Question 1: In figure, the sides BA and CA have been produced such that BA = AD and CA = AE. Prove that segment DE \parallel BC.

Solution:

Sides BA and CA have been produced such that BA = AD and CA = AE.

To prove: DE \parallel BC

Consider \triangle BAC and \triangle DAE,

BA = AD and CA = AE \quad \text{(Given)}

\angle BAC = \angle DAE \quad \text{(vertically opposite angles)}

By SAS congruence criterion, we have

\triangle BAC \cong \triangle DAE

We know, corresponding parts of congruent triangles are equal

So, BC = DE and \angle DEA = \angle BCA, \angle EDA = \angle CBA

Now, DE and BC are two lines intersected by a transversal DB such that
\angle DEA = \angle BCA \quad \text{(alternate angles are equal)}

Therefore, DE \parallel BC. Proved.
Question 2: In a PQR, if PQ = QR and L, M and N are the mid-points of the sides PQ, QR and RP respectively. Prove that LN = MN.

Solution:

Draw a figure based on given instruction,

In $\triangle PQR$, PQ = QR and L, M, N are midpoints of the sides PQ, QP and RP respectively (Given)

To prove : LN = MN

As two sides of the triangle are equal, so $\triangle PQR$ is an isosceles triangle

$PQ = QR$ and $\angle QPR = \angle QRP$ .... (i)

Also, L and M are midpoints of PQ and QR respectively

$PL = LQ = QM = MR = QR/2$

Now, consider $\triangle LPN$ and $\triangle MRN$,

$LP = MR$

$\angle LPN = \angle MRN$ [From (i)]

$\angle QPR = \angle LPN$ and $\angle QRP = \angle MRN$

$PN = NR$ [N is midpoint of PR]

By SAS congruence criterion, $\triangle LPN \simeq \triangle MRN$
We know, corresponding parts of congruent triangles are equal.

So \( LN = MN \)

Proved.

**Question 3:** In figure, \( PQRS \) is a square and \( SRT \) is an equilateral triangle. Prove that (i) \( PT = QT \) (ii) \( \angle TQR = 15^\circ \)

**Solution:**

Given: \( PQRS \) is a square and \( SRT \) is an equilateral triangle.

To prove:

(i) \( PT = QT \) and (ii) \( \angle TQR = 15^\circ \)

Now,

\( PQRS \) is a square:

\( PQ = QR = RS = SP \) \( \ldots \ldots \) (i)

And \( \angle SPQ = \angle PQR = \angle QRS = \angle RSP = 90^\circ \)

Also, \( \triangle SRT \) is an equilateral triangle:

\( SR = RT = TS \) \( \ldots \ldots \) (ii)

And \( \angle TSR = \angle SRT = \angle RTS = 60^\circ \)

From (i) and (ii)

\( PQ = QR = SP = SR = RT = TS \) \( \ldots \ldots \) (iii)
From figure,
\[ \angle TSP = \angle TSR + \angle RSP = 60^\circ + 90^\circ = 150^\circ \]
\[ \angle TRQ = \angle TRS + \angle SRQ = 60^\circ + 90^\circ = 150^\circ \]
=> \[ \angle TSR = \angle TRQ = 150^\circ \] \hspace{1cm} \text{(iv)}

By SAS congruence criterion, \( \Delta TSP \cong \Delta TRQ \)

We know, corresponding parts of congruent triangles are equal
So, \( PT = QT \)
Proved part (i).

Now, consider \( \Delta TQR \).
\( QR = TR \) \hspace{1cm} \text{[From (iii)]}
\( \Delta TQR \) is an isosceles triangle.
\( \angle QTR = \angle TQR \) \hspace{1cm} \text{[angles opposite to equal sides]}
Sum of angles in a triangle = 180°
=> \[ \angle QTR + \angle TQR + \angle TRQ = 180^\circ \]
=> \[ 2 \angle TQR + 150^\circ = 180^\circ \] \hspace{1cm} \text{[From (iv)]}
=> \[ 2 \angle TQR = 30^\circ \]
=> \[ \angle TQR = 15^\circ \]
Hence proved part (ii).

Question 4: Prove that the medians of an equilateral triangle are equal.

Solution:
Consider an equilateral \( \triangle ABC \), and let \( D, E, F \) are midpoints of \( BC, CA \) and \( AB \).
Here, AD, BE and CF are medians of \( \triangle ABC \).

Now,

D is midpoint of BC \( \Rightarrow \) BD = DC

Similarly, CE = EA and AF = FB

Since \( \triangle ABC \) is an equilateral triangle

\[ AB = BC = CA \quad \ldots \ldots \quad (i) \]

\[ BD = DC = CE = EA = AF = FB \quad \ldots \ldots \quad (ii) \]

And also, \( \angle ABC = \angle BCA = \angle CAB = 60^\circ \) \( \ldots \ldots \quad (iii) \)

Consider \( \triangle ABD \) and \( \triangle BCE \)

\[ AB = BC \quad \text{[From (i)]} \]

\[ BD = CE \quad \text{[From (ii)]} \]

\[ \angle ABD = \angle BCE \quad \text{[From (iii)]} \]

By SAS congruence criterion,

\[ \triangle ABD \cong \triangle BCE \]

\[ \Rightarrow AD = BE \quad \ldots \ldots \quad (iv) \]

[Corresponding parts of congruent triangles are equal in measure]
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Now, consider Δ BCE and Δ CAF,

- BC = CA \ [\text{From (i)}]
- \angle BCE = \angle CAF \ [\text{From (ii)}]
- CE = AF \ [\text{From (ii)}]

By SAS congruence criterion,

\[ \Delta BCE \cong \Delta CAF \]

=> \( BE = CF \) \hspace{1cm} (v)

[Corresponding parts of congruent triangles are equal]

From (iv) and (v), we have

\[ AD = BE = CF \]

Median AD = Median BE = Median CF

The medians of an equilateral triangle are equal.

Hence proved

Question 5: In a Δ ABC, if \( \angle A = 120^\circ \) and AB = AC. Find \( \angle B \) and \( \angle C \).

Solution:

To find: \( \angle B \) and \( \angle C \).

Here, Δ ABC is an isosceles triangle since AB = AC

\[ \angle B = \angle C \] \hspace{1cm} (i)

[Angles opposite to equal sides are equal]

We know, sum of angles in a triangle = 180°
\( \angle A + \angle B + \angle C = 180^\circ \)

\( \angle A + \angle B + \angle B = 180^\circ \) (using (i))

\( 120^0 + 2\angle B = 180^0 \)

\( 2\angle B = 180^0 - 120^0 = 60^0 \)

\( \angle B = 30^0 \)

Therefore, \( \angle B = \angle C = 30^\circ \)

**Question 6:** In a \( \Delta ABC \), if \( AB = AC \) and \( \angle B = 70^\circ \), find \( \angle A \).

**Solution:**

Given: In a \( \Delta ABC \), \( AB = AC \) and \( \angle B = 70^\circ \)

\( \angle B = \angle C \) [Angles opposite to equal sides are equal]

Therefore, \( \angle B = \angle C = 70^\circ \)

Sum of angles in a triangle = \( 180^\circ \)

\( \angle A + \angle B + \angle C = 180^\circ \)

\( \angle A + 70^\circ + 70^\circ = 180^\circ \)

\( \angle A = 180^\circ - 140^\circ \)

\( \angle A = 40^\circ \)
Question 1: In figure, it is given that RT = TS, $\angle 1 = 2 \angle 2$ and $\angle 4 = 2(\angle 3)$. Prove that $\triangle RBT \cong \triangle SAT$.

Solution:

In the figure,

RT = TS \hspace{1cm} \text{(i)}

$\angle 1 = 2 \angle 2 $ \hspace{1cm} \text{(ii)}

And $\angle 4 = 2 \angle 3$ \hspace{1cm} \text{(iii)}

To prove: $\triangle RBT \cong \triangle SAT$

Let the point of intersection RB and SA be denoted by O.

$\angle AOR = \angle BOS$ \hspace{1cm} \text{[Vertically opposite angles]}

or $\angle 1 = \angle 4$

$2 \angle 2 = 2 \angle 3$ \hspace{1cm} \text{[From (ii) and (iii)]}

or $\angle 2 = \angle 3$ \hspace{1cm} \text{(iv)}

Now in $\triangle TRS$, we have RT = TS
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=> Δ TRS is an isosceles triangle

∠ TRS = ∠ TSR ......(v)

But, ∠ TRS = ∠ TRB + ∠ 2 ......(vi)

∠ TSR = ∠ TSA + ∠ 3 ......(vii)

Putting (vi) and (vii) in (v) we get

∠ TRB + ∠ 2 = ∠ TSA + ∠ B

=> ∠ TRB = ∠ TSA [From (iv)]

Consider Δ RBT and Δ SAT

RT = ST [From (i)]

∠ TRB = ∠ TSA [From (iv)]

By ASA criterion of congruence, we have

Δ RBT ≅ Δ SAT

Question 2: Two lines AB and CD intersect at O such that BC is equal and parallel to AD. Prove that the lines AB and CD bisect at O.

Solution: Lines AB and CD Intersect at O

Such that BC ∥ AD and

BC = AD ......(i)
To prove : AB and CD bisect at O.

First we have to prove that \( \triangle AOD \cong \triangle BOC \)

- \( \angle OCB = \angle ODA \) [AD || BC and CD is transversal]
- \( AD = BC \) [from (i)]
- \( \angle OBC = \angle OAD \) [AD || BC and AB is transversal]

By \( \text{ASA Criterion:} \)

\( \triangle AOD \cong \triangle BOC \)

\( OA = OB \) and \( OD = OC \) (By c.p.c.t.)

Therefore, AB and CD bisect each other at O.

Hence Proved.

**Question 3:** BD and CE are bisectors of \( \angle B \) and \( \angle C \) of an isosceles \( \triangle ABC \) with \( AB = AC \). Prove that \( BD = CE \).

**Solution:**

\( \triangle ABC \) is isosceles with \( AB = AC \) and BD and CE are bisectors of \( \angle B \) and \( \angle C \)

We have to prove \( BD = CE \). (Given)

\[ \text{Since } AB = AC \]

\[ \Rightarrow \angle ABC = \angle ACB \quad \ldots \text{(i)} \]

[Angles opposite to equal sides are equal]

Since BD and CE are bisectors of \( \angle B \) and \( \angle C \)

\[ \angle ABD = \angle DBC = \angle BCE = \angle ECA = \frac{\angle B}{2} = \frac{\angle C}{2} \quad \ldots \text{(ii)} \]
Now, Consider \( \triangle EBC = \triangle DCB \)

\( \angle EBC = \angle DCB \) [From (i)]

\( BC = BC \) [Common side]

\( \angle BCE = \angle CBD \) [From (ii)]

By ASA congruence criterion, \( \triangle EBC \cong \triangle DCB \)

Since corresponding parts of congruent triangles are equal.

=> CE = BD

or, BD = CE

Hence proved.
### Exercise 10.3

#### Question 1:

In two right triangles one side an acute angle of one are equal to the corresponding side and angle of the other. Prove that the triangles are congruent.

**Solution:**

In two right triangles one side and acute angle of one are equal to the corresponding side and angles of the other. (Given)

Consider two right triangles such that

∠ B = ∠ E = 90°  
AB = DE  
∠ C = ∠ F

Here we have two right triangles, △ ABC and △ DEF

From (i), (ii) and (iii),  
By AAS congruence criterion, we have △ ABC ≅ △ DEF

Both the triangles are congruent. Hence proved.

#### Question 2:

If the bisector of the exterior vertical angle of a triangle be parallel to the base. Show that the triangle is isosceles.

**Solution:**
Let \( \triangle ABC \) be a triangle such that \( AD \) is the angular bisector of exterior vertical angle, \( \angle EAC \) and \( AD \parallel BC \).

From the figure,

\[
\angle 1 = \angle 2 \quad \text{[AD is a bisector of \( \angle EAC \)]}
\]
\[
\angle 1 = \angle 3 \quad \text{[Corresponding angles]}
\]
and
\[
\angle 2 = \angle 4 \quad \text{[alternative angle]}
\]

From above, we have

\[
\angle 3 = \angle 4
\]

This implies, \( AB = AC \)

Two sides \( AB \) and \( AC \) are equal.

\( \Rightarrow \) \( \triangle ABC \) is an isosceles triangle.

**Question 3:** In an isosceles triangle, if the vertex angle is twice the sum of the base angles, calculate the angles of the triangle.

**Solution:**

Let \( \triangle ABC \) be isosceles where \( AB = AC \) and \( \angle B = \angle C \)

Given: Vertex angle \( A \) is twice the sum of the base angles \( B \) and \( C \). i.e., \( \angle A = 2(\angle B + \angle C) \)

\[
\angle A = 2(\angle B + \angle B)
\]
\[
\angle A = 2(2 \angle B)
\]
\[
\angle A = 4(\angle B)
\]
Now, We know that sum of angles in a triangle =180°

∠ A + ∠ B + ∠ C =180°

4 ∠ B + ∠ B + ∠ B = 180°

6 ∠ B =180°

∠ B = 30°

Since, ∠ B = ∠ C

∠ B = ∠ C = 30°

And ∠ A = 4 ∠ B

∠ A = 4 x 30° = 120°

Therefore, angles of the given triangle are 30° and 30° and 120°.

Question 4: PQR is a triangle in which PQ = PR and is any point on the side PQ. Through S, a line is drawn parallel to QR and intersecting PR at T. Prove that PS = PT.

Solution: Given that PQR is a triangle such that PQ = PR and S is any point on the side PQ and ST || QR.

To prove: PS = PT

Since, PQ= PR, so △PQR is an isosceles triangle.

∠ PQR = ∠ PRQ

Now, ∠ PST = ∠ PQR and ∠ PTS = ∠ PRQ
[Corresponding angles as ST parallel to QR]
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Since, \( \angle PQR = \angle PRQ \)

\( \angle PST = \angle PTS \)

In \( \triangle PST \),
\( \angle PST = \angle PTS \)

\( \triangle PST \) is an isosceles triangle.

Therefore, \( PS = PT \).

Hence proved.
Exercise 10.4

Question 1: In figure, it is given that AB = CD and AD = BC. Prove that ΔADC ≅ ΔCBA.

Solution:

From the figure, AB = CD and AD = BC.

To prove: ΔADC ≅ ΔCBA

Consider ΔADC and ΔCBA.

AB = CD \quad [\text{Given}]
BC = AD \quad [\text{Given}]
AC = AC \quad [\text{Common side}]

So, by SSS congruence criterion, we have

ΔADC ≅ ΔCBA

Hence proved.

Question 2: In a Δ PQR, if PQ = QR and L, M and N are the mid-points of the sides PQ, QR and RP respectively. Prove that LN = MN.

Solution:

Given: In Δ PQR, PQ = QR and L, M and N are the mid-points of the sides PQ, QR and RP respectively

To prove: LN = MN
Join L and M, M and N, N and L

We have PL = LQ, QM = MR and RN = NP

[Since, L, M and N are mid-points of PQ, QR and RP respectively]

And also PQ = QR

PL = LQ = QM = MR = PN = LR \ldots (i)

[Using mid-point theorem]

MN \parallel PQ and MN = PQ/2

MN = PL = LQ \ldots (ii)

Similarly, we have

LN \parallel QR and LN = (1/2)QR

LN = QM = MR \ldots (iii)

From equation (i), (ii) and (iii), we have

PL = LQ = QM = MR = MN = LN

This implies, LN = MN

Hence Proved.
Question 1: ABC is a triangle and D is the mid-point of BC. The perpendiculars from D to AB and AC are equal. Prove that the triangle is isosceles.

Solution:
Given: D is the midpoint of BC and PD = DQ in a triangle ABC.

To prove: ABC is isosceles triangle.

In $\triangle BDP$ and $\triangle CDQ$
- PD = QD (Given)
- BD = DC (D is mid-point)
- $\angle BPD = \angle CQD = 90^\circ$

By RHS Criterion: $\triangle BDP \cong \triangle CDQ$

BP = CQ ... (i) (By CPCT)

In $\triangle APD$ and $\triangle AQD$
- PD = QD (given)
- AD = AD (common)
- $\angle APD = \angle AQD = 90^\circ$

By RHS Criterion: $\triangle APD \cong \triangle AQD$

So, PA = QA ... (ii) (By CPCT)

Adding (i) and (ii)
BP + PA = CQ + QA

AB = AC
Two sides of the triangle are equal, so ABC is an isosceles.

**Question 2:** ABC is a triangle in which BE and CF are, respectively, the perpendiculars to the sides AC and AB. If BE = CF, prove that Δ ABC is isosceles

**Solution:**

ABC is a triangle in which BE and CF are perpendicular to the sides AC and AS respectively s.t. BE = CF.

To prove: Δ ABC is isosceles

In Δ BCF and Δ CBE,
∠ BFC = CEB = 90°  [Given]
BC = CB  [Common side]
And CF = BE  [Given]

By RHS congruence criterion: ΔBFC ≅ ΔCEB

So, ∠ FBC = ∠ EBC  [By CPCT]

=> ∠ ABC = ∠ ACB

AC = AB  [Opposite sides to equal angles are equal in a triangle]
Two sides of triangle ABC are equal.
Therefore, Δ ABC is isosceles. Hence Proved.
Question 3: If perpendiculars from any point within an angle on its arms are congruent. Prove that it lies on the bisector of that angle.

Solution:

Consider an angle ABC and BP be one of the arm within the angle.

Draw perpendiculars PN and PM on the arms BC and BA.

In Δ BPM and Δ BPN,

∠ BMP = ∠ BNP = 90° [given]
BP = BP [Common side]
MP = NP [given]

By RHS congruence criterion: ΔBPM ≅ ΔBPN

So, ∠ MBP = ∠ NBP [By CPCT]

BP is the angular bisector of ∠ABC.
Hence proved
Exercise 10.6

Question 1: In \(\triangle ABC\), if \(\angle A = 40^\circ\) and \(\angle B = 60^\circ\). Determine the longest and shortest sides of the triangle.

Solution: In \(\triangle ABC\), \(\angle A = 40^\circ\) and \(\angle B = 60^\circ\)

We know, sum of angles in a triangle = 180°

\[\angle A + \angle B + \angle C = 180^\circ\]

\[40^\circ + 60^\circ + \angle C = 180^\circ\]

\[\angle C = 180^\circ - 100^\circ = 80^\circ\]

\[\angle C = 80^\circ\]

Now, \(40^\circ < 60^\circ < 80^\circ\)

\(\Rightarrow \angle A < \angle B < \angle C\)

\(\Rightarrow \angle C\) is greater angle and \(\angle A\) is smaller angle.

Now, \(\angle A < \angle B < \angle C\)

We know, side opposite to greater angle is larger and side opposite to smaller angle is smaller.

Therefore, \(BC < AC < AB\)

\(AB\) is longest and \(BC\) is shortest side.

Question 2: In a \(\triangle ABC\), if \(\angle B = \angle C = 45^\circ\), which is the longest side?

Solution: In \(\triangle ABC\), \(\angle B = \angle C = 45^\circ\)

Sum of angles in a triangle = 180°

\[\angle A + \angle B + \angle C = 180^\circ\]

\[\angle A + 45^\circ + 45^\circ = 180^\circ\]

\[\angle A = 180^\circ - (45^\circ + 45^\circ) = 180^\circ - 90^\circ = 90^\circ\]

\[\angle A = 90^\circ\]

\(\Rightarrow \angle B = \angle C < \angle A\)

Therefore, \(BC\) is the longest side.
Question 3: In Δ ABC, side AB is produced to D so that BD = BC. If ∠ B = 60° and ∠ A = 70°. Prove that: (i) AD > CD  (ii) AD > AC

Solution: In Δ ABC, side AB is produced to D so that BD = BC.

∠ B = 60°, and ∠ A = 70°

To prove: (i) AD > CD  (ii) AD > AC

Construction: Join C and D

We know, sum of angles in a triangle = 180°

∠ A + ∠ B + ∠ C = 180°

70° + 60° + ∠ C = 180°

∠ C = 180° – (130°) = 50°

∠ C = 50°

∠ ACB = 50° ......(i)

And also in Δ BDC

∠ DBC =180° − ∠ ABC = 180 − 60° = 120°
[∠ DBA is a straight line]

and BD = BC  [given]
\( \angle BCD = \angle BDC \) [Angles opposite to equal sides are equal]

Sum of angles in a triangle = 180°

\( \angle DBC + \angle BCD + \angle BDC = 180° \)

120° + \( \angle BCD + \angle BCD = 180° \)

120° + 2\( \angle BCD = 180° \)

2\( \angle BCD = 180° - 120° = 60° \)

\( \angle BCD = 30° \)

\( \angle BCD = \angle BDC = 30° \) ....(ii)

Now, consider \( \Delta ADC \).

\( \angle DAC = 70° \) [given]

\( \angle ADC = 30° \) [From (ii)]

\( \angle ACD = \angle ACB + \angle BCD = 50° + 30° = 80° \) [From (i) and (ii)]

Now, \( \angle ADC < \angle DAC < \angle ACD \)

\( AC < DC < AD \)

[Side opposite to greater angle is longer and smaller angle is smaller]

\( AD > CD \) and \( AD > AC \)

Hence proved.

**Question 4:** Is it possible to draw a triangle with sides of length 2 cm, 3 cm and 7 cm?

**Solution:**

Lengths of sides are 2 cm, 3 cm and 7 cm.

A triangle can be drawn only when the sum of any two sides is greater than the third side.

So, let's check the rule.
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\[ 2 + 3 \not\geq 7 \] or \[ 2 + 3 < 7 \]

\[ 2 + 7 > 3 \]

and \[ 3 + 7 > 2 \]

Here \[ 2 + 3 \not\geq 7 \]

So, the triangle does not exist.
Exercise VSAQs

Question 1: In two congruent triangles ABC and DEF, if AB = DE and BC = EF. Name the pairs of equal angles.
Solution:
In two congruent triangles ABC and DEF, if AB = DE and BC = EF, then
\[ \angle A = \angle D, \angle B = \angle E \text{ and } \angle C = \angle F \]

Question 2: In two triangles ABC and DEF, it is given that \( \angle A = \angle D, \angle B = \angle E \text{ and } \angle C = \angle F \). Are the two triangles necessarily congruent?
Solution: No.
Reason: Two triangles are not necessarily congruent, because we know only angle-angle-angle (AAA) criterion. This criterion can produce similar but not congruent triangles.

Question 3: If ABC and DEF are two triangles such that AC = 2.5 cm, BC = 5 cm, \( \angle C = 75^\circ \), DE = 2.5 cm, DF = 5 cm and \( \angle D = 75^\circ \). Are two triangles congruent?
Solution: Yes.
Reason: Given triangles are congruent as AC = DE = 2.5 cm, BC = DF = 5 cm and \( \angle D = \angle C = 75^\circ \).
By SAS theorem triangle ABC is congruent to triangle EDF.

Question 4: In two triangles ABC and ADC, if AB = AD and BC = CD. Are they congruent?
Solution: Yes.
Reason: Given triangles are congruent as
AB = AD
BC = CD and
AC [common side]
By SSS theorem triangle ABC is congruent to triangle ADC.

Question 5: In triangles ABC and CDE, if AC = CE, BC = CD, \( \angle A = 60^\circ \), \( \angle C = 30^\circ \) and \( \angle D = 90^\circ \). Are two triangles congruent?
Solution: Yes.
Reason: Given triangles are congruent
Here AC = CE
BC = CD
∠B = ∠D = 90°

By SSA criteria triangle ABC is congruent to triangle CDE.

Question 6: ABC is an isosceles triangle in which AB = AC. BE and CF are its two medians. Show that BE = CF.

Solution: ABC is an isosceles triangle (given)
AB = AC (given)
BE and CF are two medians (given)

To prove: BE = CF

In △CFB and △BEC

CE = BF (Since, AC = AB = AC/2 = AB/2 = CE = BF)
BC = BC (Common)
∠ECB = ∠FBC (Angle opposite to equal sides are equal)
By SAS theorem: △CFB ≅ △BEC

So, BE = CF (By c.p.c.t)