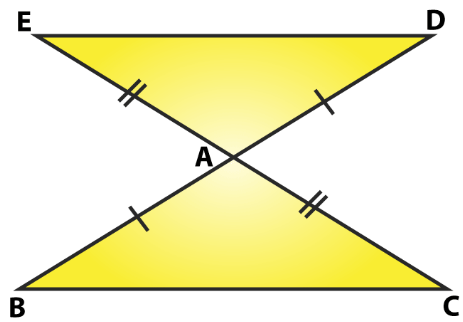
Exercise 10.1 Page No: 10.12

Question 1: In figure, the sides BA and CA have been produced such that BA = AD and CA = AE. Prove that segment DE ∥ BC.



Solution:

Sides BA and CA have been produced such that BA = AD and CA = AE.

To prove: DE ∥ BC

Consider △ BAC and △DAE,

BA = AD and CA= AE (Given)

∠BAC = ∠DAE (vertically opposite angles)

By SAS congruence criterion, we have

△ BAC ≃ △ DAE

We know, corresponding parts of congruent triangles are equal

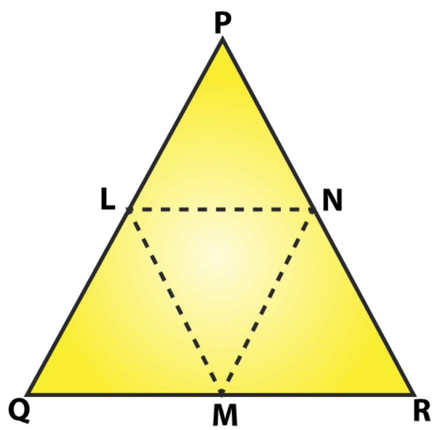
So, BC = DE and ∠DEA = ∠BCA, ∠EDA = ∠CBA

Now, DE and BC are two lines intersected by a transversal DB s.t.
∠DEA=∠BCA (alternate angles are equal)

Therefore, DE ∥ BC. Proved.

Question 2: In a PQR, if PQ = QR and L, M and N are the mid-points of the sides PQ, QR and RP respectively. Prove that LN = MN.

Solution:
Draw a figure based on given instruction,



In △PQR, PQ = QR and L, M, N are midpoints of the sides PQ, QP and RP respectively (Given)

To prove : LN = MN

As two sides of the triangle are equal, so △ PQR is an isosceles triangle

PQ = QR and ∠QPR = ∠QRP ……. (i)

Also, L and M are midpoints of PQ and QR respectively

PL = LQ = QM = MR = QR/2

Now, consider Δ LPN and Δ MRN,

LP = MR

∠LPN = ∠MRN [From (i)]

∠QPR = ∠LPN and ∠QRP = ∠MRN

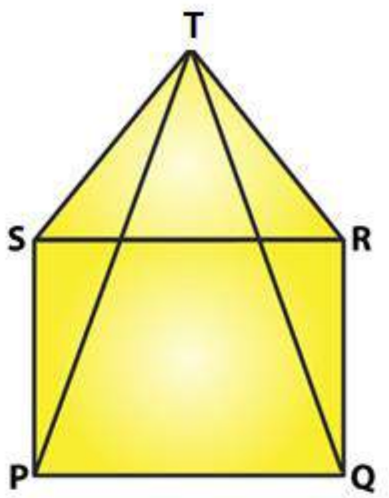
PN = NR [N is midpoint of PR]

By SAS congruence criterion,
Δ LPN ≃ Δ MRN

We know, corresponding parts of congruent triangles are equal.

So LN = MN
Proved.

Question 3: In figure, PQRS is a square and SRT is an equilateral triangle. Prove that
(i) PT = QT (ii) ∠ TQR = 15^0



Solution:
Given: PQRS is a square and SRT is an equilateral triangle.

To prove:

(i) PT =QT and (ii) ∠ TQR =15°

Now,
PQRS is a square:
PQ = QR = RS = SP ...... (i)
And ∠ SPQ = ∠ PQR = ∠ QRS = ∠ RSP = 90º

Also, △ SRT is an equilateral triangle:
SR = RT = TS ....(ii)
And ∠ TSR = ∠ SRT = ∠ RTS = 60°

From (i) and (ii)

PQ = QR = SP = SR = RT = TS ......(iii)

From figure,

∠TSP = ∠TSR + ∠ RSP = 60° + 90° = 150° and

∠TRQ = ∠TRS + ∠ SRQ = 60° + 90° = 150°

=> ∠ TSR = ∠ TRQ = 150^0 ………………… (iv)

By SAS congruence criterion, Δ TSP ≃ Δ TRQ

We know, corresponding parts of congruent triangles are equal
So, PT = QT

Proved part (i).

Now, consider Δ TQR.

QR = TR [From (iii)]

Δ TQR is an isosceles triangle.

∠ QTR = ∠ TQR [angles opposite to equal sides]

Sum of angles in a triangle = 180°

=> ∠QTR + ∠ TQR + ∠TRQ = 180°

=> 2 ∠ TQR + 150° = 180° [From (iv)]

=> 2 ∠ TQR = 30°

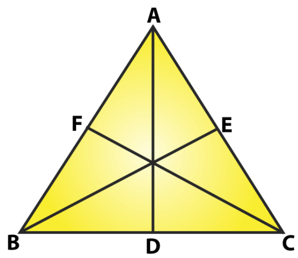
=> ∠ TQR = 15^0

Hence proved part (ii).

Question 4: Prove that the medians of an equilateral triangle are equal.

Solution:

Consider an equilateral △ABC, and Let D, E, F are midpoints of BC, CA and AB.



Here, AD, BE and CF are medians of △ABC.

Now,

D is midpoint of BC => BD = DC

Similarly, CE = EA and AF = FB

Since ΔABC is an equilateral triangle

AB = BC = CA .....(i)

BD = DC = CE = EA = AF = FB ....(ii)

And also, ∠ ABC = ∠ BCA = ∠ CAB = 60° ....(iii)

Consider Δ ABD and Δ BCE

AB = BC [From (i)]

BD = CE [From (ii)]

∠ ABD = ∠ BCE [From (iii)]

By SAS congruence criterion,

Δ ABD ≃ Δ BCE

=> AD = BE ....(iv)

[Corresponding parts of congruent triangles are equal in measure]

Now, consider Δ BCE and Δ CAF,

BC = CA [From (i)]

∠ BCE = ∠ CAF [From (ii)]

CE = AF [From (ii)]

By SAS congruence criterion,

Δ BCE ≃ Δ CAF

=> BE = CF ………….(v)
[Corresponding parts of congruent triangles are equal]

From (iv) and (v), we have

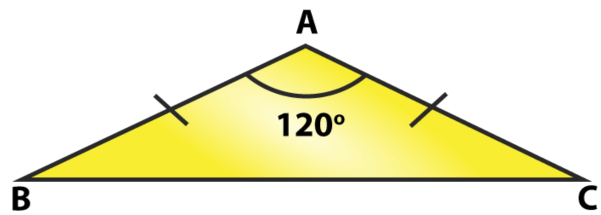
AD = BE = CF

Median AD = Median BE = Median CF

The medians of an equilateral triangle are equal.

Hence proved

Question 5: In a Δ ABC, if ∠A = 120° and AB = AC. Find ∠B and ∠C.
Solution:



To find: ∠ B and ∠ C.

Here, Δ ABC is an isosceles triangle since AB = AC

∠ B = ∠ C ……… (i)
[Angles opposite to equal sides are equal]

We know, sum of angles in a triangle = 180°

$\angle A + \angle B + \angle C = 180°$

$\angle A + \angle B + \angle B = 180°$ (using (i)

$120^0 + 2\angle B = 180^0$

$2\angle B = 180^0 - 120^0 = 60^0$

$\angle B = 30^o$

Therefore, $\angle B = \angle C = 30^\circ$

Question 6: In a Δ ABC, if AB = AC and ∠ B = 70°, find ∠ A.

Solution:

Given: In a Δ ABC, AB = AC and ∠B = 70°

∠ B = ∠ C [Angles opposite to equal sides are equal]

Therefore, $\angle B = \angle C = 70^\circ$

Sum of angles in a triangle = 180°

$\angle A + \angle B + \angle C = 180^o$

$\angle A + 70^o + 70^o = 180^o$

$\angle A = 180^o - 140^o$

$\angle A = 40^o$