EXERCISE 4.1 PAGE NO: 4.7

1. Find the cubes of the following numbers:
(i) 7 (ii) 12
(iii) 16 (iv) 21
(v) 40 (vi) 55
(vii) 100 (viii) 302
(ix) 301
Solution:
(i) 7
Cube of 7 is
7 = 7× 7 × 7 = 343

(ii) 12
Cube of 12 is
12 = 12× 12× 12 = 1728

(iii) 16
Cube of 16 is
16 = 16× 16× 16 = 4096

(iv) 21
Cube of 21 is
21 = 21 × 21 × 21 = 9261

(v) 40
Cube of 40 is
40 = 40× 40× 40 = 64000

(vi) 55
Cube of 55 is
55 = 55× 55× 55 = 166375

(vii) 100
Cube of 100 is
100 = 100× 100× 100 = 1000000

(viii) 302
Cube of 302 is

302 = 302× 302× 302 = 27543608

(ix) 301
Cube of 301 is
301 = 301× 301× 301 = 27270901

2.Write the cubes of all natural numbers between 1 and 10 and verify the following statements:
(i) Cubes of all odd natural numbers are odd.
(ii) Cubes of all even natural numbers are even.
Solutions:
Firstly let us find the Cube of natural numbers up to 10
$1^3 = 1 \times 1 \times 1 = 1$
$2^3 = 2 \times 2 \times 2 = 8$
$3^3 = 3 \times 3 \times 3 = 27$
$4^3 = 4 \times 4 \times 4 = 64$
$5^3 = 5 \times 5 \times 5 = 125$
$6^3 = 6 \times 6 \times 6 = 216$
$7^3 = 7 \times 7 \times 7 = 343$
$8^3 = 8 \times 8 \times 8 = 512$
$9^3 = 9 \times 9 \times 9 = 729$
$10^3 = 10 \times 10 \times 10 = 1000$
∴ From the above results we can say that
(i) Cubes of all odd natural numbers are odd.
(ii) Cubes of all even natural numbers are even.

3. Observe the following pattern:
$\mathbf{1^3 = 1}$
$\mathbf{1^3 + 2^3 = (1+2)^2}$
$\mathbf{1^3 + 2^3 + 3^3 = (1+2+3)^2}$
Write the next three rows and calculate the value of $\mathbf{1^3 + 2^3 + 3^3 +\ldots+ 9^3}$ by the above pattern.
Solution:
According to given pattern,
$1^3 + 2^3 + 3^3 +\ldots+ 9^3$
$1^3 + 2^3 + 3^3 +\ldots+ n^3 = (1+2+3+\ldots+n)^2$
So when n = 10
$1^3 + 2^3 + 3^3 +\ldots+ 9^3 + 10^3 = (1+2+3+\ldots+10)^2$
$= (55)^2 = 55\times55 = 3025$

4. Write the cubes of 5 natural numbers which are multiples of 3 and verify the followings:
"The cube of a natural number which is a multiple of 3 is a multiple of 27'
Solution:
We know that the first 5 natural numbers which are multiple of 3 are 3, 6, 9, 12 and 15
So now, let us find the cube of 3, 6, 9, 12 and 15
$3^3 = 3 \times 3 \times 3 = 27$
$6^3 = 6 \times 6 \times 6 = 216$
$9^3 = 9 \times 9 \times 9 = 729$
$12^3 = 12 \times 12 \times 12 = 1728$
$15^3 = 15 \times 15 \times 15 = 3375$
We found that all the cubes are divisible by 27
∴ "The cube of a natural number which is a multiple of 3 is a multiple of 27'

5.Write the cubes of 5 natural numbers which are of the form 3n + 1 (e.g. 4, 7, 10, …) and verify the following:
"The cube of a natural number of the form 3n+1 is a natural number of the same from i.e. when divided by 3 it leaves the remainder 1'
Solution:
We know that the first 5 natural numbers in the form of (3n + 1) are 4, 7, 10, 13 and 16
So now, let us find the cube of 4, 7, 10, 13 and 16
$4^3 = 4 \times 4 \times 4 = 64$
$7^3 = 7 \times 7 \times 7 = 343$
$10^3 = 10 \times 10 \times 10 = 1000$
$13^3 = 13 \times 13 \times 13 = 2197$
$16^3 = 16 \times 16 \times 16 = 4096$
We found that all these cubeswhen divided by '3' leaves remainder 1.
∴ the statement "The cube of a natural number of the form 3n+1 is a natural number of the same from i.e. when divided by 3 it leaves the remainder 1' is true.

6. Write the cubes 5 natural numbers of the from 3n+2(i.e.5,8,11….) and verify the following:
"The cube of a natural number of the form 3n+2 is a natural number of the same form i.e. when it is dividend by 3 the remainder is 2'
Solution:
We know that the first 5 natural numbers in the form (3n + 2) are 5, 8, 11, 14 and 17
So now, let us find the cubes of 5, 8, 11, 14 and 17
$5^3 = 5 \times 5 \times 5 = 125$
$8^3 = 8 \times 8 \times 8 = 512$

$11^3 = 11 \times 11 \times 11 = 1331$
$14^3 = 14 \times 14 \times 14 = 2744$
$17^3 = 17 \times 17 \times 17 = 4313$
We found that all these cubes when divided by '3' leaves remainder 2.
∴ thestatement"The cube of a natural number of the form 3n+2 is a natural number of the same form i.e. when it is dividend by 3 the remainder is 2' is true.

7.Write the cubes of 5 natural numbers of which are multiples of 7 and verity the following:
"The cube of a multiple of 7 is a multiple of 7^3.
Solution:
The first 5 natural numbers which are multiple of 7 are 7, 14, 21, 28 and 35
So, the Cube of 7, 14, 21, 28 and 35
$7^3 = 7 \times 7 \times 7 = 343$
$14^3 = 14 \times 14 \times 14 = 2744$
$21^3 = 21\times 21\times 21 = 9261$
$28^3 = 28 \times 28 \times 28 = 21952$
$35^3 = 35 \times 35 \times 35 = 42875$
We found that all these cubes are multiples of 7^3(343) as well.
∴The statement"The cube of a multiple of 7 is a multiple of 7^3 is true.

8. Which of the following are perfect cubes?
(i) 64 (ii) 216
(iii) 243 (iv) 1000
(v) 1728 (vi) 3087
(vii) 4608 (viii) 106480
(ix) 166375 (x) 456533
Solution:
(i) 64
First find the factors of 64
$64 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 2^6 = (2^2)^3 = 4^3$
Hence, it's a perfect cube.

(ii) 216
First find thefactors of 216
$216 = 2 \times 2 \times 2 \times 3 \times 3 \times 3 = 2^3 \times 3^3 = 6^3$
Hence, it's a perfect cube.

(iii) 243

First find thefactors of 243
$243 = 3 \times 3 \times 3 \times 3 \times 3 = 3^5 = 3^3 \times 3^2$
Hence, it's not a perfect cube.

(iv) 1000
First find thefactors of 1000
$1000 = 2 \times 2 \times 2 \times 5 \times 5 \times 5 = 2^3 \times 5^3 = 10^3$
Hence, it's a perfect cube.

(v) 1728
First find thefactors of 1728
$1728 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3 = 2^6 \times 3^3 = (4 \times 3)^3 = 12^3$
Hence, it's a perfect cube.

(vi) 3087
First find thefactors of 3087
$3087 = 3 \times 3 \times 7 \times 7 \times 7 = 3^2 \times 7^3$
Hence, it's not a perfect cube.

(vii) 4608
First find thefactors of 4608
$4608 = 2 \times 2 \times 3 \times 113$
Hence, it's not a perfect cube.

(viii) 106480
First find thefactors of 106480
$106480 = 2 \times 2 \times 2 \times 2 \times 5 \times 11 \times 11 \times 11$
Hence, it's not a perfect cube.

(ix) 166375
First find thefactors of 166375
$166375= 5 \times 5 \times 5 \times 11 \times 11 \times 11 = 5^3 \times 11^3 = 55^3$
Hence, it's a perfect cube.

(x) 456533
First find thefactors of 456533
$456533= 11 \times 11 \times 11 \times 7 \times 7 \times 7 = 11^3 \times 7^3 = 77^3$
Hence, it's a perfect cube.

9. Which of the following are cubes of even natural numbers?
216, 512, 729, 1000, 3375, 13824
Solution:
(i) $216 = 2^3 \times 3^3 = 6^3$
It's a cube of even natural number.

(ii) $512 = 2^9 = (2^3)^3 = 8^3$
It's a cube of even natural number.

(iii) $729 = 3^3 \times 3^3 = 9^3$
It's not a cube of even natural number.

(iv) $1000 = 10^3$
It's a cube of even natural number.

(v) $3375 = 3^3 \times 5^3 = 15^3$
It's not a cube of even natural number.

(vi) $13824 = 2^9 \times 3^3 = (2^3)^3 \times 3^3 = 8^3 \times 3^3 = 24^3$
It's a cube of even natural number.

10. Which of the following are cubes of odd natural numbers?
125, 343, 1728, 4096, 32768, 6859
Solution:
(i) $125 = 5 \times 5 \times 5 \times 5 = 5^3$
It's a cube of odd natural number.

(ii) $343 = 7 \times 7 \times 7 = 7^3$
It's a cube of odd natural number.

(iii) $1728 = 2^6 \times 3^3 = 4^3 \times 3^3 = 12^3$
It's not a cube of odd natural number. As 12 is even number.

(iv) $4096 = 2^{12} = (2^6)^2 = 64^2$
Its not a cube of odd natural number. As 64 is an even number.

(v) $32768 = 2^{15} = (2^5)^3 = 32^3$
It's not a cube of odd natural number. As 32 is an even number.

(vi) $6859 = 19 \times 19 \times 19 = 19^3$
It's a cube of odd natural number.

11. What is the smallest number by which the following numbers must be multiplied, so that the products are perfect cubes?
(i) 675 (ii) 1323
(iii) 2560 (iv) 7803
(v) 107811 (vi) 35721
Solution:
(i) 675
First find the factors of 675
$675 = 3 \times 3 \times 3 \times 5 \times 5$
$= 3^3 \times 5^2$
∴To make a perfect cube we need to multiply the product by 5.

(ii) 1323
First find the factors of 1323
$1323 = 3 \times 3 \times 3 \times 7 \times 7$
$= 3^3 \times 7^2$
∴To make a perfect cube we need to multiply the product by 7.

(iii) 2560
First find the factors of 2560
$2560 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 5$
$= 2^3 \times 2^3 \times 2^3 \times 5$
∴To make a perfect cube we need to multiply the product by $5 \times 5 = 25$.

(iv) 7803
First find the factors of 7803
$7803 = 3 \times 3 \times 3 \times 17 \times 17$
$= 3^3 \times 17^2$
∴To make a perfect cube we need to multiply the product by 17.

(v) 107811
First find the factors of 107811
$107811 = 3 \times 3 \times 3 \times 3 \times 11 \times 11 \times 11$
$= 3^3 \times 3 \times 11^3$
∴To make a perfect cube we need to multiply the product by $3 \times 3 = 9$.

(vi) 35721
First find the factors of 35721
$35721 = 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 7 \times 7$
$= 3^3 \times 3^3 \times 7^2$
∴To make a perfect cube we need to multiply the product by 7.

12. By which smallest number must the following numbers be divided so that the quotient is a perfect cube?
(i) 675 (ii) 8640
(iii) 1600 (iv) 8788
(v) 7803 (vi) 107811
(vii) 35721 (viii) 243000
Solution:
(i) 675
First find the prime factors of 675
$675 = 3 \times 3 \times 3 \times 5 \times 5$
$= 3^3 \times 5^2$
Since 675 is not a perfect cube.
To make the quotient a perfect cube we divide it by $5^2 = 25$, which gives 27 as quotient where, 27 is a perfect cube.
∴ 25 is the required smallest number.

(ii) 8640
First find the prime factors of 8640
$8640 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 5$
$= 2^3 \times 2^3 \times 3^3 \times 5$
Since 8640 is not a perfect cube.
To make the quotient a perfect cube we divide it by 5, which gives 1728 as quotient and we know that 1728 is a perfect cube.
∴5 is the required smallest number.

(iii) 1600
First find the prime factors of 1600
$1600 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 5 \times 5$
$= 2^3 \times 2^3 \times 5^2$
Since 1600 is not a perfect cube.
To make the quotient a perfect cube we divide it by $5^2 = 25$, which gives 64 as quotient and we know that 64 is a perfect cube
∴ 25 is the required smallest number.

(iv) 8788
First find the prime factors of 8788
$8788 = 2 \times 2 \times 13 \times 13 \times 13$
$= 2^2 \times 13^3$
Since 8788 is not a perfect cube.
To make the quotient a perfect cube we divide it by 4, which gives 2197 as quotient and we know that 2197 is a perfect cube
∴ 4 is the required smallest number.

(v) 7803
First find the prime factors of 7803
$7803 = 3 \times 3 \times 3 \times 17 \times 17$
$= 3^3 \times 17^2$
Since 7803 is not a perfect cube.
To make the quotient a perfect cube we divide it by $17^2 = 289$, which gives 27 as quotient and we know that 27 is a perfect cube
∴ 289 is the required smallest number.

(vi) 107811
First find the prime factors of 107811
$107811 = 3 \times 3 \times 3 \times 3 \times 11 \times 11 \times 11$
$= 3^3 \times 11^3 \times 3$
Since 107811 is not a perfect cube.
To make the quotient a perfect cube we divide it by 3, which gives 35937 as quotient and we know that 35937 is a perfect cube.
∴ 3 is the required smallest number.

(vii) 35721
First find the prime factors of 35721
$35721 = 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 7 \times 7$
$= 3^3 \times 3^3 \times 7^2$
Since 35721 is not a perfect cube.
To make the quotient a perfect cube we divide it by $7^2 = 49$, which gives 729 as quotient and we know that 729 is a perfect cube
∴ 49 is the required smallest number.

(viii) 243000
First find the prime factors of 243000
$243000 = 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 3 \times 3 \times 5 \times 5 \times 5$

$= 2^3 \times 3^3 \times 5^3 \times 3^2$
Since 243000 is not a perfect cube.
To make the quotient a perfect cube we divide it by $3^2 = 9$, which gives 27000 as quotient and we know that 27000 is a perfect cube
∴ 9 is the required smallest number.

13. Prove that if a number is trebled then its cube is 27 time the cube of the given number.
Solution:
Let us consider a number as a
So the cube of the assumed number is = a^3
Now, the number is trebled = $3 \times a = 3a$
So the cube of new number = $(3a)^3 = 27a^3$
∴New cube is 27 times of the original cube.
Hence, proved.

14. What happens to the cube of a number if the number is multiplied by
(i) 3?
(ii) 4?
(iii) 5?
Solution:
(i) 3?
Let us consider the number as a
So its cube will be = a^3
According to the question, the number is multiplied by 3
New number becomes = 3a
So the cube of new number will be = $(3a)^3 = 27a^3$
Hence, number will become 27 times the cube of the number.

(ii) 4?
Let us consider the number as a
So its cube will be = a^3
According to the question, the number is multiplied by 4
New number becomes = 4a
So the cube of new number will be = $(4a)^3 = 64a^3$
Hence, number will become 64 times the cube of the number.

(iii) 5?

Let us consider the number as a
So its cube will be = a^3
According to the question, the number is multiplied by 5
New number becomes = 5a
So the cube of new number will be = $(5a)^3 = 125a^3$
Hence, number will become 125 times the cube of the number.

15. Find the volume of a cube, one face of which has an area of $64m^2$.
Solution:
We know that the given area of one face of cube = 64 m^2
Let the length of edge of cube be 'a' metre
$a^2 = 64$
$a = \sqrt{64}$
$= 8m$
Now, volume of cube = a^3
$a^3 = 8^3 = 8 \times 8 \times 8$
$= 512m^3$
∴Volume of a cube is $512m^3$

16. Find the volume of a cube whose surface area is $384m^2$.
Solution:
We know that the surface area of cube = 384 m^2
Let us consider the length of each edge of cube be 'a' meter
$6a^2 = 384$
$a^2 = 384/6$
$= 64$
$a = \sqrt{64}$
$= 8m$
Now, volume of cube = a^3
$a^3 = 8^3 = 8 \times 8 \times 8$
$= 512m^3$
∴ Volume of a cube is $512m^3$

17. Evaluate the following:
(i) $\{(5^2 + 12^2)^{1/2}\}^3$
(ii) $\{(6^2 + 8^2)^{1/2}\}^3$
Solution:
(i) $\{(5^2 + 12^2)^{1/2}\}^3$
When simplified above equation we get,

$\{(25 + 144)^{1/2}\}^3$
$\{(169)^{1/2}\}^3$
$\{(13^2)^{1/2}\}^3$
$(13)^3$
2197

(ii) $\{(6^2 + 8^2)^{1/2}\}^3$
When simplified above equation we get,
$\{(36 + 64)^{1/2}\}^3$
$\{(100)^{1/2}\}^3$
$\{(10^2)^{1/2}\}^3$
$(10)^3$
1000

18. Write the units digit of the cube of each of the following numbers: 31, 109, 388, 4276, 5922, 77774, 44447, 125125125
Solution:
31
To find unit digit of cube of a number we perform the cube of unit digit only.
Unit digit of 31 is 1
Cube of 1 = 1^3 = 1
∴ Unit digit of cube of 31 is always 1

109
To find unit digit of cube of a number we perform the cube of unit digit only.
Unit digit of 109 is = 9
Cube of 9 = 9^3 = 729
∴ Unit digit of cube of 109 is always 9

388
To find unit digit of cube of a number we perform the cube of unit digit only.
Unit digit of 388 is = 8
Cube of 8 = 8^3 = 512
∴ Unit digit of cube of 388 is always 2

4276
To find unit digit of cube of a number we perform the cube of unit digit only.
Unit digit of 4276 is = 6
Cube of 6 = 6^3 = 216

∴ Unit digit of cube of 4276 is always 6

5922
To find unit digit of cube of a number we perform the cube of unit digit only.
Unit digit of 5922 is = 2
Cube of 2 = 2^3 = 8
∴ Unit digit of cube of 5922 is always 8

77774
To find unit digit of cube of a number we perform the cube of unit digit only.
Unit digit of 77774 is = 4
Cube of 4 = 4^3 = 64
∴ Unit digit of cube of 77774 is always 4

44447
To find unit digit of cube of a number we perform the cube of unit digit only.
Unit digit of 44447 is = 7
Cube of 7 = 7^3 = 343
∴ Unit digit of cube of 44447 is always 3

125125125
To find unit digit of cube of a number we perform the cube of unit digit only.
Unit digit of 125125125 is = 5
Cube of 5 = 5^3 = 125
∴ Unit digit of cube of 125125125 is always 5

19. Find the cubes of the following numbers by column method:
(i) 35
(ii) 56
(iii) 72
Solution:
(i) 35
We have, a = 3 and b = 5

Column I a^3	Column II $3\times a^2\times b$	Column III $3\times a\times b^2$	Column IV b^3
$3^3 = 27$	3×9×5 = 135	3×3×25 = 225	$5^3 = 125$
+15	+23	+12	12$\underline{5}$
$\underline{42}$	15$\underline{8}$	23$\underline{7}$	
42	8	7	5

∴ The cube of 35 is 42875

(ii) 56
We have, a = 5 and b = 6

Column I a^3	Column II $3\times a^2\times b$	Column III $3\times a\times b^2$	Column IV b^3
$5^3 = 125$	3×25×6 = 450	3×5×36 = 540	$6^3 = 216$
+50	+56	+21	12<u>6</u>
<u>175</u>	50<u>6</u>	56<u>1</u>	
175	6	1	6

∴ The cube of 56 is 175616

(iii) 72
We have, a = 7 and b = 2

Column I a^3	Column II $3\times a^2\times b$	Column III $3\times a\times b^2$	Column IV b^3
$7^3 = 343$	3×49×2 = 294	3×7×4 = 84	$2^3 = 8$
+30	+8	+0	<u>8</u>
<u>373</u>	30<u>2</u>	8<u>4</u>	
373	2	4	8

∴ The cube of 72 is 373248

20. Which of the following numbers are not perfect cubes?
(i) 64
(ii) 216
(iii) 243
(iv) 1728
Solution:
(i) 64
Firstly let us find the prime factors of 64
64 = 2 × 2 × 2 × 2 × 2 × 2
= $2^3 \times 2^3$

$= 4^3$
Hence, it's a perfect cube.

(ii) 216
Firstly let us find the prime factors of 216
$216 = 2 \times 2 \times 2 \times 3 \times 3 \times 3$
$= 2^3 \times 3^3$
$= 6^3$
Hence, it's a perfect cube.

(iii) 243
Firstly let us find the prime factors of 243
$243 = 3 \times 3 \times 3 \times 3 \times 3$
$= 3^3 \times 3^2$
Hence, it's not a perfect cube.

(iv) 1728
Firstly let us find the prime factors of 1728
$1728 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3$
$= 2^3 \times 2^3 \times 3^3$
$= 12^3$
Hence, it's a perfect cube.

21. For each of the non-perfect cubes in Q. No 20 find the smallest number by which it must be
(a) Multiplied so that the product is a perfect cube.
(b) Divided so that the quotient is a perfect cube.
Solution:
Only non-perfect cube in previous question was = 243
(a) Multiplied so that the product is a perfect cube.
Firstly let us find the prime factors of 243
$243 = 3 \times 3 \times 3 \times 3 \times 3 = 3^3 \times 3^2$
Hence, to make it a perfect cube we should multiply it by 3.

(b) Divided so that the quotient is a perfect cube.
Firstly let us find the prime factors of 243
$243 = 3 \times 3 \times 3 \times 3 \times 3 = 3^3 \times 3^2$
Hence, to make it a perfect cube we have to divide it by 9.

22. By taking three different, values of n verify the truth of the following statements:
(i) If n is even, then n^3 is also even.
(ii) If n is odd, then n^3 is also odd.
(ii) If n leaves remainder 1 when divided by 3, then n^3 also leaves 1 as remainder when divided by 3.
(iv) If a natural number n is of the form 3p+2 then n^3 also a number of the same type.
Solution:
(i) If n is even, then n^3 is also even.
Let us consider three even natural numbers 2, 4, 6
So now, Cubes of 2, 4 and 6 are
$2^3 = 8$
$4^3 = 64$
$6^3 = 216$
Hence, we can see that all cubes are even in nature.
Statement is verified.

(ii) If n is odd, then n^3 is also odd.
Let us consider three odd natural numbers 3, 5, 7
So now, cubes of 3, 5 and 7 are
$3^3 = 27$
$5^3 = 125$
$7^3 = 343$
Hence, we can see that all cubes are odd in nature.
Statement is verified.

(iii) If n leaves remainder 1 when divided by 3, then n^3 also leaves 1 as remainder when divided by 3.
Let us consider three natural numbers of the form (3n+1) are 4, 7 and 10
So now, cube of 4, 7, 10 are
$4^3 = 64$
$7^3 = 343$
$10^3 = 1000$
We can see that if we divide these numbers by 3, we get 1 as remainder in each case.
Hence, statement is verified.

(iv) If a natural number n is of the form 3p+2 then n^3 also a number of the same type.
Let us consider three natural numbers of the form (3p+2) are 5, 8 and 11

So now, cube of 5, 8 and 10 are
$5^3 = 125$
$8^3 = 512$
$11^3 = 1331$
Now, we try to write these cubes in form of (3p + 2)
$125 = 3 \times 41 + 2$
$512 = 3 \times 170 + 2$
$1331 = 3 \times 443 + 2$
Hence, statement is verified.

23. Write true (T) or false (F) for the following statements:
(i) 392 is a perfect cube.
(ii) 8640 is not a perfect cube.
(iii) No cube can end with exactly two zeros.
(iv) There is no perfect cube which ends in 4.
(v) For an integer a, a^3 is always greater than a^2.
(vi) If a and b are integers such that $a^2>b^2$, then $a^3>b^3$.
(vii) If a divides b, then a^3 divides b^3.
(viii) If a^2 ends in 9, then a^3 ends in 7.
(ix) If a^2 ends in an even number of zeros, then a^3 ends in 25.
(x) If a^2 ends in an even number of zeros, then a^3 ends in an odd number of zeros.
Solution:
(i) 392 is a perfect cube.
Firstly let's find the prime factors of $392 = 2 \times 2 \times 2 \times 7 \times 7 = 2^3 \times 7^2$
Hence the statement is False.

(ii) 8640 is not a perfect cube.
Prime factors of $8640 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 5 = 2^3 \times 2^3 \times 3^3 \times 5$
Hence the statement is True

(iii) No cube can end with exactly two zeros.
Statement is True.
Because a perfect cube always have zeros in multiple of 3.

(iv) There is no perfect cube which ends in 4.
We know 64 is a perfect cube $= 4 \times 4 \times 4$ and it ends with 4.
Hence the statement is False.

(v) For an integer a, a^3 is always greater than a^2.

Statement is False.
Because in case of negative integers ,
$(-2)^2 = 4$ and $(-2)^3 = -8$

(vi) If a and b are integers such that $a^2>b^2$, then $a^3>b^3$.
Statement is False.
In case of negative integers,
$(-5)^2 > (-4)^2 = 25 > 16$
But, $(-5)^3 > (-4)^3 = -125 > -64$ is not true.

(vii) If a divides b, then a^3 divides b^3.
Statement is True.
If a divides b
$b/a = k$, so $b=ak$
$b^3/a^3 = (ak)^3/a^3 = a^3k^3/a^3 = k^3$,
For each value of b and a its true.

(viii) If a^2 ends in 9, then a^3 ends in 7.
Statement is False.
Let a = 7
$7^2 = 49$ and $7^3 = 343$

(ix) If a^2 ends in an even number of zeros, then a^3 ends in 25.
Statement is False.
Since, when a = 20
$a^2 = 20^2 = 400$ and $a^3 = 8000$ (a^3 doesn't end with 25)

(x) If a^2 ends in an even number of zeros, then a^3 ends in an odd number of zeros.
Statement is False.
Since, when a = 100
$a^2 = 100^2 = 10000$ and $a^3 = 100^3 = 1000000$ (a^3 doesn't end with odd number of zeros)

EXERCISE 4.2 PAGE NO: 4.13

1. Find the cubes of:
(i) -11
(ii) -12
(iii) -21
Solution:
(i) -11
The cube of 11 is
$(-11)^3 = -11\times -11\times -11 = -1331$

(ii) -12
The cube of 12 is
$(-12)^3 = -12\times -12\times -12 = -1728$

(iii) -21
The cube of 21 is
$(-21)^3 = -21\times -21\times -21 = -9261$

2. Which of the following integers are cubes of negative integers
(i) -64
(ii) -1056
(iii) -2197
(iv) -2744
(v) -42875
Solution:
(i) -64
The prime factors of 64 are
$64 = 2 \times 2 \times 2 \times 2 \times 2 \times 2$
$= 2^3 \times 2^3$
$= 4^3$
∴ 64 is a perfect cube of negative integer – 4.

(ii) -1056
The prime factors of 1056 are
$1056 = 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 11$
1056 is not a perfect cube.
∴ -1056 is not a cube of negative integer.

(iii) -2197
The prime factors of 2197 are
$2197 = 13 \times 13 \times 13$
$= 13^3$
∴ 2197 is a perfect cube of negative integer – 13.

(iv) -2744
The prime factors of 2744 are
$2744 = 2 \times 2 \times 2 \times 7 \times 7 \times 7$
$= 2^3 \times 7^3$
$= 14^3$
2744 is a perfect cube.
∴ -2744 is a cube of negative integer – 14.

(v) -42875
The prime factors of 42875 are
$42875 = 5 \times 5 \times 5 \times 7 \times 7 \times 7$
$= 5^3 \times 7^3$
$= 35^3$
42875 is a perfect cube.
∴ -42875 is a cube of negative integer – 35.

3. Show that the following integers are cubes of negative integers. Also, find the integer whose cube is the given integer.
(i) -5832
(ii) -2744000
Solution:
(i) -5832
The prime factors of 5832 are
$5823 = 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3$
$= 2^3 \times 3^3 \times 3^3$
$= 18^3$
5832 is a perfect cube.
∴ -5832 is a cube of negative integer – 18.

(ii) -2744000
The prime factors of 2744000 are
$2744000 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 5 \times 5 \times 5 \times 7 \times 7 \times 7$
$= 2^3 \times 2^3 \times 5^3 \times 7^3$

2744000 is a perfect cube.
∴ -2744000 is a cube of negative integer – 140.

4. Find the cube of:
(i) 7/9 (ii) -8/11
(iii) 12/7 (iv) -13/8
(v) 2 2/5 (vi) 3 1/4
(vii) 0.3 (viii) 1.5
(ix) 0.08 (x) 2.1
Solution:
(i) 7/9
The cube of 7/9 is
$(7/9)^3 = 7^3/9^3 = 343/729$

(ii) -8/11
The cube of -8/11 is
$(-8/11)^3 = -8^3/11^3 = -512/1331$

(iii) 12/7
The cube of 12/7 is
$(12/7)^3 = 12^3/7^3 = 1728/343$

(iv) -13/8
The cube of -13/8 is
$(-13/8)^3 = -13^3/8^3 = -2197/512$

(v) 2 2/5
The cube of 12/5 is
$(12/5)^3 = 12^3/5^3 = 1728/125$

(vi) 3 ¼
The cube of 13/4 is
$(13/4)^3 = 13^3/4^3 = 2197/64$

(vii) 0.3
The cube of 0.3 is
$(0.3)^3 = 0.3×0.3×0.3 = 0.027$

(viii) 1.5

The cube of 1.5 is
$(1.5)^3 = 1.5\times1.5\times1.5 = 3.375$

(ix) 0.08
The cube of 0.08 is
$(0.08)^3 = 0.08\times0.08\times0.08 = 0.000512$

(x) 2.1
The cube of 2.1 is
$(2.1)^3 = 2.1\times2.1\times2.1 = 9.261$

5. Find which of the following numbers are cubes of rational numbers:
(i) 27/64
(ii) 125/128
(iii) 0.001331
(iv) 0.04
Solution:
(i) 27/64
We have,
$27/64 = (3\times3\times3)/ (4\times4\times4) = 3^3/4^3 = (3/4)^3$
∴ 27/64 is a cube of 3/4.

(ii) 125/128
We have,
$125/128 = (5\times5\times5)/ (2\times2\times2\times2\times2\times2\times2) = 5^3/ (2^3\times2^3\times2)$
∴ 125/128 is not a perfect cube.

(iii) 0.001331
We have,
$1331/1000000 = (11\times11\times11)/ (100\times100\times100) = 11^3/100^3 = (11/100)^3$
∴ 0.001331 is a perfect cube of 11/100

(iv) 0.04
We have,
$4/10 = (2\times2)/(2\times5) = 2^2/(2\times5)$
∴ 0.04 is not a perfect cube.

EXERCISE 4.3 PAGE NO: 4.21

1. Find the cube roots of the following numbers by successive subtraction of numbers:
1, 7, 19, 37, 61, 91, 127, 169, 217, 271, 331, 397, …
(i) 64
(ii) 512
(iii) 1728
Solution:
(i) 64
Let's perform subtraction
64 – 1 = 63
63 – 7 = 56
56 – 19 =37
37 – 37 = 0
Subtraction is performed 4 times.
∴ Cube root of 64 is 4.

(ii) 512
Let's perform subtraction
512 – 1 = 511
511 – 7 = 504
504 – 19 = 485
485 – 37 = 448
448 – 61 = 387
387 – 91 = 296
296 – 127 = 169
169 – 169 = 0
Subtraction is performed 8 times.
∴ Cube root of 512 is 8.

(iii) 1728
Let's perform subtraction
1728 – 1 = 1727
1727 – 7 = 1720
1720 – 19 = 1701
1701 – 37 = 1664
1664 – 91 = 1512
1512 – 127 = 1385

1385 – 169 = 1216
1216 – 217 = 999
999 – 271 = 728
728 – 331 = 397
397 – 397 = 0
Subtraction is performed 12 times.
∴ Cube root of 1728 is 12.

2. Using the method of successive subtraction examine whether or not the following numbers are perfect cubes:
(i) 130
(ii) 345
(iii) 792
(iv) 1331
Solution:
(i) 130
Let's perform subtraction
130 – 1 = 129
129 – 7 = 122
122 – 19 = 103
103 – 37 = 66
66 – 61 = 5
Next number to be subtracted is 91, which is greater than 5
∴130 is not a perfect cube.

(ii) 345
Let's perform subtraction
345 – 1 = 344
344 – 7 = 337
337 – 19 = 318
318 – 37 = 281
281 – 61 = 220
220 – 91 = 129
129 – 127 = 2
Next number to be subtracted is 169, which is greater than 2
∴ 345 is not a perfect cube

(iii) 792
Let's perform subtraction

792 – 1 = 791
791 – 7 = 784
784 – 19 = 765
765 – 37 = 728
728 – 61 = 667
667 – 91 = 576
576 – 127 = 449
449 – 169 = 280
280 – 217 = 63
Next number to be subtracted is 271, which is greater than 63
∴ 792 is not a perfect cube

(iv) 1331
Let's perform subtraction
1331 – 1 = 1330
1330 – 7 = 1323
1323 – 19 = 1304
1304 – 37 = 1267
1267 – 61 = 1206
1206 – 91 = 1115
1115 – 127 = 988
988 – 169 = 819
819 – 217 = 602
602 – 271 = 331
331 – 331 = 0
Subtraction is performed 11 times
Cube root of 1331 is 11
∴ 1331 is a perfect cube.

3. Find the smallest number that must be subtracted from those of the numbers in question 2 which are not perfect cubes, to make them perfect cubes. What are the corresponding cube roots?
Solution:
In previous question there are three numbers which are not perfect cubes.
(i) 130
Let's perform subtraction
130 – 1 = 129
129 – 7 = 122
122 – 19 = 103

103 – 37 = 66
66 – 61 = 5
Next number to be subtracted is 91, which is greater than 5
Since, 130 is not a perfect cube. So, to make it perfect cube we subtract 5 from the given number.
130 – 5 = 125 (which is a perfect cube of 5)

(ii) 345
Let's perform subtraction
345 – 1 = 344
344 – 7 = 337
337 – 19 = 318
318 – 37 = 281
281 – 61 = 220
220 – 91 = 129
129 – 127 = 2
Next number to be subtracted is 169, which is greater than 2
Since, 345 is not a perfect cube. So, to make it a perfect cube we subtract 2 from the given number.
345 – 2 = 343 (which is a perfect cube of 7)

(iii) 792
Let's perform subtraction
792 – 1 = 791
791 – 7 = 784
784 – 19 = 765
765 – 37 = 728
728 – 61 = 667
667 – 91 = 576
576 – 127 = 449
449 – 169 = 280
280 – 217 = 63
Next number to be subtracted is 271, which is greater than 63
Since, 792 is not a perfect cube. So, to make it a perfect cube we subtract 63 from the given number.
792 – 63 = 729 (which is a perfect cube of 9)

4. Find the cube root of each of the following natural numbers:
(i) 343 (ii) 2744

(iii) 4913 (iv) 1728
(v) 35937 (vi) 17576
(vii) 134217728 (viii) 48228544
(ix) 74088000 (x) 157464
(xi) 1157625 (xii) 33698267
Solution:
(i) 343
By using prime factorization method
$\sqrt[3]{343} = \sqrt[3]{(7\times7\times7)} = 7$

(ii) 2744
By using prime factorization method
$\sqrt[3]{2744} = \sqrt[3]{(2\times2\times2\times7\times7\times7)} = \sqrt[3]{(2^3\times7^3)} = 2\times7 = 14$

(iii) 4913
By using prime factorization method,
$\sqrt[3]{4913} = \sqrt[3]{(17\times17\times17)} = 17$

(iv) 1728
By using prime factorization method,
$\sqrt[3]{1728} = \sqrt[3]{(2\times2\times2\times2\times2\times2\times3\times3\times3)} = \sqrt[3]{(2^3\times2^3\times3^3)} = 2\times2\times3 = 12$

(v) 35937
By using prime factorization method,
$\sqrt[3]{35937} = \sqrt[3]{(3\times3\times3\times11\times11\times11)} = \sqrt[3]{(3^3\times11^3)} = 3\times11 = 33$

(vi) 17576
By using prime factorization method,
$\sqrt[3]{17576} = \sqrt[3]{(2\times2\times2\times13\times13\times13)} = \sqrt[3]{(2^3\times13^3)} = 2\times13 = 26$

(vii) 134217728
By using prime factorization method
$\sqrt[3]{134217728} = \sqrt[3]{(2^{27})} = 2^9 = 512$

(viii) 48228544
By using prime factorization method
$\sqrt[3]{48228544} = \sqrt[3]{(2\times2\times2\times2\times2\times2\times7\times7\times7\times13\times13\times13)} = \sqrt[3]{(2^3\times2^3\times7^3\times13^3)} = 2\times2\times7\times13 = 364$

(ix) 74088000
By using prime factorization method
$\sqrt[3]{74088000} = \sqrt[3]{(2\times2\times2\times2\times2\times2\times3\times3\times3\times5\times5\times5\times7\times7\times7)} = \sqrt[3]{(2^3\times2^3\times3^3\times5^3\times7^3)} =$
$2\times2\times3\times5\times7 = 420$

(x) 157464
By using prime factorization method
$\sqrt[3]{157464} = \sqrt[3]{(2\times2\times2\times3\times3\times3\times3\times3\times3\times3\times3\times3)} = \sqrt[3]{(2^3\times3^3\times3^3\times3^3)} = 2\times3\times3\times3 = 54$

(xi) 1157625
By using prime factorization method
$\sqrt[3]{1157625} = \sqrt[3]{(3\times3\times3\times5\times5\times5\times7\times7\times7)} = \sqrt[3]{(3^3\times5^3\times7^3)} = 3\times5\times7 = 105$

(xii) 33698267
By using prime factorization method
$\sqrt[3]{33698267} = \sqrt[3]{(17\times17\times17\times19\times19\times19)} = \sqrt[3]{(17^3\times19^3)} = 17\times19 = 323$

5. Find the smallest number which when multiplied with 3600 will make the product a perfect cube. Further, find the cube root of the product.
Solution:
Firstly let's find the prime factors for 3600
$3600 = 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 5 \times 5$
$= 2^3 \times 3^2 \times 5^2 \times 2$
Since only one triples is formed and three factors remained ungrouped in triples.
The given number 3600 is not a perfect cube.
To make it a perfect cube we have to multiply it by $(2 \times 2 \times 3 \times 5) = 60$
$3600 \times 60 = 216000$
Cube root of 216000 is
$\sqrt[3]{216000} = \sqrt[3]{(60\times60\times60)} = \sqrt[3]{(60^3)} = 60$
∴ the smallest number which when multiplied with 3600 will make the product a perfect cube is 60 and the cube root of the product is 60.

6. Multiply 210125 by the smallest number so that the product is a perfect cube. Also, find out the cube root of the product.
Solution:
The prime factors of 210125 are
$210125 = 5 \times 5 \times 5 \times 41 \times 41$
Since, one triples remained incomplete, 210125 is not a perfect cube.
To make it a perfect cube we need to multiply the factors by 41, we will get 2 triples as

23 and 41^3.
And the product become:
$210125 \times 41 = 8615125$
$8615125 = 5 \times 5 \times 5 \times 41 \times 41 \times 41$
Cube root of product = $\sqrt[3]{8615125} = \sqrt[3]{}(5\times41) = 205$

7. What is the smallest number by which 8192 must be divided so that quotient is a perfect cube? Also, find the cube root of the quotient so obtained.

Solution:
The prime factors of 8192 are
$8192 = 2\times2\times2\times2\times2\times2\times2\times2\times2\times2 = 2^3\times2^3\times2^3\times2$
Since, one triples remain incomplete, hence 8192 is not a perfect cube.
So, we divide 8192 by 2 to make its quotient a perfect cube.
$8192/2 = 4096$
$4096 = 2\times2\times2\times2\times2\times2\times2\times2\times2\times2\times2\times2 = 2^3\times2^3\times2^3\times2^3$
Cube root of 4096 = $\sqrt[3]{4096} = \sqrt[3]{}(2^3\times2^3\times2^3\times2^3) = 2\times2\times2\times2 = 16$

8. Three numbers are in the ratio 1:2:3. The sum of their cubes is 98784. Find the numbers.

Solution:
Let us consider the ratio 1:2:3 as x, 2x and 3x
According to the question,
$X^3 + (2x)^3 + (3x)^3 = 98784$
$x^3 + 8x^3 + 27x^3 = 98784$
$36x^3 = 98784$
$x^3 = 98784/36$
$= 2744$
$x = \sqrt[3]{2744} = \sqrt[3]{}(2\times2\times2\times7\times7\times7) = 2\times7 = 14$
So, the numbers are,
$x = 14$
$2x = 2 \times 14 = 28$
$3x = 3 \times 14 = 42$

9. **The volume of a cube is 9261000 m^3. Find the side of the cube.**
Given, volume of cube = 9261000 m^3
Let us consider the side of cube be 'a' metre
So, $a^3 = 9261000$
$a = \sqrt[3]{9261000} = \sqrt[3]{}(2\times2\times2\times3\times3\times3\times5\times5\times5\times7\times7\times7) = \sqrt[3]{}(2^3\times3^3\times5^3\times7^3) = 2\times3\times5\times7 = 210$
$\therefore$ the side of cube = 210 metre

EXERCISE 4.4 PAGE NO: 4.30

1. Find the cube roots of each of the following integers:
(i)-125 (ii) -5832
(iii)-2744000 (iv) -753571
(v) -32768
Solution:
(i) -125
The cube root of -125 is
$-125 = \sqrt[3]{-125} = -\sqrt[3]{125} = \sqrt[3]{(5\times5\times5)} = -5$

(ii) -5832
The cube root of -5832 is
$-5832 = \sqrt[3]{-5832} = -\sqrt[3]{5832}$
To find the cube root of 5832, we shall use the method of unit digits.
Let us consider the number 5832. Where, unit digit of 5832 = 2
Unit digit in the cube root of 5832 will be 8
After striking out the units, tens and hundreds digits of 5832,
Now we left with 5 only.
We know that 1 is the Largest number whose cube is less than or equal to 5.
So, the tens digit of the cube root of 5832 is 1.
$\sqrt[3]{-5832} = -\sqrt[3]{5832} = -18$

(iii) -2744000
$\sqrt[3]{-2744000} = -\sqrt[3]{2744000}$
We shall use the method of factorization to find the cube root of 2744000
So let's find the prime factors for 2744000
$2744000 = 2\times2\times2\times2\times2\times2\times5\times5\times5\times7\times7\times7$
Now by grouping the factors into triples of equal factors, we get,
$2744000 = (2\times2\times2) \times (2\times2\times2) \times (5\times5\times5) \times (7\times7\times7)$
Since all the prime factors of 2744000 is grouped in to triples of equal factors and no factor is left over.
So now take one factor from each group and by multiplying we get,
$2\times2\times5\times7 = 140$
Thereby we can say that 2744000 is a cube of 140
$\therefore \sqrt[3]{-2744000} = -\sqrt[3]{2744000} = -140$

(iv) -753571
$\sqrt[3]{-753571} = -\sqrt[3]{753571}$

We shall use the unit digit method,
Let us consider the number 753571, where unit digit = 1
Unit digit in the cube root of 753571 will be 1
After striking out the units, tens and hundreds digits of 753571,
Now we left with 753.
We know that 9 is the Largest number whose cube is less than or equal to $753(9^3<753<10^3)$.
So, the tens digit of the cube root of 753571 is 9.
$\sqrt[3]{753571} = 91$
$\sqrt[3]{-753571} = -\sqrt[3]{753571} = -91$

(v) -32768
$\sqrt[3]{-32765} = -\sqrt[3]{32768}$
We shall use the unit digit method,
Let us consider the Number = 32768, where unit digit = 8
Unit digit in the cube root of 32768 will be 2
After striking out the units, tens and hundreds digits of 32768,
Now we left with 32.
As we know that 9 is the Largest number whose cube is less than or equals to $32(3^3<32<4^3)$.
So, the tens digit of the cube root of 32768 is 3.
$\sqrt[3]{32765} = 32$
$\sqrt[3]{-32765} = -\sqrt[3]{32768} = -32$

2. Show that:
(i) $\sqrt[3]{27} \times \sqrt[3]{64} = \sqrt[3]{(27\times64)}$
(ii) $\sqrt[3]{(64\times729)} = \sqrt[3]{64} \times \sqrt[3]{729}$
(iii) $\sqrt[3]{(-125\times216)} = \sqrt[3]{-125} \times \sqrt[3]{216}$
(iv) $\sqrt[3]{(-125\times-1000)} = \sqrt[3]{-125} \times \sqrt[3]{-1000}$
Solution:
(i) $\sqrt[3]{27} \times \sqrt[3]{64} = \sqrt[3]{(27\times64)}$
Let us consider LHS $\sqrt[3]{27} \times \sqrt[3]{64}$
$\sqrt[3]{27} \times \sqrt[3]{64} = \sqrt[3]{(3\times3\times3)} \times \sqrt[3]{(4\times4\times4)}$
$= 3\times4$
$= 12$
Let us consider RHS $\sqrt[3]{(27\times64)}$
$\sqrt[3]{(27\times64)} = \sqrt[3]{(3\times3\times3\times4\times4\times4)}$
$= 3\times4$
$= 12$

∴ LHS = RHS, the given equation is verified.

(ii) $\sqrt[3]{(64\times729)} = \sqrt[3]{64} \times \sqrt[3]{729}$
Let us consider LHS $\sqrt[3]{(64\times729)}$
$\sqrt[3]{(64\times729)} = \sqrt[3]{(4\times4\times4\times9\times9\times9)}$
$= 4\times9$
$= 36$
Let us consider RHS $\sqrt[3]{64} \times \sqrt[3]{729}$
$\sqrt[3]{64} \times \sqrt[3]{729} = \sqrt[3]{(4\times4\times4)} \times \sqrt[3]{(9\times9\times9)}$
$= 4\times9$
$= 36$
∴ LHS = RHS, the given equation is verified.

(iii) $\sqrt[3]{(-125\times216)} = \sqrt[3]{-125} \times \sqrt[3]{216}$
Let us consider LHS $\sqrt[3]{(-125\times216)}$
$\sqrt[3]{(-125\times216)} = \sqrt[3]{(-5\times-5\times-5\times2\times2\times2\times3\times3\times3)}$
$= -5\times2\times3$
$= -30$
Let us consider RHS $\sqrt[3]{-125} \times \sqrt[3]{216}$
$\sqrt[3]{-125} \times \sqrt[3]{216} = \sqrt[3]{(-5\times-5\times-5)} \times \sqrt[3]{(2\times2\times2\times3\times3\times3)}$
$= -5\times2\times3$
$= -30$
∴ LHS = RHS, the given equation is verified.

(iv) $\sqrt[3]{(-125\times-1000)} = \sqrt[3]{-125} \times \sqrt[3]{-1000}$
Let us consider LHS $\sqrt[3]{(-125\times-1000)}$
$\sqrt[3]{(-125\times-1000)} = \sqrt[3]{(-5\times-5\times-5\times-10\times-10\times-10)}$
$= -5\times-10$
$= 50$
Let us consider RHS $\sqrt[3]{-125} \times \sqrt[3]{-1000}$
$\sqrt[3]{-125} \times \sqrt[3]{-1000} = \sqrt[3]{(-5\times-5\times-5)} \times \sqrt[3]{(-10\times-10\times-10)}$
$= -5\times-10$
$= 50$
∴ LHS = RHS, the given equation is verified.

3. Find the cube root of each of the following numbers:
(i) 8×125
(ii) -1728×216
(iii) -27×2744

(iv) -729×-15625

Solution:

(i) 8×125

We know that for any two integers a and b, $\sqrt[3]{(a\times b)} = \sqrt[3]{a} \times \sqrt[3]{b}$

By using the property

$\sqrt[3]{(8\times125)} = \sqrt[3]{8} \times \sqrt[3]{125}$

$= \sqrt[3]{(2\times2\times2)} \times \sqrt[3]{(5\times5\times5)}$

$= 2\times5$

$= 10$

(ii) -1728×216

We know that for any two integers a and b, $\sqrt[3]{(a\times b)} = \sqrt[3]{a} \times \sqrt[3]{b}$

By using the property

$\sqrt[3]{(-1728\times216)} = \sqrt[3]{-1728} \times \sqrt[3]{216}$

We shall use the unit digit method

Let us consider the number 1728, where Unit digit = 8

The unit digit in the cube root of 1728 will be 2

After striking out the units, tens and hundreds digits of the given number, we are left with the 1.

We know 1 is the largest number whose cube is less than or equal to 1.

So, the tens digit of the cube root of 1728 = 1

$\sqrt[3]{1728} = 12$

Now, let's find the prime factors for 216 = 2×2×2×3×3×3

By grouping the factors in triples of equal factor, we get,

216 = (2×2×2) × (3×3×3)

By taking one factor from each group we get,

$\sqrt[3]{216} = 2\times3 = 6$

∴ by equating the values in the given equation we get,

$\sqrt[3]{(-1728\times216)} = \sqrt[3]{-1728} \times \sqrt[3]{216}$

$= -12 \times 6$

$= -72$

(iii) -27×2744

We know that for any two integers a and b, $\sqrt[3]{(a\times b)} = \sqrt[3]{a} \times \sqrt[3]{b}$

By using the property

$\sqrt[3]{(-27\times2744)} = \sqrt[3]{-27} \times \sqrt[3]{2744}$

We shall use the unit digit method

Let us consider the number 2744, where Unit digit = 4

The unit digit in the cube root of 2744 will be 4
After striking out the units, tens and hundreds digits of the given number, we are left with the 2.
We know 2 is the largest number whose cube is less than or equal to 2.
So, the tens digit of the cube root of 2744 = 2
$\sqrt[3]{2744} = 14$

Now, let's find the prime factors for 27 = 3×3×3
By grouping the factors in triples of equal factor, we get,
27 = (3×3×3)
Cube root of 27 is
$\sqrt[3]{27} = 3$
∴ by equating the values in the given equation we get,
$\sqrt[3]{(-27\times2744)} = \sqrt[3]{-27} \times \sqrt[3]{2744}$
$= -3 \times 14$
$= -42$

(iv) -729×-15625
We know that for any two integers a and b, $\sqrt[3]{(a\times b)} = \sqrt[3]{a} \times \sqrt[3]{b}$
By using the property
$\sqrt[3]{(-729\times-15625)} = \sqrt[3]{-729} \times \sqrt[3]{-15625}$
We shall use the unit digit method
Let us consider the number 15625, where Unit digit = 5
The unit digit in the cube root of 15625 will be 5
After striking out the units, tens and hundreds digits of the given number, we are left with the 15.
We know 15 is the largest number whose cube is less than or equal to 15($2^3<15<3^3$).
So, the tens digit of the cube root of 15625 = 2
$\sqrt[3]{15625} = 25$

Now, let's find the prime factors for 729 = 9×9×9
By grouping the factors in triples of equal factor, we get,
729 = (9×9×9)
Cube root of 729 is
$\sqrt[3]{729} = 9$
∴ by equating the values in the given equation we get,
$\sqrt[3]{(-729\times-15625)} = \sqrt[3]{-729} \times \sqrt[3]{-15625}$
$= -9 \times -25$
$= 225$

4. Evaluate:
(i) $\sqrt[3]{(4^3 \times 6^3)}$
(ii) $\sqrt[3]{(8\times17\times17\times17)}$
(iii) $\sqrt[3]{(700\times2\times49\times5)}$
(iv) $125\sqrt[3]{a^6} - \sqrt[3]{125a^6}$
Solution:
(i) $\sqrt[3]{(4^3 \times 6^3)}$
We know that for any two integers a and b, $\sqrt[3]{(a\times b)} = \sqrt[3]{a} \times \sqrt[3]{b}$
By using the property
$\sqrt[3]{(4^3 \times 6^3)} = \sqrt[3]{4^3} \times \sqrt[3]{6^3}$
$= 4 \times 6$
$= 24$

(ii) $\sqrt[3]{(8\times17\times17\times17)}$
We know that for any two integers a and b, $\sqrt[3]{(a\times b)} = \sqrt[3]{a} \times \sqrt[3]{b}$
By using the property
$\sqrt[3]{(8\times17\times17\times17)} = \sqrt[3]{8} \times \sqrt[3]{17\times17\times17}$
$= \sqrt[3]{2^3} \times \sqrt[3]{17^3}$
$= 2 \times 17$
$= 34$

(iii) $\sqrt[3]{(700\times2\times49\times5)}$
Firstly let us find the prime factors for the above numbers
$\sqrt[3]{(700\times2\times49\times5)} = \sqrt[3]{(2\times2\times5\times5\times7\times2\times7\times7\times5)}$
$= \sqrt[3]{(2^3\times5^3\times7^3)}$
$= 2\times5\times7$
$= 70$

(iv) $125\sqrt[3]{a^6} - \sqrt[3]{125a^6}$
$125\sqrt[3]{a^6} - \sqrt[3]{125a^6} = 125\sqrt[3]{(a^2)^3} - \sqrt[3]{5^3(a^2)^3}$
$= 125a^2 – 5a^2$
$= 120a^2$

5. Find the cube root of each of the following rational numbers:
(i) -125/729
(ii) 10648/12167
(iii) -19683/24389
(iv) 686/-3456
(v) -39304/-42875

Solution:

(i) -125/729

Let us find the prime factors of 125 and 729

$-125/729 = -(\sqrt[3]{(5\times5\times5)}) / (\sqrt[3]{(9\times9\times9)})$

$= -(\sqrt[3]{(5^3)}) / (\sqrt[3]{(9^3)})$

$= -5/9$

(ii) 10648/12167

Let us find the prime factors of 10648 and 12167

$10648/12167 = (\sqrt[3]{(2\times2\times2\times11\times11\times11)}) / (\sqrt[3]{(23\times23\times23)})$

$= (\sqrt[3]{(2^3\times11^3)}) / (\sqrt[3]{(23^3)})$

$= (2\times11)/23$

$= 22/23$

(iii) -19683/24389

Let us find the prime factors of 19683 and 24389

$-19683/24389 = -(\sqrt[3]{(3\times3\times3\times3\times3\times3\times3\times3\times3)}) / (\sqrt[3]{(29\times29\times29)})$

$= -(\sqrt[3]{(3^3\times3^3\times3^3)}) / (\sqrt[3]{(29^3)})$

$= -(3\times3\times3)/29$

$= -27/29$

(iv) 686/-3456

Let us find the prime factors of 686 and -3456

$686/-3456 = = -(\sqrt[3]{(2\times7\times7\times7)}) / (\sqrt[3]{(2^7\times2^3)})$

$= -(\sqrt[3]{(2\times7^3)}) / (\sqrt[3]{(2^7\times2^3)})$

$= -(\sqrt[3]{(7^3)}) / (\sqrt[3]{(2^6\times2^3)})$

$= -7/(2\times2\times2)$

$= -7/8$

(v) -39304/-42875

Let us find the prime factors of -39304 and -42875

$-39304/-42875 = -(\sqrt[3]{(2\times2\times2\times17\times17\times17)}) / -(\sqrt[3]{(5\times5\times5\times7\times7\times7)})$

$= -(\sqrt[3]{(2^3\times17^3)}) / -(\sqrt[3]{(5^3\times7^3)})$

$= -(2\times17)/-(5\times7)$

$= -34/-35$

$= 34/35$

6. Find the cube root of each of the following rational numbers:

(i) 0.001728

(ii) 0.003375
(iii) 0.001
(iv) 1.331
Solution:
(i) 0.001728
0.001728 = 1728/1000000
$\sqrt[3]{}$ (0.001728) = $\sqrt[3]{1728}$ / $\sqrt[3]{1000000}$
Let us find the prime factors of 1728 and 1000000
$\sqrt[3]{(0.001728)} = \sqrt[3]{(2^3\times2^3\times3^3)} / \sqrt[3]{(100^3)}$
$= (2\times2\times3)/100$
$= 12/100$
$= 0.12$

(ii) 0.003375
0.003375 = 3375/1000000
$\sqrt[3]{}$ (0.003375) = $\sqrt[3]{3375}$ / $\sqrt[3]{1000000}$
Let us find the prime factors of 3375 and 1000000
$\sqrt[3]{(0.003375)} = \sqrt[3]{(3^3\times5^3)} / \sqrt[3]{(100^3)}$
$= (3\times5)/100$
$= 15/100$
$= 0.15$

(iii) 0.001
0.001 = 1/1000
$\sqrt[3]{}$ (0.001) = $\sqrt[3]{1}$ / $\sqrt[3]{1000}$
$= 1/ \sqrt[3]{10^3}$
$= 1/10$
$= 0.1$

(iv) 1.331
1.331 = 1331/1000
$\sqrt[3]{}$ (1.331) = $\sqrt[3]{1331}$ / $\sqrt[3]{1000}$
Let us find the prime factors of 1331 and 1000
$\sqrt[3]{(1.331)} = \sqrt[3]{(11^3)} / \sqrt[3]{(10^3)}$
$= 11/10$
$= 1.1$

7. Evaluate each of the following:
(i) $\sqrt[3]{27} + \sqrt[3]{0.008} + \sqrt[3]{0.064}$

(ii) $\sqrt[3]{1000} + \sqrt[3]{0.008} - \sqrt[3]{0.125}$
(iii) $\sqrt[3]{(729/216)} \times 6/9$
(iv) $\sqrt[3]{(0.027/0.008)} \div \sqrt[3]{(0.09/0.04)} - 1$
(v) $\sqrt[3]{(0.1\times0.1\times0.1\times13\times13\times13)}$
Solution:
(i) $\sqrt[3]{27} + \sqrt[3]{0.008} + \sqrt[3]{0.064}$
Let us simplify
$\sqrt[3]{(3\times3\times3)} + \sqrt[3]{(0.2\times0.2\times0.2)} + \sqrt[3]{(0.4\times0.4\times0.4)}$
$\sqrt[3]{(3)^3} + \sqrt[3]{(0.2)^3} + \sqrt[3]{(0.4)^3}$
$3 + 0.2 + 0.4$
3.6

(ii) $\sqrt[3]{1000} + \sqrt[3]{0.008} - \sqrt[3]{0.125}$
Let us simplify
$\sqrt[3]{(10\times10\times10)} + \sqrt[3]{(0.2\times0.2\times0.2)} - \sqrt[3]{(0.5\times0.5\times0.5)}$
$\sqrt[3]{(10)^3} + \sqrt[3]{(0.2)^3} - \sqrt[3]{(0.5)^3}$
$10 + 0.2 - 0.5$
9.7

(iii) $\sqrt[3]{(729/216)} \times 6/9$
Let us simplify
$\sqrt[3]{(9\times9\times9/6\times6\times6)} \times 6/9$
$(\sqrt[3]{(9)^3} / \sqrt[3]{(6)^3}) \times 6/9$
$9/6 \times 6/9$
1

(iv) $\sqrt[3]{(0.027/0.008)} \div \sqrt{(0.09/0.04)} - 1$
Let us simplify $\sqrt[3]{(0.027/0.008)} \div \sqrt{(0.09/0.04)}$
$\sqrt[3]{(0.3\times0.3\times0.3/0.2\times0.2\times0.2)} \div \sqrt{(0.3\times0.3/0.2\times0.2)}$
$(\sqrt[3]{(0.3)^3} / \sqrt[3]{(0.2)^3}) \div (\sqrt{(0.3)^2} / \sqrt{(0.2)^2})$
$(0.3/0.2) \div (0.3/0.2) -1$
$(0.3/0.2 \times 0.2/0.3) -1$
$1 – 1$
0

(v) $\sqrt[3]{(0.1\times0.1\times0.1\times13\times13\times13)}$
$\sqrt[3]{(0.1^3\times13^3)}$
$0.1 \times 13 = 1.3$

8. Show that:
(i) $\sqrt[3]{(729)}/\sqrt[3]{(1000)} = \sqrt[3]{(729/1000)}$
(ii) $\sqrt[3]{(-512)}/\sqrt[3]{(343)} = \sqrt[3]{(-512/343)}$
Solution:
(i) $\sqrt[3]{(729)}/\sqrt[3]{(1000)} = \sqrt[3]{(729/1000)}$
Let us consider LHS $\sqrt[3]{(729)}/\sqrt[3]{(1000)}$
$\sqrt[3]{(729)}/\sqrt[3]{(1000)} = \sqrt[3]{(9\times9\times9)}/\sqrt[3]{(10\times10\times10)}$
$= \sqrt[3]{(9^3/10^3)}$
$= 9/10$
Let us consider RHS $\sqrt[3]{(729/1000)}$
$\sqrt[3]{(729/1000)} = \sqrt[3]{(9\times9\times9/10\times10\times10)}$
$= \sqrt[3]{(9^3/10^3)}$
$= 9/10$
∴ LHS = RHS

(ii) $\sqrt[3]{(-512)}/\sqrt[3]{(343)} = \sqrt[3]{(-512/343)}$
Let us consider LHS $\sqrt[3]{(-512)}/\sqrt[3]{(343)}$
$\sqrt[3]{(-512)}/\sqrt[3]{(343)} = \sqrt[3]{-(8\times8\times8)}/\sqrt[3]{(7\times7\times7)}$
$= \sqrt[3]{-(8^3/7^3)}$
$= -8/7$
Let us consider RHS $\sqrt[3]{(-512/343)}$
$\sqrt[3]{(-512/343)} = = \sqrt[3]{-(8\times8\times8/7\times7\times7)}$
$= \sqrt[3]{-(8^3/7^3)}$
$= -8/7$
∴ LHS = RHS

9. Fill in the blanks:
(i) $\sqrt[3]{(125\times27)} = 3 \times \ldots$
(ii) $\sqrt[3]{(8\times\ldots)} = 8$
(iii) $\sqrt[3]{1728} = 4 \times \ldots$
(iv) $\sqrt[3]{480} = \sqrt[3]{3}\times2\times\sqrt[3]{..}$
(v) $\sqrt[3]{\ldots} = \sqrt[3]{7}\times\sqrt[3]{8}$
(vi) $\sqrt[3]{..} = \sqrt[3]{4}\times\sqrt[3]{5}\times\sqrt[3]{6}$
(vii) $\sqrt[3]{(27/125)} = \ldots/5$
(viii) $\sqrt[3]{(729/1331)} = 9/\ldots$
(ix) $\sqrt[3]{(512/\ldots)} = 8/13$
Solution:
(i) $\sqrt[3]{(125\times27)} = 3 \times \ldots$
Let us consider LHS $\sqrt[3]{(125\times27)}$

$\sqrt[3]{(125\times27)} = \sqrt[3]{(5\times5\times5\times3\times3\times3)}$
$= \sqrt[3]{(5^3\times3^3)}$
$= 5\times3$ or 3×5

(ii) $\sqrt[3]{(8\times\ldots)} = 8$
Let us consider LHS $\sqrt[3]{(8\times\ldots)}$
$\sqrt[3]{(8\times8\times8)} = \sqrt[3]{8^3} = 8$

(iii) $\sqrt[3]{1728} = 4 \times \ldots$
Let us consider LHS
$\sqrt[3]{1728} = \sqrt[3]{(2\times2\times2\times2\times2\times2\times3\times3\times3)}$
$= \sqrt[3]{(2^3\times2^3\times3^3)}$
$= 2\times2\times3$
$= 4\times3$

(iv) $\sqrt[3]{480} = \sqrt[3]{3}\times2\times \sqrt[3]{..}$
Let us consider LHS
$\sqrt[3]{480} = \sqrt[3]{(2\times2\times2\times2\times2\times3\times5)}$
$= \sqrt[3]{(2^3\times2^2\times3\times5)}$
$= \sqrt[3]{2^3}\times \sqrt[3]{3} \times \sqrt[3]{2\times2\times5}$
$= 2 \times \sqrt[3]{3} \times \sqrt[3]{20}$

(v) $\sqrt[3]{\ldots} = \sqrt[3]{7} \times \sqrt[3]{8}$
Let us consider RHS
$\sqrt[3]{7} \times \sqrt[3]{8} = \sqrt[3]{(7 \times 8)}$
$= \sqrt[3]{56}$

(vi) $\sqrt[3]{..}= \sqrt[3]{4} \times \sqrt[3]{5} \times \sqrt[3]{6}$
Let us consider RHS
$\sqrt[3]{4} \times \sqrt[3]{5} \times \sqrt[3]{6} = \sqrt[3]{(4 \times 5 \times 6)}$
$= \sqrt[3]{120}$

(vii) $\sqrt[3]{(27/125)} = \ldots/5$
Let us consider LHS
$\sqrt[3]{(27/125)} = \sqrt[3]{(3\times3\times3)/(5\times5\times5)}$
$= \sqrt[3]{(3^3)/(5^3)}$
$= 3/5$

(viii) $\sqrt[3]{(729/1331)} = 9/\ldots$

Let us consider LHS

$\sqrt[3]{(729/1331)} = \sqrt[3]{(9\times9\times9)/(11\times11\times11)}$

$= \sqrt[3]{(9^3)/(11^3)}$

$= 9/11$

(ix) $\sqrt[3]{(512/\ldots)} = 8/13$

Let us consider LHS

$\sqrt[3]{(512/\ldots)} = \sqrt[3]{(2\times2\times2\times2\times2\times2\times2\times2\times2)}$

$= \sqrt[3]{(2^3\times2^3\times2^3)}$

$= 2\times2\times2$

$= 8$

So, $8/\sqrt[3]{\ldots} = 8/13$

when numerators are same the denominators will also become equal.

$8 \times 13 = 8 \times \sqrt[3]{\ldots}$

$\sqrt[3]{\ldots} = 13$

$\ldots = (13)^3$

$= 2197$

10. The volume of a cubical box is 474. 552 cubic metres. Find the length of each side of the box.

Solution:

Volume of a cubical box is 474.552 cubic metres

$V = 8^3$,

Let 'S' be the side of the cube

$8^3 = 474.552$ cubic metres

$8 = \sqrt[3]{474.552}$

$= \sqrt[3]{(474552/1000)}$

Let us factorise 474552 into prime factors, we get:

$474552 = 2\times2\times2\times3\times3\times3\times13\times13\times13$

By grouping the factors in triples of equal factors, we get:

$474552 = (2\times2\times2) \times (3\times3\times3) \times (13\times13\times13)$

Now, $\sqrt[3]{474.552} = \sqrt[3]{((2\times2\times2) \times (3\times3\times3) \times (13\times13\times13))}$

$= 2\times3\times13$

$= 78$

Also,

$\sqrt[3]{1000} = \sqrt[3]{(10\times10\times10)}$

$= \sqrt[3]{(10)^3}$

$= 10$

So now let us equate in the above equation we get,

$8 = \sqrt[3]{(474552/1000)}$
$= 78/10$
$= 7.8$
∴ length of the side is 7.8m.

11. Three numbers are to one another 2:3:4. The sum of their cubes is 0.334125. Find the numbers.
Solution:
Let us consider the ratio 2:3:4 be 2a, 3a, and 4a.
So according to the question:
$(2a)^3 + (3a)^3 + (4a)^3 = 0.334125$
$8a^3 + 27\,a^3 + 64\,a^3 = 0.334125$
$99a^3 = 0.334125$
$a^3 = 334125/1000000 \times 99$
$= 3375/1000000$
$a = \sqrt[3]{(3375/1000000)}$
$= \sqrt[3]{((15\times15\times15)/\ 100\times100\times100)}$
$= 15/100$
$= 0.15$
∴ The numbers are:
$2\times0.15 = 0.30$
$3\times0.15 = 0.45$
$4\times0.15 = 0.6$

12. Find the side of a cube whose volume is $24389/216m^3$.
Solution:
Volume of the side s = 24389/216 = v
$V = 8^{.3}$
$8 = \sqrt[3]{v}$
$= \sqrt[3]{(24389/216)}$
By performing factorisation we get,
$= \sqrt[3]{(29\times29\times29/2\times2\times2\times3\times3\times3)}$
$= 29/(2\times3)$
$= 29/6$
∴ The length of the side is 29/6.

13. Evaluate:
(i) $\sqrt[3]{36} \times \sqrt[3]{384}$
(ii) $\sqrt[3]{96} \times \sqrt[3]{144}$

(iii) $\sqrt[3]{100} \times \sqrt[3]{270}$
(iv) $\sqrt[3]{121} \times \sqrt[3]{297}$
Solution:
(i) $\sqrt[3]{36} \times \sqrt[3]{384}$
We know that $\sqrt[3]{a} \times \sqrt[3]{b} = \sqrt[3]{(a\times b)}$
By using the above formula let us simplify
$\sqrt[3]{36} \times \sqrt[3]{384} = \sqrt[3]{(36\times384)}$
The prime factors of 36 and 384 are
$= \sqrt[3]{(2\times2\times3\times3) \times (2\times2\times2\times2\times2\times2\times3\times3\times3)}$
$= \sqrt[3]{(2^3\times2^3\times2^3\times3^3)}$
$= 2\times2\times2\times3$
$= 24$

(ii) $\sqrt[3]{96} \times \sqrt[3]{144}$
We know that $\sqrt[3]{a} \times \sqrt[3]{b} = \sqrt[3]{(a\times b)}$
By using the above formula let us simplify
$\sqrt[3]{96} \times \sqrt[3]{144} = \sqrt[3]{(96\times144)}$
The prime factors of 96 and 144 are
$= \sqrt[3]{(2\times2\times2\times2\times2\times3) \times (2\times2\times2\times2\times3\times3)}$
$= \sqrt[3]{(2^3\times2^3\times2^3\times3^3)}$
$= 2\times2\times2\times3$
$= 24$

(iii) $\sqrt[3]{100} \times \sqrt[3]{270}$
We know that $\sqrt[3]{a} \times \sqrt[3]{b} = \sqrt[3]{(a\times b)}$
By using the above formula let us simplify
$\sqrt[3]{100} \times \sqrt[3]{270} = \sqrt[3]{(100\times270)}$
The prime factors of 100 and 270 are
$= \sqrt[3]{(2\times2\times5\times5) \times (2\times3\times3\times3\times5)}$
$= \sqrt[3]{(2^3\times3^3\times5^3)}$
$= 2\times3\times5$
$= 30$

(iv) $\sqrt[3]{121} \times \sqrt[3]{297}$
We know that $\sqrt[3]{a} \times \sqrt[3]{b} = \sqrt[3]{(a\times b)}$
By using the above formula let us simplify
$\sqrt[3]{121} \times \sqrt[3]{297} = \sqrt[3]{(121\times297)}$
The prime factors of 121 and 297 are
$= \sqrt[3]{(11\times11) \times (3\times3\times3\times11)}$

$= \sqrt[3]{(11^3 \times 3^3)}$
$= 11 \times 3$
$= 33$

14. Find the cube roots of the numbers 3048625, 20346417, 210644875, 57066625 using the fact that
(i) 3048625 = 3375 × 729
(ii) 20346417 = 9261 × 2197
(iii) 210644875 = 42875 × 4913
(iv) 57066625 = 166375 × 343
Solution:
(i) 3048625 = 3375 × 729
By taking the cube root for the whole we get,
$\sqrt[3]{3048625} = \sqrt[3]{3375} \times \sqrt[3]{729}$
Now perform factorization
$= \sqrt[3]{3 \times 3 \times 3 \times 5 \times 5 \times 5} \times \sqrt[3]{9 \times 9 \times 9}$
$= \sqrt[3]{3^3 \times 5^3} \times \sqrt[3]{9^3}$
$= 3 \times 5 \times 9$
$= 135$

(ii) 20346417 = 9261 × 2197
By taking the cube root for the whole we get,
$\sqrt[3]{20346417} = \sqrt[3]{9261} \times \sqrt[3]{2197}$
Now perform factorization
$= \sqrt[3]{3 \times 3 \times 3 \times 7 \times 7 \times 7} \times \sqrt[3]{13 \times 13 \times 13}$
$= \sqrt[3]{3^3 \times 7^3} \times \sqrt[3]{13^3}$
$= 3 \times 7 \times 13$
$= 273$

(iii) 210644875 = 42875 × 4913
By taking the cube root for the whole we get,
$\sqrt[3]{210644875} = \sqrt[3]{42875} \times \sqrt[3]{4913}$
Now perform factorization
$= \sqrt[3]{5 \times 5 \times 5 \times 7 \times 7 \times 7} \times \sqrt[3]{17 \times 17 \times 17}$
$= \sqrt[3]{5^3 \times 7^3} \times \sqrt[3]{17^3}$
$= 5 \times 7 \times 17$
$= 595$

(iv) 57066625 = 166375 × 343

By taking the cube root for the whole we get,
$\sqrt[3]{57066625} = \sqrt[3]{166375} \times \sqrt[3]{343}$
Now perform factorization
$= \sqrt[3]{5\times5\times5\times11\times11\times11} \times \sqrt[3]{7\times7\times7}$
$= \sqrt[3]{5^3\times11^3} \times \sqrt[3]{7^3}$
$= 5\times11\times7$
$= 385$

15. Find the unit of the cube root of the following numbers:
(i) 226981
(ii) 13824
(iii) 571787
(iv) 175616
Solution:
(i) 226981
The given number is 226981.
Unit digit of 226981 = 1
The unit digit of the cube root of 226981 = 1

(ii) 13824
The given number is 13824.
Unit digit of 13824 = 4
The unit digit of the cube root of 13824 = 4

(iii) 571787
The given number is 571787.
Unit digit of 571787 = 7
The unit digit of the cube root of 571787 = 7

(iv) 175616
The given number is 175616.
Unit digit of 175616 = 6
The unit digit of the cube root of 175616 = 6

16. Find the tens digit of the cube root of each of the numbers in Q.No.15.
(i) 226981
(ii) 13824
(iii) 571787
(iv) 175616

Solution:
(i) 226981
The given number is 226981.
Unit digit of 226981 = 1
The unit digit in the cube root of 226981 = 1
After striking out the units, tens and hundreds digits of 226981, now we left with 226 only.
We know that 6 is the Largest number whose cube root is less than or equal to 226($6^3<226<7^3$).
∴ The tens digit of the cube root of 226981 is 6.

(ii) 13824
The given number is 13824.
Unit digit of 13824 = 4
The unit digit in the cube root of 13824 = 4
After striking out the units, tens and hundreds digits of 13824, now we left with 13 only.
We know that 2 is the Largest number whose cube root is less than or equal to 13($2^3<13<3^3$).
∴ The tens digit of the cube root of 13824 is 2.

(iii) 571787
The given number is 571787.
Unit digit of 571787 = 7
The unit digit in the cube root of 571787 = 3
After striking out the units, tens and hundreds digits of 571787, now we left with 571 only.
We know that 8 is the Largest number whose cube root is less than or equals to 571($8^3<571<9^3$).
∴ The tens digit of the cube root of 571787 is 8.

(iv) 175616
The given number is 175616.
Unit digit of 175616 = 6
The unit digit in the cube root of 175616 = 6
After striking out the units, tens and hundreds digits of 175616, now we left with 175 only.
We know that 5 is the Largest number whose cube root is less than or equals to 175($5^3<175<6^3$).
∴ The tens digit of the cube root of 175616 is 5.

EXERCISE 4.5 PAGE NO: 4.36

Making use of the cube root table, find the cube root of the following (correct to three decimal places):

1. 7

Solution:

As we know that 7 lies between 1 and 100 so by using cube root table we get,

$\sqrt[3]{7} = 1.913$

∴ the answer is 1.913

2. 70

Solution:

As we know that 70 lies between 1 and 100 so by using cube root table from column x We get,

$\sqrt[3]{70} = 4.121$

∴ the answer is 4.121

3. 700

Solution:

$700 = 70\times10$

By using cube root table 700 will be in the column $\sqrt[3]{10x}$ against 70.

So we get,

$\sqrt[3]{700} = 8.879$

∴ the answer is 8.879

4. 7000

Solution:

$7000 = 70\times100$

$\sqrt[3]{7000} = \sqrt[3]{(7\times1000)} = \sqrt[3]{7} \times \sqrt[3]{1000}$

By using cube root table,

We get,

$\sqrt[3]{7} = 1.913$

$\sqrt[3]{1000} = 10$

$\sqrt[3]{7000} = \sqrt[3]{7} \times \sqrt[3]{1000}$

$= 1.913 \times 10$

$= 19.13$

∴ the answer is 19.13

5. 1100

Solution:

$1100 = 11\times100$

$\sqrt[3]{1100} = \sqrt[3]{(11\times100)} = \sqrt[3]{11} \times \sqrt[3]{100}$

By using cube root table,

We get,

$\sqrt[3]{11} = 2.224$

$\sqrt[3]{100} = 4.6642$

$\sqrt[3]{1100} = \sqrt[3]{11} \times \sqrt[3]{100}$

$= 2.224 \times 4.642$

$= 10.323$

∴ the answer is 10.323

6.780

Solution:

$780 = 78\times10$

By using cube root table 780 would be in column $\sqrt[3]{10x}$ against 78.

We get,

$\sqrt[3]{780} = 9.205$

7. 7800

Solution:

$7800 = 78\times100$

$\sqrt[3]{7800} = \sqrt[3]{(78\times100)} = \sqrt[3]{78} \times \sqrt[3]{100}$

By using cube root table,

We get,

$\sqrt[3]{78} = 4.273$

$\sqrt[3]{100} = 4.6642$

$\sqrt[3]{7800} = \sqrt[3]{78} \times \sqrt[3]{100}$

$= 4.273 \times 4.642$

$= 19.835$

∴ the answer is 19.835

8. 1346

Solution:

Let us find the factors by using factorisation method,

We get,

$1346 = 2\times673$

$\sqrt[3]{1346} = \sqrt[3]{(2\times676)} = \sqrt[3]{2} \times \sqrt[3]{673}$

Since, 670<673<680 = $\sqrt[3]{670} < \sqrt[3]{673} < \sqrt[3]{680}$
By using cube root table,
$\sqrt[3]{670} = 8.750$
$\sqrt[3]{680} = 8.794$
For the difference (680-670) which is 10.
So the difference in the values = 8.794 - 8.750 = 0.044
For the difference (673-670) which is 3.
So the difference in the values = (0.044/10) × 3 = 0.0132
$\sqrt[3]{673} = 8.750 + 0.013 = 8.763$
$\sqrt[3]{1346} = \sqrt[3]{2} \times \sqrt[3]{673}$
$= 1.260 \times 8.763$
$= 11.041$
∴ the answer is 11.041

9. 250
Solution:
250 = 25×100
By using cube root table 250 would be in column $\sqrt[3]{10x}$ against 25.
We get,
$\sqrt[3]{250} = 6.3$
∴ the answer is 6.3

10. 5112
Solution:
Let us find the factors by using factorisation method,
$\sqrt[3]{5112} = \sqrt[3]{2\times2\times2\times3\times3\times71}$
$= \sqrt[3]{2^3\times3^2\times71}$
$= 2 \times \sqrt[3]{3^2} \times \sqrt[3]{71}$
$= 2 \times \sqrt[3]{9} \times \sqrt[3]{71}$
From cube root table we get,
$\sqrt[3]{9} = 2.080$
$\sqrt[3]{71} = 4.141$
$\sqrt[3]{5112} = 2 \times \sqrt[3]{9} \times \sqrt[3]{71}$
$= 2 \times 2.080 \times 4.141$
$= 17.227$
∴ the answer is 17.227

11. 9800
Solution:

$\sqrt[3]{9800} = \sqrt[3]{98} \times \sqrt[3]{100}$
From cube root table we get,
$\sqrt[3]{98} = 4.610$
$\sqrt[3]{100} = 4.642$
$\sqrt[3]{9800} = \sqrt[3]{98} \times \sqrt[3]{100}$
$= 4.610 \times 4.642$
$= 21.40$
∴ the answer is 21.40

12. 732
Solution:
$\sqrt[3]{732}$
We know that value of $\sqrt[3]{732}$ will lie between $\sqrt[3]{730}$ and $\sqrt[3]{740}$
From cube root table we get,
$\sqrt[3]{730} = 9.004$
$\sqrt[3]{740} = 9.045$
By using unitary method,
Difference between the values (740 – 730 = 10)
So, the difference in cube root values will be = 9.045 – 9.004 = 0.041
Difference between the values (732 – 730 = 2)
So, the difference in cube root values will be = (0.041/10) ×2 = 0.008
$\sqrt[3]{732} = 9.004+0.008 = 9.012$
∴ the answer is 9.012

13. 7342
Solution:
$\sqrt[3]{7342}$
We know that value of $\sqrt[3]{7342}$ will lie between $\sqrt[3]{7300}$ and $\sqrt[3]{7400}$
From cube root table we get,
$\sqrt[3]{7300} = 19.39$
$\sqrt[3]{7400} = 19.48$
By using unitary method,
Difference between the values (7400 – 7300 = 100)
So, the difference in cube root values will be = 19.48 – 19.39 = 0.09
Difference between the values (7342 – 7300 = 42)
So, the difference in cube root values will be = (0.09/100) × 42 = 0.037
$\sqrt[3]{7342} = 19.39+0.037 = 19.427$
∴ the answer is 19.427

14. 133100

Solution:

$\sqrt[3]{133100} = \sqrt[3]{(1331\times100)}$
$= \sqrt[3]{1331} \times \sqrt[3]{100}$
$= \sqrt[3]{11^3} \times \sqrt[3]{100}$
$= 11 \times \sqrt[3]{100}$

From cube root table we get,

$\sqrt[3]{100} = 4.462$

$\sqrt[3]{133100} = 11 \times \sqrt[3]{100}$
$= 11 \times 4.462$
$= 51.062$

∴ the answer is 51.062

15. 37800

Solution:

$\sqrt[3]{37800}$

Firstly let us find the factors for 37800

$\sqrt[3]{37800} = \sqrt[3]{(2\times2\times2\times3\times3\times3\times175)}$
$= \sqrt[3]{(2^3\times3^3\times175)}$
$= 6 \times \sqrt[3]{175}$

We know that value of $\sqrt[3]{175}$ will lie between $\sqrt[3]{170}$ and $\sqrt[3]{180}$

From cube root table we get,

$\sqrt[3]{170} = 5.540$

$\sqrt[3]{180} = 5.646$

By using unitary method,

Difference between the values (180 – 170 = 10)

So, the difference in cube root values will be = 5.646 – 5.540 = 0.106

Difference between the values (175 – 170 = 5)

So, the difference in cube root values will be = (0.106/10) × 5 = 0.053

$\sqrt[3]{175} = 5.540 + 0.053 = 5.593$

$\sqrt[3]{37800} = 6 \times \sqrt[3]{175}$
$= 6 \times 5.593$
$= 33.558$

∴ the answer is 33.558

16. 0.27

Solution:

$\sqrt[3]{0.27} = \sqrt[3]{(27/100)} = \sqrt[3]{27}/\sqrt[3]{100}$

From cube root table we get,

$\sqrt[3]{27} = 3$
$\sqrt[3]{100} = 4.642$
$\sqrt[3]{0.27} = \sqrt[3]{27}/\sqrt[3]{100}$
$= 3/4.642$
$= 0.646$
∴ the answer is 0.646

17. 8.6
Solution:
$\sqrt[3]{8.6} = \sqrt[3]{(86/10)} = \sqrt[3]{86}/\sqrt[3]{10}$
From cube root table we get,
$\sqrt[3]{86} = 4.414$
$\sqrt[3]{10} = 2.154$
$\sqrt[3]{8.6} = \sqrt[3]{86}/\sqrt[3]{10}$
$= 4.414/2.154$
$= 2.049$
∴ the answer is 2.049

18. 0.86
Solution:
$\sqrt[3]{0.86} = \sqrt[3]{(86/100)} = \sqrt[3]{86}/\sqrt[3]{100}$
From cube root table we get,
$\sqrt[3]{86} = 4.414$
$\sqrt[3]{100} = 4.642$
$\sqrt[3]{8.6} = \sqrt[3]{86}/\sqrt[3]{100}$
$= 4.414/4.642$
$= 0.9508$
∴ the answer is 0.951

19. 8.65
Solution:
$\sqrt[3]{8.65} = \sqrt[3]{(865/100)} = \sqrt[3]{865}/\sqrt[3]{100}$
We know that value of $\sqrt[3]{865}$ will lie between $\sqrt[3]{860}$ and $\sqrt[3]{870}$
From cube root table we get,
$\sqrt[3]{860} = 9.510$
$\sqrt[3]{870} = 9.546$
$\sqrt[3]{100} = 4.642$
By using unitary method,
Difference between the values (870 – 860 = 10)

So, the difference in cube root values will be = 9.546 – 9.510 = 0.036
Difference between the values (865 – 860 = 5)
So, the difference in cube root values will be = (0.036/10) × 5 = 0.018
$\sqrt[3]{865}$ = 9.510 + 0.018 = 9.528
$\sqrt[3]{8.65}$ = $\sqrt[3]{865}/\sqrt[3]{100}$
= 9.528/4.642
= 2.0525
∴ the answer is 2.053

20. 7532
Solution:
$\sqrt[3]{7532}$
We know that value of $\sqrt[3]{7532}$ will lie between $\sqrt[3]{7500}$ and $\sqrt[3]{7600}$
From cube root table we get,
$\sqrt[3]{7500}$ = 19.57
$\sqrt[3]{7600}$ = 19.66
By using unitary method,
Difference between the values (7600 – 7500 = 100)
So, the difference in cube root values will be = 19.66 – 19.57 = 0.09
Difference between the values (7532 – 7500 = 32)
So, the difference in cube root values will be = (0.09/100) × 32 = 0.029
$\sqrt[3]{7532}$ = 19.57 + 0.029 = 19.599
∴ the answer is 19.599

21. 833
Solution:
$\sqrt[3]{833}$
We know that value of $\sqrt[3]{833}$ will lie between $\sqrt[3]{830}$ and $\sqrt[3]{840}$
From cube root table we get,
$\sqrt[3]{830}$ = 9.398
$\sqrt[3]{840}$ = 9.435
By using unitary method,
Difference between the values (840 – 830 = 10)
So, the difference in cube root values will be = 9.435 – 9.398 = 0.037
Difference between the values (833 – 830 = 3)
So, the difference in cube root values will be = (0.037/10) ×3 = 0.011
$\sqrt[3]{833}$ = 9.398+0.011 = 9.409
∴ the answer is 9.409

22. 34.2

Solution:

$\sqrt[3]{34.2} = \sqrt[3]{(342/10)} = \sqrt[3]{342}/\sqrt[3]{10}$

We know that value of $\sqrt[3]{342}$ will lie between $\sqrt[3]{340}$ and $\sqrt[3]{350}$

From cube root table we get,

$\sqrt[3]{340} = 6.980$

$\sqrt[3]{350} = 7.047$

$\sqrt[3]{10} = 2.154$

By using unitary method,

Difference between the values (350 – 340 = 10)

So, the difference in cube root values will be = 7.047 – 6.980 = 0.067

Difference between the values (342 – 340 = 2)

So, the difference in cube root values will be = (0.067/10) × 2 = 0.013

$\sqrt[3]{342} = 6.980 + 0.013 = 6.993$

$\sqrt[3]{34.2} = \sqrt[3]{342}/\sqrt[3]{10}$

= 6.993/2.154

= 3.246

∴ the answer is 3.246

23. What is the length of the side of a cube whose volume is 275 cm^3. Make use of the table for the cube root.

Solution:

The given volume of the cube = 275cm^3

Let us consider the side of the cube as 'a'cm

$a^3 = 275$

$a = \sqrt[3]{275}$

We know that value of $\sqrt[3]{275}$ will lie between $\sqrt[3]{270}$ and $\sqrt[3]{280}$

From cube root table we get,

$\sqrt[3]{270} = 6.463$

$\sqrt[3]{280} = 6.542$

By using unitary method,

Difference between the values (280 – 270 = 10)

So, the difference in cube root values will be = 6.542 – 6.463 = 0.079

Difference between the values (275 – 270 = 5)

So, the difference in cube root values will be = (0.079/10) × 5 = 0.0395

$\sqrt[3]{275} = 6.463 + 0.0395 = 6.5025$

∴ the answer is 6.503cm