EXERCISE 2.2

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1. Write each of the following in exponential form:

(i) $(3/2)^{-1} \times (3/2)^{-1} \times (3/2)^{-1} \times (3/2)^{-1}$

(ii) $(2/5)^{-2} \times (2/5)^{-2} \times (2/5)^{-2}$

Solution:

(i) $(3/2)^{-1} \times (3/2)^{-1} \times (3/2)^{-1} \times (3/2)^{-1}$

$(3/2)^{-4}$ (we know that $a^{-n} = 1/a^n$, $a^n = a \times a \ldots n$ times)

(ii) $(2/5)^{-2} \times (2/5)^{-2} \times (2/5)^{-2}$

$(2/5)^{-6}$ (we know that $a^{-n} = 1/a^n$, $a^n = a \times a \ldots n$ times)

2. Evaluate:

(i) 5^{-2}

(ii) $(-3)^{-2}$

(iii) $(1/3)^{-4}$

(iv) $(-1/2)^{-1}$

Solution:

(i) 5^{-2}

$1/5^2 = 1/25$ (we know that $a^{-n} = 1/a^n$)

(ii) $(-3)^{-2}$

$(1/-3)^2 = 1/9$ (we know that $a^{-n} = 1/a^n$)

(iii) $(1/3)^{-4}$

$3^4 = 81$ (we know that $1/a^{-n} = a^n$)

(iv) $(-1/2)^{-1}$

$-2^1 = -2$ (we know that $1/a^{-n} = a^n$)

3. Express each of the following as a rational number in the form p/q:

(i) 6^{-1}

(ii) $(-7)^{-1}$

(iii) $(1/4)^{-1}$

(iv) $(-4)^{-1} \times (-3/2)^{-1}$

(v) $(3/5)^{-1} \times (5/2)^{-1}$

Solution:

(i) 6^{-1}

$1/6^1 = 1/6$ (we know that $a^{-n} = 1/a^n$)

(ii) $(-7)^{-1}$
$1/-7^1 = -1/7$ (we know that $a^{-n} = 1/a^n$)

(iii) $(1/4)^{-1}$
$4^1 = 4$ (we know that $1/a^{-n} = a^n$)

(iv) $(-4)^{-1} \times (-3/2)^{-1}$
$1/-4^1 \times (2/-3)^1$ (we know that $a^{-n} = 1/a^n$, $1/a^{-n} = a^n$)
$1/-2 \times -1/3$
$1/6$

(v) $(3/5)^{-1} \times (5/2)^{-1}$
$(5/3)^1 \times (2/5)^1$
$5/3 \times 2/5$
$2/3$

4. Simplify:
(i) $(4^{-1} \times 3^{-1})^2$
(ii) $(5^{-1} \div 6^{-1})^3$
(iii) $(2^{-1} + 3^{-1})^{-1}$
(iv) $(3^{-1} \times 4^{-1})^{-1} \times 5^{-1}$
(v) $(4^{-1} - 5^{-1}) \div 3^{-1}$
Solution:
(i) $(4^{-1} \times 3^{-1})^2$
$(1/4 \times 1/3)^2$ (we know that $a^{-n} = 1/a^n$)
$(1/12)^2$
$1/144$

(ii) $(5^{-1} \div 6^{-1})^3$
$(1/5 \div 1/6)^3$ (we know that $a^{-n} = 1/a^n$)
$(1/5 \times 6)^3$ (we know that $1/a \div 1/b = 1/a \times b/1$)
$(6/5)^3$
$216/125$

(iii) $(2^{-1} + 3^{-1})^{-1}$
$(1/2 + 1/3)^{-1}$ (we know that $a^{-n} = 1/a^n$)
LCM of 2 and 3 is 6

$((3+2)/6)^{-1}$
$(5/6)^{-1}$ (we know that $1/a^{-n} = a^n$)
6/5

(iv) $(3^{-1} \times 4^{-1})^{-1} \times 5^{-1}$
$(1/3 \times 1/4)^{-1} \times 1/5$ (we know that $a^{-n} = 1/a^n$)
$(1/12)^{-1} \times 1/5$ (we know that $1/a^{-n} = a^n$)
$12 \times 1/5$
12/5

(v) $(4^{-1} – 5^{-1}) \div 3^{-1}$
$(1/4 – 1/5) \div 1/3$ (we know that $a^{-n} = 1/a^n$)
LCM of 4 and 5 is 20
$(5-4)/20 \times 3/1$ (we know that $1/a \div 1/b = 1/a \times b/1$)
$1/20 \times 3$
3/20

5. Express each of the following rational numbers with a negative exponent:
(i) $(1/4)^3$
(ii) 3^5
(iii) $(3/5)^4$
(iv) $((3/2)^4)^{-3}$
(v) $((7/3)^4)^{-3}$
Solution:
(i) $(1/4)^3$
$(4)^{-3}$ (we know that $1/a^n = a^{-n}$)

(ii) 3^5
$(1/3)^{-5}$ (we know that $1/a^n = a^{-n}$)

(iii) $(3/5)^4$
$(5/3)^{-4}$ (we know that $(a/b)^{-n} = (b/a)^n$)

(iv) $((3/2)^4)^{-3}$
$(3/2)^{-12}$ (we know that $(a^n)^m = a^{nm}$)

(v) $((7/3)^4)^{-3}$
$(7/3)^{-12}$ (we know that $(a^n)^m = a^{nm}$)

6. Express each of the following rational numbers with a positive exponent:
(i) $(3/4)^{-2}$
(ii) $(5/4)^{-3}$
(iii) $4^3 \times 4^{-9}$
(iv) $((4/3)^{-3})^{-4}$
(v) $((3/2)^4)^{-2}$
Solution:
(i) $(3/4)^{-2}$
$(4/3)^2$ (we know that $(a/b)^{-n} = (b/a)^n$)

(ii) $(5/4)^{-3}$
$(4/3)^3$ (we know that $(a/b)^{-n} = (b/a)^n$)

(iii) $4^3 \times 4^{-9}$
$(4)^{3-9}$ (we know that $a^n \times a^m = a^{n+m}$)
4^{-6}
$(1/4)^6$ (we know that $1/a^n = a^{-n}$)

(iv) $((4/3)^{-3})^{-4}$
$(4/3)^{12}$ (we know that $(a^n)^m = a^{nm}$)

(v) $((3/2)^4)^{-2}$
$(3/2)^{-8}$ (we know that $(a^n)^m = a^{nm}$)
$(2/3)^8$ (we know that $1/a^n = a^{-n}$)

7. Simplify:
(i) $((1/3)^{-3} – (1/2)^{-3}) \div (1/4)^{-3}$
(ii) $(3^2 – 2^2) \times (2/3)^{-3}$
(iii) $((1/2)^{-1} \times (-4)^{-1})^{-1}$
(iv) $(((-1/4)^2)^{-2})^{-1}$
(v) $((2/3)^2)^3 \times (1/3)^{-4} \times 3^{-1} \times 6^{-1}$
Solution:
(i) $((1/3)^{-3} – (1/2)^{-3}) \div (1/4)^{-3}$
$(3^3 - 2^3) \div 4^3$ (we know that $1/a^n = a^{-n}$)
$(27-8) \div 64$
$19 \div 64$
$19 \times 1/64$ (we know that $1/a \div 1/b = 1/a \times b/1$)
$19/64$

(ii) $(3^2 – 2^2) \times (2/3)^{-3}$
$(9 - 4) \times (3/2)^3$ (we know that $1/a^n = a^{-n}$)
$5 \times (27/8)$
$135/8$

(iii) $((1/2)^{-1} \times (-4)^{-1})^{-1}$
$(2^1 \times (1/-4))^{-1}$ (we know that $1/a^n = a^{-n}$)
$(1/-2)^{-1}$ (we know that $1/a^n = a^{-n}$)
-2^1
-2

(iv) $(((-1/4)^2)^{-2})^{-1}$
$((-1/16)^{-2})^{-1}$ (we know that $1/a^n = a^{-n}$)
$((-16)^2)^{-1}$ (we know that $1/a^n = a^{-n}$)
$(256)^{-1}$ (we know that $1/a^n = a^{-n}$)
$1/256$

(v) $((2/3)^2)^3 \times (1/3)^{-4} \times 3^{-1} \times 6^{-1}$
$(4/9)^3 \times 3^4 \times 1/3 \times 1/6$ (we know that $1/a^n = a^{-n}$)
$(64/729) \times 81 \times 1/3 \times 1/6$
$(64/729) \times 27 \times 1/6$
$32/729 \times 27 \times 1/3$
$32/729 \times 9$
$32/81$

8. By what number should 5^{-1} be multiplied so that the product may be equal to $(-7)^{-1}$?
Solution:
Let us consider a number x
So, $5^{-1} \times x = (-7)^{-1}$
$1/5 \times x = 1/-7$ (we know that $1/a^n = a^{-n}$)
$x = (-1/7) / (1/5)$
$= (-1/7) \times (5/1)$ (we know that $1/a \div 1/b = 1/a \times b/1$)
$= -5/7$

9. By what number should $(1/2)^{-1}$ be multiplied so that the product may be equal to $(-4/7)^{-1}$?
Solution:
Let us consider a number x

So, $(1/2)^{-1} \times x = (-4/7)^{-1}$
$1/(1/2) \times x = 1/(-4/7)$ (we know that $1/a^n = a^{-n}$)
$x = (-7/4) / (2/1)$
$= (-7/4) \times (1/2)$ (we know that $1/a \div 1/b = 1/a \times b/1$)
$= -7/8$

10. By what number should $(-15)^{-1}$ be divided so that the quotient may be equal to $(-5)^{-1}$?
Solution:
Let us consider a number x
So, $(-15)^{-1} \div x = (-5)^{-1}$ (we know that $1/a \div 1/b = 1/a \times b/1$)
$1/-15 \times 1/x = 1/-5$ (we know that $1/a^n = a^{-n}$)
$1/x = (1\times-15)/-5$
$1/x = 3$
$x = 1/3$

11. By what number should $(5/3)^{-2}$ be multiplied so that the product may be $(7/3)^{-1}$?
Solution:
Let us consider a number x
So, $(5/3)^{-2} \times x = (7/3)^{-1}$
$1/(5/3)^2 \times x = 1/(7/3)$ (we know that $1/a^n = a^{-n}$)
$x = (3/7) / (3/5)^2$
$= (3/7) / (9/25)$
$= (3/7) \times (25/9)$ (we know that $1/a \div 1/b = 1/a \times b/1$)
$= (1/7) \times (25/3)$
$= 25/21$

12. Find x, if
(i) $(1/4)^{-4} \times (1/4)^{-8} = (1/4)^{-4x}$
Solution:
$(1/4)^{-4} \times (1/4)^{-8} = (1/4)^{-4x}$
$(1/4)^{-4-8} = (1/4)^{-4x}$ (we know that $a^n \times a^m = a^{n+m}$)
$(1/4)^{-12} = (1/4)^{-4x}$
When the bases are same we can directly equate the coefficients
$-12 = -4x$
$x = -12/-4$
$= 3$

(ii) $(-1/2)^{-19} \div (-1/2)^{8} = (-1/2)^{-2x+1}$

Solution:
$(-1/2)^{-19} \div (-1/2)^{8} = (-1/2)^{-2x+1}$
$(1/2)^{-19-8} = (1/2)^{-2x+1}$ (we know that $a^n \div a^m = a^{n-m}$)
$(1/2)^{-27} = (1/2)^{-2x+1}$
When the bases are same we can directly equate the coefficients
$-27 = -2x+1$
$-2x = -27-1$
$x = -28/-2$
$= 14$

(iii) $(3/2)^{-3} \times (3/2)^{5} = (3/2)^{2x+1}$
Solution:
$(3/2)^{-3} \times (3/2)^{5} = (3/2)^{2x+1}$
$(3/2)^{-3+5} = (3/2)^{2x+1}$ (we know that $a^n \times a^m = a^{n+m}$)
$(3/2)^{2} = (3/2)^{2x+1}$
When the bases are same we can directly equate the coefficients
$2 = 2x+1$
$2x = 2-1$
$x = 1/2$

(iv) $(2/5)^{-3} \times (2/5)^{15} = (2/5)^{2+3x}$
Solution:
$(2/5)^{-3} \times (2/5)^{15} = (2/5)^{2+3x}$
$(2/5)^{-3+15} = (2/5)^{2+3x}$ (we know that $a^n \times a^m = a^{n+m}$)
$(2/5)^{12} = (2/5)^{2+3x}$
When the bases are same we can directly equate the coefficients
$12 = 2+3x$
$3x = 12-2$
$x = 10/3$

(v) $(5/4)^{-x} \div (5/4)^{-4} = (5/4)^{5}$
Solution:
$(5/4)^{-x} \div (5/4)^{-4} = (5/4)^{5}$
$(5/4)^{-x+4} = (5/4)^{5}$ (we know that $a^n \div a^m = a^{n-m}$)
When the bases are same we can directly equate the coefficients
$-x+4 = 5$
$-x = 5-4$
$-x = 1$
$x = -1$

(vi) $(8/3)^{2x+1} \times (8/3)^5 = (8/3)^{x+2}$

Solution:

$(8/3)^{2x+1} \times (8/3)^5 = (8/3)^{x+2}$

$(8/3)^{2x+1+5} = (8/3)^{x+2}$ (we know that $a^n \times a^m = a^{n+m}$)

$(8/3)^{2x+6} = (8/3)^{x+2}$

When the bases are same we can directly equate the coefficients

$2x+6 = x+2$

$2x-x = -6+2$

$x = -4$

13. (i) If $x= (3/2)^2 \times (2/3)^{-4}$, find the value of x^{-2}.

Solution:

$x= (3/2)^2 \times (2/3)^{-4}$

$= (3/2)^2 \times (3/2)^4$ (we know that $1/a^n = a^{-n}$)

$= (3/2)^{2+4}$ (we know that $a^n \times a^m = a^{n+m}$)

$= (3/2)^6$

$x^{-2} = ((3/2)^6)^{-2}$

$= (3/2)^{-12}$

$= (2/3)^{12}$

(ii) If $x = (4/5)^{-2} \div (1/4)^2$, find the value of x^{-1}.

Solution:

$x = (4/5)^{-2} \div (1/4)^2$

$= (5/4)^2 \div (1/4)^2$ (we know that $1/a^n = a^{-n}$)

$= (5/4)^2 \times (4/1)^2$ (we know that $1/a \div 1/b = 1/a \times b/1$)

$= 25/16 \times 16$

$= 25$

$x^{-1} = 1/25$

14. Find the value of x for which $5^{2x} \div 5^{-3} = 5^5$

Solution:

$5^{2x} \div 5^{-3} = 5^5$

$5^{2x+3} = 5^5$ (we know that $a^n \div a^m = a^{n-m}$)

When the bases are same we can directly equate the coefficients

$2x+3 = 5$

$2x = 5-3$

$2x = 2$

$x = 1$