EXERCISE 7.9 PAGE NO: 7.32

Factorize each of the following quadratic polynomials by using the method of completing the square:

1. $p^2 + 6p + 8$

Solution:

We have,

$p^2 + 6p + 8$

Coefficient of p^2 is unity. So, we add and subtract square of half of coefficient of p.

$p^2 + 6p + 8 = p^2 + 6p + 3^2 – 3^2 + 8$ (Adding and subtracting 3^2)

$= (p + 3)^2 – 1^2$ (By completing the square)

By using the formula $(a^2 – b^2) = (a+b)(a-b)$

$= (p + 3 – 1)(p + 3 + 1)$

$= (p + 2)(p + 4)$

2. $q^2 – 10q + 21$

Solution:

We have,

$q^2 – 10q + 21$

Coefficient of q^2 is unity. So, we add and subtract square of half of coefficient of q.

$q^2 – 10q + 21 = q^2 – 10q + 5^2 – 5^2 + 21$ (Adding and subtracting 5^2)

$= (q – 5)^2 – 2^2$ (By completing the square)

By using the formula $(a^2 – b^2) = (a+b)(a-b)$

$= (q – 5 – 2)(q – 5 + 2)$

$= (q – 3)(q – 7)$

3. $4y^2 + 12y + 5$

Solution:

We have,

$4y^2 + 12y + 5$

$4(y^2 + 3y + 5/4)$

Coefficient of y^2 is unity. So, we add and subtract square of half of coefficient of y.

$4(y^2 + 3y + 5/4) = 4[y^2 + 3y + (3/2)^2 – (3/2)^2 + 5/4]$ (Adding and subtracting $(3/2)^2$)

$= 4[(y + 3/2)^2 – 1^2]$ (Completing the square)

By using the formula $(a^2 – b^2) = (a+b)(a-b)$

$= 4(y + 3/2 + 1)(y + 3/2 – 1)$

$= 4(y + 1/2)(y + 5/2)$ (by taking LCM)

$= 4[(2y + 1)/2][(2y + 5)/2]$

$= (2y + 1)(2y + 5)$

4. $p^2 + 6p - 16$

Solution:

We have,

$p^2 + 6p - 16$

Coefficient of p^2 is unity. So, we add and subtract square of half of coefficient of p.

$p^2 + 6p – 16 = p^2 + 6p + 3^2 – 3^2 – 16$ (Adding and subtracting 3^2)

$= (p + 3)^2 – 5^2$ (Completing the square)

By using the formula $(a^2 – b^2) = (a+b) (a-b)$

$= (p + 3 + 5) (p + 3 – 5)$

$= (p + 8) (p – 2)$

5. $x^2 + 12x + 20$

Solution:

We have,

$x^2 + 12x + 20$

Coefficient of x^2 is unity. So, we add and subtract square of half of coefficient of x.

$x^2 + 12x + 20 = x^2 + 12x + 6^2 – 6^2 + 20$ (Adding and subtracting 6^2)

$= (x + 6)^2 – 4^2$ (Completing the square)

By using the formula $(a^2 – b^2) = (a+b) (a-b)$

$= (x + 6 + 4) (x + 6 – 4)$

$= (x + 2) (x + 10)$

6. $a^2 – 14a – 51$

Solution:

We have,

$a^2 – 14a – 51$

Coefficient of a^2 is unity. So, we add and subtract square of half of coefficient of a.

$a^2 – 14a – 51 = a^2 – 14a + 7^2 – 7^2 – 51$ (Adding and subtracting 7^2)

$= (a – 7)^2 – 10^2$ (Completing the square)

By using the formula $(a^2 – b^2) = (a+b) (a-b)$

$= (a – 7 + 10) (9 – 7 – 10)$

$= (a – 17) (a + 3)$

7. $a^2 + 2a – 3$

Solution:

We have,

$a^2 + 2a – 3$

Coefficient of a^2 is unity. So, we add and subtract square of half of coefficient of a.

$a^2 + 2a – 3 = a^2 + 2a + 1^2 – 1^2 – 3$ (Adding and subtracting 1^2)
$= (a + 1)^2 – 2^2$ (Completing the square)
By using the formula $(a^2 – b^2) = (a+b)(a-b)$
$= (a + 1 + 2)(a + 1 – 2)$
$= (a + 3)(a – 1)$

8. $4x^2 - 12x + 5$
Solution:
We have,
$4x^2 - 12x + 5$
$4(x^2 - 3x + 5/4)$
Coefficient of x^2 is unity. So, we add and subtract square of half of coefficient of x.
$4(x^2 – 3x + 5/4) = 4 [x^2 – 3x + (3/2)^2 – (3/2)^2 + 5/4]$ (Adding and subtracting $(3/2)^2$)
$= 4 [(x - 3/2)^2 – 1^2]$ (Completing the square)
By using the formula $(a^2 – b^2) = (a+b)(a-b)$
$= 4 (x - 3/2 + 1)(x - 3/2 – 1)$
$= 4 (x – 1/2)(x – 5/2)$ (by taking LCM)
$= 4 [(2x-1)/2] [(2x - 5)/2]$
$= (2x - 5)(2x - 1)$

9. $y^2 - 7y + 12$
Solution:
We have,
$y^2 - 7y + 12$
Coefficient of y^2 is unity. So, we add and subtract square of half of coefficient of y.
$y^2 - 7y + 12 = y^2 – 7y + (7/2)^2 – (7/2)^2 + 12$ [Adding and subtracting $(7/2)^2$]
$= (y – 7/2)^2 – (7/2)^2$ (Completing the square)
By using the formula $(a^2 – b^2) = (a+b)(a-b)$
$= (y – (7/2- 1/2))(y – (7/2 + 1/2))$
$= (y – 3)(y – 4)$

10. $z^2 – 4z - 12$
Solution:
We have,
$z^2 – 4z – 12$
Coefficient of z^2 is unity. So, we add and subtract square of half of coefficient of z.
$z^2 – 4z – 12 = z^2 – 4z + 2^2 – 2^2 – 12$ [Adding and subtracting 2^2]
$= (z – 2)^2 – 4^2$ (Completing the square)
By using the formula $(a^2 – b^2) = (a+b)(a-b)$

$= (z - 2 + 4)(z - 2 - 4)$

$= (z - 6)(z + 2)$