

Exercise 2.2

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1. Verify that the numbers given alongside of the cubic polynomials below are their zeroes. Also, verify the relationship between the zeros and coefficients in each of the following cases:

(i) $f(x) = 2x^3 + x^2 - 5x + 2$; $1/2, 1, -2$

Solution:

Given, $f(x) = 2x^3 + x^2 - 5x + 2$, where $a = 2, b = 1, c = -5$ and $d = 2$

For $x = 1/2$

$$f(1/2) = 2(1/2)^3 + (1/2)^2 - 5(1/2) + 2$$

$$= 1/4 + 1/4 - 5/2 + 2 = 0$$

$\Rightarrow f(1/2) = 0$, hence $x = 1/2$ is a root of the given polynomial.

For $x = 1$

$$f(1) = 2(1)^3 + (1)^2 - 5(1) + 2$$

$$= 2 + 1 - 5 + 2 = 0$$

$\Rightarrow f(1) = 0$, hence $x = 1$ is also a root of the given polynomial.

For $x = -2$

$$f(-2) = 2(-2)^3 + (-2)^2 - 5(-2) + 2$$

$$= -16 + 4 + 10 + 2 = 0$$

$\Rightarrow f(-2) = 0$, hence $x = -2$ is also a root of the given polynomial.

Now,

Sum of zeros = $-b/a$

$$1/2 + 1 - 2 = -(1)/2$$

$$-1/2 = -1/2$$

Sum of the products of the zeros taken two at a time = c/a

$$(1/2 \times 1) + (1 \times -2) + (1/2 \times -2) = -5/2$$

$$1/2 - 2 + (-1) = -5/2$$

$$-5/2 = -5/2$$

Product of zeros = $-d/a$

$$1/2 \times 1 \times (-2) = -(2)/2$$

$$-1 = -1$$

Hence, the relationship between the zeros and coefficients is verified.

(ii) $g(x) = x^3 - 4x^2 + 5x - 2$; $2, 1, 1$

Solution:

Given, $g(x) = x^3 - 4x^2 + 5x - 2$, where $a = 1, b = -4, c = 5$ and $d = -2$

For $x = 2$

$$g(2) = (2)^3 - 4(2)^2 + 5(2) - 2$$

$$= 8 - 16 + 10 - 2 = 0$$

$\Rightarrow g(2) = 0$, hence $x = 2$ is a root of the given polynomial.

For $x = 1$

$$g(1) = (1)^3 - 4(1)^2 + 5(1) - 2$$

$$= 1 - 4 + 5 - 2 = 0$$

$\Rightarrow g(1) = 0$, hence $x = 1$ is also a root of the given polynomial.

Now,

Sum of zeros = $-b/a$

$$1 + 1 + 2 = -(-4)/1$$

$$4 = 4$$

Sum of the products of the zeros taken two at a time = c/a

$$(1 \times 1) + (1 \times 2) + (2 \times 1) = 5/1$$

$$1 + 2 + 2 = 5$$

$$5 = 5$$

Product of zeros = $-d/a$

$$1 \times 1 \times 2 = -(-2)/1$$

$$2 = 2$$

Hence, the relationship between the zeros and coefficients is verified.

2. Find a cubic polynomial with the sum, sum of the product of its zeroes taken two at a time, and product of its zeros as 3, -1 and -3 respectively.

Solution:

Generally,

A cubic polynomial say, $f(x)$ is of the form $ax^3 + bx^2 + cx + d$.

And, can be shown w.r.t its relationship between roots as.

$$\Rightarrow f(x) = k [x^3 - (\text{sum of roots})x^2 + (\text{sum of products of roots taken two at a time})x - (\text{product of roots})]$$

Where, k is any non-zero real number.

Here,

$$f(x) = k [x^3 - (3)x^2 + (-1)x - (-3)]$$

$$\therefore f(x) = k [x^3 - 3x^2 - x + 3]$$

where, k is any non-zero real number.

3. If the zeros of the polynomial $f(x) = 2x^3 - 15x^2 + 37x - 30$ are in A.P., find them.

Solution:

Let the zeros of the given polynomial be α , β and γ . (3 zeros as it's a cubic polynomial)

And given, the zeros are in A.P.

So, let's consider the roots as

$$\alpha = a - d, \beta = a \text{ and } \gamma = a + d$$

Where, a is the first term and d is the common difference.

From given $f(x)$, $a = 2$, $b = -15$, $c = 37$ and $d = 30$

$$\Rightarrow \text{Sum of roots} = \alpha + \beta + \gamma = (a - d) + a + (a + d) = 3a = (-b/a) = -(-15/2) = 15/2$$

$$\text{So, calculating for } a, \text{ we get } 3a = 15/2 \Rightarrow a = 5/2$$

$$\Rightarrow \text{Product of roots} = (a - d) \times a \times (a + d) = a(a^2 - d^2) = -d/a = -(30)/2 = 15$$

$$\Rightarrow a(a^2 - d^2) = 15$$

Substituting the value of a , we get

$$\Rightarrow (5/2)[(5/2)^2 - d^2] = 15$$

$$\begin{aligned}\Rightarrow 5\left[\frac{25}{4} - d^2\right] &= 30 \\ \Rightarrow \frac{25}{4} - d^2 &= 6 \\ \Rightarrow 25 - 4d^2 &= 24 \\ \Rightarrow 1 &= 4d^2 \\ \therefore d &= \frac{1}{2} \text{ or } -\frac{1}{2}\end{aligned}$$

Taking $d = \frac{1}{2}$ and $a = \frac{5}{2}$
We get,
the zeros as 2, $\frac{5}{2}$ and 3

Taking $d = -\frac{1}{2}$ and $a = \frac{5}{2}$
We get,
the zeros as 3, $\frac{5}{2}$ and 2

