

## EXERCISE 4.1

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**1. Find the cubes of the following numbers:****(i) 7 (ii) 12****(iii) 16 (iv) 21****(v) 40 (vi) 55****(vii) 100 (viii) 302****(ix) 301****Solution:****(i) 7**

Cube of 7 is

$$7 = 7 \times 7 \times 7 = 343$$

**(ii) 12**

Cube of 12 is

$$12 = 12 \times 12 \times 12 = 1728$$

**(iii) 16**

Cube of 16 is

$$16 = 16 \times 16 \times 16 = 4096$$

**(iv) 21**

Cube of 21 is

$$21 = 21 \times 21 \times 21 = 9261$$

**(v) 40**

Cube of 40 is

$$40 = 40 \times 40 \times 40 = 64000$$

**(vi) 55**

Cube of 55 is

$$55 = 55 \times 55 \times 55 = 166375$$

**(vii) 100**

Cube of 100 is

$$100 = 100 \times 100 \times 100 = 1000000$$

**(viii) 302**

Cube of 302 is

$$302 = 302 \times 302 \times 302 = 27543608$$

(ix) 301

Cube of 301 is

$$301 = 301 \times 301 \times 301 = 27270901$$

**2. Write the cubes of all natural numbers between 1 and 10 and verify the following statements:**

**(i) Cubes of all odd natural numbers are odd.**

**(ii) Cubes of all even natural numbers are even.**

**Solutions:**

Firstly let us find the Cube of natural numbers up to 10

$$1^3 = 1 \times 1 \times 1 = 1$$

$$2^3 = 2 \times 2 \times 2 = 8$$

$$3^3 = 3 \times 3 \times 3 = 27$$

$$4^3 = 4 \times 4 \times 4 = 64$$

$$5^3 = 5 \times 5 \times 5 = 125$$

$$6^3 = 6 \times 6 \times 6 = 216$$

$$7^3 = 7 \times 7 \times 7 = 343$$

$$8^3 = 8 \times 8 \times 8 = 512$$

$$9^3 = 9 \times 9 \times 9 = 729$$

$$10^3 = 10 \times 10 \times 10 = 1000$$

∴ From the above results we can say that

(i) Cubes of all odd natural numbers are odd.

(ii) Cubes of all even natural numbers are even.

**3. Observe the following pattern:**

$$1^3 = 1$$

$$1^3 + 2^3 = (1+2)^2$$

$$1^3 + 2^3 + 3^3 = (1+2+3)^2$$

**Write the next three rows and calculate the value of  $1^3 + 2^3 + 3^3 + \dots + 9^3$  by the above pattern.**

**Solution:**

According to given pattern,

$$1^3 + 2^3 + 3^3 + \dots + 9^3$$

$$1^3 + 2^3 + 3^3 + \dots + n^3 = (1+2+3+\dots+n)^2$$

So when  $n = 10$

$$\begin{aligned} 1^3 + 2^3 + 3^3 + \dots + 9^3 + 10^3 &= (1+2+3+\dots+10)^2 \\ &= (55)^2 = 55 \times 55 = 3025 \end{aligned}$$

**4. Write the cubes of 5 natural numbers which are multiples of 3 and verify the followings:**

**“The cube of a natural number which is a multiple of 3 is a multiple of 27”**

**Solution:**

We know that the first 5 natural numbers which are multiple of 3 are 3, 6, 9, 12 and 15

So now, let us find the cube of 3, 6, 9, 12 and 15

$$3^3 = 3 \times 3 \times 3 = 27$$

$$6^3 = 6 \times 6 \times 6 = 216$$

$$9^3 = 9 \times 9 \times 9 = 729$$

$$12^3 = 12 \times 12 \times 12 = 1728$$

$$15^3 = 15 \times 15 \times 15 = 3375$$

We found that all the cubes are divisible by 27

∴ “The cube of a natural number which is a multiple of 3 is a multiple of 27”

**5. Write the cubes of 5 natural numbers which are of the form  $3n + 1$  (e.g. 4, 7, 10, ...) and verify the following:**

**“The cube of a natural number of the form  $3n+1$  is a natural number of the same form i.e. when divided by 3 it leaves the remainder 1”**

**Solution:**

We know that the first 5 natural numbers in the form of  $(3n + 1)$  are 4, 7, 10, 13 and 16

So now, let us find the cube of 4, 7, 10, 13 and 16

$$4^3 = 4 \times 4 \times 4 = 64$$

$$7^3 = 7 \times 7 \times 7 = 343$$

$$10^3 = 10 \times 10 \times 10 = 1000$$

$$13^3 = 13 \times 13 \times 13 = 2197$$

$$16^3 = 16 \times 16 \times 16 = 4096$$

We found that all these cubes when divided by ‘3’ leaves remainder 1.

∴ the statement “The cube of a natural number of the form  $3n+1$  is a natural number of the same form i.e. when divided by 3 it leaves the remainder 1” is true.

**6. Write the cubes 5 natural numbers of the form  $3n+2$  (i.e. 5, 8, 11, ...) and verify the following:**

**“The cube of a natural number of the form  $3n+2$  is a natural number of the same form i.e. when it is divided by 3 the remainder is 2”**

**Solution:**

We know that the first 5 natural numbers in the form  $(3n + 2)$  are 5, 8, 11, 14 and 17

So now, let us find the cubes of 5, 8, 11, 14 and 17

$$5^3 = 5 \times 5 \times 5 = 125$$

$$8^3 = 8 \times 8 \times 8 = 512$$

$$11^3 = 11 \times 11 \times 11 = 1331$$

$$14^3 = 14 \times 14 \times 14 = 2744$$

$$17^3 = 17 \times 17 \times 17 = 4313$$

We found that all these cubes when divided by '3' leaves remainder 2.

∴ the statement "The cube of a natural number of the form  $3n+2$  is a natural number of the same form i.e. when it is divided by 3 the remainder is 2' is true.

**7. Write the cubes of 5 natural numbers of which are multiples of 7 and verify the following:**

**"The cube of a multiple of 7 is a multiple of  $7^3$ .**

**Solution:**

The first 5 natural numbers which are multiple of 7 are 7, 14, 21, 28 and 35

So, the Cube of 7, 14, 21, 28 and 35

$$7^3 = 7 \times 7 \times 7 = 343$$

$$14^3 = 14 \times 14 \times 14 = 2744$$

$$21^3 = 21 \times 21 \times 21 = 9261$$

$$28^3 = 28 \times 28 \times 28 = 21952$$

$$35^3 = 35 \times 35 \times 35 = 42875$$

We found that all these cubes are multiples of  $7^3(343)$  as well.

∴ The statement "The cube of a multiple of 7 is a multiple of  $7^3$  is true.

**8. Which of the following are perfect cubes?**

**(i) 64 (ii) 216**

**(iii) 243 (iv) 1000**

**(v) 1728 (vi) 3087**

**(vii) 4608 (viii) 106480**

**(ix) 166375 (x) 456533**

**Solution:**

**(i) 64**

First find the factors of 64

$$64 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 2^6 = (2^2)^3 = 4^3$$

Hence, it's a perfect cube.

**(ii) 216**

First find the factors of 216

$$216 = 2 \times 2 \times 2 \times 3 \times 3 \times 3 = 2^3 \times 3^3 = 6^3$$

Hence, it's a perfect cube.

**(iii) 243**

First find the factors of 243

$$243 = 3 \times 3 \times 3 \times 3 \times 3 = 3^5 = 3^3 \times 3^2$$

Hence, it's not a perfect cube.

(iv) 1000

First find the factors of 1000

$$1000 = 2 \times 2 \times 2 \times 5 \times 5 \times 5 = 2^3 \times 5^3 = 10^3$$

Hence, it's a perfect cube.

(v) 1728

First find the factors of 1728

$$1728 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3 = 2^6 \times 3^3 = (4 \times 3)^3 = 12^3$$

Hence, it's a perfect cube.

(vi) 3087

First find the factors of 3087

$$3087 = 3 \times 3 \times 7 \times 7 \times 7 = 3^2 \times 7^3$$

Hence, it's not a perfect cube.

(vii) 4608

First find the factors of 4608

$$4608 = 2 \times 2 \times 3 \times 113$$

Hence, it's not a perfect cube.

(viii) 106480

First find the factors of 106480

$$106480 = 2 \times 2 \times 2 \times 2 \times 5 \times 11 \times 11 \times 11$$

Hence, it's not a perfect cube.

(ix) 166375

First find the factors of 166375

$$166375 = 5 \times 5 \times 5 \times 11 \times 11 \times 11 = 5^3 \times 11^3 = 55^3$$

Hence, it's a perfect cube.

(x) 456533

First find the factors of 456533

$$456533 = 11 \times 11 \times 11 \times 7 \times 7 \times 7 = 11^3 \times 7^3 = 77^3$$

Hence, it's a perfect cube.

**9. Which of the following are cubes of even natural numbers?**

**216, 512, 729, 1000, 3375, 13824**

**Solution:**

(i)  $216 = 2^3 \times 3^3 = 6^3$

It's a cube of even natural number.

(ii)  $512 = 2^9 = (2^3)^3 = 8^3$

It's a cube of even natural number.

(iii)  $729 = 3^3 \times 3^3 = 9^3$

It's not a cube of even natural number.

(iv)  $1000 = 10^3$

It's a cube of even natural number.

(v)  $3375 = 3^3 \times 5^3 = 15^3$

It's not a cube of even natural number.

(vi)  $13824 = 2^9 \times 3^3 = (2^3)^3 \times 3^3 = 8^3 \times 3^3 = 24^3$

It's a cube of even natural number.

**10. Which of the following are cubes of odd natural numbers?**

**125, 343, 1728, 4096, 32768, 6859**

**Solution:**

(i)  $125 = 5 \times 5 \times 5 = 5^3$

It's a cube of odd natural number.

(ii)  $343 = 7 \times 7 \times 7 = 7^3$

It's a cube of odd natural number.

(iii)  $1728 = 2^6 \times 3^3 = 4^3 \times 3^3 = 12^3$

It's not a cube of odd natural number. As 12 is even number.

(iv)  $4096 = 2^{12} = (2^6)^2 = 64^2$

Its not a cube of odd natural number. As 64 is an even number.

(v)  $32768 = 2^{15} = (2^5)^3 = 32^3$

It's not a cube of odd natural number. As 32 is an even number.

(vi)  $6859 = 19 \times 19 \times 19 = 19^3$

It's a cube of odd natural number.

**11. What is the smallest number by which the following numbers must be multiplied, so that the products are perfect cubes?**

**(i) 675 (ii) 1323**

**(iii) 2560 (iv) 7803**

**(v) 107811 (vi) 35721**

**Solution:**

**(i) 675**

First find the factors of 675

$$\begin{aligned}675 &= 3 \times 3 \times 3 \times 5 \times 5 \\ &= 3^3 \times 5^2\end{aligned}$$

∴ To make a perfect cube we need to multiply the product by 5.

**(ii) 1323**

First find the factors of 1323

$$\begin{aligned}1323 &= 3 \times 3 \times 3 \times 7 \times 7 \\ &= 3^3 \times 7^2\end{aligned}$$

∴ To make a perfect cube we need to multiply the product by 7.

**(iii) 2560**

First find the factors of 2560

$$\begin{aligned}2560 &= 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 5 \\ &= 2^8 \times 5\end{aligned}$$

∴ To make a perfect cube we need to multiply the product by  $5 \times 5 = 25$ .

**(iv) 7803**

First find the factors of 7803

$$\begin{aligned}7803 &= 3 \times 3 \times 3 \times 17 \times 17 \\ &= 3^3 \times 17^2\end{aligned}$$

∴ To make a perfect cube we need to multiply the product by 17.

**(v) 107811**

First find the factors of 107811

$$\begin{aligned}107811 &= 3 \times 3 \times 3 \times 3 \times 11 \times 11 \times 11 \\ &= 3^4 \times 11^3\end{aligned}$$

∴ To make a perfect cube we need to multiply the product by  $3 \times 3 = 9$ .

(vi) 35721

First find the factors of 35721

$$\begin{aligned}35721 &= 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 7 \times 7 \\ &= 3^3 \times 3^3 \times 7^2\end{aligned}$$

∴ To make a perfect cube we need to multiply the product by 7.

**12. By which smallest number must the following numbers be divided so that the quotient is a perfect cube?**

(i) 675 (ii) 8640

(iii) 1600 (iv) 8788

(v) 7803 (vi) 107811

(vii) 35721 (viii) 243000

**Solution:**

(i) 675

First find the prime factors of 675

$$\begin{aligned}675 &= 3 \times 3 \times 3 \times 5 \times 5 \\ &= 3^3 \times 5^2\end{aligned}$$

Since 675 is not a perfect cube.

To make the quotient a perfect cube we divide it by  $5^2 = 25$ , which gives 27 as quotient where, 27 is a perfect cube.

∴ 25 is the required smallest number.

(ii) 8640

First find the prime factors of 8640

$$\begin{aligned}8640 &= 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 5 \\ &= 2^3 \times 2^3 \times 3^3 \times 5\end{aligned}$$

Since 8640 is not a perfect cube.

To make the quotient a perfect cube we divide it by 5, which gives 1728 as quotient and we know that 1728 is a perfect cube.

∴ 5 is the required smallest number.

(iii) 1600

First find the prime factors of 1600

$$\begin{aligned}1600 &= 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 5 \times 5 \\ &= 2^3 \times 2^3 \times 5^2\end{aligned}$$

Since 1600 is not a perfect cube.

To make the quotient a perfect cube we divide it by  $5^2 = 25$ , which gives 64 as quotient and we know that 64 is a perfect cube

∴ 25 is the required smallest number.



(iv) 8788

First find the prime factors of 8788

$$\begin{aligned}8788 &= 2 \times 2 \times 13 \times 13 \times 13 \\ &= 2^2 \times 13^3\end{aligned}$$

Since 8788 is not a perfect cube.

To make the quotient a perfect cube we divide it by 4, which gives 2197 as quotient and we know that 2197 is a perfect cube

$\therefore$  4 is the required smallest number.

(v) 7803

First find the prime factors of 7803

$$\begin{aligned}7803 &= 3 \times 3 \times 3 \times 17 \times 17 \\ &= 3^3 \times 17^2\end{aligned}$$

Since 7803 is not a perfect cube.

To make the quotient a perfect cube we divide it by  $17^2 = 289$ , which gives 27 as quotient and we know that 27 is a perfect cube

$\therefore$  289 is the required smallest number.

(vi) 107811

First find the prime factors of 107811

$$\begin{aligned}107811 &= 3 \times 3 \times 3 \times 3 \times 11 \times 11 \times 11 \\ &= 3^3 \times 11^3 \times 3\end{aligned}$$

Since 107811 is not a perfect cube.

To make the quotient a perfect cube we divide it by 3, which gives 35937 as quotient and we know that 35937 is a perfect cube.

$\therefore$  3 is the required smallest number.

(vii) 35721

First find the prime factors of 35721

$$\begin{aligned}35721 &= 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 7 \times 7 \\ &= 3^3 \times 3^3 \times 7^2\end{aligned}$$

Since 35721 is not a perfect cube.

To make the quotient a perfect cube we divide it by  $7^2 = 49$ , which gives 729 as quotient and we know that 729 is a perfect cube

$\therefore$  49 is the required smallest number.

(viii) 243000

First find the prime factors of 243000

$$243000 = 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 3 \times 3 \times 5 \times 5 \times 5$$

$$= 2^3 \times 3^3 \times 5^3 \times 3^2$$

Since 243000 is not a perfect cube.

To make the quotient a perfect cube we divide it by  $3^2 = 9$ , which gives 27000 as quotient and we know that 27000 is a perfect cube

$\therefore 9$  is the required smallest number.

**13. Prove that if a number is trebled then its cube is 27 times the cube of the given number.**

**Solution:**

Let us consider a number as  $a$

So the cube of the assumed number is  $= a^3$

Now, the number is trebled  $= 3 \times a = 3a$

So the cube of new number  $= (3a)^3 = 27a^3$

$\therefore$  New cube is 27 times of the original cube.

Hence, proved.

**14. What happens to the cube of a number if the number is multiplied by**

**(i) 3?**

**(ii) 4?**

**(iii) 5?**

**Solution:**

**(i) 3?**

Let us consider the number as  $a$

So its cube will be  $= a^3$

According to the question, the number is multiplied by 3

New number becomes  $= 3a$

So the cube of new number will be  $= (3a)^3 = 27a^3$

Hence, number will become 27 times the cube of the number.

**(ii) 4?**

Let us consider the number as  $a$

So its cube will be  $= a^3$

According to the question, the number is multiplied by 4

New number becomes  $= 4a$

So the cube of new number will be  $= (4a)^3 = 64a^3$

Hence, number will become 64 times the cube of the number.

**(iii) 5?**

Let us consider the number as a

So its cube will be =  $a^3$

According to the question, the number is multiplied by 5

New number becomes =  $5a$

So the cube of new number will be =  $(5a)^3 = 125a^3$

Hence, number will become 125 times the cube of the number.

**15. Find the volume of a cube, one face of which has an area of  $64\text{m}^2$ .**

**Solution:**

We know that the given area of one face of cube =  $64\text{ m}^2$

Let the length of edge of cube be 'a' metre

$$a^2 = 64$$

$$a = \sqrt{64}$$

$$= 8\text{m}$$

Now, volume of cube =  $a^3$

$$a^3 = 8^3 = 8 \times 8 \times 8$$

$$= 512\text{m}^3$$

∴ Volume of a cube is  $512\text{m}^3$

**16. Find the volume of a cube whose surface area is  $384\text{m}^2$ .**

**Solution:**

We know that the surface area of cube =  $384\text{ m}^2$

Let us consider the length of each edge of cube be 'a' meter

$$6a^2 = 384$$

$$a^2 = 384/6$$

$$= 64$$

$$a = \sqrt{64}$$

$$= 8\text{m}$$

Now, volume of cube =  $a^3$

$$a^3 = 8^3 = 8 \times 8 \times 8$$

$$= 512\text{m}^3$$

∴ Volume of a cube is  $512\text{m}^3$

**17. Evaluate the following:**

(i)  $\{(5^2 + 12^2)^{1/2}\}^3$

(ii)  $\{(6^2 + 8^2)^{1/2}\}^3$

**Solution:**

(i)  $\{(5^2 + 12^2)^{1/2}\}^3$

When simplified above equation we get,

$$\begin{aligned} & \{(25 + 144)^{1/2}\}^3 \\ & \{(169)^{1/2}\}^3 \\ & \{(13^2)^{1/2}\}^3 \\ & (13)^3 \\ & 2197 \end{aligned}$$

$$(ii) \{(6^2 + 8^2)^{1/2}\}^3$$

When simplified above equation we get,

$$\begin{aligned} & \{(36 + 64)^{1/2}\}^3 \\ & \{(100)^{1/2}\}^3 \\ & \{(10^2)^{1/2}\}^3 \\ & (10)^3 \\ & 1000 \end{aligned}$$

**18. Write the units digit of the cube of each of the following numbers:**

**31, 109, 388, 4276, 5922, 77774, 44447, 125125125**

**Solution:**

**31**

To find unit digit of cube of a number we perform the cube of unit digit only.

Unit digit of 31 is 1

Cube of 1 =  $1^3 = 1$

∴ Unit digit of cube of 31 is always 1

**109**

To find unit digit of cube of a number we perform the cube of unit digit only.

Unit digit of 109 is = 9

Cube of 9 =  $9^3 = 729$

∴ Unit digit of cube of 109 is always 9

**388**

To find unit digit of cube of a number we perform the cube of unit digit only.

Unit digit of 388 is = 8

Cube of 8 =  $8^3 = 512$

∴ Unit digit of cube of 388 is always 2

**4276**

To find unit digit of cube of a number we perform the cube of unit digit only.

Unit digit of 4276 is = 6

Cube of 6 =  $6^3 = 216$

∴ Unit digit of cube of 4276 is always 6

### 5922

To find unit digit of cube of a number we perform the cube of unit digit only.

Unit digit of 5922 is = 2

Cube of 2 =  $2^3 = 8$

∴ Unit digit of cube of 5922 is always 8

### 77774

To find unit digit of cube of a number we perform the cube of unit digit only.

Unit digit of 77774 is = 4

Cube of 4 =  $4^3 = 64$

∴ Unit digit of cube of 77774 is always 4

### 44447

To find unit digit of cube of a number we perform the cube of unit digit only.

Unit digit of 44447 is = 7

Cube of 7 =  $7^3 = 343$

∴ Unit digit of cube of 44447 is always 3

### 125125125

To find unit digit of cube of a number we perform the cube of unit digit only.

Unit digit of 125125125 is = 5

Cube of 5 =  $5^3 = 125$

∴ Unit digit of cube of 125125125 is always 5

## 19. Find the cubes of the following numbers by column method:

(i) 35

(ii) 56

(iii) 72

**Solution:**

(i) 35

We have,  $a = 3$  and  $b = 5$

Column I $a^3$	Column II $3 \times a^2 \times b$	Column III $3 \times a \times b^2$	Column IV $b^3$
$3^3 = 27$	$3 \times 9 \times 5 = 135$	$3 \times 3 \times 25 = 225$	$5^3 = 125$
+15	+23	+12	<u>125</u>
<u>42</u>	<u>158</u>	<u>237</u>	
42	8	7	5

∴ The cube of 35 is 42875

(ii) 56

We have,  $a = 5$  and  $b = 6$

Column I $a^3$	Column II $3 \times a^2 \times b$	Column III $3 \times a \times b^2$	Column IV $b^3$
$5^3 = 125$	$3 \times 25 \times 6 = 450$	$3 \times 5 \times 36 = 540$	$6^3 = 216$
+50	+56	+21	<u>12</u> 6
<u>175</u>	50 <u>6</u>	56 <u>1</u>	
175	6	1	6

∴ The cube of 56 is 175616

(iii) 72

We have,  $a = 7$  and  $b = 2$

Column I $a^3$	Column II $3 \times a^2 \times b$	Column III $3 \times a \times b^2$	Column IV $b^3$
$7^3 = 343$	$3 \times 49 \times 2 = 294$	$3 \times 7 \times 4 = 84$	$2^3 = 8$
+30	+8	+0	<u>8</u>
<u>373</u>	30 <u>2</u>	8 <u>4</u>	
373	2	4	8

∴ The cube of 72 is 373248

**20. Which of the following numbers are not perfect cubes?**

(i) 64

(ii) 216

(iii) 243

(iv) 1728

**Solution:**

(i) 64

Firstly let us find the prime factors of 64

$$\begin{aligned}
 64 &= 2 \times 2 \times 2 \times 2 \times 2 \times 2 \\
 &= 2^3 \times 2^3
 \end{aligned}$$

$$= 4^3$$

Hence, it's a perfect cube.

(ii) 216

Firstly let us find the prime factors of 216

$$\begin{aligned} 216 &= 2 \times 2 \times 2 \times 3 \times 3 \times 3 \\ &= 2^3 \times 3^3 \\ &= 6^3 \end{aligned}$$

Hence, it's a perfect cube.

(iii) 243

Firstly let us find the prime factors of 243

$$\begin{aligned} 243 &= 3 \times 3 \times 3 \times 3 \times 3 \\ &= 3^3 \times 3^2 \end{aligned}$$

Hence, it's not a perfect cube.

(iv) 1728

Firstly let us find the prime factors of 1728

$$\begin{aligned} 1728 &= 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3 \\ &= 2^3 \times 2^3 \times 3^3 \\ &= 12^3 \end{aligned}$$

Hence, it's a perfect cube.

**21. For each of the non-perfect cubes in Q. No 20 find the smallest number by which it must be**

**(a) Multiplied so that the product is a perfect cube.**

**(b) Divided so that the quotient is a perfect cube.**

**Solution:**

Only non-perfect cube in previous question was = 243

**(a) Multiplied so that the product is a perfect cube.**

Firstly let us find the prime factors of 243

$$243 = 3 \times 3 \times 3 \times 3 \times 3 = 3^3 \times 3^2$$

Hence, to make it a perfect cube we should multiply it by 3.

**(b) Divided so that the quotient is a perfect cube.**

Firstly let us find the prime factors of 243

$$243 = 3 \times 3 \times 3 \times 3 \times 3 = 3^3 \times 3^2$$

Hence, to make it a perfect cube we have to divide it by 9.

**22. By taking three different, values of  $n$  verify the truth of the following statements:**

**(i) If  $n$  is even, then  $n^3$  is also even.**

**(ii) If  $n$  is odd, then  $n^3$  is also odd.**

**(iii) If  $n$  leaves remainder 1 when divided by 3, then  $n^3$  also leaves 1 as remainder when divided by 3.**

**(iv) If a natural number  $n$  is of the form  $3p+2$  then  $n^3$  also a number of the same type.**

**Solution:**

**(i)** If  $n$  is even, then  $n^3$  is also even.

Let us consider three even natural numbers 2, 4, 6

So now, Cubes of 2, 4 and 6 are

$$2^3 = 8$$

$$4^3 = 64$$

$$6^3 = 216$$

Hence, we can see that all cubes are even in nature.

Statement is verified.

**(ii)** If  $n$  is odd, then  $n^3$  is also odd.

Let us consider three odd natural numbers 3, 5, 7

So now, cubes of 3, 5 and 7 are

$$3^3 = 27$$

$$5^3 = 125$$

$$7^3 = 343$$

Hence, we can see that all cubes are odd in nature.

Statement is verified.

**(iii)** If  $n$  leaves remainder 1 when divided by 3, then  $n^3$  also leaves 1 as remainder when divided by 3.

Let us consider three natural numbers of the form  $(3n+1)$  are 4, 7 and 10

So now, cube of 4, 7, 10 are

$$4^3 = 64$$

$$7^3 = 343$$

$$10^3 = 1000$$

We can see that if we divide these numbers by 3, we get 1 as remainder in each case.

Hence, statement is verified.

**(iv)** If a natural number  $n$  is of the form  $3p+2$  then  $n^3$  also a number of the same type.

Let us consider three natural numbers of the form  $(3p+2)$  are 5, 8 and 11



So now, cube of 5, 8 and 10 are

$$5^3 = 125$$

$$8^3 = 512$$

$$11^3 = 1331$$

Now, we try to write these cubes in form of  $(3p + 2)$

$$125 = 3 \times 41 + 2$$

$$512 = 3 \times 170 + 2$$

$$1331 = 3 \times 443 + 2$$

Hence, statement is verified.

**23. Write true (T) or false (F) for the following statements:**

**(i) 392 is a perfect cube.**

**(ii) 8640 is not a perfect cube.**

**(iii) No cube can end with exactly two zeros.**

**(iv) There is no perfect cube which ends in 4.**

**(v) For an integer  $a$ ,  $a^3$  is always greater than  $a^2$ .**

**(vi) If  $a$  and  $b$  are integers such that  $a^2 > b^2$ , then  $a^3 > b^3$ .**

**(vii) If  $a$  divides  $b$ , then  $a^3$  divides  $b^3$ .**

**(viii) If  $a^2$  ends in 9, then  $a^3$  ends in 7.**

**(ix) If  $a^2$  ends in an even number of zeros, then  $a^3$  ends in 25.**

**(x) If  $a^2$  ends in an even number of zeros, then  $a^3$  ends in an odd number of zeros.**

**Solution:**

(i) 392 is a perfect cube.

Firstly let's find the prime factors of  $392 = 2 \times 2 \times 2 \times 7 \times 7 = 2^3 \times 7^2$

Hence the statement is False.

(ii) 8640 is not a perfect cube.

Prime factors of  $8640 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 5 = 2^3 \times 2^3 \times 3^3 \times 5$

Hence the statement is True

(iii) No cube can end with exactly two zeros.

Statement is True.

Because a perfect cube always have zeros in multiple of 3.

(iv) There is no perfect cube which ends in 4.

We know 64 is a perfect cube =  $4 \times 4 \times 4$  and it ends with 4.

Hence the statement is False.

(v) For an integer  $a$ ,  $a^3$  is always greater than  $a^2$ .

Statement is False.

Because in case of negative integers ,

$$(-2)^2 = 4 \text{ and } (-2)^3 = -8$$

(vi) If a and b are integers such that  $a^2 > b^2$ , then  $a^3 > b^3$ .

Statement is False.

In case of negative integers,

$$(-5)^2 > (-4)^2 = 25 > 16$$

But,  $(-5)^3 > (-4)^3 = -125 > -64$  is not true.

(vii) If a divides b, then  $a^3$  divides  $b^3$ .

Statement is True.

If a divides b

$$b/a = k, \text{ so } b = ak$$

$$b^3/a^3 = (ak)^3/a^3 = a^3k^3/a^3 = k^3,$$

For each value of b and a its true.

(viii) If  $a^2$  ends in 9, then  $a^3$  ends in 7.

Statement is False.

Let  $a = 7$

$$7^2 = 49 \text{ and } 7^3 = 343$$

(ix) If  $a^2$  ends in an even number of zeros, then  $a^3$  ends in 25.

Statement is False.

Since, when  $a = 20$

$$a^2 = 20^2 = 400 \text{ and } a^3 = 8000 \text{ (} a^3 \text{ doesn't end with 25)}$$

(x) If  $a^2$  ends in an even number of zeros, then  $a^3$  ends in an odd number of zeros.

Statement is False.

Since, when  $a = 100$

$$a^2 = 100^2 = 10000 \text{ and } a^3 = 100^3 = 1000000 \text{ (} a^3 \text{ doesn't end with odd number of zeros)}$$