

Exercise 21.1

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Question 1: Find the surface area of a sphere of radius:**(i) 10.5 cm (ii) 5.6 cm (iii) 14 cm****Solution:**Surface area of a sphere = $4\pi r^2$

Where, r = radius of a sphere

(i) Radius = 10.5 cmSurface area = $4 \times \frac{22}{7} \times (10.5)^2$

= 1386

Surface area is 1386 cm²**(ii) Radius = 5.6 cm**Surface area = $4 \times \frac{22}{7} \times (5.6)^2$

= 394.24

Surface area is 394.24 cm²**(iii) Radius = 14 cm**Surface area = $4 \times \frac{22}{7} \times (14)^2$

= 2464

Surface area is 2464 cm²**Question 2: Find the surface area of a sphere of diameter:****(i) 14 cm (ii) 21 cm (iii) 3.5 cm****Solution:**Surface area of a sphere = $4\pi r^2$

Where, r = radius of a sphere

(i) Diameter = 14 cm

So, Radius = Diameter/2 = 14/2 cm = 7 cm

$$\text{Surface area} = 4 \times \frac{22}{7} \times (7)^2$$

$$= 616$$

Surface area is 616 cm²

(ii) Diameter = 21cm

So, Radius = Diameter/2 = 21/2 cm = 10.5 cm

$$\text{Surface area} = 4 \times \frac{22}{7} \times (10.5)^2$$

$$= 1386$$

Surface area is 1386 cm²

(iii) Diameter = 3.5cm

So, Radius = Diameter/2 = 3.5/2 cm = 1.75 cm

$$\text{Surface area} = 4 \times \frac{22}{7} \times (1.75)^2$$

$$= 38.5$$

Surface area is 38.5 cm²

Question 3: Find the total surface area of a hemisphere and a solid hemisphere each of radius 10 cm. ($\pi=3.14$)

Solution:

Radius of a hemisphere = Radius of a solid hemisphere = 10 cm (Given)

$$\text{Surface area of the hemisphere} = 2\pi r^2$$

$$= 2 \times 3.14 \times (10)^2 \text{ cm}^2$$

$$= 628 \text{ cm}^2$$

And, surface area of solid hemisphere = $3\pi r^2$

$$= 3 \times 3.14 \times (10)^2 \text{ cm}^2$$

$$= 942 \text{ cm}^2$$

Question 4: The surface area of a sphere is 5544 cm^2 , find its diameter.

Solution:

Surface area of a sphere is 5544 cm^2

Surface area of a sphere = $4\pi r^2$

$$\text{So, } 4\pi r^2 = 5544$$

$$4 \times 22/7 \times (r)^2 = 5544$$

$$r^2 = (5544 \times 7)/88$$

$$r^2 = 441$$

$$\text{or } r = 21 \text{ cm}$$

$$\text{Now, Diameter} = 2(\text{radius}) = 2(21) = 42 \text{ cm}$$

Question 5: A hemispherical bowl made of brass has inner diameter 10.5 cm. Find the cost of tin plating it on the inside at the rate of Rs.4 per 100 cm^2 .

Solution:

Inner diameter of hemispherical bowl = 10.5 cm

$$\text{So, radius} = \text{Diameter}/2 = 10.5/2 \text{ cm} = 5.25 \text{ cm}$$

Now, Surface area of hemispherical bowl = $2\pi r^2$

$$= 2 \times 3.14 \times (5.25)^2$$

$$= 173.25$$

So, Surface area of hemispherical bowl is 173.25 cm^2

Find the cost:

Cost of tin plating 100 cm^2 area = Rs.4 (given)

Cost of tin plating 173.25 cm^2 area = Rs. $4 \times 173.25 / 100 = \text{Rs. } 6.93$

Therefore, cost of tin plating the inner side of hemispherical bowl is Rs.6.93. Answer!!

Question 6: The dome of a building is in the form of a hemisphere. Its radius is 63 dm. Find the cost of painting it at the rate of Rs. 2 per sq m.

Solution:

Radius of hemispherical dome = 63 dm or 6.3 m

Inner surface area of dome = $2\pi r^2$

$$= 2 \times 3.14 \times (6.3)^2$$

$$= 249.48$$

So, Inner surface area of dome is 249.48 m^2

Now find the cost:

Cost of painting $1 \text{ m}^2 = \text{Rs. } 2$ (given)

Therefore, cost of painting $249.48 \text{ m}^2 = \text{Rs. } (249.48 \times 2) = \text{Rs. } 498.96$.

Exercise 21.2

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Question 1: Find the volume of a sphere whose radius is:

(i) 2 cm (ii) 3.5 cm (iii) 10.5 cm.

Solution:

Volume of a sphere = $\frac{4}{3}\pi r^3$ Cubic Units

Where, r = radius of a sphere

(i) Radius = 2 cm

$$\text{Volume} = \frac{4}{3} \times \frac{22}{7} \times (2)^3$$

$$= 33.52$$

$$\text{Volume} = 33.52 \text{ cm}^3$$

(ii) Radius = 3.5cm

$$\text{Therefore volume} = \frac{4}{3} \times \frac{22}{7} \times (3.5)^3$$

$$= 179.666$$

$$\text{Volume} = 179.666 \text{ cm}^3$$

(iii) Radius = 10.5 cm

$$\text{Volume} = \frac{4}{3} \times \frac{22}{7} \times (10.5)^3$$

$$= 4851$$

$$\text{Volume} = 4851 \text{ cm}^3$$

Question 2: Find the volume of a sphere whose diameter is:

(i) 14 cm (ii) 3.5 dm (iii) 2.1 m

Solution:

Volume of a sphere = $\frac{4}{3}\pi r^3$ Cubic Units

Where, r = radius of a sphere

(i) diameter = 14 cm

So, radius = diameter/2 = 14/2 = 7cm

$$\text{Volume} = \frac{4}{3} \times \frac{22}{7} \times (7)^3$$

$$= 1437.33$$

$$\text{Volume} = 1437.33 \text{ cm}^3$$

(ii) diameter = 3.5 dm

So, radius = diameter/2 = 3.5/2 = 1.75 dm

$$\text{Volume} = \frac{4}{3} \times \frac{22}{7} \times (1.75)^3$$

$$= 22.46$$

$$\text{Volume} = 22.46 \text{ dm}^3$$

(iii) diameter = 2.1 m

So, radius = diameter/2 = 2.1/2 = 1.05 m

$$\text{Volume} = \frac{4}{3} \times \frac{22}{7} \times (1.05)^3$$

$$= 4.851$$

$$\text{Volume} = 4.851 \text{ m}^3$$

Question 3: A hemispherical tank has the inner radius of 2.8 m. Find its capacity in liters.

Solution:

Radius of hemispherical tank = 2.8 m

$$\text{Capacity of hemispherical tank} = \frac{2}{3} \pi r^3$$

$$= \frac{2}{3} \times \frac{22}{7} \times (2.8)^3 \text{ m}^3$$

$$= 45.997 \text{ m}^3$$

[Using $1 \text{ m}^3 = 1000 \text{ liters}$]

Therefore, capacity in litres = 45997 litres

Question 4: A hemispherical bowl is made of steel 0.25 cm thick. The inside radius of the bowl is 5 cm. Find the volume of steel used in making the bowl.

Solution:

Inner radius of a hemispherical bowl = 5 cm

Outer radius of a hemispherical bowl = 5 cm + 0.25 cm = 5.25 cm

Volume of steel used = Outer volume - Inner volume

$$= \frac{2}{3} \times \pi \times ((5.25)^3 - (5)^3)$$

$$= \frac{2}{3} \times \frac{22}{7} \times ((5.25)^3 - (5)^3)$$

$$= 41.282$$

Volume of steel used is 41.282 cm³

Question 5: How many bullets can be made out of a cube of lead, whose edge measures 22 cm, each bullet being 2 cm in diameter?

Solution:

Edge of a cube = 22 cm

Diameter of bullet = 2 cm

So, radius of bullet (r) = 1 cm

$$\text{Volume of the cube} = (\text{side})^3 = (22)^3 \text{ cm}^3 = 10648 \text{ cm}^3$$

And,

$$\text{Volume of each bullet which will be spherical in shape} = \frac{4}{3} \pi r^3$$

$$= \frac{4}{3} \times \frac{22}{7} \times (1)^3 \text{ cm}^3$$

$$= \frac{4}{3} \times \frac{22}{7} \text{ cm}^3$$

$$= 8821 \text{ cm}^3$$

Number of bullets = (Volume of cube) / (Volume of bullet)

$$= 10648/8821$$
$$= 2541$$

Therefore, 2541 bullets can be made.

Question 6: A shopkeeper has one laddoo of radius 5 cm. With the same material, how many laddoos of radius 2.5 cm can be made?

Solution:

$$\text{Volume of laddoo having radius 5 cm (V1)} = \frac{4}{3} \times \frac{22}{7} \times (5)^3$$

$$= 11000/21 \text{ cm}^3$$

$$\text{Also, Volume of laddoo having radius 2.5 cm (V2)} = \frac{4}{3} \pi r^3$$

$$= \frac{4}{3} \times \frac{22}{7} \times (2.5)^3 \text{ cm}^3$$

$$= 1375/21 \text{ cm}^3$$

Therefore,

$$\text{Number of laddoos of radius 2.5 cm that can be made} = V1/V2 = 11000/1375 = 8$$

Question 7: A spherical ball of lead 3 cm in diameter is melted and recast into three spherical balls. If the diameters of two balls be 3/2 cm and 2 cm, find the diameter of the third ball.

Solution:

$$\text{Volume of lead ball with radius } 3/2 \text{ cm} = \frac{4}{3} \pi r^3$$

$$= \frac{4}{3} \times \pi \times (3/2)^3$$

$$\text{Let, Diameter of first ball (d1)} = 3/2 \text{ cm}$$

$$\text{Radius of first ball (r1)} = 3/4 \text{ cm}$$

$$\text{Diameter of second ball (d2)} = 2 \text{ cm}$$

$$\text{Radius of second ball (r2)} = 2/2 \text{ cm} = 1 \text{ cm}$$

$$\text{Diameter of third ball (d3)} = d$$

$$\text{Radius of third ball (r3)} = d/2 \text{ cm}$$

Now,

$$\text{Volume of lead ball} = \frac{4}{3} \times \pi \times \left(\frac{3}{4}\right)^3 + \frac{4}{3} \times \pi \times (1)^3 + \frac{4}{3} \times \pi \times \left(\frac{d}{2}\right)^3$$

$$\frac{4}{3} \times \pi \times \left(\frac{3}{2}\right)^3 = \frac{4}{3} \times \pi \times \left(\frac{3}{4}\right)^3 + \frac{4}{3} \times \pi \times (1)^3 + \frac{4}{3} \times \pi \times \left(\frac{d}{2}\right)^3$$

$$\frac{4}{3} \pi \left[\left(\frac{3}{2}\right)^3\right] = \frac{4}{3} \pi \left[\left(\frac{3}{4}\right)^3 + (1)^3 + \left(\frac{d}{2}\right)^3\right]$$

$$\frac{27}{8} = \frac{27}{64} + 1 + \frac{d^3}{8}$$

$$d^3 = 8 \left[\frac{27}{8} - \frac{27}{64} - 1\right]$$

$$\frac{d^3}{8} = \frac{125}{64}$$

$$\frac{d}{2} = \frac{5}{4}$$

$$d = \frac{10}{4}$$

$$d = 2.5$$

So, diameter of third ball is 2.5 cm.

Question 8: A sphere of radius 5 cm is immersed in water filled in a cylinder, the level of water rises $\frac{5}{3}$ cm. Find the radius of the cylinder.

Solution:

Radius of sphere = 5 cm (Given)

Let 'r' be the radius of cylinder.

We know, Volume of sphere = $\frac{4}{3}\pi r^3$

By putting values, we get

$$= \frac{4}{3} \times \pi \times (5)^3$$

Height (h) of water rises is $\frac{5}{3}$ cm (Given)

Volume of water rises in cylinder = $\pi r^2 h$

Therefore, Volume of water rises in cylinder = Volume of sphere

$$\text{So, } \pi r^2 h = \frac{4}{3} \pi r^3$$

$$\pi r^2 \times \frac{5}{3} = \frac{4}{3} \times \pi \times (5)^3$$

$$\text{or } r^2 = 100$$

$$\text{or } r = 10$$

Therefore, radius of the cylinder is 10 cm.

Question 9: If the radius of a sphere is doubled, what is the ratio of the volume of the first sphere to that of the second sphere?

Solution:

Let r be the radius of the first sphere then $2r$ be the radius of the second sphere.

Now,

$$\frac{\text{Volume of first sphere}}{\text{Volume of second sphere}} = \frac{\frac{4}{3}\pi r^3}{\frac{4}{3}\pi(2r)^3} = \frac{1}{8}$$

Ratio of volume of the first sphere to the second sphere is 1:8.

Question 10: A cone and a hemisphere have equal bases and equal volumes. Find the ratio of their heights.

Solution:

Volume of the cone = Volume of the hemisphere (Given)

$$\frac{1}{3}\pi r^2 h = \frac{2}{3}\pi r^3$$

(Using respective formulas)

$$r^2 h = 2r^3$$

$$\text{or } h = 2r$$

Since, cone and a hemisphere have equal bases which implies they have the same radius.

$$h/r = 2$$

or $h : r = 2 : 1$

Therefore, Ratio of their heights is 2:1

Question 11: A vessel in the form of a hemispherical bowl is full of water. Its contents are emptied in a right circular cylinder. The internal radii of the bowl and the cylinder are 3.5 cm and 7 cm respectively. Find the height to which the water will rise in the cylinder.

Solution:

Volume of water in the hemispherical bowl = Volume of water in the cylinder ... (Given)

Inner radius of the bowl (r_1) = 3.5cm

Inner radius of cylinder (r_2) = 7cm

Volume of water in the hemispherical bowl = Volume of water in the cylinder

$$\frac{2}{3}\pi r_1^3 = \pi r_2^2 h$$

[Using respective formulas]

Where h be the height to which water rises in the cylinder.

$$\frac{2}{3}\pi(3.5)^3 = \pi(7)^2 h$$

$$\text{or } h = \frac{7}{12}$$

Therefore, $\frac{7}{12}$ cm be the height to which water rises in the cylinder.

Question 12: A cylinder whose height is two thirds of its diameter, has the same volume as a sphere of radius 4 cm. Calculate the radius of the base of the cylinder.

Solution:

Radius of a sphere (R) = 4 cm (Given)

Height of the cylinder = $\frac{2}{3}$ diameter (given)

We know, Diameter = 2(Radius)

Let h be the height and r be the base radius of a cylinder, then

$$h = \frac{2}{3} \times (2r) = \frac{4r}{3}$$

Volume of the cylinder = Volume of the sphere

$$\pi r^2 h = \frac{4}{3} \pi R^3$$

$$\pi \times r^2 \times \left(\frac{4r}{3}\right) = \frac{4}{3} \pi (4)^3$$

$$(r)^3 = (4)^3$$

$$\text{or } r = 4$$

Therefore, radius of the base of the cylinder is 4 cm.

Question 13: A vessel in the form of a hemispherical bowl is full of water. The contents are emptied into a cylinder. The internal radii of the bowl and cylinder are respectively 6 cm and 4 cm. Find the height of water in the cylinder.

Solution:

Radius of a bowl (R) = 6 cm (Given)

Radius of a cylinder (r) = 4 cm (given)

Let h be the height of a cylinder.

Now,

Volume of water in hemispherical bowl = Volume of cylinder

$$\frac{2}{3} \pi R^3 = \pi r^2 h$$

$$\frac{2}{3} \pi (6)^3 = \pi (4)^2 h$$

$$\text{or } h = 9$$

Therefore, height of water in the cylinder 9 cm.

Question 14: A cylindrical tub of radius 16 cm contains water to a depth of 30 cm. A spherical iron ball is dropped into the tub and thus level of water is raised by 9 cm. What is the radius of the ball?

Solution:

Let r be the radius of the iron ball.

Radius of the cylinder (R) = 16 cm (Given)

A spherical iron ball is dropped into the cylinder and thus the level of water is raised by 9 cm. So, height (h) = 9 cm

From statement,

Volume of iron ball = Volume of water raised in the hub

$$\frac{4}{3}\pi r^3 = \pi R^2 h$$

$$\frac{4}{3} r^3 = (16)^2 \times 9$$

$$\text{or } r^3 = 1728$$

$$\text{or } r = 12$$

Therefore, radius of the ball = 12cm.



Exercise VSAQs

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Question 1: Find the surface area of a sphere of radius 14 cm.**Solution:**Radius of a sphere (r) = 14 cmSurface area of a sphere = $4\pi r^2$

$$= 4 \times (22/7) \times 14^2 \text{ cm}^2$$

$$= 2464 \text{ cm}^2$$

Question 2: Find the total surface area of a hemisphere of radius 10 cm.**Solution:**Radius of a hemisphere (r) = 10 cmTotal surface area of a hemisphere = $3\pi r^2$

$$= 3 \times (22/7) \times 10^2 \text{ cm}^2$$

$$= 942 \text{ cm}^2$$

Question 3: Find the radius of a sphere whose surface area is 154 cm^2 .**Solution:**Surface area of a sphere = 154 cm^2 We know, Surface area of a sphere = $4\pi r^2$

$$\text{So, } 4\pi r^2 = 154$$

$$4 \times 22/7 \times r^2 = 154$$

$$r^2 = 49/4$$

$$\text{or } r = 7/2 = 3.5$$

Radius of a sphere is 3.5 cm.

Question 4: The hollow sphere, in which the circus motor cyclist performs his stunts, has a diameter of 7 m. Find the area available to the motorcyclist for riding.

Solution:

Diameter of hollow sphere = 7 m

So, radius of hollow sphere = $7/2$ m = 3.5 cm

Now,

Area available to the motorcyclist for riding = Surface area of a sphere = $4\pi r^2$

$$= 4 \times (22/7) \times 3.5^2 \text{ m}^2$$

$$= 154 \text{ m}^2$$

Question 5: Find the volume of a sphere whose surface area is 154 cm².

Solution:

Surface area of a sphere = 154 cm²

We know, Surface area of a sphere = $4\pi r^2$

$$\text{So, } 4\pi r^2 = 154$$

$$4 \times 22/7 \times r^2 = 154$$

$$\text{or } r^2 = 49/4$$

$$\text{or } r = 7/2 = 3.5$$

Radius (r) = 3.5 cm

Now,

Volume of sphere = $4/3 \pi r^3$

$$= (4/3) \pi \times 3.5^3$$

$$= 179.66$$

Therefore, Volume of sphere is 179.66 cm³.