EXERCISE 7.5

Factorize each of the following expressions:

1. $16x^2 – 25y^2$

Solution:

We have,

$16x^2 – 25y^2$

$(4x)^2 – (5y)^2$

By using the formula $(a^2 – b^2) = (a + b)(a - b)$ we get,

$(4x + 5y)(4x – 5y)$

2. $27x^2 – 12y^2$

Solution:

We have,

$27x^2 – 12y^2$

By taking 3 as common we get,

$3[(3x)^2 – (2y)^2]$

By using the formula $(a^2 - b^2) = (a-b)(a+b)$

$3(3x + 2y)(3x – 2y)$

3. $144a^2 – 289b^2$

Solution:

We have,

$144a^2 – 289b^2$

$(12a)^2 – (17b)^2$

By using the formula $(a^2 - b^2) = (a-b)(a+b)$

$(12a + 17b)(12a – 17b)$

4. $12m^2 – 27$

Solution:

We have,

$12m^2 – 27$

By taking 3 as common we get,

$3(4m^2 – 9)$

$3[(2m)^2 – 3^2]$

By using the formula $(a^2 - b^2) = (a-b)(a+b)$

$3(2m + 3)(2m – 3)$

5. $125x^2 – 45y^2$

Solution:
We have,
$125x^2 – 45y^2$
By taking 5 as common we get,
$5 (25x^2 – 9y^2)$
$5 [(5x)^2 – (3y)^2]$
By using the formula $(a^2 - b^2) = (a-b) (a+b)$
$5 (5x + 3y) (5x – 3y)$

6. $144a^2 – 169b^2$
Solution:
We have,
$144a^2 – 169b^2$
$(12a)^2 – (13b)^2$
By using the formula $(a^2 - b^2) = (a-b) (a+b)$
$(12a + 13b) (12a – 13b)$

7. $(2a - b)^2 – 16c^2$
Solution:
We have,
$(2a - b)^2 – 16c^2$
$(2a – b)^2 – (4c)^2$
By using the formula $(a^2 - b^2) = (a-b) (a+b)$
$(2a – b + 4c) (2a – b – 4c)$

8. $(x + 2y)^2 – 4 (2x - y)^2$
Solution:
We have,
$(x + 2y)^2 – 4 (2x - y)^2$
$(x + 2y)^2 – [2 (2x – y)]^2$
By using the formula $(a^2 – b^2)= (a + b) (a - b)$ we get,
$[(x + 2y) + 2 (2x – y)] [x + 2y – 2 (2x – y)]$
$(x + 4x + 2y – 2y) (x – 4x + 2y + 2y)$
$(5x) (4y – 3x)$

9. $3a^5 – 48a^3$
Solution:
We have,
$3a^5 – 48a^3$

By taking 3 as common we get,
$3a^3 (a^2 – 16)$
$3a^3 (a^2 – 4^2)$
By using the formula $(a^2 - b^2) = (a-b)(a+b)$
$3a^3 (a + 4)(a – 4)$

10. $a^4 – 16b^4$
Solution:
We have,
$a^4 – 16b^4$
$(a^2)^2 – (4b^2)^2$
By using the formula $(a^2 - b^2) = (a-b)(a+b)$
$(a^2 + 4b^2)(a^2 – 4b^2)$
By using the formula $(a^2 - b^2) = (a-b)(a+b)$
$(a^2 + 4b^2)(a + 2b)(a – 2b)$

11. $x^8 – 1$
Solution:
We have,
$x^8 – 1$
$(x^4)^2 – (1)^2$
By using the formula $(a^2 - b^2) = (a-b)(a+b)$
$(x^4 + 1)(x^4 – 1)$
By using the formula $(a^2 - b^2) = (a-b)(a+b)$
$(x^4 + 1)(x^2 + 1)(x – 1)(x + 1)$

12. $64 – (a + 1)^2$
Solution:
We have,
$64 – (a + 1)^2$
$8^2 – (a + 1)^2$
By using the formula $(a^2 - b^2) = (a-b)(a+b)$
$[8 + (a + 1)][8 – (a + 1)]$
$(a + 9)(7 – a)$

13. $36l^2 – (m + n)^2$
Solution:
We have,
$36l^2 – (m + n)^2$

$(6l)^2 - (m + n)^2$
By using the formula $(a^2 - b^2) = (a-b)(a+b)$
$(6l + m + n)(6l - m - n)$

14. $25x^4y^4 - 1$
Solution:
We have,
$25x^4y^4 - 1$
$(5x^2y^2)^2 - (1)^2$
By using the formula $(a^2 - b^2) = (a-b)(a+b)$
$(5x^2y^2 - 1)(5x^2y^2 + 1)$

15. $a^4 - 1/b^4$
Solution:
We have,
$a^4 - 1/b^4$
$(a^2)^2 - (1/b^2)^2$
By using the formula $(a^2 - b^2) = (a-b)(a+b)$
$(a^2 + 1/b^2)(a^2 - 1/b^2)$
By using the formula $(a^2 - b^2) = (a-b)(a+b)$
$(a^2 + 1/b^2)(a - 1/b)(a + 1/b)$

16. $x^3 - 144x$
Solution:
We have,
$x^3 - 144x$
$x[x^2 - (12)^2]$
By using the formula $(a^2 - b^2) = (a-b)(a+b)$
$x(x + 12)(x - 12)$

17. $(x - 4y)^2 - 625$
Solution:
We have,
$(x - 4y)^2 - 625$
$(x - 4y)^2 - (25)^2$
By using the formula $(a^2 - b^2) = (a-b)(a+b)$
$(x - 4y + 25)(x - 4y - 25)$

18. $9(a - b)^2 - 100(x - y)^2$

Solution:
We have,
9 (a – b)^2 – 100 (x – y)^2
[3 (a – b)]^2 – [10 (x – y)]^2
By using the formula (a^2 - b^2) = (a-b) (a+b)
[3 (a – b) + 10 (x + y)] [3 (a – b) – 10 (x – y)]
[3a – 3b + 10x – 10y] [3a – 3b – 10x + 10y]

19. (3 + 2a)^2 – 25a^2
Solution:
We have,
(3 + 2a)^2 – 25a^2
(3 + 2a)^2 – (5a)^2
By using the formula (a^2 - b^2) = (a-b) (a+b)
(3 + 2a + 5a) (3 + 2a – 5a)
(3 + 7a) (3 – 3a)
(3 + 7a) 3(1 – a)

20. (x + y)^2 – (a – b)^2
Solution:
We have,
(x + y)^2 – (a – b)^2
By using the formula (a^2 - b^2) = (a-b) (a+b)
[(x + y) + (a – b)] [(x + y) – (a – b)]
(x + y + a – b) (x + y – a + b)

21. 1/16x^{2}y^2 – 4/49y^{2}z^2
Solution:
We have,
1/16x^{2}y^2 – 4/49y^{2}z^2
(1/4xy)^2 – (2/7yz)^2
By using the formula (a^2 - b^2) = (a-b) (a+b)
(xy/4 + 2yz/7) (xy/4 - 2yz/7)
y^2 (x/4 + 2/7z) (x/4 – 2/7z)

22. 75a^{3}b^2 – 108ab^4
Solution:
We have,
75a^{3}b^2 – 108ab^4

3ab² (25a² – 36b²)
3ab² [(5a)² – (6b)²]
By using the formula (a² - b²) = (a-b) (a+b)
3ab² (5a + 6b) (5a – 6b)

23. x⁵ – 16x³
Solution:
We have,
x⁵ – 16x³
x³ (x² – 16)
x³ (x² – 4²)
By using the formula (a² - b²) = (a-b) (a+b)
x³ (x + 4) (x – 4)

24. 50/x² – 2x²/81
Solution:
We have,
50/x² – 2x²/81
2 (25/x² – x²/81)
2 [(5/x)² – (x/9)²]
By using the formula (a² - b²) = (a-b) (a+b)
2 (5/x+ x/9) (5/x – x/9)

25. 256x³ – 81x
Solution:
We have,
256x³ – 81x
x (256x⁴ – 81)
x [(16x²)² – 9²]
By using the formula (a² - b²) = (a-b) (a+b)
x (4x + 3) (4x - 3) (16x² + 9)

26. a⁴ – (2b + c)⁴
Solution:
We have,
a⁴ – (2b + c)⁴
(a²)² – [(2b + c)²]²
By using the formula (a² - b²) = (a-b) (a+b)
[a² + (2b + c)²] [a² – (2b + c)²]

By using the formula $(a^2 - b^2) = (a-b)(a+b)$
$[a^2 + (2b + c)^2] [a + 2b + c] [a – 2b – c]$

27. $(3x + 4y)^4 – x^4$
Solution:
We have,
$(3x + 4y)^4 – x^4$
$[(3x + 4y)^2]^2 – (x^2)^2$
By using the formula $(a^2 - b^2) = (a-b)(a+b)$
$[(3x + 4y)^2 + x^2] [(3x + 4y)^2 – x^2]$
$[(3x + 4y)^2 + x^2] [3x + 4y + x] [3x + 4y – x]$
$[(3x + 4y)^2 + x^2] [4x + 4y] [2x + 4y]$
$[(3x + 4y)^2 + x^2] 8[x + 2y] [x + y]$

28. $p^2q^2 – p^4q^4$
Solution:
We have,
$p^2q^2 – p^4q^4$
$(pq)^2 – (p^2q^2)^2$
By using the formula $(a^2 - b^2) = (a-b)(a+b)$
$(pq + p^2q^2) (pq – p^2q^2)$
$p^2q^2 (1 + pq) (1 – pq)$

29. $3x^3y – 24xy^3$
Solution:
We have,
$3x^3y – 24xy^3$
$3xy (x^2 – 8y^2)$
$3xy [x^2 – (8y)^2]$
By using the formula $(a^2 - b^2) = (a-b)(a+b)$
$(3xy) (x + 8y) (x – 8y)$

30. $a^4b^4 – 16c^4$
Solution:
We have,
$a^4b^4 – 16c^4$
$(a^2b^2)^2 – (4c^2)^2$
By using the formula $(a^2 - b^2) = (a-b)(a+b)$
$(a^2b^2 + 4c^2) (a^2b^2 – 4c^2)$

By using the formula $(a^2 - b^2) = (a-b) (a+b)$
$(a^2b^2 + 4c^2) (ab + 2c) (ab - 2c)$

31. $x^4 - 625$
Solution:
We have,
$x^4 - 625$
$(x^2)^2 - (25)^2$
By using the formula $(a^2 - b^2) = (a-b) (a+b)$
$(x^2 + 25) (x^2 - 25)$
$(x^2 + 25) (x^2 - 5^2)$
By using the formula $(a^2 - b^2) = (a-b) (a+b)$
$(x^2 + 25) (x + 5) (x - 5)$

32. $x^4 - 1$
Solution:
We have,
$x^4 - 1$
$(x^2)^2 - (1)^2$
By using the formula $(a^2 - b^2) = (a-b) (a+b)$
$(x^2 + 1) (x^2 - 1)$
By using the formula $(a^2 - b^2) = (a-b) (a+b)$
$(x^2 + 1) (x + 1) (x - 1)$

33. $49(a - b)^2 - 25(a + b)^2$
Solution:
We have,
$49(a - b)^2 - 25(a + b)^2$
$[7 (a - b)]^2 - [5 (a + b)]^2$
By using the formula $(a^2 - b^2) = (a-b) (a+b)$
$[7 (a - b) + 5 (a + b)] [7 (a - b) - 5 (a + b)]$
$(7a - 7b + 5a + 5b) (7a - 7b - 5a - 5b)$
$(12a - 2b) (2a - 12b)$
$2 (6a - b) 2 (a - 6b)$
$4 (6a - b) (a - 6b)$

34. $x - y - x^2 + y^2$
Solution:
We have,

x – y - x² + y²
x – y – (x² – y²)
By using the formula (a² - b²) = (a-b) (a+b)
x – y – (x + y) (x – y)
(x – y) (1 – x – y)

35. 16(2x – 1)² – 25y²
Solution:
We have,
16(2x – 1)² – 25y²
[4 (2x – 1)]² – (5y)²
By using the formula (a² - b²) = (a-b) (a+b)
(8x + 5y – 4) (8x – 5y – 4)

36. 4(xy + 1)² – 9(x – 1)²
Solution:
We have,
4(xy + 1)² – 9(x – 1)²
[2x (xy + 1)]² – [3 (x – 1)]²
By using the formula (a² - b²) = (a-b) (a+b)
(2xy + 2 + 3x – 3) (2xy + 2 – 3x + 3)
(2xy + 3x – 1) (2xy – 3x + 5)

37. (2x + 1)² – 9x⁴
Solution:
We have,
(2x + 1)² – 9x⁴
(2x + 1)² – (3x²)²
By using the formula (a² - b²) = (a-b) (a+b)
(2x + 1 + 3x²) (2x + 1 – 3x²)
(3x² + 2x + 1) (-3x² + 2x + 1)

38. x⁴ – (2y – 3z)²
Solution:
We have,
x⁴ – (2y – 3z)²
(x²)² – (2y – 3z)²
By using the formula (a² - b²) = (a-b) (a+b)
(x² + 2y – 3z) (x² – 2y + 3z)

39. $a^4 – b^2 + a – b$

Solution:

We have,

$a^4 – b^2 + a – b$

By using the formula $(a^2 - b^2) = (a-b) (a+b)$

$(a + b) (a – b) + (a – b)$

$(a – b) (a + b + 1)$

40. $16a^4 – b^4$

Solution:

We have,

$16a^4 – b^4$

$(4a^2)^2 – (b^2)^2$

$(4a^2 + b^2) (4a^2 – b^2)$

By using the formula $(a^2 - b^2) = (a-b) (a+b)$

$(4a^2 + b^2) (2a + b) (2a – b)$

41. $a^4 – 16(b – c)^4$

Solution:

We have,

$a^4 – 16(b – c)^4$

$(a^2)^2 – [4 (b – c)^2]$

By using the formula $(a^2 - b^2) = (a-b) (a+b)$

$[a^2 + 4 (b – c)^2] [a^2 – 4 (b – c)^2]$

By using the formula $(a^2 - b^2) = (a-b) (a+b)$

$[a^2 + 4 (b – c)^2] [(a + 2b – 2c) (a – 2b + 2c)]$

42. $2a^4 – 32a$

Solution:

We have,

$2a^4 – 32a$

$2a (a^4 – 16)$

$2a [(a)^2 – (4)^2]$

By using the formula $(a^2 - b^2) = (a-b) (a+b)$

$2a (a^2 + 4) (a^2 – 4)$

$2a (a^2 + 4) (a^2 – 2^2)$

By using the formula $(a^2 - b^2) = (a-b) (a+b)$

$2a (a^2 + 4) (a + 2) (a – 2)$

43. a⁴b⁴ – 81c⁴
Solution:
We have,
a⁴b⁴ – 81c⁴
(a²b²)² – (9c²)²
By using the formula (a² - b²) = (a-b) (a+b)
(a²b² + 9c²) (a²b² – 9c²)
By using the formula (a² - b²) = (a-b) (a+b)
(a²b² + 9c²) (ab + 3c) (ab – 3c)

44. xy⁹ – yx⁹
Solution:
We have,
xy⁹ – yx⁹
-xy (x⁸ – y⁸)
-xy [(x⁴)² – (y⁴)²]
By using the formula (a² - b²) = (a-b) (a+b)
-xy (x⁴ + y⁴) (x⁴ – y⁴)
By using the formula (a² - b²) = (a-b) (a+b)
-xy (x⁴ + y⁴) (x² + y²) (x² – y²)
By using the formula (a² - b²) = (a-b) (a+b)
-xy (x⁴ + y⁴) (x² + y²) (x + y) (x – y)

45. x³ – x
Solution:
We have,
x³ – x
x (x² – 1)
By using the formula (a² - b²) = (a-b) (a+b)
x (x + 1) (x – 1)

46. 18a²x² – 32
Solution:
We have,
18a²x² – 32
2 [(3ax)² – (4)²]
By using the formula (a² - b²) = (a-b) (a+b)
2 (3ax + 4) (3ax – 4)