SR = \sqrt{13^2 - 5^2} = 12$ $ar(PAS) = \frac{1}{2} ar(PQRS) = 30 \text{ cm}^2$

3. False area of $\Delta QRS = 90 \text{ cm}^2$ and $ar(ASR) < ar(QRS)$

4. True $\frac{ar(BDE)}{ar(ABC)} = \frac{\sqrt{3}(BD)^2}{\sqrt{3}(BC)^2} = \frac{1}{4}$

5. False $ar(DPC) = \frac{1}{2}(ABCD) = ar(EFGD)$

6. $10\sqrt{10}$ m

9. 25 m

10. 13 m

11. $\frac{\sqrt{3}}{a}a, \frac{\sqrt{3}}{4}a^2$

12. $4\sqrt{2}$

Miscellaneous Exercise 10

- | | | | |
|------|-------|------|------|
| 1. A | 2. D | 3. D | 4. C |
| 5. C | 6. A | 7. D | 8. D |
| 9. B | 10. B | | |

11. False: $ar(ABD) = (ACD)$ and $ar(PBD) = ar(PCD) \therefore ar(ABP) = ar(ACP)$

12. True: $ar(PQRS) = ar(LI'RS) = 2ar(MI'R)$

15. (i) 90 cm^2 ; (ii) 45 cm^2 ; (iii) 45 cm^2

19. 13 cm^3

Area of Plane Figures

11.01 Introduction

We know that a closed figure bounded by three line segments is called a triangle and a closed figure bounded by four line segments is called quadrilateral. The part of the plane enclosed by a simple closed figure is called plane region.

In the previous classes we have studied about the area of plane figures such as triangles and quadrilaterals.

In this chapter, we will find the area of triangle, quadrilateral, rectangular paths and area of four walls, with the use of Heron's formula.

11.02 Area of Triangle

We know that, area of a triangle = $\frac{1}{2} \times \text{base} \times \text{height}$. Using this formula, we can find the base and height of a triangle as :

$$\text{Base of a triangle} = \frac{2 \times \text{Area}}{\text{Height}}$$

$$\text{Height of a triangle} = \frac{2 \times \text{Area}}{\text{Base}}$$

From the above formulae, height of triangle is compulsory to find the area of a triangle. If sides of a triangle are given and height is not given then the area of triangle can be found by using Heron's formula.

Heron's Formula

(1) If a, b, c are respective three sides of any triangle

then, area of the triangle = $\sqrt{s(s-a)(s-b)(s-c)}$

where, $s = \frac{a+b+c}{2} = \frac{\text{Perimeter of triangle}}{2} = \text{Semi-perimeter of the triangle}$

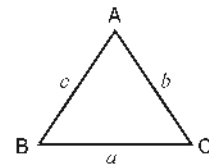


Fig. 11.01

(2) Area of an Isosceles Triangle

If the length of two equal sides of an isosceles triangle is 'a' and length of third side is b, then semi-perimeter of isosceles triangle will be :

$$s = \frac{a+a+b}{2} = \frac{2a+b}{2}$$

By Heron's formula

Area of isosceles triangle

$$\begin{aligned} &= \sqrt{\left(\frac{2a+b}{2}\right)\left(\frac{2a+b}{2}-a\right)\left[\frac{2a+b}{2}-b\right]\left[\frac{2a+b}{2}-a\right]} \\ &= \sqrt{\left(\frac{2a+b}{2}\right)\left(\frac{b}{2}\right)\left(\frac{2a-b}{2}\right)\left(\frac{b}{2}\right)} \\ &= \frac{b}{4}\sqrt{(2a+b)(2a-b)} \\ &= \frac{b}{4}\sqrt{4a^2-b^2} \end{aligned}$$

∴ Area of isosceles triangle = $\frac{b}{4}\sqrt{4a^2-b^2}$ sq. units.

(3) Area of an Equilateral Triangle

If a is the side of an equilateral triangle, then its semi-perimeter will be :

$$s = \frac{a+a+a}{2} = \frac{3a}{2}$$

∴ By Heron's formula

Area of equilateral triangle

$$\begin{aligned} &= \sqrt{\frac{3a}{2}\left[\frac{3a}{2}-a\right]\left[\frac{3a}{2}-a\right]\left[\frac{3a}{2}-a\right]} \\ &= \sqrt{\frac{3a}{2}\times\frac{a}{2}\times\frac{a}{2}\times\frac{a}{2}} \\ &= \frac{a^2\sqrt{3}}{4} \text{ sq. units} \end{aligned}$$

(4) Area of a Right-angled Triangle :

$$= \frac{1}{2} \times \text{base} \times \text{height}$$

$$= \frac{1}{2} \times (\text{Product of sides containing a right angle})$$

$$\therefore \text{Area of right angled triangle} = \frac{1}{2} \times a \times b \text{ sq. units.}$$

Illustrative Examples

Example 1. Find the area of a triangle whose sides are 8 cm, 15 cm and 17 cm.

Sol. Let $a = 8$ cm, $b = 15$ cm, $c = 17$ cm

$$\therefore \text{Semi perimeter} = s = \frac{a+b+c}{2} = \frac{8+15+17}{2} = \frac{40}{2} = 20 \text{ cm}$$

$$\begin{aligned} \therefore \text{Area of a triangle} &= \sqrt{s(s-a)(s-b)(s-c)} \\ &= \sqrt{20(20-8)(20-15)(20-17)} \text{ sq. cm} \\ &= \sqrt{20 \times 12 \times 5 \times 3} \text{ sq. cm} \\ &= \sqrt{100 \times 36} \text{ sq. cm} \\ &= 60 \text{ sq. cm} \end{aligned}$$

$$\therefore \text{Area of a triangle} = 60 \text{ sq. cm.}$$

Example 2. Each of two equal sides of an isoscles triangle is 7 cm and the third side is 6 cm. Find the area of the triangle.

Sol. Let $a = 7$ cm, $b = 6$ cm

$$\begin{aligned} \text{Area of triangle} &= \frac{b}{4} \sqrt{4a^2 - b^2} \\ &= \frac{6}{4} \sqrt{4 \times 49 - 36} \text{ sq. cm} \\ &= \frac{6}{4} \sqrt{196 - 36} \text{ sq. cm} \\ &= \frac{6}{4} \sqrt{160} \text{ sq. cm} \\ &= \frac{6}{4} \times 4 \times \sqrt{10} \text{ sq. cm} \\ &= 6\sqrt{10} \text{ sq. cm} \end{aligned}$$

Example 3. The base of a triangle is 24 cm and its height is 12 cm. Find the area of the triangle.

Sol. Base of the triangle = 24 cm and height = 12 cm

$$\begin{aligned}\text{Area of triangle} &= \frac{1}{2} \times \text{base} \times \text{height} \\ &= \frac{1}{2} \times 24 \times 12 \\ &= 144 \text{ cm}^2.\end{aligned}$$

Example 4. Find the area of an equilateral triangle whose one side is 8 cm.

Sol. Let $a = 8$ cm

$$\begin{aligned}\therefore \text{Area of equilateral triangle} &= \frac{\sqrt{3}}{4} a^2 \\ &= \frac{\sqrt{3}}{4} \times (8)^2 \\ &= \frac{\sqrt{3}}{4} \times 64 = 16\sqrt{3} \text{ cm}^2\end{aligned}$$

Example 5. Two sides of a triangle are 7 cm and 8 cm respectively. If the perimeter of the triangle is 24 cm, then find the area of the triangle.

Sol. Let $a = 7$ cm, $b = 8$ cm

$$\text{Perimeter} = a + b + c = 24 \text{ cm}$$

$$\therefore \text{Third side, } c = 24 - 7 - 8 = 9 \text{ cm}$$

$$\therefore s = \frac{a + b + c}{2} = \frac{24}{2} = 12 \text{ cm}$$

$$\begin{aligned}\therefore \text{Area of the triangle} &= \sqrt{s(s-a)(s-b)(s-c)} \\ &= \sqrt{12(12-7)(12-8)(12-9)} \text{ cm}^2 \\ &= \sqrt{12 \times 5 \times 4 \times 3} \\ &= \sqrt{12 \times 5 \times 4 \times 3} \\ &= 12\sqrt{5} \text{ cm}^2\end{aligned}$$

Exercise 11.1

1. If the base of a triangle is 20 cm and its height is 6 cm then find the area of the triangle.
2. A triangle whose sides are 15 cm, 25cm and 30cm respectively, then find the area of the triangle.
3. In an isosceles triangle each equal side is of 8 cm and third side is 4 cm, then find its area.
4. An equilateral triangle whose side is 20 cm, then find its area.
5. A triangle whose two sides are 8 cm and 15 cm and its perimeter is 40 cm then find the area of the triangle.
6. A triangular table whose sides are in the ratio 3 : 4 : 5 and perimeter is 36 m then find the area of the table.
7. A field whose shape is triangular. Its sides are 20m, 51m and 37m, then how many small beds of measure of $2 \times 3 \text{ m}^2$ can be plotted in the field?

11.03 Area of a Quadrilateral

A plane figure enclosed by four line segments is called quadrilateral. Any quadrilateral can be divided into two triangles by drawing its diagonals. In Fig 11.02 diagonal AC of a quadrilateral ABCD divides it into two triangles ABC and ACD. Therefore, area of quadrilateral ABCD will be the sum of the areas of both the triangles. From vertices B and D draw perpendicular BE and DF on diagonal AC respectively of quadrilateral ABCD.

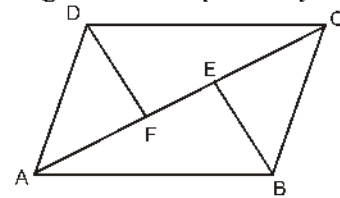


Fig. 11.02

$$\therefore \text{Area of } \Delta ABC = \frac{1}{2} \times AC \times BE$$

$$\text{and Area of } \Delta ACD = \frac{1}{2} \times AC \times DF$$

$$\therefore \text{Area of Quadrilateral ABCD} = \text{Area of } \Delta ABC + \text{Area of } \Delta ACD$$

$$= \frac{1}{2} \times AC \times BE + \frac{1}{2} \times AC \times DF$$

$$= \frac{1}{2} \times AC \times (BE + DF)$$

$$\therefore \text{Area of a quadrilateral} = \frac{1}{2} \times \text{diagonal} \times (\text{Sum of perpendiculars drawn on the diagonal})$$

the diagonal)

Area of a Parallelogram

A quadrilateral whose opposite sides are equal and parallel to each other is called parallelogram.

- (a) Area of parallelogram
= base \times height = $AB \times DE$

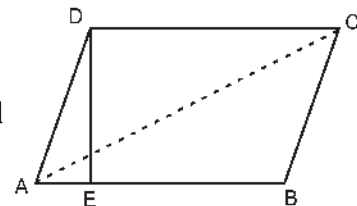


Fig. 11.03

(b) Diagonal AC divides parallelogram ABCD into two triangles ABC and ADC of equal areas.

$$\therefore \text{Area of parallelogram} = 2 \times (\text{Area of } \triangle ABC)$$

Illustrative Examples

Example 6. A diagonal of a quadrilateral is 10 cm and length of perpendiculars drawn of quadrilateral on diagonal from opposite vertices are 6 cm and 4 cm respectively, then find the area of the quadrilateral.

Sol. Area of quadrilateral

$$\begin{aligned} &= \frac{1}{2} \times \text{diagonal} \times (\text{Sum of perpendicular} \\ &\text{drawn on the diagonal}) \\ &= \frac{1}{2} \times 10 \times (6 + 4) \text{ sq. cm.} \\ &= \frac{1}{2} \times 10 \times 10 \text{ sq. cm.} \\ &= 50 \text{ sq. cm.} \end{aligned}$$

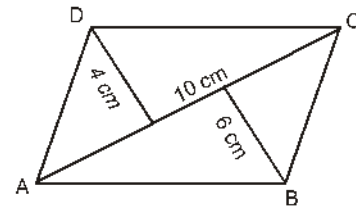


Fig. 11.04

Example 7. Find the area of a parallelogram whose height is 15 m and base is 10 m.

Sol. Base = 10 m

Height = 15 m

Area of parallelogram = base \times height

$$= 10 \times 15$$

$$= 150 \text{ m}^2$$

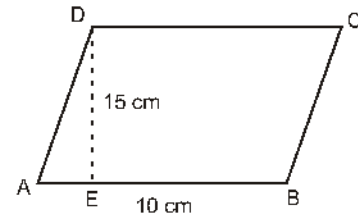


Fig. 11.05

Example 8. Find the area of a quadrilateral ABCD whose diagonal AC = 15 cm and sides AB = 7 cm, BC = 12 cm, CD = 12 cm and DA = 9 cm.

Sol. According to figure

Area of quadrilateral

$$ABCD = \text{ar}(\triangle ABC) + \text{ar}(\triangle ACD)$$

In $\triangle ABC$, AB = 7 cm, BC = 12 cm

and AC = 15 cm

$$\therefore \text{Semi-perimeter, } s = \frac{7 + 12 + 15}{2} = 17 \text{ cm}$$

$$\begin{aligned} \therefore \text{Area of } \triangle ABC &= \sqrt{17 \times (17 - 7) \times (17 - 12) \times (17 - 15)} \\ &= \sqrt{17 \times 10 \times 5 \times 2} \\ &= \sqrt{1700} \\ &= 10\sqrt{17} \end{aligned}$$

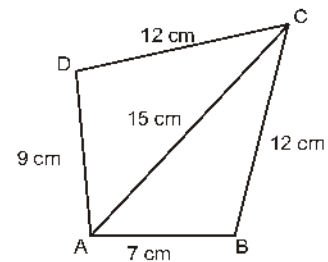


Fig. 11.06

$$= 10 \times 4.12 = 41.2 \text{ cm}^2$$

In $\triangle ACD$, $AC = 15 \text{ cm}$, $CD = 12 \text{ cm}$ and $DA = 9 \text{ cm}$

$$\therefore s = \frac{15+12+9}{2} = \frac{36}{2} = 18 \text{ cm}$$

$$\begin{aligned} \therefore \text{Area of } \triangle ACD &= \sqrt{18(18-15) \times (18-12)(18-9)} \\ &= \sqrt{18 \times 3 \times 6 \times 9} \\ &= 54 \text{ cm}^2 \end{aligned}$$

$$\begin{aligned} \therefore \text{Area of quadrilateral} &= (41.2 + 54.0) \\ &= 95.2 \text{ cm}^2 \end{aligned}$$

Example 9. Two adjacent sides of a parallelogram are 5 cm and 4 cm respectively and diagonal is 7 cm. Find the area of the parallelogram.

Sol. According to figure 11.07, area of quadrilateral $ABCD = 2 \times (\text{Area of } \triangle ABC)$

In $\triangle ABC$

$$AB = 5 \text{ cm}, BC = 4 \text{ cm}, AC = 7 \text{ cm}$$

$$\therefore s = \frac{5+4+7}{2} = \frac{16}{2} = 8 \text{ cm}$$

From Heron's formula, area of $\triangle ABC$

$$\begin{aligned} &= \sqrt{s(s-a)(s-b)(s-c)} \\ &= \sqrt{8(8-5)(8-4)(8-7)} \\ &= \sqrt{8 \times 3 \times 4 \times 1} \\ &= 4\sqrt{6} \text{ cm}^2 \end{aligned}$$

$$\begin{aligned} \therefore \text{Area of parallelogram} &= 2 \times \text{area of } \triangle ABC \\ &= 2 \times 4\sqrt{6} \text{ cm}^2 \\ &= 8\sqrt{6} \text{ cm}^2 \end{aligned}$$

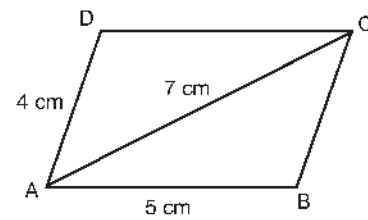


Fig. 11.07

Exercise 11.2

1. Find the area of the quadrilateral whose diagonal is 12 cm and lengths of perpendiculars drawn from opposite vertices to the diagonal are 7 cm and 8 cm respectively.
2. Area of a parallelogram shaped play ground is 2000 m^2 . If its base is 50m, then find the breadth of the play ground.
3. Find the area of a quadrilateral whose sides are $AB = 3 \text{ cm}$, $BC = 4 \text{ cm}$, $CD = 6 \text{ cm}$ and $DA = 5 \text{ cm}$ and diagonal is $AC = 5 \text{ cm}$.
4. Find the area of quadrilateral whose sides are 9 cm, 40 cm, 28 cm and 15 cm and angle between first two side is a right angle.

5. Find the area of a parallelogram whose two adjacent sides are 50 cm and 40 cm respectively and diagonal is 30 cm.
6. Find the area of a parallelogram whose a diagonal is 5.2 cm and the length of each perpendicular from the opposite vertices to the diagonal is 3.5 cm.
7. A plot of land has a shape of a parallelogram. It is to be covered by mud. Find the cost of spreading mud at the rate of ₹ 100 per square metre while the adjacent sides of the plot are 39 m and 25 m and the diagonal is 56 m.

11.04 Area of different Quadrilaterals

(1) Area of a cyclic Quadrilateral

A quadrilateral whose all four vertices are on the circumference of a circle is called cyclic quadrilateral. Opposite angles of a cyclic quadrilateral are supplementary from figure 11.08. ABCD is a cyclic quadrilateral whose sides are a, b, c and d respectively.

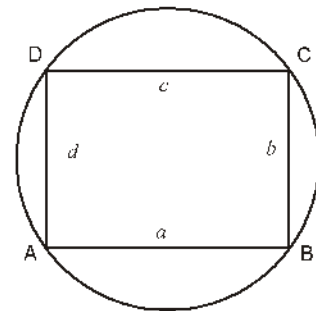


Fig. 11.08

$$\therefore \text{Semi perimeter } s = \frac{a+b+c+d}{2}$$

$$\therefore \text{Area of cyclic quadrilateral} = \sqrt{(s-a)(s-b)(s-c)(s-d)}$$

(2) Area of a Rhombus

A quadrilateral whose all four sides are equal and diagonals bisect each other at right angle, is called a rhombus.

$$\therefore \text{Area of Rhombus} = \frac{1}{2} \times (\text{Product of diagonals})$$

(3) Area of a Trapezium

A quadrilateral whose only two sides are parallel, is called trapezium. In fig. 11.09, ABCD is a trapezium whose sides AB and CD are parallel and distance between the parallel sides is DE. Here, DE is perpendicular on AB and DB is the diagonal.

$$\begin{aligned} \text{Therefore, area of trapezium ABCD} \\ &= \text{Area of } \triangle ABD + \text{Area of } \triangle BCD \\ &= \frac{1}{2} AB \times DE + \frac{1}{2} \times DC \times DE \\ &= \frac{1}{2} \times DE \times (AB + DC) \end{aligned}$$

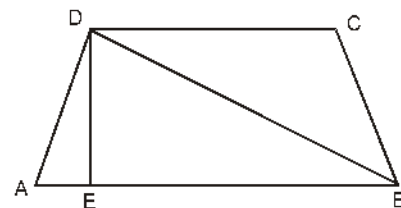


Fig. 11.09

$$\text{Area of trapezium} = \frac{1}{2} \times \text{sum of parallel sides} \times \text{distance between parallel sides}$$

Illustrative Examples

Example 10. Find the area of a cyclic quadrilateral whose sides are 36 cm, 77 cm, 75 cm

and 40 cm.

Sol. Let $a = 36$ cm, $b = 77$ cm, $c = 75$ cm and $d = 40$ cm
 \therefore Semi perimeter is a cyclic quadrilateral

$$\begin{aligned}
 s &= \frac{a+b+c+d}{2} \\
 &= \frac{36+77+75+40}{2} \\
 &= \frac{228}{2} \\
 &= 114 \text{ cm} \\
 \therefore \text{ Area of a cyclic quadrilateral} \\
 &= \sqrt{(s-a)(s-b)(s-c)(s-d)} \\
 &= \sqrt{(114-36)(114-77)(114-75)(114-40)} \\
 &= \sqrt{78 \times 37 \times 39 \times 74} \\
 &= \sqrt{2 \times 39 \times 39 \times 37 \times 37 \times 2} \\
 &= 2 \times 39 \times 37 \\
 &= 2886 \text{ cm}^2
 \end{aligned}$$

Example 11. The length of diagonals of a rhombus are 20 cm and 30 cm respectively, then find its area.

Sol. Area of rhombus = $\frac{1}{2} \times$ (product of length of diagonals)

$$\begin{aligned}
 &= \frac{1}{2} \times 20 \times 30 \\
 &= 300 \text{ cm}^2
 \end{aligned}$$

Example 12. Find the area of a trapezium whose parallel sides are 65 cm and 50 cm respectively and non-parallel sides are 20 cm and 25 cm respectively.

Sol. In fig. 11.10 ABCD is a trapezium whose parallel sides are $AB = 65$ cm and $DC = 50$ cm and non-parallel sides are $AD = 20$ cm and $BC = 25$ cm

Here $EB = AB - AE = AB - DC$
 $= (65 - 50)$ cm
 $= 15$ cm

\therefore Semi perimeter of $\triangle BEC$

$$s = \frac{15+20+25}{2} = 30 \text{ cm}$$

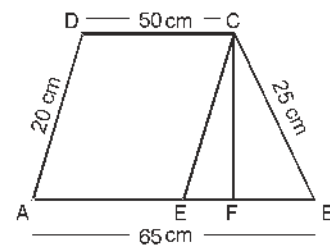


Fig. 11.10

$$\begin{aligned}
\therefore \text{Area of } \triangle BEC &= \sqrt{s(s-a)(s-b)(s-c)} \\
&= \sqrt{30(30-15)(30-20)(30-25)} \\
&= \sqrt{30 \times 15 \times 10 \times 5} \\
&= \sqrt{22500} \\
&= 150 \text{ cm}^2
\end{aligned}$$

$$\text{Height of } \triangle BEC = CF = \frac{2 \times \text{Area of } \triangle BCE}{\text{base } BE}$$

$$= \frac{2 \times 150}{15} = 20 \text{ cm}$$

$$\begin{aligned}
\therefore \text{Area of parallelogram AECD} &= AE \times CF \\
&= 50 \times 20 \\
&= 1000 \text{ cm}^2
\end{aligned}$$

$$\begin{aligned}
\therefore \text{Area of trapezium ABCD} &= \text{area of parallelogram AECD} + \text{area of } \triangle BEC \\
&= 1000 \text{ cm}^2 + 150 \text{ cm}^2 \\
&= 1150 \text{ cm}^2
\end{aligned}$$

Example 13. Find the area of trapezium whose parallel sides are 32 cm and 37 cm respectively and the distance between parallel sides is 20 cm.

Sol. Area of trapezium = $\frac{1}{2} \times (\text{Sum of parallel lines}) \times (\text{distance between parallel lines})$

$$= \frac{1}{2} \times (32 + 37) \times 20$$

$$= \frac{1}{2} \times 69 \times 20$$

$$= 690 \text{ cm}^2$$

Exercise 11.3

1. Sides of a cyclic quadrilateral shaped ground are 72 m, 154 m, 80 m and 150 m respectively. Find its area. Find the total expenditure of the cost of tiling is ₹ 5 per square metre.
2. Diagonals of a rhombus are 25 cm and 42 cm. Find its area and perimeter.
3. Perimeter of a rhombus is 40 m and length of its diagonal is 12 m, then find its area.
4. A trapezium shaped field whose parallel sides are 42 m and 30 m and the other sides are 18 m and 18 m. Find its area.
5. If the area of a trapezium is 350 cm² and lengths of its parallel sides are 26 cm and 44 cm, then find the distance between the parallel sides.

6. A table has a shape of a trapezium. The parallel side of the table are 8 m and 16 m. If area of the table is 108 m^2 then find the width (distance between parallel sides) of the table.

11.5 Area of a Rectangle, Square and four walls

1. In fig. 11.11 length of the rectangular is a and breadth is b , therefore perimeter of rectangle = total length of all four sides

$$= 2 \times (\text{length} + \text{breadth})$$

$$= 2(a + b)$$

$$\text{Area of rectangle} = \text{length} \times \text{breadth}$$

$$= a \times b$$

2. If side of a square is ' a ', then perimeter of square = $4 \times \text{side} = 4a$

$$\text{area of square} = (\text{side})^2 = a^2$$

3. Area of four walls of a room or rectangular tank = $2 \times (\text{length} + \text{breadth}) \times \text{height}$

$$\therefore \text{Area of four walls of a room} = \text{Perimeter} \times \text{height}$$

$$\text{Perimeter of floor of a room} = 2 \times (\text{length} + \text{breadth})$$

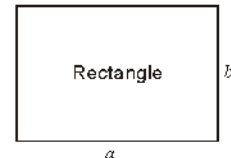


Fig. 11.11

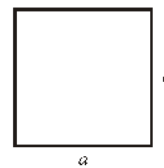


Fig. 11.12



Fig. 11.13

Illustrative Examples

Example 14. The length of a rectangular ground is 75 m and breadth is 45 m, then find the area and perimeter of the ground.

Sol. Length = 75 m and breadth = 45 m

$$\therefore \text{Area of a rectangular ground}$$

$$= \text{length} \times \text{breadth}$$

$$= 75 \times 45$$

$$= 3375 \text{ m}^2$$

$$\text{Perimeter of rectangular ground} = 2(\text{length} + \text{breadth})$$

$$= 2(75 + 45)$$

$$= 2 \times 120$$

$$= 240 \text{ m}$$

Example 15. Distance covered to take 5 rounds of a rectangular field is 600 m. If the breadth of the field is 25 m, then find its length.

Sol. Perimeter of field = $\frac{600}{5} \text{ m}$

$$= 120 \text{ m}$$

$$\text{or } 2(a + b) = 120$$

$$\text{or } 2(a + 25) = 120$$

$$\text{or } a + 25 = 60$$

$$\text{or } a = 60 - 25$$

$$\text{or } a = 35 \text{ m}$$

$$\therefore \text{Length of field} = 35 \text{ m.}$$

Example 16. Length of a square field is 120 m. Find its perimeter and area.

Sol. Side of a square field = 120 m

$$\therefore \text{Perimeter of field} = 4 \times \text{side}$$

$$= 4 \times 120$$

$$= 480 \text{ m}$$

$$\text{Area of field} = (\text{side})^2$$

$$= (120)^2$$

$$= 14400 \text{ m}^2$$

Example 17. Length of a rectangular field is 35 m and breadth is 20m. It is to be tiled. If the measures of a tile is 7 cm \times 5cm then how many tiles will be required?

Sol. Length of rectangular field = 35 m

Breadth of rectangular field = 20 m

$$\therefore \text{Area of field} = 35 \times 20$$

$$= 700 \text{ m}^2$$

Length of tile = 7 cm = 0.07 m

Breadth of tile = 5 cm = 0.05 m

$$\therefore \text{Area of a tile} = 0.07 \times 0.05$$

$$= 0.0035 \text{ m}^2$$

\therefore Required number of tiles

$$= \frac{\text{area of field}}{\text{area of tile}}$$

$$= \frac{700}{0.0035}$$

$$= \frac{7000000}{35}$$

$$= 200000.$$

Example 18. Area of a square ground is 625 m². There is 2.5 m wide path outside around it. Find the area of the path. If 50 cm long square pieces are to be paved in the path, then find how many square pieces are required.

Sol. As per question

Area of square ground = 625 m²

$$\therefore \text{Length of the ground} = \sqrt{\text{Area}} = \sqrt{625} = 25 \text{ m}$$

In Fig. 11.14, there is 2.5 m path surrounding the ground outside

$$\therefore \text{Outer length with path} = 25 + 2.5 + 2.5 = 30 \text{ m}$$

$$\therefore \text{Area of ground with path} = (30)^2 = 900 \text{ m}^2$$

$$\begin{aligned} \therefore \text{Area of path} &= \text{Area of ground with path} \\ &\quad - \text{Area of ground} \\ &= 900 - 625 \\ &= 275 \text{ m}^2 \end{aligned}$$

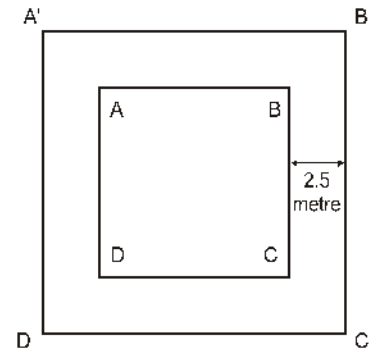


Fig. 11.14

Length of the square piece to be paved in path = 50 cm = 0.50 m

$$\therefore \text{Area of a square piece} = 0.50 \times 0.50 = 0.2500 \text{ m}^2$$

\therefore Required number of square pieces

$$= \frac{\text{area of path}}{\text{area of a square piece}}$$

$$= \frac{275}{0.25} = \frac{27500}{25}$$

$$= 1100$$

Example 19. A rectangular ground is 40 m long and 30 m wide. A 3m wide path, parallel to its length and width, is made in the middle of the ground. find the expenditure of paving the concrete on this path at the rate of Rs 200 per square meter.

Sol. As per question

Length of ground = 40 m

Width of ground = 30 m

$$\therefore \text{Area of ground} = 40 \times 30 = 1200 \text{ m}^2$$

In fig. 11.15

Area of path IJKL parallel to length

$$40 \times 3 = 120 \text{ m}^2$$

Area of path EFGH parallel to breadth = $30 \times 3 = 90 \text{ m}^2$

Area of shaded part

$$PQRS = 3 \times 3 = 9 \text{ m}^2$$

$$\text{Area of total path} = 120 + 90 - 9 = 201 \text{ m}^2$$

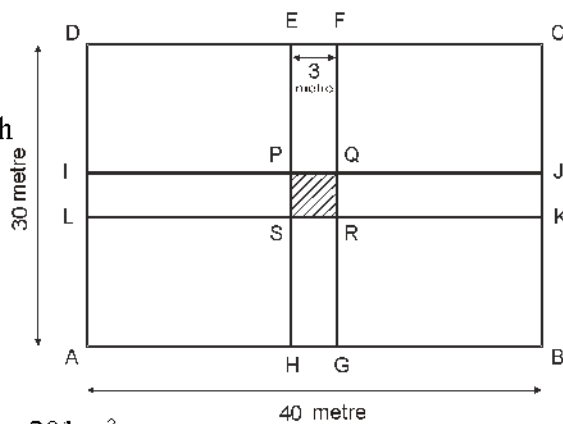


Fig. 11.15

$$\begin{aligned}
 \therefore \text{Expenditure at the rate of ₹ 200 per square metre of paving concrete} \\
 &= ₹ 200 \times 201 \\
 &= ₹ 40200 \\
 \therefore \text{Expenditure} &= ₹ 40200
 \end{aligned}$$

Example 20. Find the area of a field based on given measure in figure.

Sol. We can divide the given figure in the form of rectangular, therefore

$$\begin{aligned}
 \text{(i) Area of rectangle GFEP} &= EF \times GF \\
 &= 5 \times 2 \\
 &= 10 \text{ m}^2
 \end{aligned}$$

$$\begin{aligned}
 \text{(ii) Area of rectangle BCDQ} &= QB \times BC \\
 &= 5 \times 3 \\
 &= 15 \text{ m}^2
 \end{aligned}$$

$$\begin{aligned}
 \text{(iii) Area of rectangle HPQA} &= HA \times HP \\
 &= 25 \times 10 \\
 &= 250 \text{ m}^2
 \end{aligned}$$

$$\begin{aligned}
 \therefore \text{Area of given field} &= \text{area GFEP} + \\
 &\text{area BCDQ} + \text{area HPQA} \\
 &= (10 + 15 + 250) \text{ m}^2 \\
 &= 275 \text{ m}^2.
 \end{aligned}$$

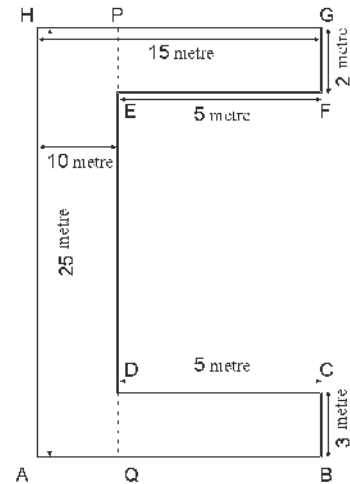


Fig. 11.16

Example 21. A rectangular room is 7m long, 6 m wide and 3.5m high. If there are 3 doors of size $2\text{m} \times 1.5\text{m}$ and 4 windows of size $1.25\text{m} \times 1\text{m}$ then find the expenditure of colouring of the walls at the rate of ₹ 4 and 45 paise per square metre.

Sol. Length of room = 7 m

Breadth of room = 6 m

and height of the room = 3.5m

$$\begin{aligned}
 \therefore \text{Area of four walls of room} &= 2(l + b) \times h \\
 &= 2(7 + 6) \times 3.5 \text{ m}^2 \\
 &= 91 \text{ m}^2
 \end{aligned}$$

$$\text{Area of one door} = 2 \times 1.5 = 3 \text{ m}^2$$

$$\begin{aligned}
 \text{Area of 3 door} &= 3 \times 3 \text{ m}^2 \\
 &= 9 \text{ m}^2
 \end{aligned}$$

$$\text{Area of one window} = 1.25 \times 1 \text{ m}^2$$

$$\begin{aligned}
 \text{Area of 4 windows} &= 1.25 \times 4 \text{ m}^2 \\
 &= 5.00 \text{ m}^2
 \end{aligned}$$

$$\begin{aligned}
 \therefore \text{Remaining area of four walls excluding doors and windows} \\
 &= [91 - (9+5)] \\
 &= (91 - 9 - 5)
 \end{aligned}$$

$$\begin{aligned}
 &= (91 - 14) \\
 &= 77 \text{ m}^2 \\
 \therefore \text{Expenditure of colouring of the walls} \\
 &= ₹ 77 \times 4.50 \\
 &= ₹ 346.50
 \end{aligned}$$

Example 22. A rectangular tank of water is 12 m long, 6 m wide and 2 m deep. Find the cost of repairing its four walls and floor at the rate of ₹ 15 per square metre.

Sol. Length of rectangular tank = 12 m
 Breadth of rectangular tank = 6 m
 Depth of rectangular tank = 2 m

$$\begin{aligned}
 \therefore \text{Area of four walls of tank} \\
 &= 2 \times (\text{length} + \text{breadth}) \times \text{depth} \\
 &= 2 \times (12 + 6) \times 2 \\
 &= 72 \text{ m}^2 \\
 \text{Area of floor of tank} &= \text{length} \times \text{breadth} \\
 &= 12 \times 6 \text{ m}^2 \\
 &= 72 \text{ m}^2 \\
 \therefore \text{Total area of four walls and floor} \\
 &= (72 + 72) \text{ m}^2 \\
 &= 144 \text{ m}^2 \\
 \therefore \text{Cost of repairing the four walls and floor} &= 144 \times 15 \\
 &= ₹ 2160
 \end{aligned}$$

Exercise - 11.4

1. Find the area and perimeter of rectangles of following measure:
 - (i) length = 9.5 m, breadth = 7.5 m
 - (ii) length = 125 m, breadth = 75 m
 - (iii) length = 12.5 cm, breadth = 7.5 cm
2. Find the area and perimeter of the square of following measure of a side :
 - (i) 5.3 m (ii) 8.5 m (iii) 9.6 m
3. After running 4 rounds to a square field, a distance covered is 16 km. Find the length and area of the ground.
4. A rectangular field is 75 m long and 45 m wide. How many beds 5 m long and 3 m wide can be made in it?
5. The length of a room is 10 m and breadth is 5 m. How many square pieces of area 50 cm² are required on its floor? Find the expenditure of paving the square pieces on the floor at the rate of ₹ 20 per piece.

6. Area of a square ground is 2025 m^2 . There is 3.5 m wide path outside around it then find the area of the path.
7. A rectangular room is 40 m long and 25 m wide. There is a road 2.5 m wide around it. Tiles of size $50 \text{ cm} \times 70 \text{ cm}$ are to be paved on it. Find the number of pieces.
8. A rectangular ground is 60 m long and 40 m wide. There is a 4 m wide path parallel to its length and width in side the ground. find the expenditure of spreading mud on the path at the rate of ₹ 100 per square metre.
9. Find the area and perimeter of the given diagram

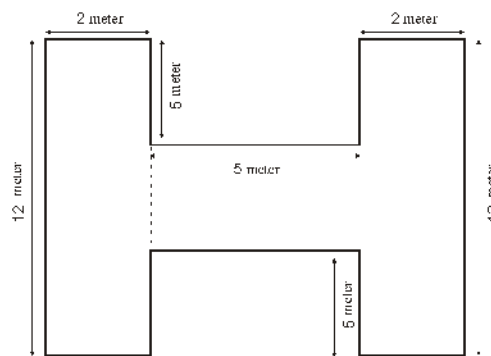


Fig. 11.17

10. The length, breadth and height of a room are respectively 15.35 m , 4.65 m and 6.50 m . If there are 4 doors of size $1.5 \text{ m} \times 1.3 \text{ m}$ and 3 windows of size $1.5 \text{ m} \times 1.2 \text{ m}$, then find the expenditure on whitewashing the room at the rate of ₹ 4.30 per square metre.
11. A water tank is 10 m long, 8 m wide and 2 m deep. Find the expenditure of repairing its four walls and floor at the rate of ₹ 15 per square metre.

Important Points

1. Area of triangle = $\frac{1}{2} \times \text{base} \times \text{height}$
2. Area of triangle by Heron's formula
= $\sqrt{s(s-a)(s-b)(s-c)}$ sq. units
where $s = \frac{a+b+c}{2}$
= $\frac{\text{perimeter of triangle}}{2}$
= semi perimeter, where a, b, c are side of a triangle.
3. Area of an isosceles triangle = $\frac{b}{4} \sqrt{4a^2 - b^2}$ sq. units
Where a is one of the equal sides of a triangle and b is other side.
4. Area of an equilateral triangle = $\frac{a^2 \sqrt{3}}{4}$ sq. units, where a is the side of triangle.
5. Area of right angled triangle = $\frac{1}{2} \times (\text{product of sides containing a right angle})$
= $\frac{1}{2} \times \text{base} \times \text{height}$
6. Area of quadrilateral = $\frac{1}{2} \times \text{diagonal} \times (\text{Sum of perpendiculars drawn on diagonal})$
7. Area of parallelogram = base \times height
8. Area of a cyclic quadrilateral = $\sqrt{(s-a)(s-b)(s-c)(s-d)}$
where a, b, c, d are sides and $s = \frac{a+b+c+d}{2}$
9. Area of rhombus = $\frac{1}{2} \times \text{Products of lengths of diagonals}$
10. Area of trapezium = $\frac{1}{2} \times (\text{Sum of parallel sides}) \times \text{distance between two parallel sides.}$
11. Area of rectangle = length \times breadth
12. Area of a square = (side)²
13. Area of four walls = 2 \times (length + breadth) \times height

Miscellaneous Exercise - II

- If the side of an equilateral triangle is 8 cm, then area of the triangle will be :
(A) $16\sqrt{3}$ cm² (B) $8\sqrt{3}$ cm²
(C) $64\sqrt{3}$ cm² (D) $4\sqrt{3}$ cm² ()
- The sides of a triangle are 40 cm, 70 cm and 90 cm, area of triangle will be :
(A) $600\sqrt{5}$ cm² (B) $500\sqrt{6}$ cm²
(C) $482\sqrt{5}$ cm² (D) $60\sqrt{5}$ cm² ()
- Each equal side of an isosceles triangle is 6 cm and the other sides is 8 cm, then area of triangle will be :
(A) $8\sqrt{5}$ cm² (B) $5\sqrt{8}$ cm²
(C) $3\sqrt{55}$ cm² (D) $3\sqrt{8}$ cm² ()
- The perimeter of an equilateral triangle is 60 cm, then its area will be :
(A) $400\sqrt{3}$ cm² (B) $100\sqrt{3}$ cm²
(C) $50\sqrt{3}$ cm² (D) $200\sqrt{3}$ cm² ()
- Area of a right- triangle is 6 cm² and its base is 9 cm, then the length of perpendicular will be :
(A) 8cm (B) 4 cm (C) 16 cm (D) 32 cm ()
- If the side of a square is 10 cm, then its perimeter will be :
(A) 20 cm (B) 10 cm (C) 40 cm (D) 30 cm ()
- If the diagonals of a rhombus are 8 cm and 6 cm, then its area will be :
(A) 48 cm² (B) 24 cm² (C) 12 cm² (D) 96 cm² ()
- If the perimeter of room is 40 m and height is 4m, then area of its four walls will be:
(A) 40 m² (B) 80 m² (C) 120 m² (D) 160 m² ()
- What will be the length of side of an equilateral triangle whose area is $9\sqrt{3}$ cm²

- Write the formula to find the area of cyclic quadrilateral.

- Area of a square is 144 m² are, then write its perimeter.

- If the base of a parallelogram is 18m and area is 174.60 m², then write its height.

- Write the area of a quadrilateral whose diagonal is 6 cm and the sum of internal perpendiculars is 12 cm.

14. The ratio of the sides of a triangle is 25 : 17 : 12 and perimeter is 540 m, then find the area of triangle.
-
15. Area of an isosceles triangle is 12 cm^2 Find its base, if the length of its equal sides is 5 cm.
-
16. The perimeter of any triangle is 40 cm. If its two sides are 8 cm and 15 cm respectively, then find its area and also find the length of perpendicular, drawn from a vertex to the longest side.
-
17. The perimeter of a rhombus is 146 cm and length of a diagonal is 55 cm, then find the area of the rhombus.
18. There is sufficient grass for eating in a rhombus shaped grass field for 18 cows. If each side of this rhombus is 30 m and longest diagonal is 48 m, how much area of this field will be available for each cow to eat grass?
19. An umbrella is made by joining 10 triangular pieces of two different coloured cloths. The measures of each piece is 20 cm, 50 cm and 50 cm. How much cloth is used in umbrella.
20. Ratio of two parallel sides of a trapezium is 16 : 5. Which is cut out of a rectangle whose sides are 63 m and 5m. If the area of a trapezium is $\frac{4}{15}$ th part of the area of a rectangle trapezium.
21. Area of a rectangular field is 4356 m^2 and length of the ground is 99m. There is a 4.5 m wide road in the middle of the ground parallel to length and breadth. How many square pieces of side 1.50 m will be required to cover the road.
22. A room is 8m50 cm long and 6m 50cm wide. How long a carpet 25 cm wide required to cover its floor? Find the cost of carpet at the rate of Rs. 20 per meter.

Answer

Exercise 11.1

1. 60 square cm
2. $50\sqrt{14}$ cm²
3. $4\sqrt{15}$ square cm
4. $100\sqrt{3}$ cm²
5. 60 cm²
6. 54 m²
7. 51 beds

Exercise 11.2

1. 90 square cm
2. 40 m
3. 18 square cm
4. 306 square cm
5. 1200 square cm
6. 18.2 cm²
7. ₹ 84000

Exercise 11.3

1. 11544 m², ₹ 57720
2. 525 cm², 97.72 cm
3. 96 m²
4. 610.9 m²
5. 10 cm
6. 9 m

Exercise 11.4

1. (i) 71.25 m², 34 m
(ii) 93.75 cm², 40 cm
2. (i) 28.09 m², 21.2 m
(ii) 72.25 m², 34 m
(iii) 92.16 m², 38.4 m
3. 250 m, 62500 m²
4. 225 beds
5. 1000 square piece, ₹ 20000
6. 679 m²
7. 1000 square piece
8. ₹ 38400
9. 58 m²
10. ₹ 839.12
11. ₹ 2280

Miscellaneous Exercise - 11

- | | |
|---|------------------------------------|
| 1. (A) | 2. (A) |
| 3. (A) | 4. (B) |
| 5. (A) | 6. (C) |
| 7. (B) | 8. (D) |
| 9. 6 cm | 11. 480 m |
| 12. 9.7 m | 13. 36 cm ² |
| 14. 9000 m ² | 15. 8 cm |
| 16. 60 cm ² , $7\frac{1}{17}$ cm | 17. 1320 cm ² |
| 18. 48 m ² | 19. $2000\sqrt{6}$ cm ² |
| 20. 25.6 m, 8 m | 21. 277 square pieces |
| 22. 221 m, ₹ 4420 | |

Surface Area and Volume of Cube and Cuboid

12.01 Introduction

In the previous chapters, we have learnt to find the areas of some plane figures like triangle, quadrilateral and rectangle. In this chapter, we will learn to find the surface area and volume of the solid figures like brick, match box, chalk box and iron box etc.

The shape and volume of all these objects are definite. These figures are three dimensional solid figures, it means these figures have length, breadth and height. The surface area of any solid figures means sum of areas of all surfaces. The space occupied by any solid is called volume. The area is measured in square unit and volume is measured in cubic unit.

12.02 Cuboid

A bundle of rectangular sheet of paper placed one by one to a height, takes a shape that's called a cuboid. Each face of it is rectangular therefore a cuboid is also called a rectangular solid. There are 6 faces, 8 vertices and 12 edges in cuboid. For example room, box and brick etc. Opposite faces of a cuboid are equal to each other. To find the surface area of a cuboid, we have to find the area of its 6 faces.

Area of the face ABCD = Area of the face A' B' C' D' = length \times breadth

Area of the face ADD'A' = Area of the face BCC'B' = height \times breadth

Area of the face ABB'A' = Area of the face DCC'B' = length \times height

\therefore Total surface area of cuboid

= 2 [length \times breadth + height \times breadth + length \times height] square units

= 2 [$l \times b + b \times h + h \times l$]

= 2 [$l b + bh + hl$]

12.03 Cube

If each face of a cuboid is of the shape of a square means its length, breadth and

height all are equal, then it is called a cube.

Every face of a cube is square.

$$\therefore \text{Surface area of a face of the cube} = (\text{side})^2$$

$$\therefore \text{Surface area of the 6 faces} = 6 (\text{side})^2$$

$$\Rightarrow \text{Total surface area of a cube} = 6 (\text{side})^2 \text{ square unit}$$

12.04 Diagonal of Cube and Cuboid

A line joining the opposite vertices of two parallel faces of either cube or cuboid is called diagonal. There are four diagonal in a cube or cuboid.

Length of diagonal of a cuboid =

$$= \sqrt{(\text{length})^2 + (\text{breadth})^2 + (\text{height})^2}$$

$$= \sqrt{\ell^2 + b^2 + h^2} \text{ unit}$$

$$\text{Length of diagonal of a cube} = \sqrt{3} (\text{length of side}) \text{ unit}$$

Illustrative Examples

Example 1. The length, breadth and height of a room are 5 m, 3.5m and 4 m respectively. Find the total surface area of the room.

Solution :

Length of the room = 5 m
Breadth of the room = 3.5 m
Height of the room = 4m

$$\begin{aligned} \therefore \text{Total surface area of the room} &= 2 [l \times b + b \times h + h \times l] \\ &= 2 [5 \times 3.5 + 3.5 \times 4 + 4 \times 5] \\ &= 2 [51.5] \text{ cm}^2 \\ &= 103 \text{ cm}^2 \end{aligned}$$

Example 2. Find the maximum length of a rod that can be placed inside a 12 m long, 9 m wide and 8 m high room.

Solution :

Length of room = 12 m
Breadth of room = 9 m
Height of room = 8 m

$$\therefore \text{Maximum length of rod, that can be placed inside the room will be equal to its diagonal.}$$

so, length of rod = diagonal of room = $\sqrt{(\ell^2) + (b^2) + (h^2)}$

$$\begin{aligned}
&= \sqrt{(12)^2 + (9)^2 + (8)^2} \text{ m} \\
&= \sqrt{144 + 81 + 64} \text{ m} \\
&= \sqrt{289} \text{ m} \\
&= 17 \text{ m}
\end{aligned}$$

Example 3. The side of a cube is 12 cm. Find the total surface area of cube.

Solution :

$$\begin{aligned}
\text{Total surface area of a cube} &= 6 (\text{side})^2 \\
&= 6 \times (12)^2 \\
&= 6 \times 144 \text{ cm}^2 \\
&= 864 \text{ cm}^2
\end{aligned}$$

Example 4. A box is 1 m long, 60 cm wide and 40 cm high. Find the expenditure of colouring its all outer side without its bottom at the rate of Rs. 20 per square meter.

Solution :

$$\begin{aligned}
\text{Length of box} &= 1\text{m} = 100 \text{ cm} \\
\text{Breadth of box} &= 60 \text{ cm} \\
\text{Height of box} &= 40 \text{ cm} \\
\text{Total surface area of box except bottom} \\
&= (l \times b) + 2 [b \times h + h \times l] \\
&= (100 \times 60) + 2 [60 \times 40 + 40 \times 100] \\
&= 6000 + 2 [2400 + 4000] \\
&= 6000 + 2 [6400] \\
&= 6000 + 12800 \text{ cm}^2 \\
&= 18800 \text{ cm}^2 \\
&= 1.88 \text{ m}^2
\end{aligned}$$

Rate of colouring = Rs. 20 per square meter

Thus, total expenditure on colouring

$$= \text{Rs. } 1.88 \times 20$$

$$= \text{Rs. } 37.60$$

$$= \text{Rs. } 37.60$$

Example 5. The length, breadth and height of a box of tin is 15 cm, 10 cm and 8 cm respectively. Such 20 boxes are to be made. Find the area of tin used in it. If the cost of tin is Rs. 50 per square meter, then find the total cost of tin used in boxes.

Solution :

$$\begin{aligned}
\text{Length of a box} &= 15 \text{ cm} \\
\text{Breadth of a box} &= 10 \text{ cm} \\
\text{Height of box} &= 8 \text{ cm}
\end{aligned}$$

$$\begin{aligned}
\text{Total surface area of a box} &= 2 [l \times b + b \times h + h \times l] \\
&= 2 [15 \times 10 + 10 \times 8 + 8 \times 15] \text{cm}^2 \\
&= 2 [150 + 80 + 120] \text{cm}^2 \\
&= 2 [350] \text{cm}^2 \\
&= 700 \text{cm}^2
\end{aligned}$$

Area of tin used in 20 such boxes

$$\begin{aligned}
&= 20 \times 700 \text{cm}^2 \\
&= 14000 \text{cm}^2 \\
&= 1.4 \text{m}^2
\end{aligned}$$

\therefore Cost of tin used in 20 boxes = Rs. 1.4×50 = Rs. 70

Exercise 12.1

1. The length, breadth and height of a wooden box is 1m, 60m and 40 cm respectively. find the outer surface area of the wooden box.
2. The measure of a box are 40 cm, 30 cm and 20 cm respectively. How much square cm cloth is required to make the cover of box?
3. The length of a room is 5m, breadth is 3.5 m and height is 4 cm. Find the total expenditure of white washing on the four walls and roof at the rate of ₹ 15 per square meter.
4. The side of a cubical chalk box is 4 cm. Find the total surface area of chalk box and length of its diagonal.
5. Total surface area of a cube is 1014m^2 . find the length of the side of cube.
6. A wooden box with lid is made of 2.5 cm thick wood. Inner length, breadth and height of box are 1 m, 65 cm and 55 cm. Find the total expenditure of colouring its outer surface area at the rate of ₹ 15 per square meter.
7. Each face of a cube is 100cm^2 . If cube is cut by a plane, parallel to its base, in two equal parts, then find the total surface area of each equal part.
8. A box without lid, is made of 3 cm thick wood whose external length, breadth and height are 146 cm, 116 cm and 83 cm respectively. find total surface area of inner side.

12.05 Volume of Cube and Cuboid

We know that each solid object occupies some space. Measure of this occupied space is called volume of that object. If object is hollow, then it can be filled with air or liquid. This liquid takes shape of that object (pot). In this condition, the amount of liquid filled inside the pot, is called the capacity of the pot.

Capacity of a pot, is the volume of the liquid that can be filled inside the pot. Its unit is cubic unit.

Volume of cube and cuboid can be find by following formulae.

Volume of cuboid = (length \times breadth \times height) cubic unit

Volume of cube = (side)³ cubic unit

Illustrative Examples

Example 6. Volume of a cube is 216 cubic meter. Find its side

Solution : Let side of cube is x meter

$$\therefore \text{Volume of cube} = (\text{side})^3$$

$$\Rightarrow 216 = x^3$$

$$\Rightarrow x^3 = 6 \times 6 \times 6 = (6)^3$$

$$\Rightarrow x = 6$$

$$\therefore \text{side of cube} = 6 \text{ cm}$$

Example 7. A water tank is 6m long, 5 m wide and 4.5 m deep. How many litres of water can be filled in it. (1 litre = 1000 cubic cm)

Solution : Length of water tank = 6 m = 600 cm

Breadth of water tank = 5 m = 500 cm

Height of water tank = 4.5 m = 450 cm

$$\therefore \text{volume of tank} = 600 \times 500 \times 450 \text{ cm}^3$$

$$= \frac{600 \times 500 \times 450}{1000} \text{ litres}$$

$$= 135000 \text{ litres}$$

$$\therefore 13500 \text{ litres water can be filled in the tank.}$$

Example 8. The length of a box is 30 cm, breadth 20 cm and height is 6 cm. How many cassetts of size 10 cm \times 5 cm \times 2 cm can be placed inside it.

Solution : Length of box = 30 cm

Breadth = 20 cm

Height = 6 cm

$$\therefore \text{Volume of box} = 30 \times 20 \times 6 \text{ cm}^3$$

$$\text{Volume of a casset} = 10 \times 5 \times 2 = 100 \text{ cm}^3$$

$$\therefore \text{Number of cassetts} = \frac{\text{Volume of box}}{\text{Volume of a casset}}$$

$$= \frac{30 \times 20 \times 6}{10 \times 5 \times 2}$$

$$= 36$$

$$\therefore \text{Number of cassetts} = 36$$

Example 9. A wooden box is made of 1 cm thick wood. If its outer length, breadth and height are 75 cm, 60 cm and 40 cm, then find the volume of the wood used in the box.

Solution :

$$\begin{aligned} \text{Length of the box} &= 75 \text{ cm} \\ \text{Breadth} &= 60 \text{ cm} \\ \text{Height} &= 40 \text{ cm} \\ \text{Outer volume of the box} &= 75 \times 60 \times 40 \\ &= 180000 \text{ cm}^3 \\ \text{Thickness of wood} &= 1 \text{ cm} \\ \therefore \text{Inner length of the box} &= (75 - 2 \times 1) \text{ cm} \\ &= 73 \text{ cm} \\ \text{Inner breadth of the box} &= (60 - 2 \times 1) \text{ cm} \\ &= 58 \text{ cm} \\ \text{Inner height of the box} &= (40 - 2 \times 1) \text{ cm} \\ &= 38 \text{ cm} \\ \therefore \text{Inner volume of the box} &= 73 \times 58 \times 38 \\ &= 160892 \text{ cm}^3 \\ \therefore \text{Volume of wood used in the box} \\ &= \text{Outer volume} - \text{Inner volume} \\ &= 180000 - 160892 \\ &= 19108 \text{ cm}^3 \end{aligned}$$

Example 10. How many bricks of measure 25 cm × 16cm × 10cm are required to make a wall of size 20 m long, 5m wide and 50 cm thick while there is a door of size 2m × 1.5m and two windows of size 1.5m × 1m? Find the cost of bricks at the rate of ₹ 280 per thousand.

Solution :

$$\begin{aligned} \text{Volume of wall} &= 20 \times 5 \times 0.5 = 50 \text{ m}^3 \\ \text{Volume of blank place left for a door and two windows} \\ &= (\text{length} \times \text{breadth} \times \text{thickness}) \text{ of door} + \\ &2 (\text{length} \times \text{breadth} \times \text{thickness}) \text{ of windows} \\ &= [2 \times 1.5 \times 0.5 + 2 (1.5 \times 1 \times 0.5)] \text{m}^3 \\ &= [1.5 + 1.5] \text{m}^3 \\ &= 3 \text{m}^3 \\ \therefore \text{Volume of wall where bricks will be used} &= 50 - 3 = 47 \text{ m}^3 \\ \text{Volume of a brick} &= (25 \times 16 \times 10) \text{ cm}^3 \\ &= \left(\frac{25}{100} \times \frac{16}{100} \times \frac{10}{100} \right) \text{ m}^3 \end{aligned}$$

$$= \frac{4000}{1000000} = \frac{1}{2500} \text{ m}^3$$

$$\therefore \text{Number of bricks} = \frac{47}{\frac{1}{2500}} = 47 \times 2500 = 117500$$

$$\therefore \text{Cost of bricks} = 117500 \times \frac{280}{1000} = ₹ 3290$$

Exercise 12.2

1. The measure of a match box is $3\text{cm} \times 2\text{cm} \times 1\text{cm}$. Find the volume of a packet of such 12 match boxes.
2. Perimeter of a face of a cube is 24 cm. Find the volume of the cube.
3. Core of three cubes of metal are 3 cm, 5cm and 4cm respectively, By melting all there, a new cube is made. Find the volume of new cube and the length of the core of this cube.
4. The length of a water tank is 2.5m and breadth in 2m. It contains 1500 litres water in it. Find the deapth of the tank.
5. The length of a wall is 4m, breadth 15 cm and height is 3m. How many bricks of size $20\text{cm} \times 10\text{cm} \times 8\text{cm}$ are required to make a wall. If the cost of bricks is ₹ 120 per thousand, then find the total cost of bricks.
6. There is a water tank of size $20\text{m} \times 15\text{m} \times 6\text{m}$ in a village. How many litres of water can be filled in it? If 1000 litre of water is used daily from it. Then for how many days it will be sufficient?
7. The length of a wall in 8m and height is 4m. Wall is 35 cm thick. There is one door of size $2\text{m} \times 1\text{m}$ and two windows each of size $1.20\text{m} \times 1\text{m}$. Find the expenditure of making wall at the rate of ₹ 1500 per cubic meter.
8. How many bricks of size $25\text{cm} \times 15\text{cm} \times 6\text{cm}$ are required to make a wall 5 m long, 2m high and 50cm thick, if 10% place is occupied by cement?
9. The mud taken out from a pond, is spread equally in a field. If the pit dug out in pond is 200 m long, 50m wide and 0.75m deep, then how much high, will be the lend of field ? The area of the field is 1500 square meter.
10. The outer length, breadth and height of the wooden box with lid is 1.25m, 0.80 m and 0.55m. Thickness of wood is 2.5 cm. If the weight of 1 cubic metre wood is 250 kg. Find the toal weight of box.

Important Points

1. If the side of a cube is a , then total surface area of cube = $6a^2$
Volume of cube = a^3
Diagonal of cube = $a\sqrt{3}$
2. If length is a , breadth is b and height is c of a cuboid, then total surface area of a cuboid
 $= 2[ab + bc + ac]$
Volume of cuboid = $a \times b \times c$
Diagonal of cuboid = $\sqrt{a^2 + b^2 + c^2}$
3. Units related to volume are :
1 litre = 1000 cubic centimetre
1000 litre = 1 cubic metre = 1 kilo litre
1 cubic centimetre = 1000 cubic millimetre
1 cubic metre = 1000000 centimetre

Miscellaneous Exercise - 12

1. If the volume of a cube is 125m^3 , then side of a cube will be :
(A) 7m (B) 6m (C) 5m (D) 2m
2. If the volume of a cube is 1331 cubic centimeter, then surface area of cube is -
(A) 762cm^2 (B) 726cm^2 (C) 426cm^2 (D) 468cm^2
3. Length, breadth and height of a cuboid is 4m, 3m and 2m, surface area of cuboid will be
(A) 25m^2 (B) 26m^2 (C) 52m^2 (D) 62m^2
4. Diagonal of a cuboid of measure $8\text{m} \times 7\text{m} \times 6\text{m}$ is
(A) 12.2m (B) 12.02m (C) 14.2m (D) 14.02m
5. Side of a cube is 5 cm. Diagonal of cube is
(A) $4\sqrt{3}$ cm (B) $2\sqrt{3}$ cm (C) $5\sqrt{3}$ cm (D) 5cm
6. If the volume of a cuboid is 400 cm^3 and area of its base is 80 cm^2 , then height of cuboid is
(A) 7 cm (B) 6 cm (C) 4 cm (D) 5 cm

7. Measures of a cuboid are $15\text{ cm} \times 12\text{ cm} \times 6\text{ cm}$. How many cubes of side 3 cm can be made by melting this cuboid?

8. Edge of two cubical dice is 2 cm . A solid is made by pasting a face of each dice. Find the total surface area of solid.

9. If the edges of three cubes are 3 cm , 4 cm and 5 cm respectively, then find side of the cube made by these cubes.

10. An empty cistern is 4 m long and 3 m wide. How many cubic meter of water is filled in it so that the height of water become 2 m ?

11. There is 8 litre water in a cubical pot. Find the total surface area of pot.
12. The measures of any godown are $60\text{ m} \times 25\text{ m} \times 10\text{ m}$. How many maximum wooden crates can be put inside the store of size $1.5\text{ m} \times 1.25\text{ m} \times 0.5\text{ m}$?
13. A river of 3 m deep and 40 m wide, is flowing at the rate of 2 km/hour and falls into the sea. How much water will fell in a minute, in sea?
14. If length, breadth and height of a right angular parallel hexagon are in the ratio $6 : 5 : 4$ and its total surface area is 33300 square centimeter. Find the volume of right angular parallel hexagon.
15. A plot is 20 m long and 15 m wide. Digging the land 10 m , breadth 6 m and depth 5 m , from outside of the plot, is spread over the plot. Find the height of the mud spread over in the plot.

Answers

Exercise 12.1

1. 2.28 m^2
2. 5200 cm^2
3. ₹1282.5
4. 96 cm^2 , $4\sqrt{3} \text{ cm}$
5. 13 cm
6. ₹ 53.55
7. 400 cm^2
8. 55400 cm^2

Exercise 12.2

1. 72 cm^3
2. 216 cm^3
3. 216 cm^3 , 6 cm
4. 30 cm
5. 1125 bricks, Rs. 135
6. 1800000 litres, 1800 days
7. ₹ 14490
8. 2000 bricks
9. 50 cm
10. 25 kg.

Miscellaneous Exercise - 12

1. (C)
2. (B)
3. (C)
4. (A)
5. (C)
6. (D)
7. 40 cubes
8. 80 cm^2
9. 6 cm
10. 24 m^3
11. 2400 cm^2
12. 16000 crate
13. 4000 m^3
14. 405000 cm^3
15. 1m

Angles and their Measurement

13.01 Trigonometry

The word "trigonometry" is derived from a combination of three words 'tri', 'gon', and metron. 'Tri' means three, 'gon' means sides and 'metron' means 'a measure'. Thus, trigonometry deals with the measurement of sides (and angles) of a triangle. The relations between sides and angles of a triangle are used to find distance, height and areas etc. which can not be easily measured. We used the trigonometry in finding the distance of the earth from the sun and moon. Field like physics. Navy or Engineering the knowledge of trigonometry is very useful.

13.02 Positive and Negative distances

XOX' and YOY' are two mutually perpendicular lines, intersecting at O . Thus, the plane is divided into four parts. In such case

- (i) The distance measured from O along OX are considered positive and the distance measured along OX' are considered negative.
- (ii) The distances measured from O along OY are considered positive and those measured along OY' are considered negative. XOY , YOX' , $X'OY'$ and $Y'OX$ are called the first, second, third and fourth quadrants respectively. It should be noted that this order is in anticlockwise direction.

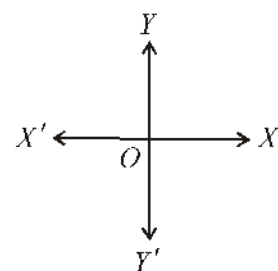


Fig. 13.01

13.03 Angle

The amount of rotation produced by the revolving the ray in moving from its initial position OX to the present position OA is called angle. Thus, in fig. 13.02, XOA is an

angle. We use the symbol \angle to denote an angle. Generally the capital letters A, B, C, ... of the English alphabet are used to denote the vertices of the angles and the measures of the angles are denoted by α (alpha), β (beta), γ (gamma), θ (theta), ϕ (phi), ψ (psi) e t c .

Positive and Negative Angles :

If the ray OA, starting from its initial position OX revolves in the anticlockwise direction, the angle so formed is considered positive. In fig. 13.02, $\angle XOA$ is a positive angle.

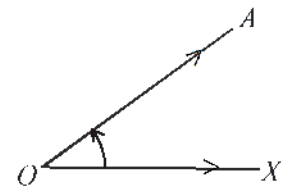


Fig. 13.02

If the ray OA, starting from its initial position OX revolves in clockwise direction, the angle so formed is considered negative. In fig. 13.03, $\angle XOA'$ is a negative angle.

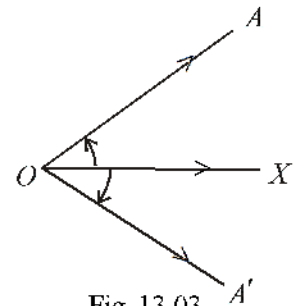
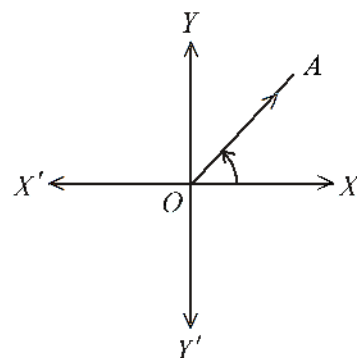


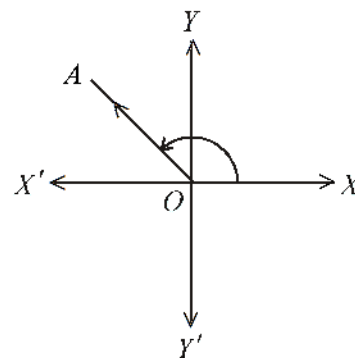
Fig. 13.03

13.04 Angles of any magnitude :

- (i) If the revolving ray OA, starting from its initial position OX revolving in anticlockwise direction and makes certain angle in the first quadrant, then this angle lies between 0° and 90° . For example $\angle XOA$ in fig. 13.04 (i).
- (ii) The angle in which the revolving ray OA, starting from its initial position OX and revolving in anticlockwise direction, makes in the second quadrant lies between 90° and 180° . For example $\angle XOA$ in fig. 13.04 (ii).
- (iii) The angle in which the revolving ray OA, starting from its initial position OX and revolving in anticlockwise direction, makes in the third quadrant lies between 180° and 270° . For example $\angle XOA$ in fig. 13.04 (iii).
- (iv) The angle in which the revolving ray OA, starting from its initial position OX and revolving in anticlockwise direction makes, in the fourth quadrant lies between 270° and 360° . For example $\angle XOA$ in fig. 13.04 (iv).



(i)



(ii)

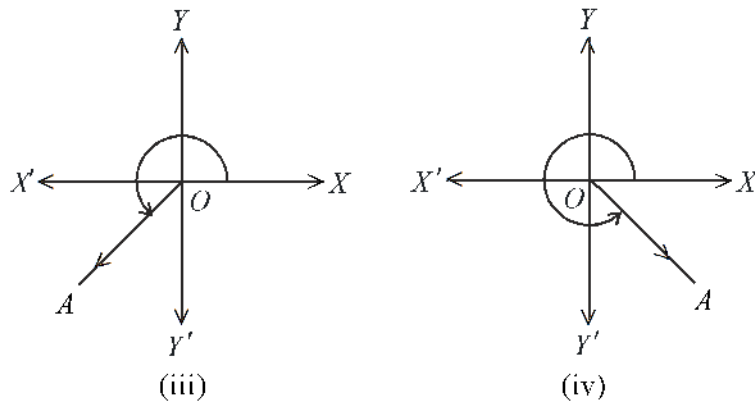


Fig. 13.04

If the ray OA , moving in anticlockwise direction, makes a complete revolution and comes back to its original position OX , then it describes an angle of 360° .

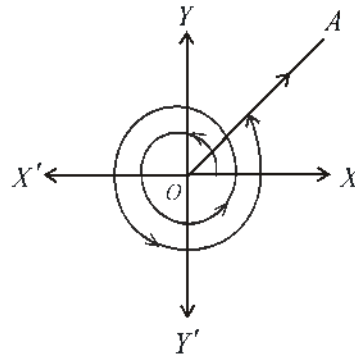


Fig. 13.05

So far we have seen that the maximum measure of an angle can be 360° or 4 right angles but angles greater than 360° are also possible. The revolving ray, revolving about its original position, described an angle of 360° in each complete rotation.

If the revolving ray, starting from its original position OX and revolving about the point O , make two complete rotations, then it describes an angle of $2 \times 360^\circ = 720^\circ$. If, after two complete rotations, come back to the position OA again then the angle so described $= 2 \times 360^\circ + \angle XOA$ (Fig. 13.05).

Example 1. Display an angle of 390° with the help of a figure.

Solution : $390^\circ = 1 \times 360^\circ + 30^\circ$

The revolving ray OA , starting from its initial position OX and revolving in anticlockwise direction point O makes one

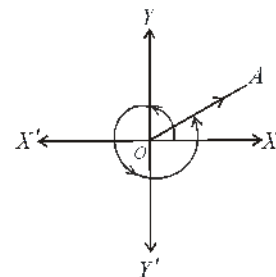


Fig. 13.06

complete rotation and moves through 30° in the same direction and comes to the position OA in the first quadrant, as shown in figure 13.06.

Example 2. Display an angle of -750° with the help of a figure.

Solution : $-750^\circ = -(2 \times 360^\circ) - 30^\circ$

The revolving ray OA, starting from its initial position OX and revolving in clockwise direction about the point O, makes two complete rotation and moves in the same direction through an angle of 30° . Thus its final position in the fourth quadrant will be as shown in the figure 13.07.

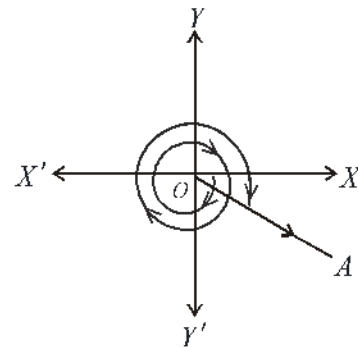


Fig. 13.07

13.05 Measurement of Angles

- (i) Sexagesimal system
- (ii) Centesimal system
- (iii) Circular system

(i) Sexagesimal system : In this system, angles are measured in degrees, minutes and seconds. They are related as follows :

$$1 \text{ right angle} = \text{ninety degrees} = 90^\circ$$

$$1 \text{ degree } (1^\circ) = \text{sixty minutes} = 60'$$

$$1 \text{ minute } (1') = \text{sixty seconds} = 60''$$

(ii) Centesimal system : This system is also known as French system. In this system, the angles are measured in grades, minutes and seconds. They are related as follows :

$$1 \text{ right angle} = 100 \text{ grade} = 100^g$$

$$1 \text{ grade } (1^g) = 100 \text{ minutes} = 100'$$

$$\text{one minute } (1') = 100 \text{ seconds} = 100''$$

Note : This system is not in practice.

(iii) Circular system : In this system the unit of angle is radian. "The angle subtended by an arc of a circle, whose length is equal to radius, at the centre of the circle, is called one radian". Let O be the centre and r be the radius of the circle. Draw an arc AB whose length is r. Thus the $\angle AOB$ is called one radian. The angle of 1 radian is denoted as 1^c . Hence in Fig. 13.08, $\angle AOB = 1^c$.

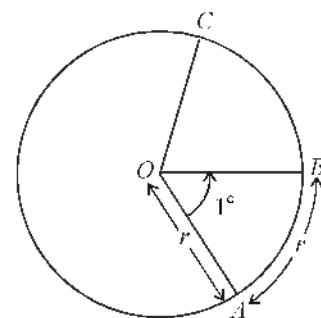


Fig. 13.08

Let C be any other point on the circumference of the circle.

Then

$$\frac{\angle AOC}{\angle AOB} = \frac{\text{arc } AC}{\text{arc } AB}$$

or
$$\frac{\angle AOC}{\text{arc } AC} = \frac{\angle AOB}{\text{arc } AB}$$

$$\begin{aligned}\angle AOC &= \frac{\text{arc } AC}{\text{arc } AB} \times \angle AOB \\ &= \frac{\text{arc } AC}{\text{arc } AB} \times 1^\circ\end{aligned}$$

$$= \frac{\text{arc } AC}{\text{arc } AB} \text{ radian} \quad \dots(1)$$

If $\angle AOC = \theta^\circ$ (θ radian) and $\text{arc } AC = x$ then from equation (1)

$$\theta^\circ = \frac{x}{r} = \frac{\text{arc}}{\text{radius}} \text{ radian}$$

13.06 The Value of π

The ratio of the circumference and the diameter of a circle is always constant. This constant quantity is denoted by the Greek letter π upto 8 places of decimal is 3.14159265.

In fractional form, its value is considered as $\frac{22}{7}$.

13.07 Value of 1 radian

We know that

$$\pi = \frac{\text{circumference of the circle}}{\text{diameter of the circle}}$$

If the radius of the circle is r , its diameter is $2r$ and its circumference is $2\pi r$. We also know that there is a definite relationship between the length of an arc and the angle subtended by that arc at the centre from Fig. 13.08.

$$\frac{\angle AOB}{360^\circ} = \frac{\text{arc } AB}{\text{circumference of the circle}}$$

or
$$\frac{\angle AOB}{\text{arc } AB} = \frac{360^\circ}{\text{circumference}}$$

or
$$\frac{1^\circ}{r} = \frac{360^\circ}{2\pi r}$$

or
$$1^\circ = \frac{180^\circ}{\pi} \quad \dots(1)$$

or $1^\circ = 57^\circ 17' 45''$ approximately

(Taking $\pi = 3.1416$)

Thus we find that the value of radian does not depend on the radius of the circle. Hence the value of one radian is constant for all circles.

From equation (1)

$$\pi^c = 180^\circ$$

In practice, we leave 'c' and instead of writing π^c , we simply write π .

Hence $180^\circ = \pi$.

Example 3. Convert 60° into radians.

Solution : We know that $180^\circ = \pi$ radian

$$\therefore 1^\circ = \frac{\pi}{180} \text{ radian}$$

$$\therefore 60^\circ = \frac{\pi}{3} \text{ radian.}$$

Example 4. Convert $\frac{\pi}{4}$ radian into degrees.

Solution : We know that $1^\circ = \frac{180^\circ}{\pi}$

$$\therefore \frac{\pi}{4} = \frac{180^\circ}{4} = 45^\circ$$

Example 5. Find the length of the arc subtending an angle of $\frac{\pi}{3}$ radians at the centre of the circle.

Solution : If r be the radius of the circle and x be the length of the arc subtending an angle of $\frac{\pi}{3}$ radians at the centre, then

$$\frac{\pi}{3} = \frac{x}{r}$$

$$\text{or } x = \frac{\pi}{3} r.$$

Example 6. Find the angle subtended by the whole of the circumference at the centre of the circle.

Solution : We know that the circumference of a circle of radius r is $2\pi r$ and the angle subtended by an arc of length r at the centre is 1 radian.

\therefore Angle subtended at the centre by an arc of length $r = 1$ radian

\therefore Angle subtended at the centre by an arc of length $l = \frac{l}{r}$ radian

\therefore Angle subtended at the centre by an arc of length $2\pi r = \frac{1}{r} \times 2\pi r$ radian
 $= 2\pi$ radian.

Hence the whole circumference of a circle subtends an angle of 2π radians at the centre.

Example 7. How much time does the minute hand of a watch take to describe an angle of $\frac{3\pi}{2}$ radians.

Solution : We know that

$$4 \text{ right angles} = 2\pi \text{ radians}$$

The time taken by the minute hand of the watch to describe an angle of 2π radians = 1 hour

\therefore The time for describing 1 radian $= \frac{1}{2\pi}$ hours

\therefore Time for describing an angle of $\frac{3\pi}{2}$ radian $= \frac{1}{2\pi} \times \frac{3\pi}{2}$ hours
 $= \frac{3}{4}$ hours $= \frac{3}{4} \times 60$ minutes = 45 minutes

Important Points

1. When the revolving ray revolves in anticlockwise direction, the angle thus formed is positive and if it revolves in clockwise direction, the angle so formed is negative.
2. The following systems are used to measure angles :
 - (i) Sexagesimal system
 - (ii) Centesimal system
 - (iii) Circular system.
3. In sexagesimal system, the unit of angle is 'degree'
1 right angle $= 90^\circ$, $1^\circ = 60'$ and $1' = 60''$.
4. The angle subtended at the centre of the circle by an arc whose length is equal to its radius is called an angle of one radian.
5. The angle subtended at the centre by the circumference of the circle is 2π radians.
6. The relation between the sexagesimal and circular system is $D = \left(\frac{180^\circ}{\pi}\right)R$, where D and R are the measures of the angle in degrees and radians respectively.

Miscellaneous Exercise 13

Objective questions [1-5]

- The line describing an angle of 750° , lies in :
(A) First quadrant (B) Second quadrant
(C) Third quadrant (D) Fourth quadrant
- The number of radians in angle 30° is :
(A) $\frac{\pi}{3}$ radian (B) $\frac{\pi}{4}$ radian
(C) $\frac{\pi}{6}$ radian (D) $\frac{3\pi}{4}$ radian
- The value of $\frac{3\pi}{4}$ in sexagesimal system is :
(A) 75° (B) 135°
(C) 120° (D) 220°
- How much time the minute hand of a watch will take to describe an angle of $\frac{\pi}{6}$ radians :
(A) 10 minutes (B) 20 minutes
(C) 15 minutes (D) 5 minutes
- The value of the angle, in radian, subtended at the centre of the circle of radius 100 meters by an arc of length 25π meters is :
(A) $\frac{\pi}{4}$ (B) $\frac{\pi}{3}$
(C) $\frac{\pi}{6}$ (D) $\frac{3\pi}{4}$
- In which quadrant does the revolving ray lie when it makes the following angles :
(i) 240° (ii) 425° (iii) -580°
(iv) 1280° (v) -980°
- Convert the following angles in radians :
(i) 45° (ii) 120°
(iii) 135° (iv) 540°

8. Express the following angles in sexagesimal system :

(i) $\frac{\pi}{2}$

(ii) $\frac{2\pi}{5}$

(iii) $\frac{5}{6}\pi$

(iv) $\frac{\pi}{15}$

9. Find the angle in radians, subtended at the centre of a circle of radius 5 cm by an arc of the circle whose length is 12cm.

10. How much time the minute hand of a watch will take to describe an angle of $\frac{3\pi}{2}$ radians ?

11. How much time the minute hand of a watch will take to describe an angle of 120° ?

12. In a circle, the angle subtended at the centre by an arc of length 10 cm is 60° . Find the radius of the circle.

13. If the minute hand of a watch has revolved through 30 right angles, just after the mid day, then what is the time by the watch ?

14. The angles of a triangle are in ratio 2 : 3 : 4. Find the all three angles in radians.

15. Express $\frac{3}{5}\pi$ into sexagesimal system.

Answers

Miscellaneous Exercise 13

1. (A) 2. (C) 3. (B)
4. (D) 5. (A)
6. (i) Third (ii) First (iii) Second (iv) Third (v) Second
7. (i) $\frac{\pi}{4}$ (ii) $\frac{2\pi}{3}$ (iii) $\frac{3\pi}{4}$ (iv) 3π
8. (i) 90° (ii) 72° (iii) 150° (iv) 12°
9. $\frac{12}{5}$ radian
10. 45 minute
11. 20 minute
12. $\frac{30}{\pi}$ cm
13. 7.30 P.M.
14. $\frac{2\pi}{9}, \frac{\pi}{3}, \frac{4\pi}{9}$
15. 108°



Trigonometric Ratios of Acute Angles

14.01 Right Angled Triangle :

In previous chapters we have studied about the triangle in which one angle is right angle, such triangles are called right triangles. Fig. 14.01 is a right triangle in which $\angle B$ is a right angle. The side opposite the right angle is called hypotenuse. Hence in right angled triangle $\triangle ABC$, AC is hypotenuse. Regarding other two angles of the right angled triangle, the side which makes angle with the hypotenuse is called base or adjacent side and the side opposite to this angle is called perpendicular. In Fig. 14.01 for $\angle C$, side CB is base AB is perpendicular. Similarly for $\angle A$ side AB is base and side BC is perpendicular. In right angled triangle the angles other than the right angle are acute angles. The relation between the sides of a right triangle is "Square on hypotenuse is equal to the sum of squares on other two sides" known as Baudhayan Theorem. In brief the above theorem may be read as

$$AC^2 = AB^2 + BC^2$$

Clearly if out of three sides AB , BC , and AC two are given then third can easily be obtained.

Example 1. In right angled triangle ABC give the names of sides corresponding to angle θ and ϕ .

Solution : In $\triangle ABC$, $\angle B$ is a right angle, so AC is hypotenuse. Now for angle θ BC is base and AB is perpendicular. Similarly for angle ϕ side AB is base and side BC is perpendicular.

Example 2. In triangle ABC , $\angle B$ is right angle. If $AB = 4$ cm and $AC = 5$ cm then find BC .

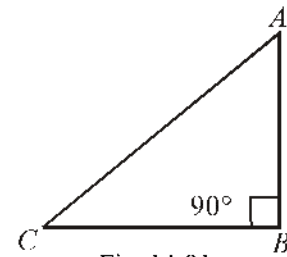


Fig. 14.01

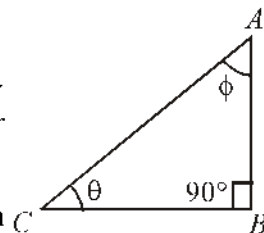


Fig. 14.02

Solution : Draw rough figure of ΔABC as per the given specification.

$$\angle B = 90^\circ$$

$$AC = 5 \text{ cm}$$

$$AB = 4 \text{ cm}$$

From Baudhayan theorem

$$AC^2 = AB^2 + BC^2$$

$$\text{or } (5)^2 = (4)^2 + BC^2$$

$$\text{or } BC^2 = 25 - 16$$

$$\text{or } BC^2 = 9$$

$$\text{Hence } BC = 3 \text{ cm.}$$

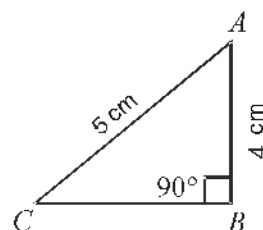


Fig. 14.03

14.02 Trigonometric Ratios of an Acute Angle :

In right angled triangle the ratio of any two sides is called trigonometrical ratio.

Let triangle ABC be a right angled triangle in which $\angle ABC$ is a right angle and $\angle CAB = \theta$, then

$$(i) \frac{\text{Perpendicular}}{\text{Hypotenuse}} = \frac{BC}{AC} = \text{sine } \theta \text{ or } \sin \theta$$

In brief sine θ is written as $\sin \theta$

$$(ii) \frac{\text{Base}}{\text{Hypotenuse}} = \frac{AB}{AC} = \text{cosine } \theta \text{ or } \cos \theta$$

In brief cosine θ or is written as $\cos \theta$.

$$(iii) \frac{\text{Perpendicular}}{\text{Base}} = \frac{BC}{AB} = \text{tangent } \theta \text{ or } \tan \theta$$

In brief tangent θ is written as $\tan \theta$

$$(iv) \frac{\text{Base}}{\text{Perpendicular}} = \frac{AB}{BC} = \text{cotangent } \theta \text{ or } \cot \theta$$

In brief cotangent θ is written as $\cot \theta$.

$$(v) \frac{\text{Hypotenuse}}{\text{Base}} = \frac{AC}{AB} = \text{secant } \theta \text{ or } \sec \theta$$

In brief secant θ is written as $\sec \theta$.

$$(vi) \frac{\text{Hypotenuse}}{\text{Perpendicular}} = \frac{AC}{BC} = \text{cosecant } \theta \text{ or } \text{cosec } \theta$$

In brief cosecant θ is written as $\text{cosec } \theta$.

Note : (i) Suppose the revolving line AX moves in anticlock wise direction keeping the vertex A fixed and makes an acute angle $\angle XAP = \theta$. Draw perpendiculars CB, C_1B_1 and C_2B_2 from the points C, C_1, C_2 respectively on AX .

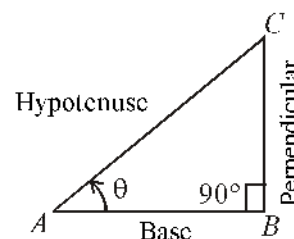


Fig. 14.04

We find that the triangle CAB, C_1AB_1, C_2AB_2 etc. are similar. Therefore

$$\sin \theta = \frac{BC}{AC} = \frac{B_1C_1}{AC_1} = \frac{B_2C_2}{AC_2} = \dots$$

$$\text{and } \cos \theta = \frac{AB}{AC} = \frac{AB_1}{AC_1} = \frac{AB_2}{AC_2} = \dots$$

From this we observe that the value of $\sin \theta$ or $\cos \theta$ remains unchanged in every case, i.e. these values do not depend on the position of the point P on the revolving line. Similarly the other trigonometrical ratios also do not depend on the position of point P on the revolving line.

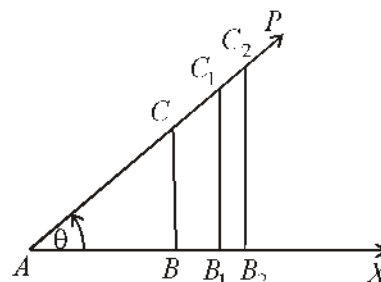


Fig. 14.05

Hence the trigonometrical ratio depend on the acute angle θ not on the size of the right angled triangle. Since for every acute angle θ , the value of the trigonometrical ratio is unique, therefore, trigonometrical ratios are also called as trigonometrical functions.

(ii) $\sin \theta, \cos \theta, \tan \theta, \dots$ does not mean the multiplication of \sin or \cos or \tan or \dots by θ

$$\begin{aligned} \text{i.e., } \sin \theta &\neq \sin \times \theta \\ \cos \theta &\neq \cos \times \theta \\ \tan \theta &\neq \tan \times \theta \end{aligned}$$

(iii) Trigonometrical ratios of any positive acute angle are always positive.

Illustrative Examples

Example 3. In triangle ABC angle B is a right angle, find all trigonometrical ratios of the angle A .

Solution : From Fig. 14.06, side AC is hypotenuse and side opposite to $\angle A$ is BC . Therefore in $\triangle ABC$ side AB is base, BC is perpendicular and AC is hypotenuse.

$$\therefore \sin A = \frac{\text{Perpendicular}}{\text{Hypotenuse}} = \frac{BC}{AC}$$

$$\cos A = \frac{\text{Base}}{\text{Hypotenuse}} = \frac{AB}{AC}$$

$$\tan A = \frac{\text{Perpendicular}}{\text{Base}} = \frac{BC}{AB}$$

$$\cot A = \frac{\text{Base}}{\text{Perpendicular}} = \frac{AB}{BC}$$

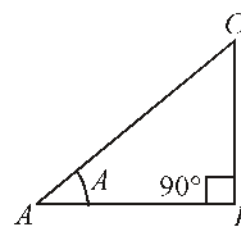


Fig. 14.06

$$\operatorname{sec} A = \frac{\text{Hypotenuse}}{\text{Base}} = \frac{AC}{AB}$$

$$\operatorname{cosec} A = \frac{\text{Hypotenuse}}{\text{Perpendicular}} = \frac{AC}{BC}$$

Example 4. In triangle ABC angle C is a right angle and $AB = 25$ cm and $BC = 24$ cm then find all trigonometric ratios of the angle A.

Solution : $\because \Delta ABC$ is a right triangle.

$$\begin{aligned} \therefore AC &= \sqrt{AB^2 - BC^2} \\ &= \sqrt{(25)^2 - (24)^2} \\ &= \sqrt{49} \\ &= 7 \text{ cm} \end{aligned}$$

$$\therefore \sin A = \frac{BC}{AB} = \frac{24}{25}$$

$$\cos A = \frac{AC}{AB} = \frac{7}{25}$$

$$\tan A = \frac{BC}{AC} = \frac{24}{7}$$

$$\cot A = \frac{AC}{BC} = \frac{7}{24}$$

$$\operatorname{sec} A = \frac{AB}{AC} = \frac{25}{7}$$

$$\operatorname{cosec} A = \frac{AB}{BC} = \frac{25}{24}$$

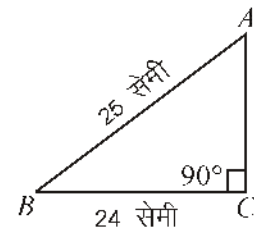


Fig. 14.07

Example 5. If $\sin \theta = \frac{3}{5}$, then find remaining trigonometrical ratios of θ .

Solution : We draw a right ΔABC where $\sin \theta = \frac{3}{5}$ (Fig. 14.08) i.e., perpendicular AB and hypotenuse AC are in the ratio 3 : 5. Let $AB = 3k$ and $AC = 5k$, where $k > 0$, which is proportionality constant.

Hence from Baudhayan theorem

$$BC^2 = AC^2 - AB^2 = (5k)^2 - (3k)^2 = 16k^2$$

$$\therefore BC = \pm 4k$$

Hence angle θ is an acute angle, therefore BC will be positive.

$$BC = 4k$$

$$\therefore \cos \theta = \frac{BC}{AC} = \frac{4k}{5k} = \frac{4}{5}$$

$$\tan \theta = \frac{AB}{BC} = \frac{3k}{4k} = \frac{3}{4}$$

$$\cot \theta = \frac{BC}{AB} = \frac{4k}{3k} = \frac{4}{3}$$

$$\sec \theta = \frac{AC}{BC} = \frac{5k}{4k} = \frac{5}{4}$$

$$\operatorname{cosec} \theta = \frac{AC}{AB} = \frac{5k}{3k} = \frac{5}{3}$$

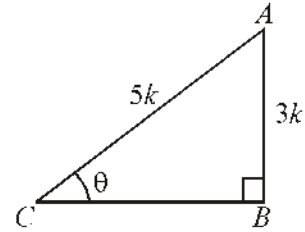


Fig. 14.08

Example 6. If $\sec \theta = \frac{13}{12}$ then find the value of $\frac{1 - \tan \theta}{1 + \tan \theta}$.

Solution : Draw a right angled triangle ABC where $\sec \theta = \frac{13}{12}$ (Fig. 14.09)

i.e., hypotenuse AC and base BC are in the ratio of 13 : 12.

Let $AC = 13k$, $BC = 12k$, where $k > 0$, which is proportionality constant.

Hence form Baudhayan theorem

$$AB^2 = AC^2 - BC^2 = (13k)^2 - (12k)^2 = 25k^2$$

$$\therefore AB = \pm 5k$$

Since angle θ is an acute angle, therefore AB will be positive.

$$\therefore AB = 5k$$

$$\therefore \tan \theta = \frac{AB}{BC} = \frac{5k}{12k} = \frac{5}{12}$$

$$\text{Now } \frac{1 - \tan \theta}{1 + \tan \theta} = \frac{1 - \frac{5}{12}}{1 + \frac{5}{12}} = \frac{7}{17}$$

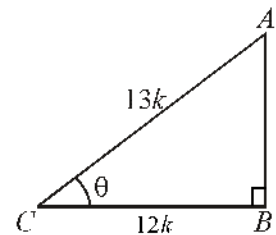


Fig. 14.09

Example 7. If $\operatorname{cosec} A = 2$, then find the value of $\cot A + \frac{\sin A}{1 + \cos A}$.

Solution : $\because \operatorname{cosec} A = \frac{2}{1}$

Draw a right angled triangle ABC in which hypotenuse AC and perpendicular BC are

in the ratio 2 : 1 (Fig. 14.10).

Let $AC = 2k$, $BC = k$

where $k > 0$, is the proportionality constant.

Hence from Baudhayan theorem

$$AB^2 = AC^2 - BC^2$$

$$\text{or } AB^2 = (2k)^2 - k^2 = 3k^2$$

$$\text{or } AB = \pm\sqrt{3}k$$

Since angle A is an acute angle, therefore AB will be positive.

$$\therefore AB = \sqrt{3}k$$

$$\therefore \sin A = \frac{BC}{AC} = \frac{k}{2k} = \frac{1}{2}$$

$$\cos A = \frac{AB}{AC} = \frac{\sqrt{3}k}{2k} = \frac{\sqrt{3}}{2}$$

$$\cot A = \frac{AB}{BC} = \frac{\sqrt{3}k}{k} = \sqrt{3}$$

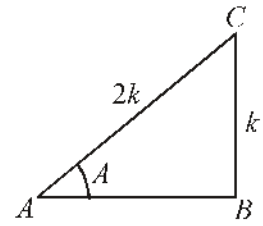


Fig. 14.10

Now

$$\begin{aligned} \cot A + \frac{\sin A}{1 + \cos A} &= \sqrt{3} + \frac{\frac{1}{2}}{1 + \frac{\sqrt{3}}{2}} \\ &= \sqrt{3} + \frac{1}{2 + \sqrt{3}} \times \frac{(2 - \sqrt{3})}{(2 - \sqrt{3})} \\ &= \sqrt{3} + \frac{2 - \sqrt{3}}{(2)^2 - (\sqrt{3})^2} = \sqrt{3} + 2 - \sqrt{3} = 2 \end{aligned}$$

Exercise 14.1

1. In $\triangle ABC$, $\angle A = 90^\circ$, $a = 25$ cm, $b = 7$ cm then find all the trigonometric ratio of $\angle B$ and $\angle C$.

2. In $\triangle ABC$, $\angle B = 90^\circ$, $a = 12$ cm, $b = 13$ cm then find all the trigonometrical ratios of $\angle A$ and $\angle C$.
3. If $\tan A = \sqrt{2} - 1$ then prove that $\sin A \cos A = \frac{1}{2\sqrt{2}}$.
4. If $\sin A = \frac{1}{3}$ then find the value of $\cos A \operatorname{cosec} A + \tan A \sec A$.
5. If $\cos \theta = \frac{8}{17}$ then find all the remaining trigonometrical ratios.
6. If $\cos A = \frac{5}{13}$ then find the value of $\frac{\operatorname{cosec} A}{\cos A + \operatorname{cosec} A}$.
7. If $5 \tan \theta = 4$ then find the value of $\frac{5 \sin \theta - 3 \cos \theta}{\sin \theta + 2 \cos \theta}$.
8. In $\triangle ABC$, $\angle C = 90^\circ$ and if $\cot A = \sqrt{3}$ and $\cot B = \frac{1}{\sqrt{3}}$ then prove that $\sin A \cos B + \cos A \sin B = 1$.
9. If $16 \cot A = 12$ then find the value of $\frac{\sin A + \cos A}{\sin A - \cos A}$.
10. In Fig. 14.13, $AD = DB$ and $\angle B = 90^\circ$ then find the value of the following:
 - (i) $\sin \theta$
 - (ii) $\cos \theta$
 - (iii) $\tan \theta$

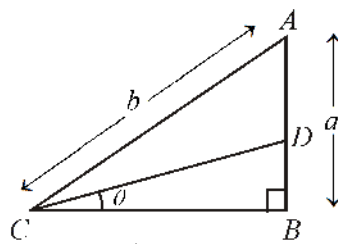


Fig. 14.13

14.03 Relation between Trigonometric Ratios :

In any right angle triangle OMP, PM is perpendicular, OM is base and OP is hypotenuse for the $\angle \theta$.

(i) $\sin \theta \operatorname{cosec} \theta = 1$

$$\sin \theta = \frac{PM}{OP} \quad \dots (1)$$

and $\operatorname{cosec} \theta = \frac{OP}{PM} \quad \dots (2)$

Multiplying (1) and (2)

$$\sin \theta \cdot \operatorname{cosec} \theta = \frac{PM}{OP} \times \frac{OP}{PM} = 1$$

i.e., $\sin \theta \operatorname{cosec} \theta = 1$

$$\Rightarrow \sin \theta = \frac{1}{\operatorname{cosec} \theta}$$

or $\operatorname{cosec} \theta = \frac{1}{\sin \theta}$

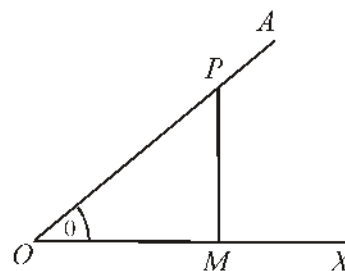


Fig. 14.14

Hence $\sin \theta$ and $\operatorname{cosec} \theta$ are reciprocal of each other.

(ii) $\cos \theta \cdot \sec \theta = 1$

From Fig. (14.14)

$$\cos \theta = \frac{\text{Base}}{\text{Hypotenuse}} = \frac{OM}{OP} \quad \dots (3)$$

and $\sec \theta = \frac{\text{Hypotenuse}}{\text{Base}} = \frac{OP}{OM} \quad \dots (4)$

Multiplying (3) and (4)

$$\cos \theta \cdot \sec \theta = \frac{OM}{OP} \cdot \frac{OP}{OM} = 1$$

i.e., $\cos \theta \cdot \sec \theta = 1$

$$\Rightarrow \cos \theta = \frac{1}{\sec \theta} \quad \text{या} \quad \sec \theta = \frac{1}{\cos \theta}$$

Hence $\cos \theta$ and $\sec \theta$ are reciprocal of each other.

(iii) $\tan \theta \cdot \cot \theta = 1$

From Fig. 14.14

$$\tan \theta = \frac{\text{Perpendicular}}{\text{Base}} = \frac{PM}{OM} \quad \dots (5)$$

and $\cot \theta = \frac{\text{Base}}{\text{Perpendicular}} = \frac{OM}{PM} \quad \dots (6)$

Multiplying (5) and (6)

$$\tan \theta \cdot \cot \theta = \frac{PM}{OM} \times \frac{OM}{PM} = 1$$

i.e., $\tan \theta \cdot \cot \theta = 1$

$$\Rightarrow \tan \theta = \frac{1}{\cot \theta} \text{ or } \cot \theta = \frac{1}{\tan \theta}$$

Hence $\tan \theta$ and $\cot \theta$ are reciprocal of each other.

$$(iv) \tan \theta = \frac{\sin \theta}{\cos \theta}$$

From equation (1) and (3)

$$\frac{\sin \theta}{\cos \theta} = \frac{\overset{PM}{OP}}{\overset{OM}{OP}} = \frac{PM}{OP} \times \frac{OP}{OM} = \frac{PM}{OM} = \frac{\text{Perpendicular}}{\text{Base}} = \tan \theta$$

$$i.e., \tan \theta = \frac{\sin \theta}{\cos \theta}$$

$$(v) \cot \theta = \frac{\cos \theta}{\sin \theta}$$

From equation (1) and (3)

$$\frac{\cos \theta}{\sin \theta} = \frac{\overset{OM}{OP}}{\overset{PM}{OP}} = \frac{OM}{OP} \times \frac{OP}{PM} = \frac{OM}{PM} = \frac{\text{Base}}{\text{Perpendicular}} = \cot \theta$$

$$i.e., \cot \theta = \frac{\cos \theta}{\sin \theta}$$

$$(vi) \sin^2 \theta + \cos^2 \theta = 1$$

From equation (1) and (3)

$$\sin \theta = \frac{PM}{OP} \text{ वरुन } \cos \theta = \frac{OM}{OP}$$

Squaring and adding

$$\begin{aligned} \sin^2 \theta + \cos^2 \theta &= \left(\frac{PM}{OP}\right)^2 + \left(\frac{OM}{OP}\right)^2 \\ &= \frac{PM^2 + OM^2}{OP^2} = \frac{OP^2}{OP^2} = 1 \quad [\text{from Baudhayan theorem (Fig. 14.14)}] \end{aligned}$$

$$(vii) 1 + \tan^2 \theta = \sec^2 \theta$$

From Fig. 14.14

$$\tan \theta = \frac{PM}{OM}$$

$$\begin{aligned} \therefore 1 + \tan^2 \theta &= 1 + \frac{PM^2}{OM^2} = \frac{OM^2 + PM^2}{OM^2} \\ &= \frac{OP^2}{OM^2} \quad [\text{From Baudhayab theroem}] \end{aligned}$$

$$[\because \sec \theta = \frac{OP}{OM}, \text{ Fig. (14.14)}]$$

$$\text{or } 1 + \tan^2 \theta = \sec^2 \theta$$

Aliter :

We know that

$$\sin^2 \theta + \cos^2 \theta = 1$$

Dividing both sides by $\cos^2 \theta$

$$\left(\frac{\sin \theta}{\cos \theta} \right)^2 + 1 = \frac{1}{\cos^2 \theta}$$

$$\Rightarrow \tan^2 \theta + 1 = \sec^2 \theta \quad \left(\because \frac{\sin \theta}{\cos \theta} = \tan \theta, \frac{1}{\cos \theta} = \sec \theta \right)$$

$$\text{(viii) } 1 + \cot^2 \theta = \operatorname{cosec}^2 \theta$$

From Fig. 14.14

$$\cot \theta = \frac{OM}{PM}$$

$$\therefore 1 + \cot^2 \theta = 1 + \left(\frac{OM}{PM} \right)^2 = \frac{PM^2 + OM^2}{PM^2}$$

$$= \frac{OP^2}{PM^2} \quad (\text{from Baudhayayan theorem})$$

$$= \operatorname{cosec}^2 \theta \quad \left(\because \operatorname{cosec} \theta = \frac{OP}{PM} \text{ from fig. 14.14} \right)$$

Aliter :

We know that

$$\cos^2 \theta + \sin^2 \theta = 1$$

Dividing both sides by $\sin^2 \theta$

$$\left(\frac{\cos \theta}{\sin \theta} \right)^2 + 1 = \frac{1}{\sin^2 \theta}$$

$$\Rightarrow \cot^2 \theta + 1 = \operatorname{cosec}^2 \theta \left(\because \frac{\cos \theta}{\sin \theta} = \cot \theta, \frac{1}{\sin \theta} = \operatorname{cosec} \theta \right)$$

Note : $(\sin \theta)^2$ is always written as $\sin^2 \theta$ and same is read as 'sign square theta'

$$i.e., (\sin \theta)^2 = \sin^2 \theta \neq \sin \theta^2$$

Other trigonometrical ratios are also to be treated in the same manner

Illustrative Examples

Example 8. If $\cos \theta = \frac{5}{13}$ then find the value of $\sin \theta$, $\tan \theta$ with the help of relations between the trigonometrical ratios when θ is an acute angle.

Solution : We know that

$$\sin^2 \theta + \cos^2 \theta = 1$$

$$\text{Putting } \cos \theta = \frac{5}{13}$$

$$\sin^2 \theta + \left(\frac{5}{13}\right)^2 = 1$$

$$\Rightarrow \sin^2 \theta = 1 - \frac{25}{169}$$

$$\Rightarrow \sin^2 \theta = \frac{144}{169}$$

$$\Rightarrow \sin \theta = \pm \frac{12}{13}$$

$$\Rightarrow \sin \theta = \frac{12}{13} \quad (\because \theta \text{ is an acute angle})$$

$$\therefore \tan \theta = \frac{\sin \theta}{\cos \theta} = \frac{12/13}{5/13} = \frac{12}{5}$$

Example 9. If $\tan \theta = \sqrt{3}$ then find all the trigonometric ratios using relations between the trigonometric ratios, when θ is an acute angle.

Solution : we know that

$$1 + \tan^2 \theta = \sec^2 \theta$$

$$\text{or } \sec^2 \theta = 1 + (\sqrt{3})^2 \\ = 1 + 3 = 4$$

$$\Rightarrow \sec \theta = \pm 2$$

or $\sec \theta = 2$ ($\because \theta$ is an acute angle)

$$\therefore \cos \theta = \frac{1}{\sec \theta} = \frac{1}{2}$$

Now $\sin \theta = \tan \theta \cdot \cos \theta$

$$\Rightarrow \sin \theta = \sqrt{3} \cdot \frac{1}{2} = \frac{\sqrt{3}}{2},$$

$$\operatorname{cosec} \theta = \frac{1}{\sin \theta} = \frac{1}{(\sqrt{3}/2)} = \frac{2}{\sqrt{3}},$$

$$\cot \theta = \frac{1}{\tan \theta} = \frac{1}{\sqrt{3}}.$$

Example 10. If $\operatorname{cosec} A = \sqrt{10}$ then find $\cot A, \sin A, \cos A$ using the relations between the trigonometrical ratios, when θ is an acute angle.

Solution: We know that

$$1 + \cot^2 A = \operatorname{cosec}^2 A$$

$$\Rightarrow \cot^2 A = \operatorname{cosec}^2 A - 1$$

$$\Rightarrow \cot^2 A = (\sqrt{10})^2 - 1 = 10 - 1 = 9$$

$$\Rightarrow \cot A = 3 \quad (\because \theta \text{ is an acute angle})$$

Now $\sin A = \frac{1}{\operatorname{cosec} A} = \frac{1}{\sqrt{10}}$

and $\cos A = \cot A \cdot \sin A$
 $= 3 \cdot \frac{1}{\sqrt{10}} = \frac{3}{\sqrt{10}}$

Example 11. If $\sec \theta = \frac{17}{8}$ then find all the trigonometrical ratios with the help of relation between the trigonometrical ratios, when θ is an acute angle.

Solution: We know that

$$1 + \tan^2 \theta = \sec^2 \theta$$

or $\tan^2 \theta = \sec^2 \theta - 1$

$$\begin{aligned}\text{or } \tan^2 \theta &= \left(\frac{17}{8}\right)^2 - 1 \\ &= \frac{289 - 64}{64} = \frac{225}{64}\end{aligned}$$

$$\Rightarrow \tan \theta = \frac{15}{8} \quad (\because \theta \text{ is an acute angle})$$

$$\therefore \cot \theta = \frac{1}{\tan \theta} = \frac{1}{(15/8)} = \frac{8}{15}$$

$$\text{and } \cos \theta = \frac{1}{\sec \theta} = \frac{1}{17/8} = \frac{8}{17}$$

$$\begin{aligned}\text{Now } \sin \theta &= \tan \theta \cos \theta \\ &= \frac{15}{8} \cdot \frac{8}{17} \\ &= \frac{15}{17}\end{aligned}$$

$$\text{and } \operatorname{cosec} \theta = \frac{1}{\sin \theta} = \frac{1}{\left(\frac{15}{17}\right)} = \frac{17}{15}$$

Example 12. If $\sin \theta = \frac{a^2 - b^2}{a^2 + b^2}$ then find the value of $\cos \theta$ and $\tan \theta$ using the relation between the trigonometrical ratios, when θ is an acute angle.

Solution: $\sin \theta = \frac{a^2 - b^2}{a^2 + b^2}$

$$\text{Now } \sin^2 \theta + \cos^2 \theta = 1$$

$$\Rightarrow \cos^2 \theta = 1 - \sin^2 \theta$$

$$\begin{aligned}\text{or } \cos^2 \theta &= 1 - \frac{(a^2 - b^2)^2}{(a^2 + b^2)^2} \\ &= \frac{(a^2 + b^2)^2 - (a^2 - b^2)^2}{(a^2 + b^2)^2} \\ &= \frac{4a^2b^2}{(a^2 + b^2)^2}\end{aligned}$$

$$\Rightarrow \cos \theta = \pm \frac{2ab}{(a^2 + b^2)}$$

$$\text{or } \cos \theta = \frac{2ab}{(a^2 + b^2)} \quad (\because \theta \text{ is an acute angle})$$

$$\text{Now } \tan \theta = \frac{\sin \theta}{\cos \theta} = \frac{\left(\frac{a^2 - b^2}{a^2 + b^2} \right)}{\left(\frac{2ab}{a^2 + b^2} \right)} = \frac{a^2 - b^2}{2ab}$$

Exercise 14.2

Solve with the help of relation between trigonometric ratio [Q. 1 to 10]

1. If $\operatorname{cosec} A = \frac{5}{4}$ then find the value $\cot A, \sin A, \cos A$.
2. If $\tan A = \frac{20}{21}$ then find the value $\cos A$ and $\sin A$.
3. If $\sin A = \frac{3}{5}$ then find the value $\cos A$ and $\tan A$.
4. If $\cos B = \frac{1}{3}$ then find remaining trigonometric ratios.
5. If $\sin A = \frac{5}{13}$ then find the value $\cos A$ and $\tan A$.
6. If $\tan A = \sqrt{2} - 1$ then prove that $\sin A \cos A = \frac{1}{2\sqrt{2}}$.
7. If $\tan A = 2$ then find the value $\sec A \sin A + \tan^2 A - \operatorname{cosec} A$.
8. If $\sin \theta = \frac{4}{5}$ then find the value $\frac{4 \tan \theta - 5 \cos \theta}{\sec \theta + 4 \cot \theta}$.
9. If $\cos \theta = \frac{1}{\sqrt{2}}$ then find the value $\sin \theta$ and $\cot \theta$.
10. If $\sec \theta = 2$ then evaluate $\tan \theta, \cos \theta$ and $\sin \theta$.

14.04 Trigonometric Identities :

Such trigonometric relations which are always true for the angles involved and for those angles for which trigonometrical ratios are defined are called trigonometric identities.

The relations defined in article 14.02 and 14.03 are the true for all values of angle θ . Thus these relations are called basic identities.

In trigonometry all relations are not identities, for example $\sin \theta = \cos \theta$ is an equation because this is not true for all values of θ .

To prove the trigonometrical identities one should take care of the following points :

- (i) Always start from the difficult side of the identity and making use of basic identities and find second side of the identity.
- (ii) If the identity contains the trigonometrical ratios then it is always better to convert these ratios in term of sines and cosines
- (iii) If there exists any radical sign then it should be removed.
- (iv) In some problems we may use rationalisation.
- (v) If it is not possible to obtain one side from the other then simplify both the sides as far as possible and prove them identically equal.

Illustrative Examples

Example 13. Prove the identity :

$$(\sec \theta + \cos \theta)(\sec \theta - \cos \theta) = \tan^2 \theta + \sin^2 \theta.$$

Solution : L.H.S = $\sec^2 \theta - \cos^2 \theta$

$$= 1 + \tan^2 \theta - (1 - \sin^2 \theta) \left[\begin{array}{l} \because \sec^2 \theta = 1 + \tan^2 \theta \text{ and} \\ \cos^2 \theta = 1 - \sin^2 \theta \end{array} \right]$$

$$= \tan^2 \theta + \sin^2 \theta$$

$$= \text{R.H.S}$$

Example 14. Prove the identity :

$$(\operatorname{cosec} \theta - \sin \theta)(\sec \theta - \cos \theta)(\tan \theta + \cot \theta) = 1.$$

Solution : L.H.S = $(\operatorname{cosec} \theta - \sin \theta)(\sec \theta - \cos \theta)(\tan \theta + \cot \theta)$

$$= \left(\frac{1}{\sin \theta} - \sin \theta \right) \left(\frac{1}{\cos \theta} - \cos \theta \right) \left(\frac{\sin \theta}{\cos \theta} + \frac{\cos \theta}{\sin \theta} \right)$$

(converting all ratios into sine and cosine θ)

$$= \left(\frac{1 - \sin^2 \theta}{\sin \theta} \right) \left(\frac{1 - \cos^2 \theta}{\cos \theta} \right) \left(\frac{\sin^2 \theta + \cos^2 \theta}{\sin \theta \cos \theta} \right)$$

$$= \frac{\cos^2 \theta}{\sin \theta} \cdot \frac{\sin^2 \theta}{\cos \theta} \cdot \frac{1}{\sin \theta \cos \theta} \quad (\text{Using basic identities})$$

$$= 1$$

$$= \text{R.H.S}$$

Example 15. Prove the identity :

$$\sqrt{\sec^2 \theta + \operatorname{cosec}^2 \theta} = \tan \theta + \cot \theta.$$

Solution : L.H.S = $\sqrt{\sec^2 \theta + \operatorname{cosec}^2 \theta}$

$$= \sqrt{(1 + \tan^2 \theta) + (1 + \cot^2 \theta)} \quad [\text{use of trigonometric identities}]$$
$$= \sqrt{\tan^2 \theta + 2 + \cot^2 \theta}$$
$$= \sqrt{\tan^2 \theta + 2 \tan \theta \cot \theta + \cot^2 \theta} \quad [\because \tan \theta \cot \theta = 1]$$
$$= \sqrt{(\tan \theta + \cot \theta)^2}$$
$$= \tan \theta + \cot \theta$$
$$= \text{R.H.S}$$

Example 16. Prove the identity :

$$\sqrt{\frac{1 + \cos \theta}{1 - \cos \theta}} = \operatorname{cosec} \theta + \cot \theta.$$

Solution : To remove radical sign multiplying numerator and denominator by $\sqrt{1 + \cos \theta}$ in the L.H.S.

$$\begin{aligned} \text{L.H.S.} &= \sqrt{\frac{1 + \cos \theta}{1 - \cos \theta}} \times \sqrt{\frac{1 + \cos \theta}{1 + \cos \theta}} \\ &= \sqrt{\frac{(1 + \cos \theta)^2}{1 - \cos^2 \theta}} \\ &= \sqrt{\frac{(1 + \cos \theta)^2}{\sin^2 \theta}} \quad [\because 1 - \cos^2 \theta = \sin^2 \theta] \\ &= \frac{1 + \cos \theta}{\sin \theta} \\ &= \frac{1}{\sin \theta} + \frac{\cos \theta}{\sin \theta} \\ &= \operatorname{cosec} \theta + \cot \theta \\ &= \text{R.H.S.} \end{aligned}$$

Example 17. Prove the identity :

$$\frac{\sec \theta - \tan \theta}{\sec \theta + \tan \theta} = 1 - 2 \sec \theta \tan \theta + 2 \tan^2 \theta.$$

Solution : L.H.S. = $\frac{1 - \sin \theta}{\cos \theta} \cdot \frac{\cos \theta}{1 + \sin \theta}$

$$= \frac{1 - \sin \theta}{\cos \theta} \times \frac{\cos \theta}{1 + \sin \theta}$$

$$= \frac{1 - \sin \theta}{1 + \sin \theta} \quad \dots (1)$$

R.H.S. = $1 - 2 \cdot \frac{1}{\cos \theta} \cdot \frac{\sin \theta}{\cos \theta} + 2 \cdot \frac{\sin^2 \theta}{\cos^2 \theta}$

$$= \frac{\cos^2 \theta - 2 \sin \theta + 2 \sin^2 \theta}{\cos^2 \theta}$$

$$= \frac{(\cos^2 \theta + \sin^2 \theta) - 2 \sin \theta + \sin^2 \theta}{\cos^2 \theta}$$

$$= \frac{1 - 2 \sin \theta + \sin^2 \theta}{1 - \sin^2 \theta}$$

$$= \frac{(1 - \sin \theta)^2}{(1 - \sin \theta)(1 + \sin \theta)}$$

$$= \frac{1 - \sin \theta}{1 + \sin \theta}$$

From equation (1) and (2)

\therefore L.H.S. = R.H.S.

Example 18. Prove the identity :

$$\frac{1}{\sec \theta - \tan \theta} - \frac{1}{\cos \theta} = \frac{1}{\cos \theta} - \frac{1}{\sec \theta + \tan \theta}.$$

Solution : On rearranging both the sides

$$\frac{1}{\sec \theta - \tan \theta} + \frac{1}{\sec \theta + \tan \theta} = \frac{2}{\cos \theta} = 2 \sec \theta$$

$$\begin{aligned}
\text{Now, L.H.S.} &= \frac{\sec \theta + \tan \theta + \sec \theta - \tan \theta}{(\sec \theta - \tan \theta)(\sec \theta + \tan \theta)} \\
&= \frac{2 \sec \theta}{\sec^2 \theta - \tan^2 \theta} \\
&= \frac{2 \sec \theta}{1 + \tan^2 \theta - \tan^2 \theta} \\
&= 2 \sec \theta \\
&= \text{R.H.S.}
\end{aligned}$$

Example 19. Prove the identity :

$$\sec^6 \theta - \tan^6 \theta = 1 + 3 \tan^2 \theta + 3 \tan^4 \theta.$$

Solution : We know that

$$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

$$\begin{aligned}
\text{Now, L.H.S.} &= (\sec^2 \theta)^3 - (\tan^2 \theta)^3 \\
&= (\sec^2 \theta - \tan^2 \theta)(\sec^4 \theta + \sec^2 \theta \tan^2 \theta + \tan^4 \theta) \\
&= (1 + \tan^2 \theta - \tan^2 \theta) \{ \sec^2 \theta (\sec^2 \theta + \tan^2 \theta) + \tan^4 \theta \} \\
&= 1 \cdot \{ (1 + \tan^2 \theta)(1 + \tan^2 \theta + \tan^2 \theta) + \tan^4 \theta \} \\
&= (1 + \tan^2 \theta)(1 + 2 \tan^2 \theta) + \tan^4 \theta \\
&= 1 + 3 \tan^2 \theta + 3 \tan^4 \theta \\
&= \text{R.H.S.}
\end{aligned}$$

Exercise 14.3

Prove the following identities :

1. $\cos \theta \cdot \tan \theta = \sin \theta$
2. $(1 - \sin^2 \theta) \tan^2 \theta = \sin^2 \theta$
3. $\frac{\cos^2 \theta}{\sin \theta} + \sin \theta = \operatorname{cosec} \theta$
4. $(\sin \theta + \cos \theta)^2 + (\sin \theta - \cos \theta)^2 = 2$
5. $\operatorname{cosec}^6 \theta - \cot^6 \theta = 1 + 3 \operatorname{cosec}^2 \theta \cot^2 \theta$

$$6. \quad \sin^2 \theta \cos \theta + \tan \theta \sin \theta + \cos^3 \theta = \sec \theta$$

$$7. \quad \frac{\cos \theta}{1 - \tan \theta} + \frac{\sin \theta}{1 - \cot \theta} = \sin \theta + \cos \theta$$

$$8. \quad \frac{\operatorname{cosec} \theta}{\operatorname{cosec} \theta - 1} + \frac{\operatorname{cosec} \theta}{\operatorname{cosec} \theta + 1} = 2 \sec^2 \theta$$

$$9. \quad \frac{\sin \theta}{1 - \cos \theta} = \frac{1 + \cos \theta}{\sin \theta}$$

$$10. \quad \frac{\sin \theta}{1 + \cos \theta} + \frac{1 + \cos \theta}{\sin \theta} = 2 \operatorname{cosec} \theta$$

$$11. \quad \sqrt{\frac{1 - \sin \theta}{1 + \sin \theta}} = \frac{1 - \sin \theta}{\cos \theta}$$

$$12. \quad \sqrt{\frac{\sec \theta + 1}{\sec \theta - 1}} = \cot \theta + \operatorname{cosec} \theta$$

$$13. \quad \frac{\sqrt{(\operatorname{cosec}^2 \theta - 1)}}{\operatorname{cosec} \theta} = \cos \theta$$

$$14. \quad (1 + \cot \theta - \operatorname{cosec} \theta)(1 + \tan \theta + \sec \theta) = 2$$

Important Points

1. In right angled triangle

$$(i) \quad \sin \theta = \frac{\text{Perpendicular}}{\text{Hypotenuse}}$$

$$(ii) \quad \cos \theta = \frac{\text{Base}}{\text{Hypotenuse}}$$

$$(iii) \quad \tan \theta = \frac{\text{Perpendicular}}{\text{Base}}$$

$$(iv) \quad \cot \theta = \frac{\text{Base}}{\text{Perpendicular}}$$

$$(v) \quad \sec \theta = \frac{\text{Hypotenuse}}{\text{Base}}$$

$$(vi) \quad \operatorname{cosec} \theta = \frac{\text{Hypotenuse}}{\text{Perpendicular}}$$

$$2. \quad (i) \quad \tan \theta = \frac{\sin \theta}{\cos \theta} \quad (ii) \quad \cot \theta = \frac{\cos \theta}{\sin \theta}$$

$$(iii) \quad \operatorname{cosec} \theta = \frac{1}{\sin \theta} \quad (iv) \quad \sec \theta = \frac{1}{\cos \theta}$$

$$3. \quad \sin^2 \theta + \cos^2 \theta = 1$$

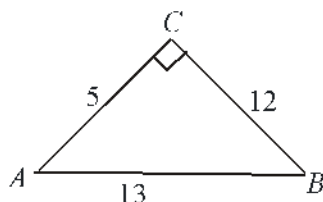
$$4. \quad 1 + \tan^2 \theta = \sec^2 \theta$$

$$5. \quad 1 + \cot^2 \theta = \operatorname{cosec}^2 \theta$$

Miscellaneous Exercise 14

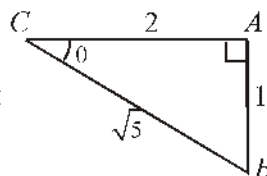
1. If $\tan \theta = \sqrt{3}$ then the value of $\sin \theta$ is :
 (A) $\frac{1}{\sqrt{3}}$ (B) $\frac{\sqrt{3}}{2}$ (C) $\frac{2}{\sqrt{3}}$ (D) 1
2. If $\sin \theta = \frac{5}{13}$ then the value of $\tan \theta$ is :
 (A) $\frac{5}{12}$ (B) $\frac{12}{13}$ (C) $\frac{13}{12}$ (D) $\frac{12}{5}$
3. If $\sqrt{3} \cos A = \sin A$ then the value of $\cot A$ is :
 (A) $\sqrt{3}$ (B) 1 (C) $\frac{1}{\sqrt{3}}$ (D) 2

4. In given $\triangle ABC$ the value of $\cot A$ is :



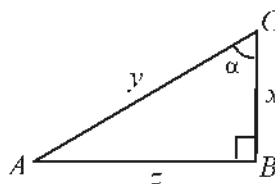
- (A) $\frac{12}{13}$ (B) $\frac{5}{12}$ (C) $\frac{5}{13}$ (D) $\frac{13}{5}$

5. In given $\triangle ABC$ the value of $\tan \theta$ is :



- (A) 2 (B) $\frac{1}{\sqrt{5}}$ (C) $\frac{2}{\sqrt{5}}$ (D) $\frac{1}{2}$

6. In given $\triangle ABC$ the value of $\operatorname{cosec} \alpha$ is :



- (A) $\frac{y}{x}$ (B) $\frac{y}{z}$ (C) $\frac{x}{z}$ (D) $\frac{x}{y}$

7. The value of $\sin^2 30^\circ + \cos^2 30^\circ$ is :

- (A) 0 (B) 2 (C) 3 (D) 1

8. The value of $\operatorname{cosec}^2 55^\circ - \cot^2 55^\circ$ is :
 (A) 1 (B) 2 (C) 3 (D) 0
9. If $\cot \phi = \frac{20}{21}$ then the value of $\operatorname{cosec} \phi$ is :
 (A) $\frac{21}{20}$ (B) $\frac{20}{29}$ (C) $\frac{29}{21}$ (D) $\frac{21}{29}$
10. If in $\triangle ABC$, $\angle B = 90^\circ$, $c = 12$ cm $a = 9$ cm then the value of $\cos C$ is :
 (A) $\frac{3}{5}$ (B) $\frac{3}{4}$ (C) $\frac{5}{3}$ (D) $\frac{4}{5}$
11. The value of $(\sec 40^\circ + \tan 40^\circ)(\sec 40^\circ - \tan 40^\circ)$ is :
 (A) -1 (B) 1 (C) $\cos 40^\circ$ (D) $\sin 40^\circ$
12. The value of $\frac{1}{\sin \theta - \tan \theta}$ is :
 (A) $\frac{\cot \theta}{\cos \theta - 1}$ (B) $\frac{\cot \theta}{\cot \theta - \operatorname{cosec} \theta}$
 (C) $\operatorname{cosec} \theta - \cot \theta$ (D) $\cot \theta$
13. The value of $\frac{\sec A - 1}{\sec A + 1}$ is:
 (A) $\frac{1 + \cos A}{1 - \cos A}$ (B) $\frac{\cos A - 1}{1 + \cos A}$ (C) $\frac{1 - \cos A}{1 + \cos A}$ (D) $\frac{\cos A - 1}{1 - \cos A}$
14. The value of $\cot^2 \theta - \frac{1}{\sin^2 \theta}$ is :
 (A) 2 (B) 1 (C) 0 (D) -1
15. If $\operatorname{cosec} \theta = \frac{41}{40}$ then find the value of $\tan \theta$ and $\cos \theta$.

16. If in $\triangle ABC$, $\angle B$ is right angle and $AB = 12$ cm and $BC = 5$ cm then find the value of $\sin A$, $\tan A$, $\sin C$ and $\cot C$.

17. If $\cos \theta = \frac{3}{5}$ then evaluate $\frac{\sin \theta - \cot \theta}{2 \tan \theta}$

18. If $\cos \theta = \frac{21}{29}$ then evaluate $\frac{\sec \theta}{\tan \theta - \sin \theta}$

19. If $\cot A = \sqrt{3}$ then prove that $\sin A \cos B + \cos A \sin B = 1$.

Using relation between trigonometric ratios prove the following [Q. 20-24].

20. If $\tan \theta = \frac{4}{3}$ then evaluate $\frac{3 \sin \theta + 2 \cos \theta}{3 \sin \theta - 2 \cos \theta}$.

21. If $\cot \theta = \frac{b}{a}$ then evaluate $\frac{\cos \theta + \sin \theta}{\cos \theta - \sin \theta}$.

22. If $\operatorname{cosec} A = 2$ then evaluate $\cot A + \frac{\sin A}{1 + \cos A}$.

23. If $\cot \theta = \frac{1}{\sqrt{3}}$ then evaluate $\frac{1 - \cos^2 \theta}{2 - \sin^2 \theta} = \frac{3}{5}$.

24. If $\sin A = \frac{1}{3}$ then evaluate $\cos A \operatorname{cosec} A + \tan A \sec A$.

Prove the following [Q. 25-27]

25. $\sqrt{\sec^2 A + \operatorname{cosec}^2 A} = \tan A + \cot A$

26. $\frac{1 + \sec \theta}{\sec \theta} = \frac{\sin^2 \theta}{1 - \cos \theta}$

27. $\frac{\tan A + \sec A - 1}{\tan A - \sec A + 1} = \tan A + \sec A$

Prove the following identities [Q. 28-29]

28. $\frac{\tan \alpha + \tan \beta}{\cot \alpha + \cot \beta} = \tan \alpha \tan \beta$

29. $\sin \theta = \frac{\tan \theta}{\sqrt{1 + \tan^2 \theta}}$

Prove the following [Q. 30-35]

30. $\cos^4 \theta - \sin^4 \theta = 1 - 2 \sin^2 \theta$

31. $\sec^2 \theta - \operatorname{cosec}^2 \theta = \tan^2 \theta - \cot^2 \theta$

32. $\frac{\sin A - \sin B}{\cos A + \cos B} + \frac{\cos A - \cos B}{\sin A + \sin B} = 0$

33. $(\sin A + \operatorname{cosec} A)^2 + (\cos A + \sec A)^2 = \tan^2 A + \cot^2 A + 7$

$$34. \frac{\tan A + \sec A - 1}{\tan A - \sec A + 1} = \frac{1 + \sin A}{\cos A}$$

Answers

Exercise 14.1

$$1. \quad \begin{array}{lll} \sin B = \frac{7}{25}, & \cos B = \frac{24}{25}, & \tan B = \frac{7}{24} \\ \operatorname{cosec} B = \frac{25}{7}, & \sec B = \frac{25}{24}, & \cot B = \frac{24}{7} \\ \sin C = \frac{24}{25}, & \cos C = \frac{7}{25}, & \tan C = \frac{24}{7} \\ \operatorname{cosec} C = \frac{25}{24}, & \sec C = \frac{25}{7}, & \cot C = \frac{7}{24} \end{array}$$

$$2. \quad \begin{array}{lll} \sin A = \frac{12}{13}, & \cos A = \frac{5}{13}, & \tan A = \frac{12}{5} \\ \operatorname{cosec} A = \frac{13}{12}, & \sec A = \frac{13}{5}, & \cot A = \frac{5}{12} \\ \sin C = \frac{5}{13}, & \cos C = \frac{12}{13}, & \tan C = \frac{5}{12} \\ \operatorname{cosec} C = \frac{13}{5}, & \sec C = \frac{13}{12}, & \cot C = \frac{12}{5} \end{array}$$

$$4. \quad 2\sqrt{2} + \frac{3}{8}$$

$$5. \quad \begin{array}{lll} \sin \theta = \frac{15}{17}, & \tan \theta = \frac{15}{8}, & \operatorname{cosec} \theta = \frac{17}{15}, \\ \sec \theta = \frac{17}{8}, & \cot \theta = \frac{8}{15} \end{array}$$

$$6. \quad \frac{169}{229}$$

$$7. \quad \frac{5}{14}$$

$$9. \quad 7$$

$$10. \quad \text{(i) } \frac{a}{\sqrt{4b^2 - 3a^2}} \quad \text{(ii) } \frac{2\sqrt{b^2 - a^2}}{\sqrt{4b^2 - 3a^2}} \quad \text{(iii) } \frac{a}{2\sqrt{b^2 - a^2}}$$

Exercise 14.2

1. $\cot A = \frac{3}{4}$, $\sin A = \frac{4}{5}$, $\cos A = \frac{3}{5}$
2. $\cos A = \frac{21}{29}$, $\sin A = \frac{20}{29}$
3. $\cos A = \frac{4}{5}$, $\tan A = \frac{3}{4}$
4. $\sin B = \frac{2\sqrt{2}}{3}$, $\tan B = 2\sqrt{2}$, $\cot B = \frac{1}{2\sqrt{2}}$,
 $\sec B = 3$, $\operatorname{cosec} B = \frac{3}{2\sqrt{2}}$
5. $\cos A = \frac{12}{13}$, $\tan A = \frac{5}{12}$
7. $\frac{12 - \sqrt{5}}{2}$
8. $\frac{1}{2}$
9. $\sin \theta = \frac{1}{\sqrt{2}}$, $\cot \theta = 1$
10. $\cos \theta = \frac{1}{2}$, $\sin \theta = \frac{\sqrt{3}}{2}$, $\tan \theta = \sqrt{3}$

Miscellaneous Exercise 14

1. (B) 2. (A) 3. (C) 4. (B) 5. (D) 6. (B)
7. (D) 8. (A) 9. (C) 10. (A) 11. (B) 12. (A)
13. (C) 14. (D)
15. $\tan \theta = \frac{40}{9}$, $\cos \theta = \frac{9}{41}$ 16. $\frac{5}{13}$, $\frac{5}{12}$, $\frac{12}{13}$, $\frac{5}{12}$ 17. $\frac{3}{160}$
18. $\frac{841}{160}$ 20. 3 21. $\frac{b+a}{b-a}$ 22. 2 24. $\frac{16\sqrt{2}+3}{8}$

□

15.01 Introduction :

The branch of science known as Statistics has been used in India from ancient times. Which is evident from the following examples.

In Mahabharata period, king Rituparna while going alone with King Nala for the Swaymbara of Damyanti, estimated accurately the number of leaves and fruits on the basis of sample of a tree. The description of administrative setup based on registration of births and deaths during the region of Chandragupta Maurya (324-300 B.C.) is found in Kautilya's Arthashastra. Similarly many examples of use of statistics in administrative setup and in wars are found. When we observe or notice in newspapers, channels of electronic media, magazines and communication, then we get the factual and comparative informations about the temperature of cities, position of rainfall.

In this chapter we shall try to study, in detail, about collection, classification, tabulation, mean, median and mode of such data. Situation of shares of different companies. We use the data in our whole life. It is very important for us that we know to produce meaningful information from the data.

The branch of mathematics in which we learn about these meaningful information is known as statistics.

15.02 Collection of data :

On the basis of method of collection, data can be divided into two categories :

(1) Primary data (2) Secondary data.

(1) Primary data :

Data which are collected for the first time by the statistical investigator or with the help of his workers is called primary data. e.g. The weight and height of the student in class 9th of school. We collect the primary data by following methods.

- (i) **Direct personal investigation :** In this method, the investigator himself contacts the person from whose he/she is to collect information.
- (ii) **Indirect investigation :** When the field of investigation is wide it is not possible

for the investigator to go and contact everyone personally. In such a situation he/she may try to collect the information in the following ways :

- (a) **Schedule to be filled in by enumerators :** In this method, the investigator prepares the schedules, gives it to the trained enumerators. Then the enumerators ask the questions to concerned informants and write the answer in schedules.
- (b) **Information through questionnaires to be filled in by informants :** In this method investigator prepares a questionnaire related to investigation and sends to the informants with the object to get information and assurance of information's secrecy.
- (c) **Information through local sources or correspondents :** When collection of daily information is required from different places, then investigator appoints local persons or correspondents for reporting the informations.
- (d) **Indirect investigation through experts :** In this method the information is not collected from the people directly involved in the investigation. Rather it is collected from people called witnesses who are indirectly involved. If the annual evaluation of the students is to be done without examination, then this can be done by the concerned teacher.

(2) Secondary data :

If this method the data are already collected by a person or a society and these may be in published or unpublished form. These are generally obtained from the following two sources :

- (i) **Published sources :** Government, non-government and other investigators collect data on different subjects from time to time and get it published, their main sources are as follows :
 - (a) **International publication :** Organisations like United Nations Organisation (U.N.O.), International Labour Organisation (I.L.O.), International Monetary Fund (I.M.F.) collect the data related with them and get them published for their members.
 - (b) **Government publication :** A number of Ministries and Departments (The department of Statistics) of central and state government collect data concerning them and published them.
 - (c) **Semi Government publication :** Municipalities, Corporations, Zila Parishads, Village Panchayats publish data on Education, Electricity, Birth and death and revenue records from time to time.

Similarly Universities, Research Institutes and teachers organisations compile different types of data and get them published.

(ii) **Unpublished sources** : Sometimes data on important subjects is collected by government and non-government institutions but they remain unpublished. Such unpublished matter can be obtained from office files, records, registers or the diaries of research scholars.

Exercise 15.1

1. What are primary and secondary data ? Clarify the difference.
2. Describe the methods of collection of primary data.

15.03 Presentation of Data :

After collection of data, an investigator has to select the suitable method to present the data in a form which is meaningful, easy to understand and gives its main features at a glance.

Data collected in original form is called '**raw data**', e.g. marks obtained by 10 students in a test of science are :

62 75 65 40 35 70 25 20 36 55

Can you obtain maximum and minimum marks from this data? If we arrange the data in ascending order or descending order, then we can easily obtain maximum and minimum marks. Ascending order of data are follows :

20 25 35 36 40 55 62 65 70 75

Here, it is clear that minimum marks is 20 and maximum marks is 75. Difference of maximum and minimum marks of data is called **range**. Thus, here

$75 - 20 = 55$ is the range.

If data are large, then it should be written in tabular form rather than in ascending or descending order.

Presentation of Data in Tabular Form :

In a school, following are the marks obtained by 20 students of class X in half yearly examination, out of 10 marks :

5 4 3 4 5 8 4 3 8 2
4 5 4 3 8 5 3 4 2 8

By seeing this raw data, it is difficult to estimate the level of class, but if we arrange

the data in tabular form, then this task will become very easy. "The number of times an observation occurs in the given data, is called the frequency of observation." It is denoted as " f ".

Following is the method to prepare a table from the above data :

- (i) In the first column of the table, we write all the marks from lowest to highest.
- (ii) We now look at the first value in the given raw data and put vertical line known as tally mark, in the second column opposite to it.
- (iii) When four lines are made opposite any mark, don't make the fifth line in the same way but make a line across the first four (||||)
- (iv) If a number occurs six times, then draw again a vertical line and repeat the process. (|||||)
- (v) After completing the entries of second column, count the number of tally marks against each observation and write in the third column.

Table

Marks	Tally Marks	Frequency (f)
2		2
3		4
4		6
5		4
8		4

$$\underline{\underline{\sum f = 20}}$$

This type of frequency distribution is called **ungrouped frequency distribution table**.

If number of observations is very large, then we divide the data in groups. These groups are called **class** and their measure is known as **class-interval** or **class size** or **class width**. Lower number of each class is known as **lower class limit** and maximum number is **upper class limit**. In next example we will understand the presentation in tabular form of data.

During Van Mahotsava 50 plants had planted in each school out of 30 schools.

After one month, following was the number of remaining plants.

22 6 48 0 28 22 17 10 32 6
 22 22 28 26 17 36 10 22 28 0
 28 22 48 32 10 48 25 36 6 32

Table

No. of Remaining Plants	Tally Marks	No. of students (frequency)
0 — 10		8
11 — 20		2
21 — 30		12
31 — 40		5
41 — 50		3
Total		$\underline{\underline{\sum f = 30}}$

Presentation of data, by this method is called '**Grouped frequency distribution**'.

We can easily estimate and conclude by inspection of this table.

Exercises 15.2

1. Given below are the weights (in kg.) of 30 students of class X of a school :

34 34 36 37 38 33 34 35 36 37 38 33 34 35 34
 33 37 35 34 36 38 36 35 34 35 37 38 34 35 35

Prepare a frequency distribution table for the above data.

2. Given below were the weights (in kg.) of 30 children born in a village :

3.4 3.6 3.0 3.8 3.6 3.8 2.9 3.4 2.9 3.4
 3.0 3.4 3.2 3.1 3.2 3.2 3.1 3.2 3.4 3.0
 3.1 3.2 3.5 3.7 3.1 3.0 2.9 3.0 3.1 3.2

Prepare a frequency distribution table for the above data.

3. Three coins were tossed 30 times. Each time the number of heads occurring was noted down as follow :

0 1 2 2 1 2 3 1 3 0
 1 3 1 1 2 2 0 1 2 1
 3 0 0 1 1 2 3 2 2 0

Prepare a frequency distribution table for the data given above.

4. The blood groups of 30 students of class X are recorded as follows :

AB O O A B O A O B A O B A O O
 A AB O A A O O AB B A O B A B O

Present this data in the form of a frequency distribution table. Find out which is the most common and which is the rarest blood group among these students.

5. Following are the marks obtained by 30 students of class IX in an examination. Use these marks to prepare a frequency table of 5 class size 10.

19 27 40 3 33 41 18 8 20 0 23 49 16 36 14
 39 6 12 29 28 22 24 37 10 23 38 35 9 49 23

6. Prepare a frequency distribution table for the following distribution of taking class interval 5.

13 11 8 19 0 44 27 10 8 35 13
 27 30 17 43 23 19 43 17 7

7. The value of π upto 50 decimal places is given below :

3.14159265358979323846264338327950288419716939937510

- (i) Prepare a frequency distribution table of the digits from 0 to 9 which comes after the decimal point.
 (ii) What are the most and the least frequently occurring digits ?

8. The distances from the residence of 40 engineers to their working palce (in km) are as follows :

5 3 10 20 25 11 13 7 12 31
 19 10 12 17 18 11 32 17 16 2
 3 18 15 12 5 3 8 7 9 7
 14 12 9 2 15 6 7 15 12 6

By taking first class interval 0-5 (5 not included) prepare a frequency distribution table of class size 5 for the above data . What main features do you observe from this tabular representation ?

9. It is asked to 30 students that how many hours they have studied in last week ? Result obtained are as follows :

2 3 5 8 6 9 8 7 14 12
 6 17 1 15 8 2 12 4 3 10
 3 2 6 1 12 5 8 5 8 4

- (i) Prepare a frequency distribution table from the above data, taking class size as 5.
- (ii) How many students have studied upto 15 hours or more than in one week ?

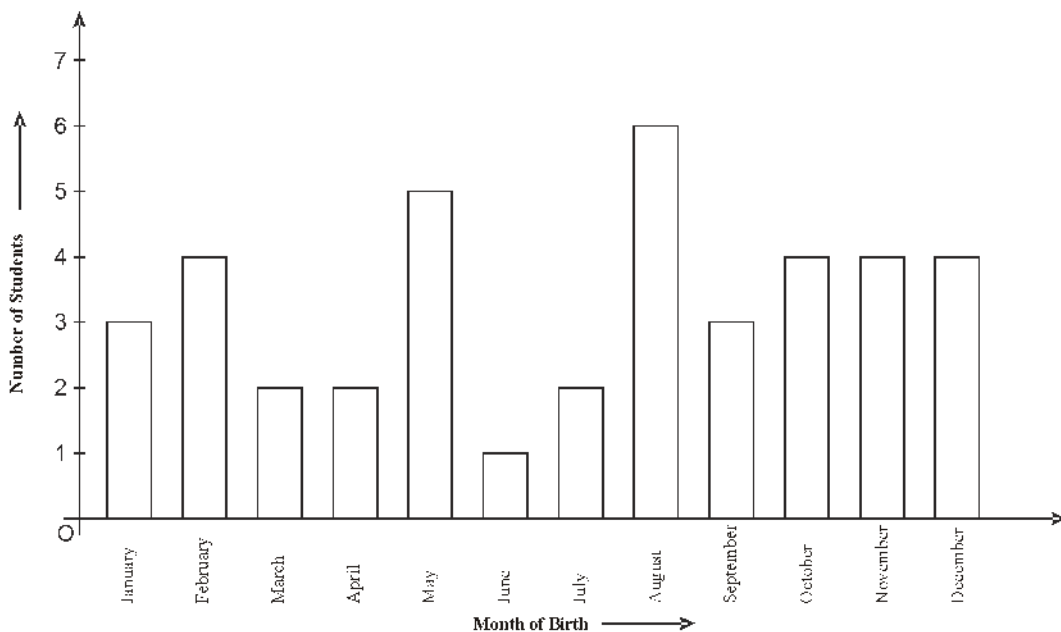
15.04 Graphical Representation of Data

After the creation of universe, man has developed science and mathematics according to his needs. In this development, statistics also developed and figures are being used for sending information which is known as graphical representation. Graphs are eye-catching and easier to understand than the actual data. We shall study the following three types of graphical representation of the data :

- (i) Bar Graph
- (ii) Histograms
- (iii) Frequency polygon

(i) Bar graphs : A bar graph is a pictorial representation of the numerical data, in which bars or rectangles of uniform width are drawn with equal spaces in between them on the axis. The height of bar is proportional to the numerical data it represents, we can understand this by the following example.

Example 1. In class IX, 40 students were asked about the months of their birth and the following graph was prepared for the data so obtained :



- (i) How many students were born in the month of May ?
- (ii) In which month maximum number of students were born ?
- (iii) In which month minimum number of students were born ?

Solution : Here variable is 'month of birth' and the value of variable is "number of students born".

- (i) 5 students born in the month of May.
- (ii) In August, the maximum number of students were born.
- (iii) In June, the minimum number of students were born.

Example 2. The family has planned the following expenditures per month under various heads : monthly income of a family is 20 thousand.

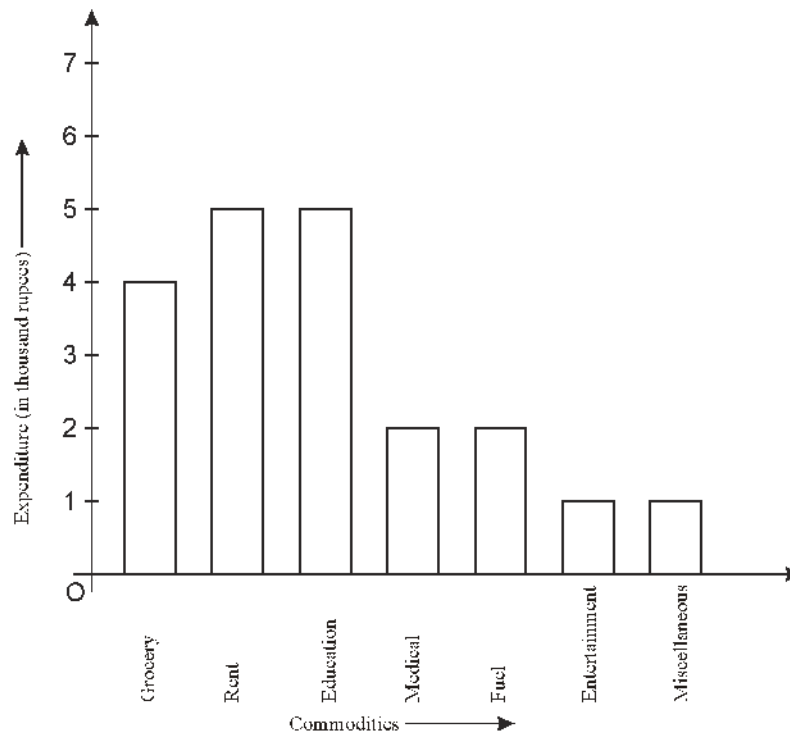
Table - 3

Commodities (variable)	Expenditure (in thousand rupees)
Grocery	4
Rent	5
Children's education	5
Medicine	2
Fuel	2
Entertainment	1
Miscellaneous	1

Preapre a bar graph for the above data.

Solution : Following are the steps to prepare a bar graph for the above data :

1. Take any scale and mark the variables on the horizontal axis. Since, the width of the bar is not important, but for clarity, we take equal width for all bars and maintain equal gaps in between. Let one variable be represented by one unit.
2. We repercent the expenditure on the vertical axis. Since the maximum expenditure is 5 thousand rupees, so we can choose the scale as 1 unit = 1000 rupees.
3. To represent first variable (Grocery), we draw a rectangle bar of width 1 unit and height 4 units.
4. Similarly, other variable are represented by giving the unit space in between consecutive bars. (See Fig. 15.02)



Here, we can easily compare the expenditures on different commodities. Thus, this is a best way instead of presentation of data in tabular form.

(ii) Histogram

A histogram is a graphical representation of a frequency distribution in the form of rectangle with class intervals as bases and heights proportional to corresponding frequencies. Choosing a suitable scale, mark class-limits on X-axis and frequencies on Y-axis, such that area of so formed rectangles should remain proportional to the corresponding frequencies. Here, we will study the construction of histograms related to four different types of frequency distribution.

- When frequency distribution is grouped and continuous and with equal class-intervals.
- When frequency distribution is grouped and continuous but with unequal class-intervals.
- When frequency distribution is grouped but not continuous.
- When frequency distribution is ungrouped and mid-point of class-intervals are given.

Now, we will clarify the above statements by the following examples.

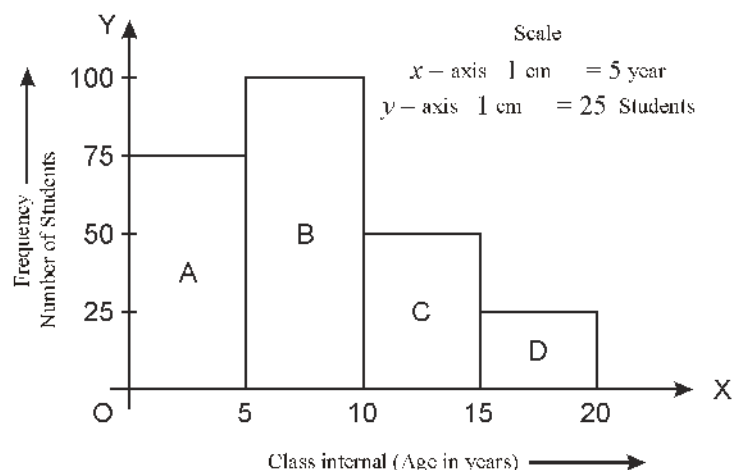
Example 3. Draw a histogram for the following frequency distribution:

Table-4

Class-interval (Age in Years)	0 — 5	5 — 10	10 — 15	15 — 20
No. of students	72	103	50	25

Solution : Here, frequency distribution is grouped and continuous and class-size is also same. Therefore, class-interval, *i.e.*, age in years (scale 1cm = 5 years) will be marked on x-axis.

Now, since number of students in class-interval 0-5 is 72. Taking class-interval 0-5 as base and the corresponding frequency as height, we construct rectangle. Similarly, we will construct rectangles B, C, D.



So, it is clear that all the rectangles have same base (1 cm) and height equal to frequency therefore area of rectangles should be proportional to frequency.

Example 4. The weekly wages of the workers of a company is given in the following table. Draw workers histogram for this data.

Table-5

Weekly Wages	1000 — 2000	2000 — 2500	2500 — 3000	3000 — 5000	5000 — 5500
Number of Workers	26	30	20	16	1

Solution : Here, frequency distribution is grouped and continuous but the class-interval is unequal, so to find heights of rectangles following method will be used, in which heights remains proportional to frequencies.

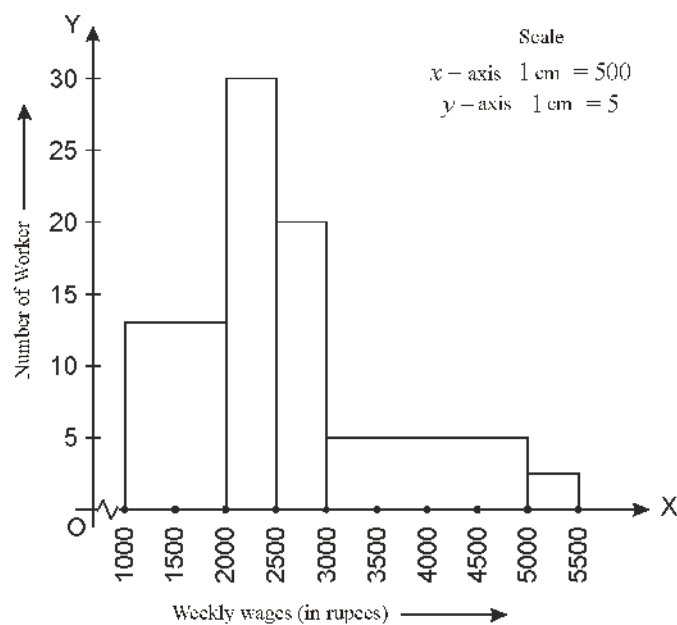
- (a) Write minimum class size (h) of class-interval, here $h = 500$
- (b) Compute the adjusted frequencies of each class by using the following formula :

$$\text{Adjusted frequency of a class} = \frac{h}{\text{class size}} \times \text{frequency of the class}$$

Thus, we obtain a new table as :

Table-6

Weekly Wages (in Rupees)	No. of Workers	Adjusted Frequencies (Adjusted No. of Workers)
1000 — 2000	26	$\frac{500}{1000} \times 26 = 13$
2000 — 2500	30	$\frac{500}{500} \times 30 = 30$
2500 — 3000	20	$\frac{500}{500} \times 20 = 20$
3000 — 5000	16	$\frac{500}{2000} \times 16 = 4$
5000 — 5500	1	$\frac{500}{500} \times 1 = 1$



Draw class interval on X -axis (scale 1 cm = ₹ 500) and number of workers on Y -axis (scale 1 cm = 5 workers). Since, the first class interval (1000-2000) is starting from 1000 and not zero, so we show it on the graph by making a *kink* or a break on the axis. Now, we construct rectangles A, B, C, D, E with class-limits as bases and respective adjusted frequencies as heights.

Example 5. Draw a histogram for the frequency distribution :

Class-interval	10 — 19	20 — 29	30 — 39	40 — 49	50 — 59
Frequency	2	3	5	7	3

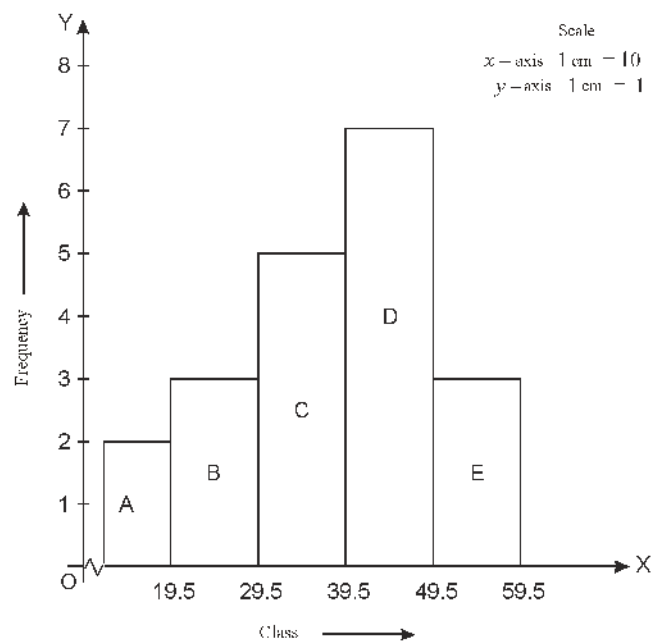
Solution : Here, frequency distribution is grouped but not continuous so we first convert it into continuous frequency distribution. After making certain modifications, we get the following Table-8.

Table - 8

Class-interval	9.5 — 19.5	19.5 — 29.5	29.5 — 39.5	39.5 — 49.5	49.5 — 59.5
Frequency	2	3	5	7	3

Therefore, we will draw class-interval (scale 1cm = 10) on x -axis and frequencies on y -axis (scale 1 cm = 1).

Since, the first class interval (9.5-19.5) is starting from 9.5 and not zero, so we show it on the graph by making a kink or a break on the axis. Thus, the required histogram is shown above.



Example 6. Draw a histogram for the following frequency distribution :

Table - 9

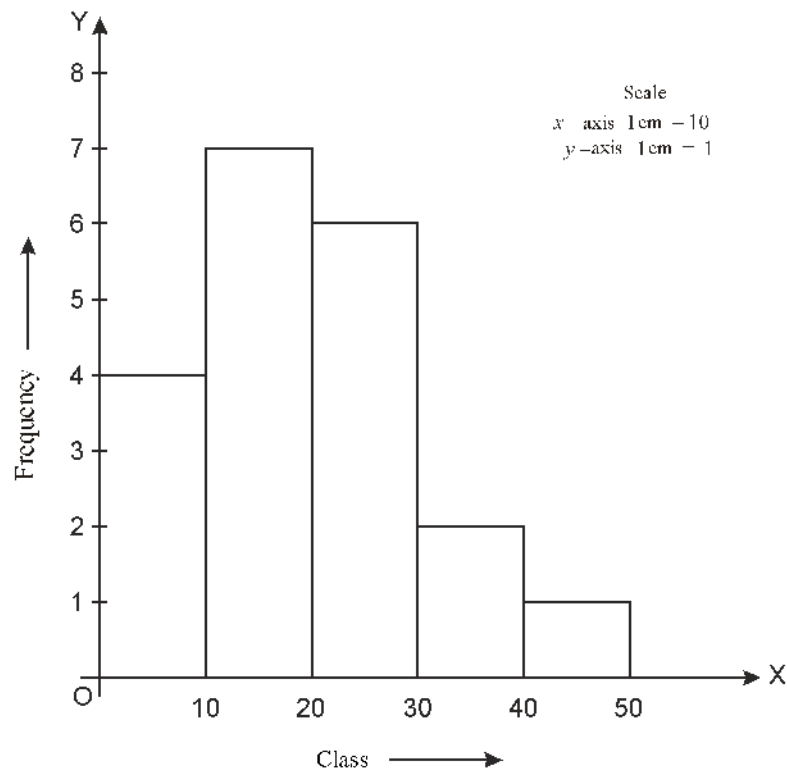
Mid point	5	15	25	35	45
Frequency	4	7	6	2	1

Solution : Here frequency distribution is ungrouped and mid points of the distribution are given. So, after converting in grouped frequency distribution. We get the following table :

Table - 10

Class	0 — 10	10 — 20	20 — 30	30 — 40	40 — 50
Frequency	4	7	6	2	1

∴ This frequency distribution is grouped and continuous and class-size is also same. So, we will draw class-intervals on X -axis (scale : 1 cm = 10) and frequencies on Y -axis (scale : 1 cm = 1) and obtained the following histogram.



(3) Frequency Polygon

Frequency polygon is another method of representing frequency distributions graphically. Frequency polygon can be drawn in two ways :

1. By using histogram
2. Without using histogram

1. In order to draw a frequency polygon by using histogram, we may follow the following process :

- (i) Draw a histogram for the given frequency distribution.
- (ii) Obtain the mid points of the upper horizontal side of each rectangle.
- (iii) Join these mid points respectively, by straight lines.
- (iv) The mid points at each end are joined to the immediately lower or higher mid-point at zero frequency, *i.e.*, on the X -axis.
- (v) Join the mid point of last rectangle to its next possible mid point of class-interval on X -axis.

The figure so obtained is called a frequency polygon.

Remark :

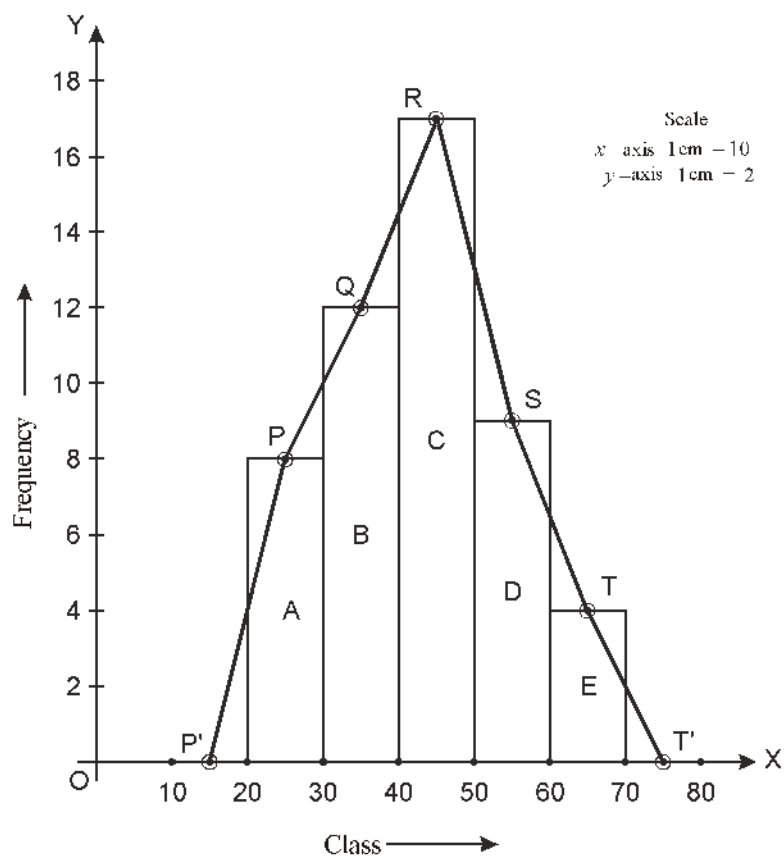
If there is no possibility of part (iv) & (v) for example, in case of marks obtained by the students in a test, we cannot go below zero and beyond maximum marks on the two sides. In such cases the extreme line-segments are only partly drawn and are brought down vertically so that they meet with the vertical sides of the first and last rectangles.

Example 7. Prepare a frequency polygon by making histogram for the following frequency distribution :

Table-11

Class-interval	20 — 30	30 — 40	40 — 50	50 — 60	60 — 70
Frequency	8	12	17	9	4

Solution : Given frequency distribution is grouped and continuous so we draw histogram by the method mentioned earlier. Join the mid points P, Q, R, S, T of the upper horizontal side of rectangle A, B, C, D, E respectively, by straight lines.



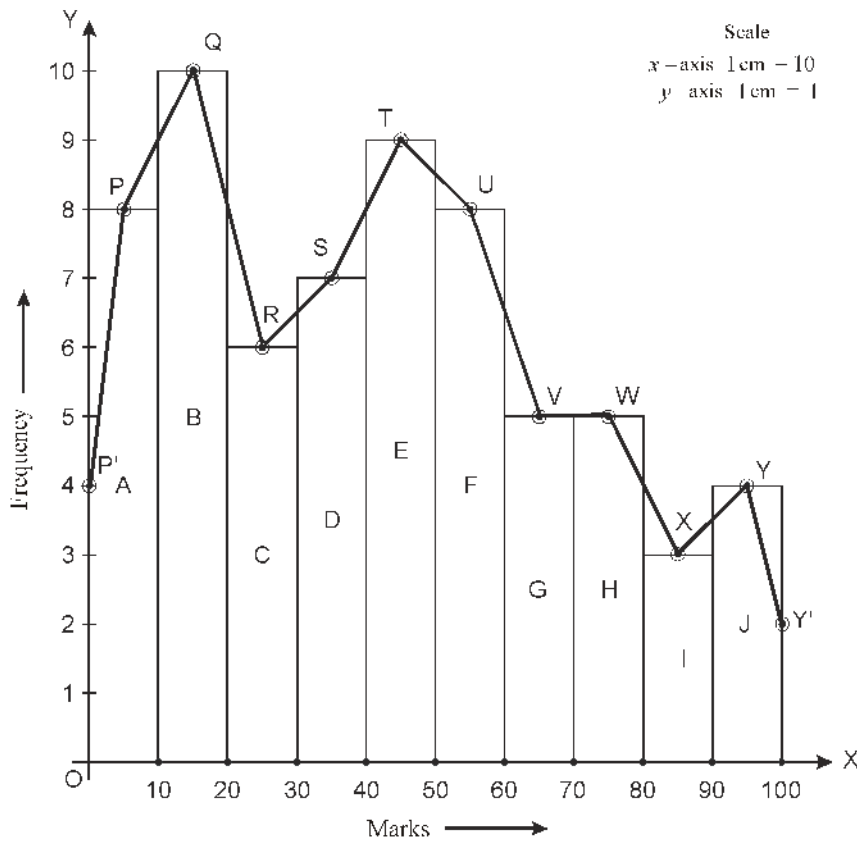
Since, here nothing is said about imagined classes so by joining mid points of class intervals from the beginning and end points, we get frequency polygon $P'PQRSTT'$. Thus, the required frequency polygon is shown in fig. 15.7.

Example 8. Make a frequency polygon by drawing a histogram for following frequency distribution.

Table - 12

Marks	0	10	10	20	20	30	30	40	40	50	50	60	60	70	70	80	80	90	90	100
No. of Students		8		10		6		7		9		8		8		6		3		4

Solution : Given frequency distribution is grouped and continuous, so we draw histogram according to the method learnt earlier.



Join the mid-points $P, Q, R, S, T, U, V, W, X, Y$ of the upper horizontal side of rectangles $A, B, C, D, E, F, G, H, I, J$ respectively, by straight lines.

Since, here marks obtained by students cannot be below 0 and beyond 100 so imaginary intervals do not exist. The first mid-point, i.e., P is joined to zero frequency the point where this line segment meets the vertical axis is marked as P' . Let Y be the mid-point of the class succeeding the last class of the given data. Thus, $OP'PQRSTUVWXY$ is the frequency polygon, which is shown in Fig. 15.08.

2. Following is the working method to draw a frequency polygon without making histogram:

- (i) If frequency distribution is grouped, then find class mark. Now this frequency distribution will be converted into ungrouped.
- (ii) Represent class marks on x-axis on a suitable scale.
- (iii) Represent class marks on y-axis on a suitable scale.

- (iv) Plot the points $(x_1, f_1), (x_2, f_2) \dots$
- (v) Join these points by line segments.
- (vi) Join these ends by mid points of the imagined classes adjacent to them and thus obtained frequency polygon for given frequency distribution.
- (v) Join these points by line segments.
- (vi) Join these ends by mid points of the imagined classes adjacent to them and thus obtained frequency polygon for given frequency distribution.

Remark:

1. For any type of frequency distribution, polygon can be made easily.
2. If to make imaginary classes is not possible, then draw vertical lines at the starting point and end point of these class-intervals and join them with the mid point of class-interval. Thus, we will get required frequency polygon.

Example 9: Draw frequency polygon for given frequency distribution.

Table-13

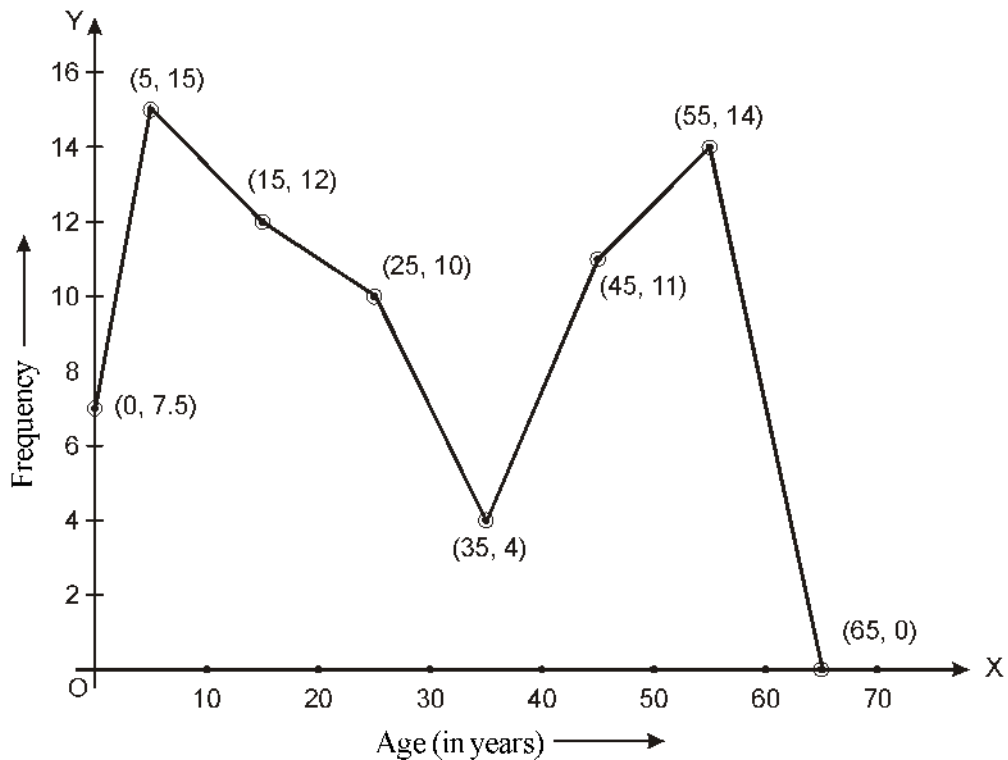
Age (in years)	0 — 10	10 — 20	20 — 30	30 — 40	40 — 50	50 — 60
Frequency	15	12	10	4	11	14

Solution : The frequency distribution is grouped and continuous. So, we obtained the following table on the basis of class.

Table-14

Age (in years)	0 — 10	10 — 20	20 — 30	30 — 40	40 — 50	50 — 60
Class mark	5	15	25	35	45	55
Frequency	15	12	10	4	11	14

Now, mark the points $(5, 15), (15, 12), (25, 10), (35, 4), (45, 11), (55, 14)$ assuming suitable scale.



Since, age cannot be negative, so instead of imaginary classes, draw vertical lines at the starting point and end point of these class intervals and join them with the mid points of class interval. Thus, we will get required frequency polygon for given frequency distribution as shown in the figure.

Example 10. Draw frequency polygon for the following frequency distribution.

Table-15

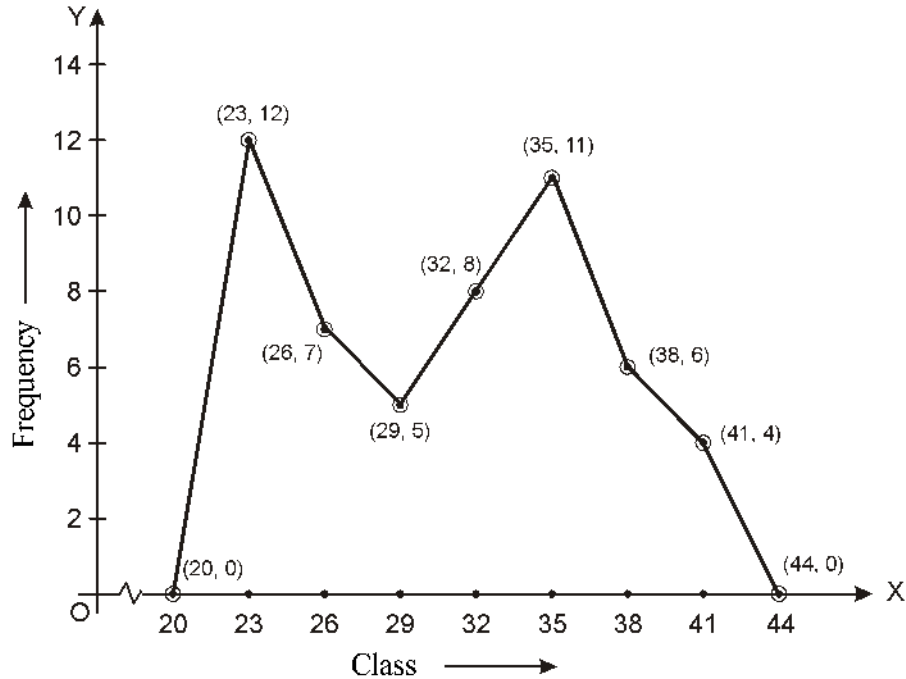
Class	22 — 24	25 — 27	28 — 30	31 — 33	34 — 36	37 — 39	40 — 42
Frequency	12	7	5	8	11	6	4

Solution : Here, frequency distribution is grouped but not continuous because draw after making continuous class there is not change in class mark. Thus, instead of making continuous, we will find class mark and then draw frequency polygon.

Table - 16

Class	22 — 24	25 — 27	28 — 30	31 — 33	34 — 36	37 — 39	40 — 42
Class mark	23	26	29	32	35	38	41
Frequency	12	7	5	8	11	6	4

By assuming suitable scale, we will mark the points (23, 12), (26, 7), (29, 5), (32, 8), (35, 11), (38, 6), (41, 4) on graph paper.



Join points (23, 12) to (20, 0) and (41, 4) to point (44, 0). Thus, we get the required frequency polygon.

Exercise 15.3

1. An organisation have conducted a survey in the whole world to find out the reason behind the diseases and death of females of age group 15-44 (in years) and obtained the following data (in %).

Table -17

S. No.	Reasons	Female death rate (%)
1.	Fertility health stage	31.8
2.	Nerve Psychiatric stage	25.4
3.	Loss	12.4
4.	Heart vessel stage	4.3
5.	Breathign stage	4.1
6.	Other reasons	22.0

- (i) Express above informations by bar graph.
- (ii) Which stage is the major cause of bad health and death of females in the whole world?

2. The following data on the number of girls (to the nearest ten) per thousand boys in different sections of Indian society is given below:

Table - 18

S.No.	Area	No. of Girls per thousand boys
1.	Scheduled Caste	940
2.	Scheduled Tribe	970
3.	Non SC/Tribe	920
4.	Backward districts	950
5.	Non-backward districts	920
6.	Rural	930
7.	Urban	910

- (i) Represent the above informations by bar graph.
 (ii) Discuss, what conclusion can be drawn from this graph?
3. In an assembly elections, results of winning seats by various parties are as follows:
- | | | | | | | |
|-----------------|----|----|----|----|----|----|
| Political Party | A | B | C | D | E | F |
| Winning Seats | 75 | 55 | 37 | 29 | 10 | 37 |
- (i) Draw a bar graph to represent the results of election.
 (ii) Which party won maximum seats?
 From the following tables draw a histogram (4 to 8)

Table - 20

4. Class	0 — 5	5 — 10	10 — 15	15 — 20	20 — 25
Frequency	18	15	14	8	10

Table - 21

5. Class	0 — 20	20 — 40	40 — 60	60 — 80	80 — 100
Frequency	5	6	12	4	3

Table - 22

6. Class	3 — 6	6 — 12	12 — 13	13 — 14	14 — 15
Frequency	150	420	100	110	50

Table - 23

7. Class	5 — 9	10 — 14	15 — 19	20 — 24
Frequency	3	5	8	2

Table - 24

8.	Class	8	14	20	26	32
	Frequency	10	15	25	9	6

9. Draw a frequency polygon with the help of a histogram for the following distribution. Maximum number is 10.

Table - 25

Marks	0 — 5	5 — 10	10 — 15	15 — 20	20 — 25	25 — 30	30 — 35
Frequency	1	2	4	6	5	3	2

10. Draw a frequency polygon with the help of a histogram for the following distribution. Maximum number is 10.

Table - 26

Marks	0 — 2	2 — 4	4 — 6	6 — 8	8 — 10
Number of students	7	8	4	9	2

11. Draw a frequency polygon for the following frequency distribution.

Table - 27

Variable x	5	10	15	20	25	30
Frequency f	2	6	4	1	5	2

12. Draw a frequency polygon for the following frequency distribution.

Table - 28

Production (in tonnes)	0 — 10	10 — 20	20 — 30	30 — 40	40 — 50	50 — 60	60 — 70
Frequency	8	18	23	37	47	26	16

Measures of Central Tendency

In this chapter, we have studied the presentation of data in various forms - by frequency distribution tables, bar graphs, histograms and frequency polygons.

In order to make these data meaningful, there is always need to study all the data. This can be done with the help of measures of central tendency or averages.

Let us consider a situation, in which two students Parveen and Akash have received their test copies.

In test, there were five questions of 10 marks each. Following are the marks obtained by them.

Table - 29

Question Numbers	1	2	3	4	5
Marks obtained by Praveen	10	8	9	8	7
Marks obtained by Akash	4	7	10	10	10

$$\text{Average marks obtained by Praveen} = \frac{42}{5} = 8.4$$

$$\text{Average marks obtained by Akash} = \frac{41}{5} = 8.2$$

Since average marks of Praveen was greater than average marks of Akash. So Praveen says his performance is good but Akash was not agree with this. He kept marks obtained by both in ascending order and then obtained mid marks.

Table - 30

Marks obtained by Praveen	7	8	8	9	10
Marks obtained by Akash	4	10	10	10	10

Akash says his mid marks obtained was 10 whereas of Praveen's was 8. So his performance was better than Praveen, but Praveen was not agree with Akash to convince Praveen, Akash make a trick. He says, he had got 10 marks 3 times whereas Praveen have got only once. Thus his performance is better.

In first stage, Praveen has got average marks, that is '**Mean**'. Mid marks which was used by Akash in his discussion is **Median**. In second discussion Akash has maximum marks got maximum times, is **mode**.

Now, we will discuss deeply on Mean.

The arithmetic mean of a set of observations is equal to their sum divided by the total number of observations and is denoted by \bar{x} and read as (\bar{x} bar).

Example 11. Daily wages of 5 workers are ₹ 250, ₹ 200, ₹ 225, ₹ 300, ₹ 275. Find their mean.

Solution : Mean of observations

$$(\bar{x}) = \frac{\text{Sum of all observations}}{\text{Total no. of observations}} = \frac{x_1 + x_2 + x_3 + x_4 + x_5}{5}$$

$$= \frac{250 + 200 + 225 + 300 + 275}{5} = \frac{1250}{5} = ₹250$$

Thus, average wages of 5 persons = ₹ 250

If we want to find the mean of 30 persons, we will write it as : $x_1 + x_2 + x_3 \dots + x_{30}$ which is a lengthy process. We use Σ for summations. So, $x_1 + x_2 + x_3 \dots + x_{30}$ can be

written as $\sum_{j=1}^{30} x_j$

$$\bar{x} = \frac{\sum_{j=1}^{30} x_j}{30}$$

Similarly, if number of observation = n

$$\bar{x} = \frac{\sum_{j=1}^n x_j}{n}$$

Median : Median of the given number of observations is the value of the variable which divides it exactly into two equal parts. So, when data is written in ascending or descending order, then median of ungrouped data is calculated as:

(i) If number of observation (n) is odd, then

$$\text{Median} = \text{value of } \left(\frac{n+1}{2}\right)^{\text{th}} \text{ observation}$$

For example, if $n = 13$ then

Median will be the value of $\left(\frac{13+1}{2}\right)^{\text{th}}$ observation, i.e., value of 7th observation.

(ii) If number of observation (n) is even, then

$$\text{Median} = \frac{\text{value of } \left(\frac{n}{2}\right)^{\text{th}} \text{ observation} + \text{value of } \left(\frac{n}{2} + 1\right)^{\text{th}} \text{ observation}}{2}$$

For example, if $n = 16$, then

$$\text{Median} = \frac{\text{value of } 8^{\text{th}} \text{ observation} + \text{value of } 9^{\text{th}} \text{ observation}}{2}$$

Example 12 : Following are the heights (in cm) of 9 students of a class. Find the median of these data:

155 160 145 149 150 147 152 144 148

Solution : Arranging the given data in ascending order, we have

144 145 147 148 149 150 152 155 160

Here, number of students is 9 i.e., which is an odd number

$$\begin{aligned} \therefore \text{Median} &= \text{value of } \left(\frac{n+1}{2}\right)^{\text{th}} \text{ observation} \\ &= \text{value of } \left(\frac{9+1}{2}\right)^{\text{th}} \text{ observation} \\ &= \text{value of } 5^{\text{th}} \text{ observation} \\ &= 149 \text{ cm} \end{aligned}$$

Example 13 : Following are the points obtained by a Kabaddi team in various matches:

17 2 7 27 15 5 14 8 10 24
48 10 8 7 18 28

Find the median of the points obtained by the team.

Solution : Arranging the given data in ascending order, we have

2 5 7 7 8 8 10 10 14 15
17 18 24 27 28 48

Here, number of observations is 16 which is an even number.

$$\begin{aligned} \therefore \text{Median} &= \frac{\left(\frac{16}{2}\right)^{\text{th}} \text{ observation} + \left(\frac{16}{2} + 1\right)^{\text{th}} \text{ observation}}{2} \\ &= \frac{8^{\text{th}} + 9^{\text{th}} \text{ observation}}{2} \\ &= \frac{10 + 14}{2} = \frac{24}{2} = 12 \end{aligned}$$

Mode:

Mode is the value of the observation which occurs most frequently. It means a variable with maximum frequency is called mode.

This measure of central tendency is frequently used in readymade garment and shoe industry. With the help of mode, these industries take decision that production of which product would be increased.

Example 14 : Find the mode of the marks obtained by 20 students (out of 10), from the followign data:

4, 6, 5, 9, 3, 2, 7, 7, 6, 5, 4, 9, 10, 10, 3, 4, 7, 6, 9, 9

Solution : Arranging the given data in ascending order, we have

2, 3, 3, 4, 4, 4, 5, 5, 6, 6, 6, 7, 7, 7, 9, 9, 9, 9, 10, 10

Here, 9 occurs maximum number of time, i.e., 4 times.

Thus, mode is 9.

Exercise 15.4

1. The following number of goals were scored by a team in a series of 10 matches:

2 3 4 5 0 1 3 3 4 3

Find the mean, median and mode of these scores.

2. In a mathematics test given to 15 students, the following marks (out of 100) are recorded:

41 39 48 52 46 62 54 40 96 52
98 40 42 52 60

Find the mean, median and mode for above data.

3. The following observations are arranged in ascending order. If the median of the data is 63, then find the value of x :

29 32 48 50 x $x + 2$ 72 78 84 95

4. Find the mode of 14 25 14 28 18 17 18 14
23 22 14 18

5. Find the mean salary of 60 workers, working in a factory, from the following table:

Table - 31

Salary (in ₹)	3000	4000	5000	6000	7000	8000	9000	10000
No. of workers	16	12	10	8	6	4	3	1

Miscellaneous Exercise - 15

1. In a distribution 5, 5, 6, 4, 9, 5, 3, 2, 7, 6, 3, 8, 4 frequency of class interval 3 - 5 is

(A) 3 (B) 4 (C) 6 (D) 7

2. Range of the following distribution will be

3.2 2.8 3.1 2.1 3.2 2.4 2.1 2.8 2.7 2.7

(A) 2.7 (B) 3.1 (C) 2.4 (D) 1.1

3. In the following distribution, number of students of age less than 25 years is

Age (in years)	5-10	10 - 15	15 - 20	20 - 25	25 - 30
No. of students	3	6	8	8	2

(A) 8 (B) 16 (C) 9 (D) 25

4. In a bar graph, height of rectangle is

(A) inversely proportional to frequency of class

(B) proportional to frequency of class

(C) proportional to class-interval

(D) inversely proportional to class-interval

5. The examination result of any class of a school can be comparatively studied:

(A) by circular diagram (B) by bar graph

(C) by linear graph (D) all of these

6. The range of distribution 6, 1, 2, 3, 9, 8, 3, 4, 8, 2, 3 will be

(A) 4 (B) 8 (C) 7 (D) 6

7. In the distribution 5, 1, 5, 2, 3, 6, 5, 4 frequency of 5 will be
(A) 1 (B) 2 (C) 3 (D) 4
8. The median of 11, 2, 7, 8, 9, 3, 5 will be
(A) 7 (B) 9 (C) 5 (D) 11
9. The mean of 15, 0, 10, 5 will be
(A) 15 (B) 10 (C) 5 (D) 7.5
10. In 4, 3, 4, 5, 4, 2, 4, 1 mode will be
(A) 1 (B) 2 (C) 5 (D) 4

Very Short Answer Type Question

11. Write the frequency of class-interval 0 - 5 from the following distribution:
3, 2, 0, 10, 8, 5, 13, 5, 6, 6, 0, 14
12. If 7 is the mean of the numbers 5, 8, 4, x , 6, 9 then find the value of x
13. What is range?
14. What is histogram?
15. Make a frequency table of 9, 7, 9, 8, 3, 9, 8, 3, 5, 7, 5, 3.
16. If arithmetic mean of a frequency distribution is 15 and $\sum f = 20$ then find the value of $\sum fx$
17. Write median of distribution 5, 2, 3, 7, 5, 4, 3, 2, 1
18. Find median of distribution 12, 1, 6, 4, 10, 8, 1, 4
19. Find mode of distribution 4, 3, 4, 1, 2, 4, 7, 5, 3

Importants Points

1. The data are of two types : (i) primary data (ii) secondary data.
2. The methods of collecting the primary data
(i) By direct personal (individual) investigation, (ii) By indirect investigation.
3. Indirectly data is obtained by the following ways:
(i) Filling the list by statisticians, (ii) Filling the list by informer,
(iii) By local sources of correspondent,
(iv) Direct oral investigation by the specialists.
4. Data which have already collected whether published or not, is called secondary data.
5. Secondary data may be published or not.
6. Following are the main sources of published data:
(i) International organisation, (ii) Government publication,
(iii) Semi-government publication, (iv) Publication of business organisation,
(v) Publication of research centres, (vi) Paper-magazines,
(vii) Published papers of research scholars.
7. Generally, x is used for marks is frequency distribution.
8. Range is the difference between maximum and minimum value of variable x .
9. Maximum and minimum value of a class is known as its upper and lower limit.
10. Average of upper and lower limit of a class is called mid vlaue of class or class mark and is represented as x
11. The following graphs are mainly used in statistical analysis:
(a) Bar graph, (b) Histogram, (c) Frequency polygon
12. Three measures of central tendency for ungrouped data are:
 - (i) **Mean** : The mean of a set of observation is equal to their sum divided by the total number of observations, i.e.,

$$\bar{x} = \frac{\sum f x}{\sum f}$$

- (ii) **Median** : Middle most value (observation) is called median.
- (iii) **Mode** : Observation which occurs maximum times is called mode.

Answer

Exercise 15.1

1. **Primary data :** When an investigator collects data himself with a definite plan or design in his mind, it is called primary data.

Secondary data : Data which are not originally collected rather obtained from published or unpublished sources is known as secondary data.

Exercise 15.2

1.

Variable (x)	33	34	35	36	37	38
Tally makrs						
Frequency (f)	3	8	7	4	4	4

2.

Variable (x)	2.9	3.0	3.1	3.2	3.3	3.4	3.5	3.6	3.7	3.8
Tally makrs					—					
Frequency (f)	3	5	5	6	—	5	1	2	1	2

3.

No. of ends	Tally makrs	Frequency (f)
0		6
1		10
2		9
3		5
Total		30

4.

Blood group	Tally makrs	No. of students
A		9
B		6
O		12
AB		3
Total		30

5.

Class interval	Tally makrs	Frequency
0 — 10		5
10 — 20		6

20 — 30		9
30 — 40		6
40 — 50		4

6.

Class	Tally Marks	Frequency (<i>f</i>)
0 — 5		1
5 — 10		3
10 — 15		4
15 — 20		4
20 — 25		1
25 — 30		2
30 — 35		1
35 — 40		1
40 — 45		3

7. (i)

Digits	0	1	2	3	4	5	6	7	8	9	Total
Frequency	2	5	5	8	4	5	4	4	5	8	50

(ii) 3 and 9 are the numbers which occur maximum and number 0 occurs minimum

8.

Distance (in km)	Tally Marks	Frequency
0 — 5		5
5 — 10		11
10 — 15		11
15 — 20		9
20 — 25		1
25 — 30		1
30 — 35		2

9. Number of hours	Frequency
0 — 5	10
5 — 10	13
10 — 15	5
15 — 20	2
Total	30

(ii) 2 Children

Exercise 15.3

- (ii) Fertility health stage
- (ii) Party A

Exercise 15.4

- Mean = 2.8, Median = 3, Mode = 3
- Mean = 54.8, Median = 52, Mode = 52
- $x = 62$
- 14
- Mean salary of 60 workers is ₹ 5083.33.

Miscellaneous Exercise - 15

- (b)
- (d)
- (d)
- (d)
- (b)
- (b)
- (c)
- (a)
- (d)
- (d)
- 4
- 10

13. Range is the difference between the maximum and the minimum value of variable x .

14. Histogram is a rectangular form of grouped and continuous frequency distribution.

15.

x	3	5	7	8	9
f	3	2	2	2	3

- 300
- 3
- 5
- 4

Percentages

Objective :

Concept of percentage is brought out through the number of fatal accidents.

Content :

In 2009, a total of 7516 accidents were recorded in which 2325 persons lost their lives and 6936 were injured. Calculate percentage of total road accident victims who killed. Of fatal accidents, 1170 pedestrians and 2677 were injured. Calculate percentage of pedestrians-

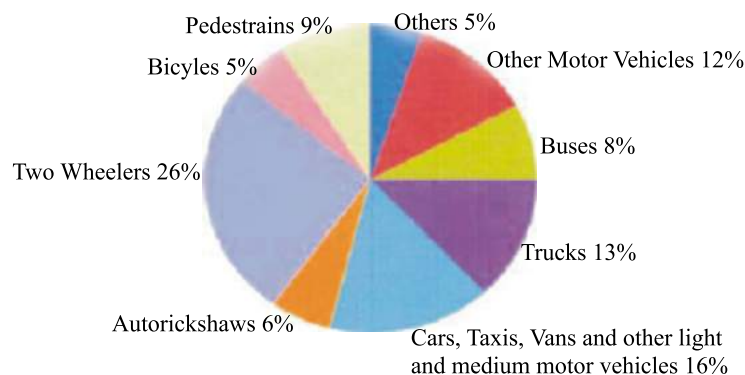
- (a) Killed (b) Injured

Pedestrians are the most vulnerable road users. Can you suggest some measures of safety for them? 2-wheeler drivers accounted for 691 of all those killed and 2358 of all the injured victims in 2009. Calculate percentage of two-wheeler killed and injured. Can you suggest some measures of road safety for these drivers?

In 2009, 1993 males were killed and 158 females were killed. By what percentage are more males killed. If 174 children were killed, what is the percentage of children who have lost their lives on Delhi Roads?

Activity :

Profile of people killed on roads in detail is given in the form of a pie chart.



- Express each percentage in decimal form.
- If 1,33,938 persons were killed, calculate how many pedestrians were killed?

Circles

Objective :

Circular road signs give order. Identification of common circular road signs.

Content :

You have already studied that -

- Triangles warn
- Rectangles inform

Now we study about a circle. A circular traffic sign gives us order.



Blue circle gives a positive instruction. Draw one such traffic sign.



Red circle gives a negative instruction. Draw one such traffic sign.

Find out two traffic sign which do not follow these rules.



Activity :

Locate all the traffic signs in your locality. Find out what percentage of the traffic signs give order.

Statistics

Objective :

Circular road signs give order. Identification of common circular road signs.

Content :

Bar graphs are pictorial representation of numerical data.

Exercise :

The data of various vehicles for the year 1980 and 1990 in percentage is given below:

Vehicle	1980	1990
Private Car	22.48	21.74
Motor Cycle/Scooter	64.13	67.50
Taxi	1.20	0.58
MCR/TSR	3.82	3.51
Goods Vehicles	6.85	5.61
Buses	1.52	1.06

Draw a bar graph of the above data.

1. Compare the percentage of taxis in the year 1980 and 1990.
2. What is the difference in the percentage of private cars and taxis in the year 1990?



Quadrilaterals

Objective :

Using road signs to determine the area of quadrilaterals.

Content :

Students are introduced to the topic quadrilaterals in which they can differentiate between a square, a rectangle, a parallelogram, a rhombus etc. traffic signs are in various shapes.



Size of rectangle is 20 inches x 18 inches.

- Q1.** Find the cost of making two rectangular boards @ ₹10 per square inch.
An informatory sign board, rectangular in shape, blue in colour is given below.



- Q2.** It has a white square with Red Cross in it. Rectangle is 30"x25" and square is 20"x20". Find the area of blue region.

Activity :

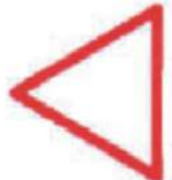
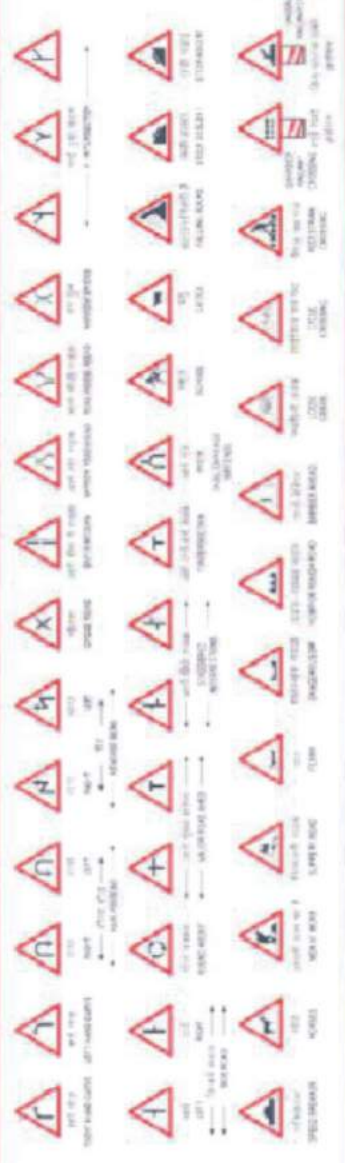
Visit any traffic training institute and look at the board which show traffic signs of all shapes.

Draw these signs on a chart paper and state which of them are squares and which are rectangles. Also state which of them are parallelograms?

ROAD SIGN Conforming to IRC : 67-1977 Standard



Mandatory



Warning



Information

Probability

Objective :

Children will be given the knowledge of probability.

$$P(e) = \text{number of favourable outcomes} / \text{total outcomes}$$



Exercise :

Traffic lights remain green, orange or red for 90 seconds on a busy road crossing. Suresh, while travelling to his office noted that sometimes the traffic signal was red, sometimes green and sometimes orange. He noted this for 10 days and found the following:

SINGLE	RED	ORANGE	GREEN
10	3	3	4

Q1. Find the probability of Suresh getting:

1. Green light
2. Red light at red light crossing

Q2. What is the sequence of traffic lights?



Data

Laws and regulations help us to be aware of the road safety system. A specific eligibility of 18 years is given for obtaining a licence and children should not break this law. Over 500 minors were prosecuted in 2009. Find out how many students drive in your class. What is the fraction of the students driving to the overall strength of the class? Compare it with students of other sections and make a bar graph.

The fine for underage driving is Rs. 300 and if a minor is caught driving a vehicle, the owner of the vehicle can be fined an amount of Rs. 1,000. If an underage driver causes any accident, he can be looked under IPC 304 A or IPC 337, and can be sent to juvenile home.



Seat belts help a driver to lead a safe and secure life. 2,55,686 persons were prosecuted for not wearing a helmet in Delhi. **Find out penalty for this violation. What is total challan collected for this offence this year?**

11,084 persons were prosecuted for not wearing a seat belt in Delhi. **Find out the penalty for this violation. What is the total challan collected for this offence this year?**



Section 177 of Motor Vehicle Act can punish a driver who is not wearing seatbelt with a fine upto Rs. 100.

Drunken driving is a major cause of fatal accidents. 8296 people were prosecuted in 2008 but 12,784 in 2009 in Delhi. **Find out increase in the number of such cases.**



Section 185 of Motor Vehicle Act can punish a driver under the influence of alcohol with a fine upto Rs. 2000 or imprisonment with a term which may extend upto 6 months. With a subsequent offence within 3 years, imprisonment increases to 2 years and a fine of Rs. 3,000.

