

EXERCISE 5.3
PAGE NO: 5.30

Solve each of the following Cryptarithms:

1.

$$\begin{array}{r} 37 \\ +AB \\ \hline 9A \end{array}$$

Solution:

Firstly let us solve for unit's place,

$$7 + B = A$$

And for ten's place,

$$3 + A = 9$$

Which means that $A = 6$ and $B = -1$ which is not possible.

So, there should be one carry in ten's place which means $7 + B > 9$

Now solving for ten's place with one carry,

$$3 + A + 1 = 9$$

$$A = 9 - 1 - 3 = 5$$

For unit's place subtracting 10 as one carry is given to ten's place,

$$7 + B - 10 = 5$$

$$B = 5 + 10 - 7 = 8$$

$$\therefore A = 5 \text{ and } B = 8$$

2.

$$\begin{array}{r} AB \\ +37 \\ \hline 9A \end{array}$$

Solution:

Firstly let us solve for unit's place,

$$B + 7 = A$$

And for ten's place,

$$A + 3 = 9$$

Which means that $A = 6$ and $B = -1$ which is not possible.

So, there should be one carry in ten's place, which means $B + 7 > 9$

Now solving for ten's place with one carry,

$$A + 3 + 1 = 9$$

$$A = 9 - 4 = 5$$

For unit's place subtracting 10 as one carry is given to ten's place,

$$B + 7 - 10 = 5$$

$$B = 5 + 10 - 7 = 8$$

$$\therefore A = 5 \text{ and } B = 8$$

3.

$$\begin{array}{r} A \ 1 \\ +1 \ B \\ \hline B \ 0 \end{array}$$

Solution:

Firstly let us solve for unit's place,

$$1 + B = 0$$

Which means that $B = -1$ which is not possible.

So, there should be one carry in ten's place,

$$A + 1 + 1 = B \text{ ---- (1)}$$

For unit's place, we need to subtract 10 as one carry is given in ten's place,

$$1 + B - 10 = 0$$

$$B = 10 - 1 = 9$$

Substituting $B = 9$ in (1),

$$A + 1 + 1 = 9$$

$$A = 9 - 1 - 1 = 7$$

$$\therefore A = 7 \text{ and } B = 9$$

4.

$$\begin{array}{r} 2 \ A \ B \\ +A \ B \ 1 \\ \hline B \ 1 \ 8 \end{array}$$

Solution:

Firstly let us solve for unit's place,

$$B + 1 = 8$$

$$B = 7$$

Now let us solve for ten's place,

$$A + B = 1$$

$$A + 7 = 1$$

$A = -6$ which is not possible.

Hence, $A + B > 9$

We know that now there should be one carry in hundred's place and so we need to subtract 10 from ten's place,

$$\text{i.e., } A + B - 10 = 1$$

$$A + 7 = 11$$

$$A = 11 - 7 = 4$$

Now to check whether our values of A and B are correct, we should solve for hundred's place.

$$2 + A + 1 = B$$

$$2 + 4 + 1 = 7$$

$$7 = 7$$

i.e., RHS = LHS

$$\therefore A = 4 \text{ and } B = 7$$

5.

$$\begin{array}{r} 1 \ 2 \ A \\ +6 \ A \ B \\ \hline A \ 0 \ 9 \end{array}$$

Solution:

Firstly let us solve for unit's place,

$$A + B = 9 \text{ ---- (1)}$$

With this condition we know that sum of 2 digits can be greater than 18.

So, there is no need to carry one from ten's place.

Now let us solve for ten's place,

$$2 + A = 0$$

Which means $A = -2$ which is never possible

$$\text{Hence, } 2 + A > 9$$

Now, there should be one carry in hundred's place and hence we need to subtract 10 from ten's place,

$$\text{i.e., } 2 + A - 10 = 0$$

$$A = 10 - 2 = 8$$

Now, substituting $A=8$ in 1,

$$A + B = 9$$

$$8 + B = 9$$

$$B = 9 - 8$$

$$B = 1$$

$$\therefore A = 8 \text{ and } B = 1$$

6.

$$\begin{array}{r} A \ B \ 7 \\ +7 \ A \ B \\ \hline 9 \ 8 \ A \end{array}$$

Solution:

Firstly let us solve for unit's place,

We have two conditions here, $7 + B \leq 9$ and $7 + B > 9$

For $7 + B \leq 9$

$$7 + B = A$$

$$A - B = 7 \text{ ---- (1)}$$

Now let us solve for ten's place,

$$B + A = 8 \text{ ---- (2)}$$

Solving 1 and 2 simultaneously,

$2A = 15$ which means $A = 7.5$ which is not possible

So, our condition $7 + B \leq 9$ is wrong.

$\therefore 7 + B > 9$ is correct condition

Hence, there should be one carry in ten's place and subtracting 10 from unit's place,

$$7 + B - 10 = A$$

$$B - A = 3 \text{ ---- (3)}$$

For ten's place,

$$B + A + 1 = 8$$

$$B + A = 8 - 1$$

$$B + A = 7 \text{ ---- (4)}$$

Solving (3) and (4) simultaneously,

$$2B = 10$$

$$B = 10/2 = 5$$

Substituting the value of B in equation 4

$$B + A = 7$$

$$5 + A = 7$$

$$A = 7 - 5 = 2$$

$\therefore B = 5$ and $A = 2$

7. Show that the Cryptarithm $4 \times \overline{AB} = \overline{CAB}$ does not have any solution.

Solution:

If B is multiplied by 4 then only 0 satisfies the above condition.

So, for unit place to satisfy the above condition, we should have $B = 0$.

Similarly for ten's place, only 0 satisfies the above condition.

But, AB cannot be 00 as 00 is not a two digit number.

So, A and B cannot be equal to 0

\therefore there is no solution satisfying the condition $4 \times \overline{AB} = \overline{CAB}$.