Question 1: In two right triangles one side an acute angle of one are equal to the corresponding side and angle of the other. Prove that the triangles are congruent.

Solution:

In two right triangles one side and acute angle of one are equal to the corresponding side and angles of the other. (Given)

To prove: Both the triangles are congruent.

Consider two right triangles such that

\[ \angle B = \angle E = 90^\circ \quad \text{...(i)} \]
\[ AB = DE \quad \text{...(ii)} \]
\[ \angle C = \angle F \quad \text{...(iii)} \]

Here we have two right triangles, \( \triangle ABC \) and \( \triangle DEF \)

From (i), (ii) and (iii),
By AAS congruence criterion, we have \( \triangle ABC \cong \triangle DEF \)

Both the triangles are congruent. Hence proved.

Question 2: If the bisector of the exterior vertical angle of a triangle be parallel to the base. Show that the triangle is isosceles.

Solution:
Let ABC be a triangle such that AD is the angular bisector of exterior vertical angle, ∠EAC and AD || BC.

From figure,

∠1 = ∠2  \[AD \text{ is a bisector of } ∠EAC\]

∠1 = ∠3  \[Corresponding angles\]

and ∠2 = ∠4  \[alternative angle\]

From above, we have ∠3 = ∠4

This implies, AB = AC

Two sides AB and AC are equal.

=> Δ ABC is an isosceles triangle.

**Question 3:** In an isosceles triangle, if the vertex angle is twice the sum of the base angles, calculate the angles of the triangle.

**Solution:**

Let Δ ABC be isosceles where AB = AC and ∠B = ∠C

Given: Vertex angle A is twice the sum of the base angles B and C. i.e., ∠A = 2(∠B + ∠C)

∠A = 2(∠B + ∠B)  
∠A = 2(2∠B)  
∠A = 4(∠B)
Now, We know that sum of angles in a triangle $= 180^\circ$

$\angle A + \angle B + \angle C = 180^\circ$

$4 \angle B + \angle B + \angle B = 180^\circ$

$6 \angle B = 180^\circ$

$\angle B = 30^\circ$

Since, $\angle B = \angle C$

$\angle B = \angle C = 30^\circ$

And $\angle A = 4 \angle B$

$\angle A = 4 \times 30^\circ = 120^\circ$

Therefore, angles of the given triangle are $30^\circ$ and $30^\circ$ and $120^\circ$.

**Question 4:** PQR is a triangle in which PQ = PR and S is any point on the side PQ. Through S, a line is drawn parallel to QR and intersecting PR at T. Prove that PS = PT.

**Solution:** Given that PQR is a triangle such that PQ = PR and S is any point on the side PQ and ST || QR.

To prove: PS = PT

Since, PQ = PR, so $\triangle$PQR is an isosceles triangle.

$\angle PQR = \angle PRQ$

Now, $\angle PST = \angle PQR$ and $\angle PTS = \angle PRQ$

[Corresponding angles as ST parallel to QR]
Since, \( \angle PQR = \angle PRQ \)
\( \angle PST = \angle PTS \)

In \( \triangle PST \),
\( \angle PST = \angle PTS \)

\( \triangle PST \) is an isosceles triangle.

Therefore, \( PS = PT \).

Hence proved.