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1. Find the cube roots of each of the following integers:
(i)-125 (ii) -5832
(iii)-2744000 (iv) -753571
(v) -32768
Solution:
(i) -125
The cube root of -125 is
$-125 = \sqrt[3]{-125} = -\sqrt[3]{125} = \sqrt[3]{(5\times5\times5)} = -5$

(ii) -5832
The cube root of -5832 is
$-5832 = \sqrt[3]{-5832} = -\sqrt[3]{5832}$
To find the cube root of 5832, we shall use the method of unit digits.
Let us consider the number 5832. Where, unit digit of 5832 = 2
Unit digit in the cube root of 5832 will be 8
After striking out the units, tens and hundreds digits of 5832,
Now we left with 5 only.
We know that 1 is the Largest number whose cube is less than or equal to 5.
So, the tens digit of the cube root of 5832 is 1.
$\sqrt[3]{-5832} = -\sqrt[3]{5832} = -18$

(iii) -2744000
$\sqrt[3]{-2744000} = -\sqrt[3]{2744000}$
We shall use the method of factorization to find the cube root of 2744000
So let's find the prime factors for 2744000
$2744000 = 2\times2\times2\times2\times2\times2\times5\times5\times5\times7\times7\times7$
Now by grouping the factors into triples of equal factors, we get,
$2744000 = (2\times2\times2) \times (2\times2\times2) \times (5\times5\times5) \times (7\times7\times7)$
Since all the prime factors of 2744000 is grouped in to triples of equal factors and no factor is left over.
So now take one factor from each group and by multiplying we get,
$2\times2\times5\times7 = 140$
Thereby we can say that 2744000 is a cube of 140
$\therefore \sqrt[3]{-2744000} = -\sqrt[3]{2744000} = -140$

(iv) -753571
$\sqrt[3]{-753571} = -\sqrt[3]{753571}$

We shall use the unit digit method,
Let us consider the number 753571, where unit digit = 1
Unit digit in the cube root of 753571 will be 1
After striking out the units, tens and hundreds digits of 753571,
Now we left with 753.
We know that 9 is the Largest number whose cube is less than or equal to 753(9^3<753<10^3).
So, the tens digit of the cube root of 753571 is 9.
$\sqrt[3]{753571}$ = 91
$\sqrt[3]{-753571}$ = -$\sqrt[3]{753571}$ = -91

(v) -32768
$\sqrt[3]{-32765}$ = -$\sqrt[3]{32768}$
We shall use the unit digit method,
Let us consider the Number = 32768, where unit digit = 8
Unit digit in the cube root of 32768 will be 2
After striking out the units, tens and hundreds digits of 32768,
Now we left with 32.
As we know that 9 is the Largest number whose cube is less than or equals to 32(3^3<32<4^3).
So, the tens digit of the cube root of 32768 is 3.
$\sqrt[3]{32765}$ = 32
$\sqrt[3]{-32765}$ = -$\sqrt[3]{32768}$ = -32

2. Show that:
(i) $\sqrt[3]{27} \times \sqrt[3]{64} = \sqrt[3]{(27\times64)}$
(ii) $\sqrt[3]{(64\times729)} = \sqrt[3]{64} \times \sqrt[3]{729}$
(iii) $\sqrt[3]{(-125\times216)} = \sqrt[3]{-125} \times \sqrt[3]{216}$
(iv) $\sqrt[3]{(-125\times-1000)} = \sqrt[3]{-125} \times \sqrt[3]{-1000}$
Solution:
(i) $\sqrt[3]{27} \times \sqrt[3]{64} = \sqrt[3]{(27\times64)}$
Let us consider LHS $\sqrt[3]{27} \times \sqrt[3]{64}$
$\sqrt[3]{27} \times \sqrt[3]{64} = \sqrt[3]{(3\times3\times3)} \times \sqrt[3]{(4\times4\times4)}$
= 3×4
= 12
Let us consider RHS $\sqrt[3]{(27\times64)}$
$\sqrt[3]{(27\times64)} = \sqrt[3]{(3\times3\times3\times4\times4\times4)}$
= 3×4
= 12

∴ LHS = RHS, the given equation is verified.

(ii) $\sqrt[3]{(64\times729)} = \sqrt[3]{64} \times \sqrt[3]{729}$
Let us consider LHS $\sqrt[3]{(64\times729)}$
$\sqrt[3]{(64\times729)} = \sqrt[3]{(4\times4\times4\times9\times9\times9)}$
$= 4\times9$
$= 36$
Let us consider RHS $\sqrt[3]{64} \times \sqrt[3]{729}$
$\sqrt[3]{64} \times \sqrt[3]{729} = \sqrt[3]{(4\times4\times4)} \times \sqrt[3]{(9\times9\times9)}$
$= 4\times9$
$= 36$
∴ LHS = RHS, the given equation is verified.

(iii) $\sqrt[3]{(-125\times216)} = \sqrt[3]{-125} \times \sqrt[3]{216}$
Let us consider LHS $\sqrt[3]{(-125\times216)}$
$\sqrt[3]{(-125\times216)} = \sqrt[3]{(-5\times-5\times-5\times2\times2\times2\times3\times3\times3)}$
$= -5\times2\times3$
$= -30$
Let us consider RHS $\sqrt[3]{-125} \times \sqrt[3]{216}$
$\sqrt[3]{-125} \times \sqrt[3]{216} = \sqrt[3]{(-5\times-5\times-5)} \times \sqrt[3]{(2\times2\times2\times3\times3\times3)}$
$= -5\times2\times3$
$= -30$
∴ LHS = RHS, the given equation is verified.

(iv) $\sqrt[3]{(-125\times-1000)} = \sqrt[3]{-125} \times \sqrt[3]{-1000}$
Let us consider LHS $\sqrt[3]{(-125\times-1000)}$
$\sqrt[3]{(-125\times-1000)} = \sqrt[3]{(-5\times-5\times-5\times-10\times-10\times-10)}$
$= -5\times-10$
$= 50$
Let us consider RHS $\sqrt[3]{-125} \times \sqrt[3]{-1000}$
$\sqrt[3]{-125} \times \sqrt[3]{-1000} = \sqrt[3]{(-5\times-5\times-5)} \times \sqrt[3]{(-10\times-10\times-10)}$
$= -5\times-10$
$= 50$
∴ LHS = RHS, the given equation is verified.

3. Find the cube root of each of the following numbers:
(i) 8×125
(ii) -1728×216
(iii) -27×2744

(iv) -729×-15625
Solution:
(i) 8×125
We know that for any two integers a and b, $\sqrt[3]{(a\times b)} = \sqrt[3]{a} \times \sqrt[3]{b}$
By using the property
$\sqrt[3]{(8\times125)} = \sqrt[3]{8} \times \sqrt[3]{125}$
$= \sqrt[3]{(2\times2\times2)} \times \sqrt[3]{(5\times5\times5)}$
$= 2\times5$
$= 10$

(ii) -1728×216
We know that for any two integers a and b, $\sqrt[3]{(a\times b)} = \sqrt[3]{a} \times \sqrt[3]{b}$
By using the property
$\sqrt[3]{(-1728\times216)} = \sqrt[3]{-1728} \times \sqrt[3]{216}$
We shall use the unit digit method
Let us consider the number 1728, where Unit digit = 8
The unit digit in the cube root of 1728 will be 2
After striking out the units, tens and hundreds digits of the given number, we are left with the 1.
We know 1 is the largest number whose cube is less than or equal to 1.
So, the tens digit of the cube root of 1728 = 1
$\sqrt[3]{1728} = 12$

Now, let's find the prime factors for 216 = 2×2×2×3×3×3
By grouping the factors in triples of equal factor, we get,
216 = (2×2×2) × (3×3×3)
By taking one factor from each group we get,
$\sqrt[3]{216} = 2\times3 = 6$
∴ by equating the values in the given equation we get,
$\sqrt[3]{(-1728\times216)} = \sqrt[3]{-1728} \times \sqrt[3]{216}$
$= -12 \times 6$
$= -72$

(iii) -27×2744
We know that for any two integers a and b, $\sqrt[3]{(a\times b)} = \sqrt[3]{a} \times \sqrt[3]{b}$
By using the property
$\sqrt[3]{(-27\times2744)} = \sqrt[3]{-27} \times \sqrt[3]{2744}$
We shall use the unit digit method
Let us consider the number 2744, where Unit digit = 4

The unit digit in the cube root of 2744 will be 4
After striking out the units, tens and hundreds digits of the given number, we are left with the 2.
We know 2 is the largest number whose cube is less than or equal to 2.
So, the tens digit of the cube root of 2744 = 2
$\sqrt[3]{2744} = 14$

Now, let's find the prime factors for 27 = 3×3×3
By grouping the factors in triples of equal factor, we get,
27 = (3×3×3)
Cube root of 27 is
$\sqrt[3]{27} = 3$
∴ by equating the values in the given equation we get,
$\sqrt[3]{(-27\times2744)} = \sqrt[3]{-27} \times \sqrt[3]{2744}$
$= -3 \times 14$
$= -42$

(iv) -729×-15625
We know that for any two integers a and b, $\sqrt[3]{(a\times b)} = \sqrt[3]{a} \times \sqrt[3]{b}$
By using the property
$\sqrt[3]{(-729\times-15625)} = \sqrt[3]{-729} \times \sqrt[3]{-15625}$
We shall use the unit digit method
Let us consider the number 15625, where Unit digit = 5
The unit digit in the cube root of 15625 will be 5
After striking out the units, tens and hundreds digits of the given number, we are left with the 15.
We know 15 is the largest number whose cube is less than or equal to 15($2^3<15<3^3$).
So, the tens digit of the cube root of 15625 = 2
$\sqrt[3]{15625} = 25$

Now, let's find the prime factors for 729 = 9×9×9
By grouping the factors in triples of equal factor, we get,
729 = (9×9×9)
Cube root of 729 is
$\sqrt[3]{729} = 9$
∴ by equating the values in the given equation we get,
$\sqrt[3]{(-729\times-15625)} = \sqrt[3]{-729} \times \sqrt[3]{-15625}$
$= -9 \times -25$
$= 225$

4. Evaluate:
(i) $\sqrt[3]{(4^3 \times 6^3)}$
(ii) $\sqrt[3]{(8\times17\times17\times17)}$
(iii) $\sqrt[3]{(700\times2\times49\times5)}$
(iv) $125\sqrt[3]{a^6} - \sqrt[3]{125a^6}$
Solution:
(i) $\sqrt[3]{(4^3 \times 6^3)}$
We know that for any two integers a and b, $\sqrt[3]{(a\times b)} = \sqrt[3]{a} \times \sqrt[3]{b}$
By using the property
$\sqrt[3]{(4^3 \times 6^3)} = \sqrt[3]{4^3} \times \sqrt[3]{6^3}$
$= 4 \times 6$
$= 24$

(ii) $\sqrt[3]{(8\times17\times17\times17)}$
We know that for any two integers a and b, $\sqrt[3]{(a\times b)} = \sqrt[3]{a} \times \sqrt[3]{b}$
By using the property
$\sqrt[3]{(8\times17\times17\times17)} = \sqrt[3]{8} \times \sqrt[3]{17\times17\times17}$
$= \sqrt[3]{2^3} \times \sqrt[3]{17^3}$
$= 2 \times 17$
$= 34$

(iii) $\sqrt[3]{(700\times2\times49\times5)}$
Firstly let us find the prime factors for the above numbers
$\sqrt[3]{(700\times2\times49\times5)} = \sqrt[3]{(2\times2\times5\times5\times7\times2\times7\times7\times5)}$
$= \sqrt[3]{(2^3\times5^3\times7^3)}$
$= 2\times5\times7$
$= 70$

(iv) $125\sqrt[3]{a^6} - \sqrt[3]{125a^6}$
$125\sqrt[3]{a^6} - \sqrt[3]{125a^6} = 125\sqrt[3]{(a^2)^3} - \sqrt[3]{5^3(a^2)^3}$
$= 125a^2 – 5a^2$
$= 120a^2$

5. Find the cube root of each of the following rational numbers:
(i) -125/729
(ii) 10648/12167
(iii) -19683/24389
(iv) 686/-3456
(v) -39304/-42875

Solution:

(i) -125/729

Let us find the prime factors of 125 and 729

$-125/729 = -(\sqrt[3]{(5\times5\times5)}) / (\sqrt[3]{(9\times9\times9)})$

$= -(\sqrt[3]{(5^3)}) / (\sqrt[3]{(9^3)})$

$= -5/9$

(ii) 10648/12167

Let us find the prime factors of 10648 and 12167

$10648/12167 = (\sqrt[3]{(2\times2\times2\times11\times11\times11)}) / (\sqrt[3]{(23\times23\times23)})$

$= (\sqrt[3]{(2^3\times11^3)}) / (\sqrt[3]{(23^3)})$

$= (2\times11)/23$

$= 22/23$

(iii) -19683/24389

Let us find the prime factors of 19683 and 24389

$-19683/24389 = -(\sqrt[3]{(3\times3\times3\times3\times3\times3\times3\times3\times3)}) / (\sqrt[3]{(29\times29\times29)})$

$= -(\sqrt[3]{(3^3\times3^3\times3^3)}) / (\sqrt[3]{(29^3)})$

$= -(3\times3\times3)/29$

$= -27/29$

(iv) 686/-3456

Let us find the prime factors of 686 and -3456

$686/-3456 = = -(\sqrt[3]{(2\times7\times7\times7)}) / (\sqrt[3]{(2^7\times2^3)})$

$= -(\sqrt[3]{(2\times7^3)}) / (\sqrt[3]{(2^7\times2^3)})$

$= -(\sqrt[3]{(7^3)}) / (\sqrt[3]{(2^6\times2^3)})$

$= -7/(2\times2\times2)$

$= -7/8$

(v) -39304/-42875

Let us find the prime factors of -39304 and -42875

$-39304/-42875 = -(\sqrt[3]{(2\times2\times2\times17\times17\times17)}) / -(\sqrt[3]{(5\times5\times5\times7\times7\times7)})$

$= -(\sqrt[3]{(2^3\times17^3)}) / -(\sqrt[3]{(5^3\times7^3)})$

$= -(2\times17)/-(5\times7)$

$= -34/-35$

$= 34/35$

6. Find the cube root of each of the following rational numbers:

(i) 0.001728

(ii) 0.003375
(iii) 0.001
(iv) 1.331
Solution:
(i) 0.001728
0.001728 = 1728/1000000
$\sqrt[3]{(0.001728)} = \sqrt[3]{1728} / \sqrt[3]{1000000}$
Let us find the prime factors of 1728 and 1000000
$\sqrt[3]{(0.001728)} = \sqrt[3]{(2^3 \times 2^3 \times 3^3)} / \sqrt[3]{(100^3)}$
$= (2\times2\times3)/100$
$= 12/100$
$= 0.12$

(ii) 0.003375
0.003375 = 3375/1000000
$\sqrt[3]{(0.003375)} = \sqrt[3]{3375} / \sqrt[3]{1000000}$
Let us find the prime factors of 3375 and 1000000
$\sqrt[3]{(0.003375)} = \sqrt[3]{(3^3 \times 5^3)} / \sqrt[3]{(100^3)}$
$= (3\times5)/100$
$= 15/100$
$= 0.15$

(iii) 0.001
0.001 = 1/1000
$\sqrt[3]{(0.001)} = \sqrt[3]{1} / \sqrt[3]{1000}$
$= 1/ \sqrt[3]{10^3}$
$= 1/10$
$= 0.1$

(iv) 1.331
1.331 = 1331/1000
$\sqrt[3]{(1.331)} = \sqrt[3]{1331} / \sqrt[3]{1000}$
Let us find the prime factors of 1331 and 1000
$\sqrt[3]{(1.331)} = \sqrt[3]{(11^3)} / \sqrt[3]{(10^3)}$
$= 11/10$
$= 1.1$

7. Evaluate each of the following:
(i) $\sqrt[3]{27} + \sqrt[3]{0.008} + \sqrt[3]{0.064}$

(ii) $\sqrt[3]{1000} + \sqrt[3]{0.008} - \sqrt[3]{0.125}$
(iii) $\sqrt[3]{(729/216)} \times 6/9$
(iv) $\sqrt[3]{(0.027/0.008)} \div \sqrt[3]{(0.09/0.04)} - 1$
(v) $\sqrt[3]{(0.1\times0.1\times0.1\times13\times13\times13)}$
Solution:
(i) $\sqrt[3]{27} + \sqrt[3]{0.008} + \sqrt[3]{0.064}$
Let us simplify
$\sqrt[3]{(3\times3\times3)} + \sqrt[3]{(0.2\times0.2\times0.2)} + \sqrt[3]{(0.4\times0.4\times0.4)}$
$\sqrt[3]{(3)^3} + \sqrt[3]{(0.2)^3} + \sqrt[3]{(0.4)^3}$
$3 + 0.2 + 0.4$
3.6

(ii) $\sqrt[3]{1000} + \sqrt[3]{0.008} - \sqrt[3]{0.125}$
Let us simplify
$\sqrt[3]{(10\times10\times10)} + \sqrt[3]{(0.2\times0.2\times0.2)} - \sqrt[3]{(0.5\times0.5\times0.5)}$
$\sqrt[3]{(10)^3} + \sqrt[3]{(0.2)^3} - \sqrt[3]{(0.5)^3}$
$10 + 0.2 - 0.5$
9.7

(iii) $\sqrt[3]{(729/216)} \times 6/9$
Let us simplify
$\sqrt[3]{(9\times9\times9/6\times6\times6)} \times 6/9$
$(\sqrt[3]{(9)^3} / \sqrt[3]{(6)^3})\times 6/9$
$9/6 \times 6/9$
1

(iv) $\sqrt[3]{(0.027/0.008)} \div \sqrt{(0.09/0.04)} - 1$
Let us simplify $\sqrt[3]{(0.027/0.008)} \div \sqrt{(0.09/0.04)}$
$\sqrt[3]{(0.3\times0.3\times0.3/0.2\times0.2\times0.2)} \div \sqrt{(0.3\times0.3/0.2\times0.2)}$
$(\sqrt[3]{(0.3)^3} / \sqrt[3]{(0.2)^3}) \div (\sqrt{(0.3)^2} / \sqrt{(0.2)^2})$
$(0.3/0.2) \div (0.3/0.2) -1$
$(0.3/0.2 \times 0.2/0.3) -1$
$1 – 1$
0

(v) $\sqrt[3]{(0.1\times0.1\times0.1\times13\times13\times13)}$
$\sqrt[3]{(0.1^3\times13^3)}$
$0.1 \times 13 = 1.3$

8. Show that:
(i) $\sqrt[3]{(729)}/\sqrt[3]{(1000)} = \sqrt[3]{(729/1000)}$
(ii) $\sqrt[3]{(-512)}/\sqrt[3]{(343)} = \sqrt[3]{(-512/343)}$
Solution:
(i) $\sqrt[3]{(729)}/\sqrt[3]{(1000)} = \sqrt[3]{(729/1000)}$
Let us consider LHS $\sqrt[3]{(729)}/\sqrt[3]{(1000)}$
$\sqrt[3]{(729)}/\sqrt[3]{(1000)} = \sqrt[3]{(9\times9\times9)}/\sqrt[3]{(10\times10\times10)}$
$= \sqrt[3]{(9^3/10^3)}$
$= 9/10$
Let us consider RHS $\sqrt[3]{(729/1000)}$
$\sqrt[3]{(729/1000)} = \sqrt[3]{(9\times9\times9/10\times10\times10)}$
$= \sqrt[3]{(9^3/10^3)}$
$= 9/10$
∴ LHS = RHS

(ii) $\sqrt[3]{(-512)}/\sqrt[3]{(343)} = \sqrt[3]{(-512/343)}$
Let us consider LHS $\sqrt[3]{(-512)}/\sqrt[3]{(343)}$
$\sqrt[3]{(-512)}/\sqrt[3]{(343)} = \sqrt[3]{-(8\times8\times8)}/\sqrt[3]{(7\times7\times7)}$
$= \sqrt[3]{-(8^3/7^3)}$
$= -8/7$
Let us consider RHS $\sqrt[3]{(-512/343)}$
$\sqrt[3]{(-512/343)} = = \sqrt[3]{-(8\times8\times8/7\times7\times7)}$
$= \sqrt[3]{-(8^3/7^3)}$
$= -8/7$
∴ LHS = RHS

9. Fill in the blanks:
(i) $\sqrt[3]{(125\times27)} = 3 \times \ldots$
(ii) $\sqrt[3]{(8\times\ldots)} = 8$
(iii) $\sqrt[3]{1728} = 4 \times \ldots$
(iv) $\sqrt[3]{480} = \sqrt[3]{3}\times2\times\sqrt[3]{..}$
(v) $\sqrt[3]{\ldots} = \sqrt[3]{7}\times\sqrt[3]{8}$
(vi) $\sqrt[3]{..} = \sqrt[3]{4}\times\sqrt[3]{5}\times\sqrt[3]{6}$
(vii) $\sqrt[3]{(27/125)} = \ldots/5$
(viii) $\sqrt[3]{(729/1331)} = 9/\ldots$
(ix) $\sqrt[3]{(512/\ldots)} = 8/13$
Solution:
(i) $\sqrt[3]{(125\times27)} = 3 \times \ldots$
Let us consider LHS $\sqrt[3]{(125\times27)}$

$\sqrt[3]{(125\times27)} = \sqrt[3]{(5\times5\times5\times3\times3\times3)}$
$= \sqrt[3]{(5^3\times3^3)}$
$= 5\times3$ or 3×5

(ii) $\sqrt[3]{(8\times\ldots)} = 8$
Let us consider LHS $\sqrt[3]{(8\times\ldots)}$
$\sqrt[3]{(8\times8\times8)} = \sqrt[3]{8^3} = 8$

(iii) $\sqrt[3]{1728} = 4 \times \ldots$
Let us consider LHS
$\sqrt[3]{1728} = \sqrt[3]{(2\times2\times2\times2\times2\times2\times3\times3\times3)}$
$= \sqrt[3]{(2^3\times2^3\times3^3)}$
$= 2\times2\times3$
$= 4\times3$

(iv) $\sqrt[3]{480} = \sqrt[3]{3}\times2\times \sqrt[3]{..}$
Let us consider LHS
$\sqrt[3]{480} = \sqrt[3]{(2\times2\times2\times2\times2\times3\times5)}$
$= \sqrt[3]{(2^3\times2^2\times3\times5)}$
$= \sqrt[3]{2^3}\times \sqrt[3]{3} \times \sqrt[3]{2\times2\times5}$
$= 2 \times \sqrt[3]{3} \times \sqrt[3]{20}$

(v) $\sqrt[3]{\ldots} = \sqrt[3]{7} \times \sqrt[3]{8}$
Let us consider RHS
$\sqrt[3]{7} \times \sqrt[3]{8} = \sqrt[3]{(7 \times 8)}$
$= \sqrt[3]{56}$

(vi) $\sqrt[3]{..} = \sqrt[3]{4} \times \sqrt[3]{5} \times \sqrt[3]{6}$
Let us consider RHS
$\sqrt[3]{4} \times \sqrt[3]{5} \times \sqrt[3]{6} = \sqrt[3]{(4 \times 5 \times 6)}$
$= \sqrt[3]{120}$

(vii) $\sqrt[3]{(27/125)} = \ldots/5$
Let us consider LHS
$\sqrt[3]{(27/125)} = \sqrt[3]{(3\times3\times3)/(5\times5\times5)}$
$= \sqrt[3]{(3^3)/(5^3)}$
$= 3/5$

(viii) $\sqrt[3]{(729/1331)} = 9/\ldots$

Let us consider LHS

$\sqrt[3]{(729/1331)} = \sqrt[3]{(9\times9\times9)/(11\times11\times11)}$

$= \sqrt[3]{(9^3)/(11^3)}$

$= 9/11$

(ix) $\sqrt[3]{(512/\ldots)} = 8/13$

Let us consider LHS

$\sqrt[3]{(512/\ldots)} = \sqrt[3]{(2\times2\times2\times2\times2\times2\times2\times2\times2)}$

$= \sqrt[3]{(2^3\times2^3\times2^3)}$

$= 2\times2\times2$

$= 8$

So, $8/\sqrt[3]{\ldots} = 8/13$

when numerators are same the denominators will also become equal.

$8 \times 13 = 8 \times \sqrt[3]{\ldots}$

$\sqrt[3]{\ldots} = 13$

$\ldots = (13)^3$

$= 2197$

10. The volume of a cubical box is 474. 552 cubic metres. Find the length of each side of the box.

Solution:

Volume of a cubical box is 474.552 cubic metres

$V = 8^3$,

Let 'S' be the side of the cube

$8^3 = 474.552$ cubic metres

$8 = \sqrt[3]{474.552}$

$= \sqrt[3]{(474552/1000)}$

Let us factorise 474552 into prime factors, we get:

$474552 = 2\times2\times2\times3\times3\times3\times13\times13\times13$

By grouping the factors in triples of equal factors, we get:

$474552 = (2\times2\times2) \times (3\times3\times3) \times (13\times13\times13)$

Now, $\sqrt[3]{474.552} = \sqrt[3]{((2\times2\times2) \times (3\times3\times3) \times (13\times13\times13))}$

$= 2\times3\times13$

$= 78$

Also,

$\sqrt[3]{1000} = \sqrt[3]{(10\times10\times10)}$

$= \sqrt[3]{(10)^3}$

$= 10$

So now let us equate in the above equation we get,

$8 = \sqrt[3]{(474552/1000)}$
$= 78/10$
$= 7.8$
∴ length of the side is 7.8m.

11. Three numbers are to one another 2:3:4. The sum of their cubes is 0.334125. Find the numbers.
Solution:
Let us consider the ratio 2:3:4 be 2a, 3a, and 4a.
So according to the question:
$(2a)^3 + (3a)^3 + (4a)^3 = 0.334125$
$8a^3 + 27a^3 + 64a^3 = 0.334125$
$99a^3 = 0.334125$
$a^3 = 334125/1000000 \times 99$
$= 3375/1000000$
$a = \sqrt[3]{(3375/1000000)}$
$= \sqrt[3]{((15\times15\times15)/100\times100\times100)}$
$= 15/100$
$= 0.15$
∴ The numbers are:
$2\times0.15 = 0.30$
$3\times0.15 = 0.45$
$4\times0.15 = 0.6$

12. Find the side of a cube whose volume is 24389/216m^3.
Solution:
Volume of the side s = 24389/216 = v
$V = 8^{.3}$
$8 = \sqrt[3]{v}$
$= \sqrt[3]{(24389/216)}$
By performing factorisation we get,
$= \sqrt[3]{(29\times29\times29/2\times2\times2\times3\times3\times3)}$
$= 29/(2\times3)$
$= 29/6$
∴ The length of the side is 29/6.

13. Evaluate:
(i) $\sqrt[3]{36} \times \sqrt[3]{384}$
(ii) $\sqrt[3]{96} \times \sqrt[3]{144}$

(iii) $\sqrt[3]{100} \times \sqrt[3]{270}$
(iv) $\sqrt[3]{121} \times \sqrt[3]{297}$
Solution:
(i) $\sqrt[3]{36} \times \sqrt[3]{384}$
We know that $\sqrt[3]{a} \times \sqrt[3]{b} = \sqrt[3]{(a\times b)}$
By using the above formula let us simplify
$\sqrt[3]{36} \times \sqrt[3]{384} = \sqrt[3]{(36\times384)}$
The prime factors of 36 and 384 are
$= \sqrt[3]{(2\times2\times3\times3) \times (2\times2\times2\times2\times2\times2\times3\times3\times3)}$
$= \sqrt[3]{(2^3\times2^3\times2^3\times3^3)}$
$= 2\times2\times2\times3$
$= 24$

(ii) $\sqrt[3]{96} \times \sqrt[3]{144}$
We know that $\sqrt[3]{a} \times \sqrt[3]{b} = \sqrt[3]{(a\times b)}$
By using the above formula let us simplify
$\sqrt[3]{96} \times \sqrt[3]{144} = \sqrt[3]{(96\times144)}$
The prime factors of 96 and 144 are
$= \sqrt[3]{(2\times2\times2\times2\times2\times3) \times (2\times2\times2\times2\times3\times3)}$
$= \sqrt[3]{(2^3\times2^3\times2^3\times3^3)}$
$= 2\times2\times2\times3$
$= 24$

(iii) $\sqrt[3]{100} \times \sqrt[3]{270}$
We know that $\sqrt[3]{a} \times \sqrt[3]{b} = \sqrt[3]{(a\times b)}$
By using the above formula let us simplify
$\sqrt[3]{100} \times \sqrt[3]{270} = \sqrt[3]{(100\times270)}$
The prime factors of 100 and 270 are
$= \sqrt[3]{(2\times2\times5\times5) \times (2\times3\times3\times3\times5)}$
$= \sqrt[3]{(2^3\times3^3\times5^3)}$
$= 2\times3\times5$
$= 30$

(iv) $\sqrt[3]{121} \times \sqrt[3]{297}$
We know that $\sqrt[3]{a} \times \sqrt[3]{b} = \sqrt[3]{(a\times b)}$
By using the above formula let us simplify
$\sqrt[3]{121} \times \sqrt[3]{297} = \sqrt[3]{(121\times297)}$
The prime factors of 121 and 297 are
$= \sqrt[3]{(11\times11) \times (3\times3\times3\times11)}$

$= \sqrt[3]{(11^3 \times 3^3)}$
$= 11 \times 3$
$= 33$

14. Find the cube roots of the numbers 3048625, 20346417, 210644875, 57066625 using the fact that
(i) 3048625 = 3375 × 729
(ii) 20346417 = 9261 × 2197
(iii) 210644875 = 42875 × 4913
(iv) 57066625 = 166375 × 343
Solution:
(i) 3048625 = 3375 × 729
By taking the cube root for the whole we get,
$\sqrt[3]{3048625} = \sqrt[3]{3375} \times \sqrt[3]{729}$
Now perform factorization
$= \sqrt[3]{3 \times 3 \times 3 \times 5 \times 5 \times 5} \times \sqrt[3]{9 \times 9 \times 9}$
$= \sqrt[3]{3^3 \times 5^3} \times \sqrt[3]{9^3}$
$= 3 \times 5 \times 9$
$= 135$

(ii) 20346417 = 9261 × 2197
By taking the cube root for the whole we get,
$\sqrt[3]{20346417} = \sqrt[3]{9261} \times \sqrt[3]{2197}$
Now perform factorization
$= \sqrt[3]{3 \times 3 \times 3 \times 7 \times 7 \times 7} \times \sqrt[3]{13 \times 13 \times 13}$
$= \sqrt[3]{3^3 \times 7^3} \times \sqrt[3]{13^3}$
$= 3 \times 7 \times 13$
$= 273$

(iii) 210644875 = 42875 × 4913
By taking the cube root for the whole we get,
$\sqrt[3]{210644875} = \sqrt[3]{42875} \times \sqrt[3]{4913}$
Now perform factorization
$= \sqrt[3]{5 \times 5 \times 5 \times 7 \times 7 \times 7} \times \sqrt[3]{17 \times 17 \times 17}$
$= \sqrt[3]{5^3 \times 7^3} \times \sqrt[3]{17^3}$
$= 5 \times 7 \times 17$
$= 595$

(iv) 57066625 = 166375 × 343

By taking the cube root for the whole we get,
$\sqrt[3]{57066625} = \sqrt[3]{166375} \times \sqrt[3]{343}$
Now perform factorization
$= \sqrt[3]{5\times5\times5\times11\times11\times11} \times \sqrt[3]{7\times7\times7}$
$= \sqrt[3]{5^3\times11^3} \times \sqrt[3]{7^3}$
$= 5\times11\times7$
$= 385$

15. Find the unit of the cube root of the following numbers:
(i) 226981
(ii) 13824
(iii) 571787
(iv) 175616
Solution:
(i) 226981
The given number is 226981.
Unit digit of 226981 = 1
The unit digit of the cube root of 226981 = 1

(ii) 13824
The given number is 13824.
Unit digit of 13824 = 4
The unit digit of the cube root of 13824 = 4

(iii) 571787
The given number is 571787.
Unit digit of 571787 = 7
The unit digit of the cube root of 571787 = 7

(iv) 175616
The given number is 175616.
Unit digit of 175616 = 6
The unit digit of the cube root of 175616 = 6

16. Find the tens digit of the cube root of each of the numbers in Q.No.15.
(i) 226981
(ii) 13824
(iii) 571787
(iv) 175616

Solution:

(i) 226981

The given number is 226981.

Unit digit of 226981 = 1

The unit digit in the cube root of 226981 = 1

After striking out the units, tens and hundreds digits of 226981, now we left with 226 only.

We know that 6 is the Largest number whose cube root is less than or equal to 226($6^3<226<7^3$).

∴ The tens digit of the cube root of 226981 is 6.

(ii) 13824

The given number is 13824.

Unit digit of 13824 = 4

The unit digit in the cube root of 13824 = 4

After striking out the units, tens and hundreds digits of 13824, now we left with 13 only.

We know that 2 is the Largest number whose cube root is less than or equal to 13($2^3<13<3^3$).

∴ The tens digit of the cube root of 13824 is 2.

(iii) 571787

The given number is 571787.

Unit digit of 571787 = 7

The unit digit in the cube root of 571787 = 3

After striking out the units, tens and hundreds digits of 571787, now we left with 571 only.

We know that 8 is the Largest number whose cube root is less than or equals to 571($8^3<571<9^3$).

∴ The tens digit of the cube root of 571787 is 8.

(iv) 175616

The given number is 175616.

Unit digit of 175616 = 6

The unit digit in the cube root of 175616 = 6

After striking out the units, tens and hundreds digits of 175616, now we left with 175 only.

We know that 5 is the Largest number whose cube root is less than or equals to 175($5^3<175<6^3$).

∴ The tens digit of the cube root of 175616 is 5.