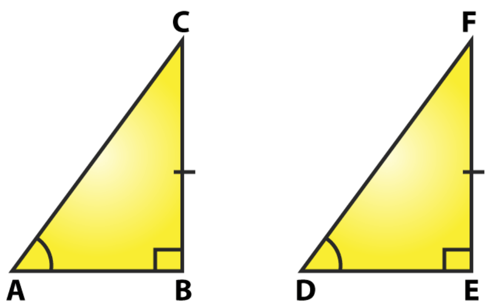
Exercise 10.3 Page No: 10.38

Question 1: In two right triangles one side an acute angle of one are equal to the corresponding side and angle of the other. Prove that the triangles are congruent.

Solution:

In two right triangles one side and acute angle of one are equal to the corresponding side and angles of the other. (Given)



To prove: Both the triangles are congruent.

Consider two right triangles such that

$\angle B = \angle E = 90^{o}$ ....(i)

AB = DE ....(ii)

$\angle C = \angle F$ ......(iii)

Here we have two right triangles, △ ABC and △ DEF

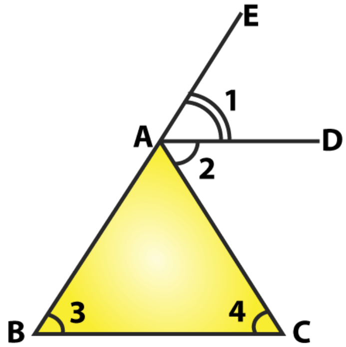
From (i), (ii) and (iii),
By AAS congruence criterion, we have Δ ABC ≅ Δ DEF

Both the triangles are congruent. Hence proved.

Question 2: If the bisector of the exterior vertical angle of a triangle be parallel to the base. Show that the triangle is isosceles.

Solution:

Let ABC be a triangle such that AD is the angular bisector of exterior vertical angle, ∠EAC and AD ∥ BC.



From figure,

∠1 = ∠2 [AD is a bisector of ∠ EAC]

∠1 = ∠3 [Corresponding angles]

and ∠2 = ∠4 [alternative angle]

From above, we have ∠3 = ∠4

This implies, AB = AC

Two sides AB and AC are equal.

=> Δ ABC is an isosceles triangle.

Question 3: In an isosceles triangle, if the vertex angle is twice the sum of the base angles, calculate the angles of the triangle.

Solution:

Let Δ ABC be isosceles where AB = AC and ∠ B = ∠ C

Given: Vertex angle A is twice the sum of the base angles B and C. i.e., ∠ A = 2(∠ B + ∠ C)

∠ A = 2(∠ B + ∠ B)

∠ A = 2(2 ∠ B)

∠ A = 4(∠ B)

Now, We know that sum of angles in a triangle =180°

$\angle A + \angle B + \angle C = 180°$

$4\angle B + \angle B + \angle B = 180°$

$6\angle B = 180°$

$\angle B = 30°$

Since, $\angle B = \angle C$

$\angle B = \angle C = 30°$

And $\angle A = 4\angle B$

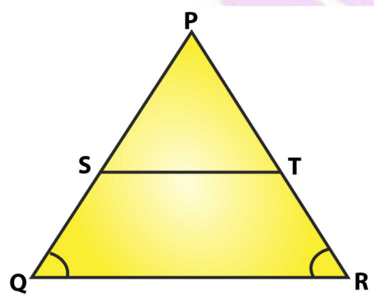
$\angle A = 4 \times 30° = 120°$

Therefore, angles of the given triangle are 30° and 30° and 120°.

Question 4: PQR is a triangle in which PQ = PR and is any point on the side PQ. Through S, a line is drawn parallel to QR and intersecting PR at T. Prove that PS = PT.

Solution: Given that PQR is a triangle such that PQ = PR and S is any point on the side PQ and ST ∥ QR.

To prove: PS = PT



Since, PQ= PR, so △PQR is an isosceles triangle.

$\angle PQR = \angle PRQ$

Now, $\angle PST = \angle PQR$ and $\angle PTS = \angle PRQ$
[Corresponding angles as ST parallel to QR]

Since, ∠ PQR = ∠ PRQ

∠ PST = ∠ PTS

In Δ PST,
∠ PST = ∠ PTS

Δ PST is an isosceles triangle.

Therefore, PS = PT.

Hence proved.