

EXERCISE 2.1

PAGE NO: 2.8

1. Express each of the following as a rational number of the form p/q , where p and q are integers and $q \neq 0$:

(i) 2^{-3}

(ii) $(-4)^{-2}$

(iii) $1/(3)^{-2}$

(iv) $(1/2)^{-5}$

(v) $(2/3)^{-2}$

Solution:

(i) $2^{-3} = 1/2^3 = 1/2 \times 2 \times 2 = 1/8$ (we know that $a^{-n} = 1/a^n$)

(ii) $(-4)^{-2} = 1/4^2 = 1/4 \times 4 = 1/16$ (we know that $a^{-n} = 1/a^n$)

(iii) $1/(3)^{-2} = 3^2 = 3 \times 3 = 9$ (we know that $1/a^{-n} = a^n$)

(iv) $(1/2)^{-5} = 2^5 / 1^5 = 2 \times 2 \times 2 \times 2 \times 2 = 32$ (we know that $a^{-n} = 1/a^n$)

(v) $(2/3)^{-2} = 3^2 / 2^2 = 3 \times 3 / 2 \times 2 = 9/4$ (we know that $a^{-n} = 1/a^n$)

2. Find the values of each of the following:

(i) $3^{-1} + 4^{-1}$

(ii) $(3^0 + 4^{-1}) \times 2^2$

(iii) $(3^{-1} + 4^{-1} + 5^{-1})^0$

(iv) $((1/3)^{-1} - (1/4)^{-1})^{-1}$

Solution:

(i) $3^{-1} + 4^{-1}$

$1/3 + 1/4$ (we know that $a^{-n} = 1/a^n$)

LCM of 3 and 4 is 12

$(1 \times 4 + 1 \times 3)/12$

$(4+3)/12$

$7/12$

(ii) $(3^0 + 4^{-1}) \times 2^2$

$(1 + 1/4) \times 4$ (we know that $a^{-n} = 1/a^n$, $a^0 = 1$)

LCM of 1 and 4 is 4

$(1 \times 4 + 1 \times 1)/4 \times 4$

$(4+1)/4 \times 4$

$$\frac{5}{4} \times 4$$
$$5$$

(iii) $(3^{-1} + 4^{-1} + 5^{-1})^0$
(We know that $a^0 = 1$)
 $(3^{-1} + 4^{-1} + 5^{-1})^0 = 1$

(iv) $((\frac{1}{3})^{-1} - (\frac{1}{4})^{-1})^{-1}$
 $(3^1 - 4^1)^{-1}$ (we know that $1/a^{-n} = a^n$, $a^{-n} = 1/a^n$)
 $(3-4)^{-1}$
 $(-1)^{-1}$
 $1/-1 = -1$

3. Find the values of each of the following:

(i) $(\frac{1}{2})^{-1} + (\frac{1}{3})^{-1} + (\frac{1}{4})^{-1}$

(ii) $(\frac{1}{2})^{-2} + (\frac{1}{3})^{-2} + (\frac{1}{4})^{-2}$

(iii) $(2^{-1} \times 4^{-1}) \div 2^{-2}$

(iv) $(5^{-1} \times 2^{-1}) \div 6^{-1}$

Solution:

(i) $(\frac{1}{2})^{-1} + (\frac{1}{3})^{-1} + (\frac{1}{4})^{-1}$
 $2^1 + 3^1 + 4^1$ (we know that $1/a^{-n} = a^n$)
 $2+3+4 = 9$

(ii) $(\frac{1}{2})^{-2} + (\frac{1}{3})^{-2} + (\frac{1}{4})^{-2}$
 $2^2 + 3^2 + 4^2$ (we know that $1/a^{-n} = a^n$)
 $2 \times 2 + 3 \times 3 + 4 \times 4$
 $4+9+16 = 29$

(iii) $(2^{-1} \times 4^{-1}) \div 2^{-2}$
 $(\frac{1}{2^1} \times \frac{1}{4^1}) / (\frac{1}{2^2})$ (we know that $a^{-n} = 1/a^n$)
 $(\frac{1}{2} \times \frac{1}{4}) \times \frac{4}{1}$ (we know that $1/a \div 1/b = 1/a \times b/1$)
 $\frac{1}{2}$

(iv) $(5^{-1} \times 2^{-1}) \div 6^{-1}$
 $(\frac{1}{5^1} \times \frac{1}{2^1}) / (\frac{1}{6^1})$ (we know that $a^{-n} = 1/a^n$)
 $(\frac{1}{5} \times \frac{1}{2}) \times \frac{6}{1}$ (we know that $1/a \div 1/b = 1/a \times b/1$)
 $\frac{3}{5}$

4. Simplify:

(i) $(4^{-1} \times 3^{-1})^2$

(ii) $(5^{-1} \div 6^{-1})^3$

(iii) $(2^{-1} + 3^{-1})^{-1}$

(iv) $(3^{-1} \times 4^{-1})^{-1} \times 5^{-1}$

Solution:

(i) $(4^{-1} \times 3^{-1})^2$ (we know that $a^{-n} = 1/a^n$)

$(1/4 \times 1/3)^2$

$(1/12)^2$

$(1 \times 1 / 12 \times 12)$

$1/144$

(ii) $(5^{-1} \div 6^{-1})^3$

$((1/5) / (1/6))^3$ (we know that $a^{-n} = 1/a^n$)

$((1/5) \times 6)^3$ (we know that $1/a \div 1/b = 1/a \times b/1$)

$(6/5)^3$

$6 \times 6 \times 6 / 5 \times 5 \times 5$

$216/125$

(iii) $(2^{-1} + 3^{-1})^{-1}$

$(1/2 + 1/3)^{-1}$ (we know that $a^{-n} = 1/a^n$)

LCM of 2 and 3 is 6

$((1 \times 3 + 1 \times 2)/6)^{-1}$

$(5/6)^{-1}$

$6/5$

(iv) $(3^{-1} \times 4^{-1})^{-1} \times 5^{-1}$

$(1/3 \times 1/4)^{-1} \times 1/5$ (we know that $a^{-n} = 1/a^n$)

$(1/12)^{-1} \times 1/5$

$12/5$

5. Simplify:

(i) $(3^2 + 2^2) \times (1/2)^3$

(ii) $(3^2 - 2^2) \times (2/3)^{-3}$

(iii) $((1/3)^{-3} - (1/2)^{-3}) \div (1/4)^{-3}$

(iv) $(2^2 + 3^2 - 4^2) \div (3/2)^2$

Solution:

(i) $(3^2 + 2^2) \times (1/2)^3$

$(9 + 4) \times 1/8 = 13/8$

$$\begin{aligned} \text{(ii)} & (3^2 - 2^2) \times (2/3)^{-3} \\ & (9-4) \times (3/2)^3 \\ & 5 \times (27/8) \\ & 135/8 \end{aligned}$$

$$\begin{aligned} \text{(iii)} & ((1/3)^{-3} - (1/2)^{-3}) \div (1/4)^{-3} \\ & (3^3 - 2^3) \div 4^3 \text{ (we know that } 1/a^{-n} = a^n) \\ & (27-8) \div 64 \\ & 19 \times 1/64 \text{ (we know that } 1/a \div 1/b = 1/a \times b/1) \\ & 19/64 \end{aligned}$$

$$\begin{aligned} \text{(iv)} & (2^2 + 3^2 - 4^2) \div (3/2)^2 \\ & (4 + 9 - 16) \div (9/4) \\ & (-3) \times 4/9 \text{ (we know that } 1/a \div 1/b = 1/a \times b/1) \\ & -4/3 \end{aligned}$$

6. By what number should 5^{-1} be multiplied so that the product may be equal to $(-7)^{-1}$?

Solution:

Let us consider a number x

$$\text{So, } 5^{-1} \times x = (-7)^{-1}$$

$$1/5 \times x = 1/-7$$

$$\begin{aligned} x &= (-1/7) / (1/5) \\ &= (-1/7) \times (5/1) \\ &= -5/7 \end{aligned}$$

7. By what number should $(1/2)^{-1}$ be multiplied so that the product may be equal to $(-4/7)^{-1}$?

Solution:

Let us consider a number x

$$\text{So, } (1/2)^{-1} \times x = (-4/7)^{-1}$$

$$1/(1/2) \times x = 1/(-4/7)$$

$$\begin{aligned} x &= (-7/4) / (2/1) \\ &= (-7/4) \times (1/2) \\ &= -7/8 \end{aligned}$$

8. By what number should $(-15)^{-1}$ be divided so that the quotient may be equal to $(-5)^{-1}$?

Solution:

Let us consider a number x

$$\text{So, } (-15)^{-1} \div x = (-5)^{-1}$$

$$1/-15 \times 1/x = 1/-5$$

$$1/x = (1 \times -15)/-5$$

$$1/x = 3$$

$$x = 1/3$$

