

EXERCISE 7.6
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Factorize each of the following algebraic expressions:

1. $4x^2 + 12xy + 9y^2$

Solution:

We have,

$$4x^2 + 12xy + 9y^2$$

By using the formula $(x + y)^2 = x^2 + y^2 + 2xy$

$$(2x)^2 + (3y)^2 + 2(2x)(3y)$$

$$(2x + 3y)^2$$

$$(2x + 3y)(2x + 3y)$$

2. $9a^2 - 24ab + 16b^2$

Solution:

We have,

$$9a^2 - 24ab + 16b^2$$

By using the formula $(x - y)^2 = x^2 + y^2 - 2xy$

Here $x = 3a$, $y = 4b$ So,

$$(3a)^2 + (4b)^2 - 2(3a)(4b)$$

$$(3a - 4b)^2$$

$$(3a - 4b)(3a - 4b)$$

3. $p^2q^2 - 6pqr + 9r^2$

Solution:

We have,

$$p^2q^2 - 6pqr + 9r^2$$

By using the formula $(a - b)^2 = a^2 + b^2 - 2ab$

$$(pq)^2 + (3r)^2 - 2(pq)(3r)$$

$$(pq - 3r)^2$$

$$(pq - 3r)(pq - 3r)$$

4. $36a^2 + 36a + 9$

Solution:

We have,

$$36a^2 + 36a + 9$$

$$9(4a^2 + 4a + 1)$$

$$9[(2a)^2 + 2(2a) + 1]$$

$$9(2a + 1)^2$$

$$9(2a + 1)(2a + 1)$$

5. $a^2 + 2ab + b^2 - 16$

Solution:

We have,

$$a^2 + 2ab + b^2 - 16$$

By using the formula $(a - b)^2 = a^2 + b^2 - 2ab$

$$(a + b)^2 - 4^2$$

By using the formula $(a^2 - b^2) = (a+b)(a-b)$

$$(a + b + 4)(a + b - 4)$$

6. $9z^2 - x^2 + 4xy - 4y^2$

Solution:

We have,

$$9z^2 - x^2 + 4xy - 4y^2$$

$$(3z)^2 - [x^2 - 2(x)(2y) + (2y)^2]$$

By using the formula $(a - b)^2 = a^2 + b^2 - 2ab$

$$(3z)^2 - (x - 2y)^2$$

By using the formula $(a^2 - b^2) = (a+b)(a-b)$

$$[(x - 2y) + 3z] [-x + 2y + 3z]$$

7. $9a^2 - 24a^2b^2 + 16b^4 - 256$

Solution:

We have,

$$9a^2 - 24a^2b^2 + 16b^4 - 256$$

$$(3a^2)^2 - 2(4a^2)(3b^2) + (4b^2)^2 - (16)^2$$

By using the formula $(a - b)^2 = a^2 + b^2 - 2ab$

$$(3a^2 - 4b^2)^2 - (16)^2$$

By using the formula $(a^2 - b^2) = (a+b)(a-b)$

$$(3a^2 - 4b^2 + 16)(3a^2 - 4b^2 - 16)$$

8. $16 - a^6 + 4a^3b^3 - 4b^6$

Solution:

We have,

$$16 - a^6 + 4a^3b^3 - 4b^6$$

$$4^2 - [(a^3)^2 - 2(a^3)(2b^3) + (2b^3)^2]$$

By using the formula $(a - b)^2 = a^2 + b^2 - 2ab$

$$4^2 - (a^3 - 2b^3)^2$$

By using the formula $(a^2 - b^2) = (a+b)(a-b)$

$$[4 + (a^3 - 2b^3)][4 - (a^3 - 2b^3)]$$

9. $a^2 - 2ab + b^2 - c^2$

Solution:

We have,

$$a^2 - 2ab + b^2 - c^2$$

By using the formula $(a - b)^2 = a^2 + b^2 - 2ab$

$$(a - b)^2 - c^2$$

By using the formula $(a^2 - b^2) = (a+b)(a-b)$

$$(a - b + c)(a - b - c)$$

10. $x^2 + 2x + 1 - 9y^2$

Solution:

We have,

$$x^2 + 2x + 1 - 9y^2$$

By using the formula $(a - b)^2 = a^2 + b^2 - 2ab$

$$(x + 1)^2 - (3y)^2$$

By using the formula $(a^2 - b^2) = (a+b)(a-b)$

$$(x + 3y + 1)(x - 3y + 1)$$

11. $a^2 + 4ab + 3b^2$

Solution:

We have,

$$a^2 + 4ab + 3b^2$$

By using factors for 3 i.e., 3 and 1

$$a^2 + ab + 3ab + 3b^2$$

By grouping we get,

$$a(a + b) + 3b(a + b)$$

$$(a + 3b)(a + b)$$

12. $96 - 4x - x^2$

Solution:

We have,

$$96 - 4x - x^2$$

$$-x^2 - 4x + 96$$

By using factors for 96 i.e., 12 and 8

$$-x^2 - 12x + 8x + 96$$

By grouping we get,

$$-x(x + 12) + 8(x + 12)$$

$$(x + 12)(-x + 8)$$

13. $a^4 + 3a^2 + 4$
Solution:

We have,

$$a^4 + 3a^2 + 4$$

$$(a^2)^2 + (a^2)^2 + 2(2a^2) + 4 - a^2$$

$$(a^2 + 2)^2 + (-a^2)$$

 By using the formula $(a^2 - b^2) = (a+b)(a-b)$

$$(a^2 + 2 + a)(a^2 + 2 - a)$$

$$(a^2 + a + 2)(a^2 - a + 2)$$

14. $4x^4 + 1$
Solution:

We have,

$$4x^4 + 1$$

$$(2x^2)^2 + 1 + 4x^2 - 4x^2$$

$$(2x^2 + 1)^2 - 4x^2$$

 By using the formula $(a^2 - b^2) = (a+b)(a-b)$

$$(2x^2 + 1 + 2x)(2x^2 + 1 - 2x)$$

$$(2x^2 + 2x + 1)(2x^2 - 2x + 1)$$

15. $4x^4 + y^4$
Solution:

We have,

$$4x^4 + y^4$$

$$(2x^2)^2 + (y^2)^2 + 4x^2y^2 - 4x^2y^2$$

$$(2x^2 + y^2)^2 - 4x^2y^2$$

 By using the formula $(a^2 - b^2) = (a+b)(a-b)$

$$(2x^2 + y^2 + 2xy)(2x^2 + y^2 - 2xy)$$

16. $(x + 2)^4 - 6(x + 2) + 9$
Solution:

We have,

$$(x + 2)^4 - 6(x + 2) + 9$$

$$(x^2 + 2^2)^2 - 6x - 12 + 9$$

$$(x^2 + 2^2 + 2(2)(x)) - 6x - 12 + 9$$

$$x^2 + 4 + 4x - 6x - 12 + 9$$

$$x^2 - 2x + 1$$

 By using the formula $(a - b)^2 = a^2 + b^2 - 2ab$

$$(x - 1)^2$$

17. $25 - p^2 - q^2 - 2pq$

Solution:

We have,

$$25 - p^2 - q^2 - 2pq$$

$$25 - (p^2 + q^2 + 2pq)$$

$$(5)^2 - (p + q)^2$$

By using the formula $(a^2 - b^2) = (a+b)(a-b)$

$$(5 + p + q)(5 - p - q)$$

$$-(p + q + 5)(p + q - 5)$$

18. $x^2 + 9y^2 - 6xy - 25a^2$

Solution:

We have,

$$x^2 + 9y^2 - 6xy - 25a^2$$

$$(x - 3y)^2 - (5a)^2$$

By using the formula $(a^2 - b^2) = (a+b)(a-b)$

$$(x - 3y + 5a)(x - 3y - 5a)$$

19. $49 - a^2 + 8ab - 16b^2$

Solution:

We have,

$$49 - a^2 + 8ab - 16b^2$$

$$49 - (a^2 - 8ab + 16b^2)$$

$$49 - (a - 4b)^2$$

By using the formula $(a^2 - b^2) = (a + b)(a - b)$

$$(7 + a - 4b)(7 - a + 4b)$$

$$-(a - 4b + 7)(a - 4b - 7)$$

20. $a^2 - 8ab + 16b^2 - 25c^2$

Solution:

We have,

$$a^2 - 8ab + 16b^2 - 25c^2$$

$$(a - 4b)^2 - (5c)^2$$

By using the formula $(a^2 - b^2) = (a+b)(a-b)$

$$(a - 4b + 5c)(a - 4b - 5c)$$

21. $x^2 - y^2 + 6y - 9$

Solution:

We have,

$$x^2 - y^2 + 6y - 9$$

$$x^2 + 6y - (y^2 - 6y + 9)$$

$$x^2 - (y - 3)^2$$

By using the formula $(a^2 - b^2) = (a+b)(a-b)$

$$(x + y - 3)(x - y + 3)$$

22. $25x^2 - 10x + 1 - 36y^2$

Solution:

We have,

$$25x^2 - 10x + 1 - 36y^2$$

$$(5x)^2 - 2(5x) + 1 - (6y)^2$$

$$(5x - 1)^2 - (6y)^2$$

By using the formula $(a^2 - b^2) = (a+b)(a-b)$

$$(5x - 6y - 1)(5x + 6y - 1)$$

23. $a^2 - b^2 + 2bc - c^2$

Solution:

We have,

$$a^2 - b^2 + 2bc - c^2$$

$$a^2 - (b^2 - 2bc + c^2)$$

$$a^2 - (b - c)^2$$

By using the formula $(a^2 - b^2) = (a+b)(a-b)$

$$(a + b - c)(a - b + c)$$

24. $a^4 + 2b + b^2 - c^2$

Solution:

We have,

$$a^4 + 2b + b^2 - c^2$$

$$(a + b)^2 - c^2$$

By using the formula $(a^2 - b^2) = (a+b)(a-b)$

$$(a + b + c)(a + b - c)$$

25. $49 - x^2 - y^2 + 2xy$

Solution:

We have,

$$49 - x^2 - y^2 + 2xy$$

$$49 - (x^2 + y^2 - 2xy)$$

$$7^2 - (x - y)^2$$

By using the formula $(a^2 - b^2) = (a+b)(a-b)$

$$[7 + (x - y)] [7 - x + y]$$

$$(x - y + 7)(y - x + 7)$$

26. $a^2 + 4b^2 - 4ab - 4c^2$

Solution:

We have,

$$a^2 + 4b^2 - 4ab - 4c^2$$

$$a^2 - 2(a)(2b) + (2b)^2 - (2c)^2$$

$$(a - 2b)^2 - (2c)^2$$

By using the formula $(a^2 - b^2) = (a+b)(a-b)$

$$(a - 2b + 2c)(a - 2b - 2c)$$

27. $x^2 - y^2 - 4xz + 4z^2$

Solution:

We have,

$$x^2 - y^2 - 4xz + 4z^2$$

$$x^2 - 2(x)(2z) + (2z)^2 - y^2$$

As $(a-b)^2 = a^2 + b^2 - 2ab$

$$(x - 2z)^2 - y^2$$

By using the formula $(a^2 - b^2) = (a+b)(a-b)$

$$(x + y - 2z)(x - y - 2z)$$