Exercise 3.2

Solve the following system of equations graphically:

1. **x + y = 3**

2x + 5y = 12

Solution:

Given,

x + y = 3……. (i)

2x + 5y = 12……. (ii)

For equation (i),

When y = 0, we have x =3

When x= 0, we have y =3

Thus we have the following table giving points on the line x + y = 3

x	0	3
y	3	0

For equation (ii),

We solve for y:

⇒ y = (12 – 2x)/5

So, when x = 1

y = (12 – 2(1))/5 = 2

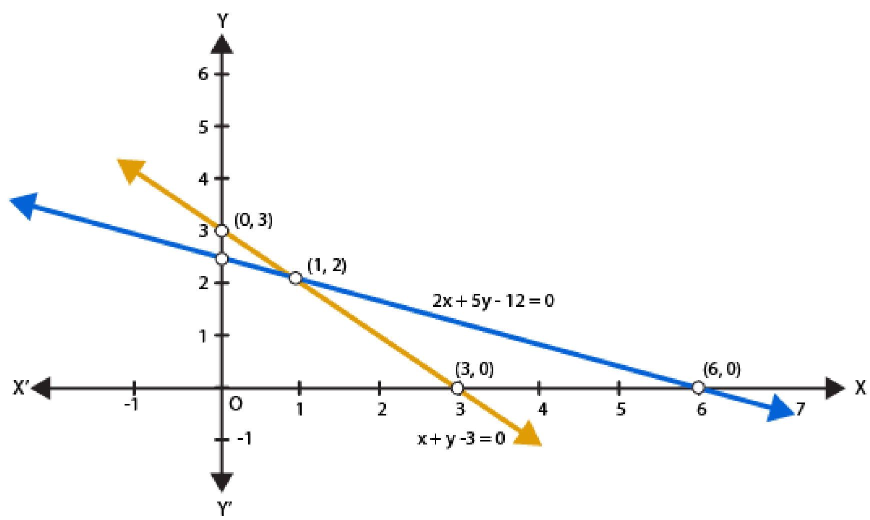
And, when x = 6

⇒ y = (12 – 2(6))/5 = 0

Thus we have the following table giving points on the line 2x + 5y = 12

x	1	6
y	2	0

Graph of the equations (i) and (ii) is as below:



Clearly the two lines intersect at a single point P (1, 2)
Hence, x= 1 and y = 2

2. x - 2y = 5
2x + 3y = 10
Solution:

Given,
x - 2y = 5……. (i)
2x + 3y = 10……. (ii)

For equation (i),
⇒ y = (x - 5)/2
When y = 0, we have x = 5
When x = -1, we have y = -2
Thus we have the following table giving points on the line x - 2y = 5

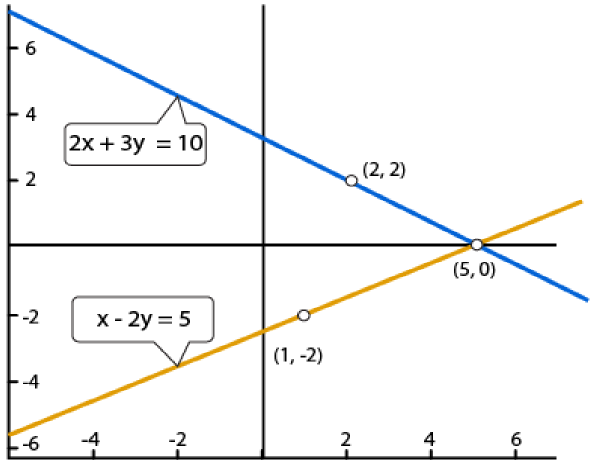
x	5	-1
y	0	-2

For equation (ii),
We solve for y:
⇒ y = (10 – 2x)/3

So, when x = 5
y = (10 – 2(5))/3 = 0
And, when x = 2
⇒ y = (10 – 2(2))/3 =2
Thus we have the following table giving points on the line 2x + 3y = 10

x	5	2
y	0	2

Graph of the equations (i) and (ii) is as below:



Clearly the two lines intersect at a single point P (5, 0)
Hence, x= 5 and y = 0

3. 3x+ y + 1 = 0
2x - 3y + 8 = 0
Solution:

Given,
3x+ y + 1 = 0 ……. (i)
2x - 3y + 8 = 0……. (ii)

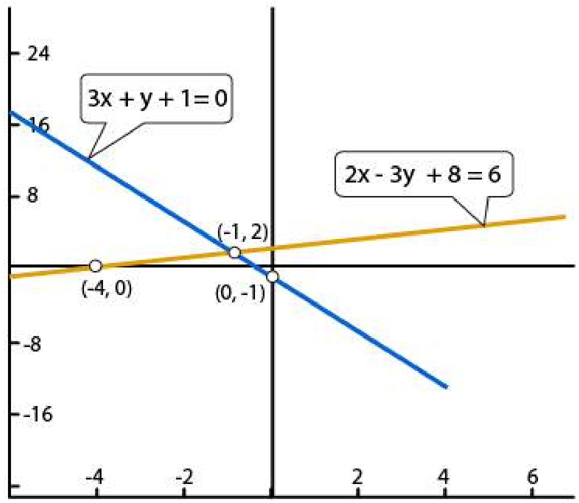
For equation (i),
⇒ y = -(1 + 3x)
When x = 0, we have y = -1
When x = -1, we have y = 2
Thus we have the following table giving points on the line 3x+ y + 1 = 0

x	-1	0
y	2	-1

For equation (ii),
We solve for y:
⇒ y = (2x + 8)/ 3
So, when x = -4
y = (2(-4) + 8)/3 = 0
And, when x = -1
⇒ y = (2(-1) + 8)/3 = 2
Thus we have the following table giving points on the line 2x - 3y + 8 = 0

x	-4	-1
y	0	2

Graph of the equations (i) and (ii) is as below:



Clearly the two lines intersect at a single point P (-1, 2)
Hence, x= -4 and y = 0

4. $2x + y - 3 = 0$
$2x - 3y - 7 = 0$
Solution:

Given,
$2x + y - 3 = 0$……. (i)
$2x - 3y - 7 = 0$……. (ii)

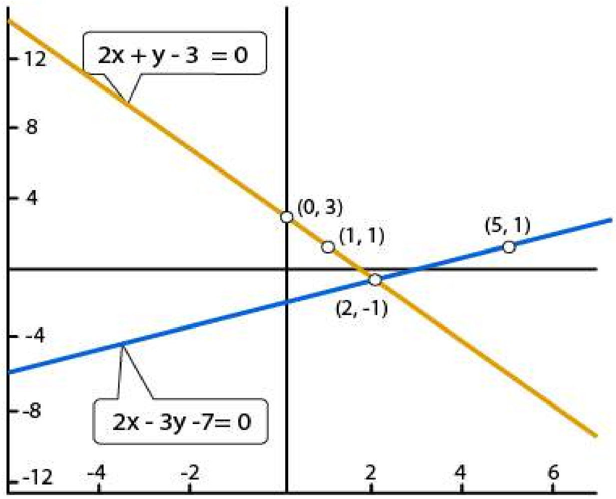
For equation (i),
$\Rightarrow y = (3 - 2x)$
When x = 0, we have $y = (3 – 2(0)) = 3$
When x = 1, we have $y = (3 – 2(1)) = 1$
Thus we have the following table giving points on the line $2x + y - 3 = 0$

x	0	1
y	3	1

For equation (ii),
We solve for y:
$\Rightarrow y = (2x - 7)/ 3$
So, when x = 2
$y = (2(2) - 7)/3 = -1$
And, when x = 5
$\Rightarrow y = (2(5) - 7)/3 = 1$
Thus we have the following table giving points on the line $2x - 3y - 7 = 0$

x	2	5
y	-1	1

Graph of the equations (i) and (ii) is as below:



Clearly the two lines intersect at a single point P (2, -1)
Hence, x= 2 and y = -1

5. **x + y = 6**
x - y = 2
Solution:

Given,
x + y = 6……. (i)
x - y = 2……. (ii)

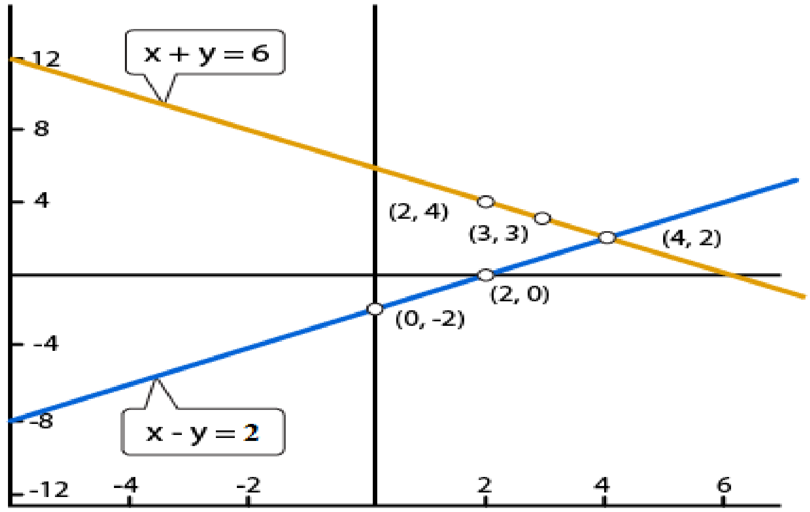
For equation (i),
⇒ y = (6 - x)
When x = 2, we have y = (6 – 2)) = 4
When x = 3, we have y = (6 – 3) = 3
Thus we have the following table giving points on the line **x + y = 6**

x	2	3
y	4	3

For equation (ii),
We solve for y:
⇒ y = (x - 2)
So, when x = 2
y = (0 - 2) = -2
And, when x = 5
⇒ y = (2 - 2) = 0
Thus we have the following table giving points on the line **x - y = 2**

x	0	2
y	-2	0

Graph of the equations (i) and (ii) is as below:



Clearly the two lines intersect at a single point P (4, 2)
Hence, x= 4 and y = 2

6. x - 2y = 6
3x - 6y = 0
Solution:

Given,
x - 2y = 6……. (i)
3x - 6y = 0……. (ii)

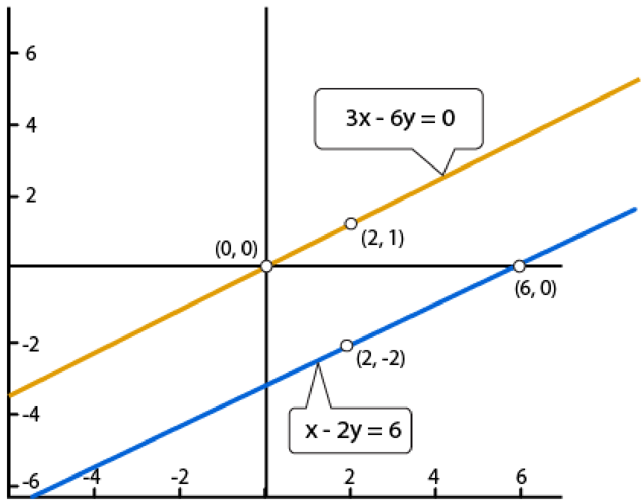
For equation (i),
⇒ y = (x - 6)/2
When x = 2, we have y = (2 – 6)/2 = -2
When x = 0, we have y = (0 – 6)/2 = -3
Thus we have the following table giving points on the line x - 2y = 6

x	2	0
y	-2	-3

For equation (ii),
We solve for y:
⇒ y = x/ 2
So, when x = 0
y = 0/2 = 0
And, when x = 2
⇒ y = 2/2 = 1
Thus we have the following table giving points on the line 3x - 6y = 0

x	0	2
y	0	1

Graph of the equations (i) and (ii) is as below:



Clearly the two lines are parallel to each other. So, the two lines do not intersect.
Hence, the given system has no solutions.

7. x + y = 4
2x - 3y = 3
Solution:

Given,
x + y = 4……. (i)
2x - 3y = 3……. (ii)

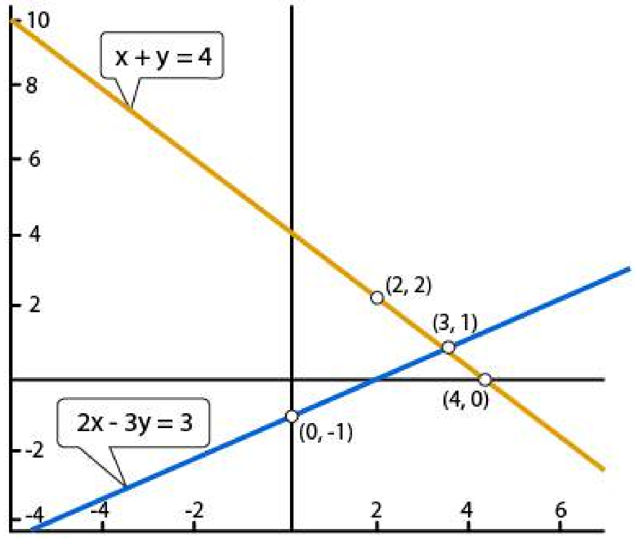
For equation (i),
⇒ y = (4 - x)
When x = 4, we have y = (4 – 4) = 0
When x = 2, we have y = (4 – 2) = 2
Thus we have the following table giving points on the line x + y = 4

x	4	2
y	0	2

For equation (ii),
We solve for y:
⇒ y = (2x - 3)/3
So, when x = 3
y = (2(3) - 3)/3 = 1
And, when x = 0
⇒ y = (2(0) - 3)/3 = -1
Thus we have the following table giving points on the line 2x - 3y = 3

x	3	0
y	1	-1

Graph of the equations (i) and (ii) is as below:



Clearly the two lines intersect at a single point P (3, 1)
Hence, x= 3 and y = 1

8. 2x + 3y = 4
x – y + 3 = 0
Solution:

Given,
2x + 3y = 4……. (i)
x – y + 3 = 0……. (ii)

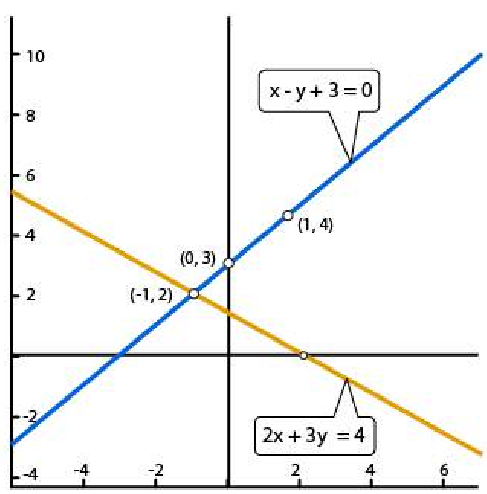
For equation (i),
⇒ y = (4 - 2x) /3
When x = -1, we have y = (4 - 2(-1))/3 = 2
When x = 2, we have y = (4 - 2(2))/3 = 0
Thus we have the following table giving points on the line 2x + 3y = 4

x	-1	2
y	2	0

For equation (ii),
We solve for y:
⇒ y = (x + 3)
So, when x = 0
y = (0 + 3) = 3
And, when x = 1
⇒ y = (1 + 3) = 4
Thus we have the following table giving points on the line x – y + 3 = 0

x	0	1
y	3	4

Graph of the equations (i) and (ii) is as below:



Clearly the two lines intersect at a single point P (-1, 2)
Hence, x= -1 and y = 2

9. 2x - 3y + 13 = 0
3x - 2y + 12 = 0
Solution:

Given,
2x - 3y + 13 = 0……. (i)
3x - 2y + 12 = 0……. (ii)

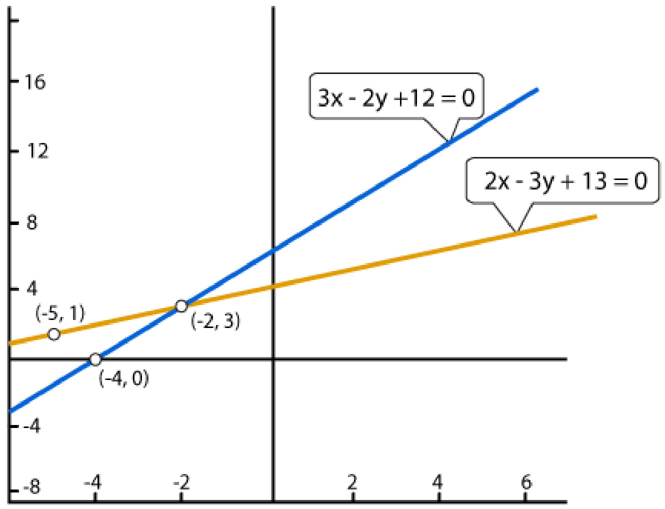
For equation (i),
⇒ y = (2x + 13) /3
When x = -5, we have y = (2(-5) + 13))/3 = 1
When x = -2, we have y = (2(-2) + 13))/3 = 3
Thus we have the following table giving points on the line 2x - 3y + 13 = 0

x	-5	-2
y	1	3

For equation (ii),
We solve for y:
⇒ y = (3x + 12)/2
So, when x = -4
y = (3(-4) + 12)/2 = 0
And, when x = -2
⇒ y = (3(-2) + 12)/2 = 3
Thus we have the following table giving points on the line 3x - 2y + 12 = 0

x	-4	-2
y	0	3

Graph of the equations (i) and (ii) is as below:



Clearly the two lines intersect at a single point P (-2, 3)
Hence, x= -2 and y = 3

10. 2x + 3y + 5 = 0
3x + 2y - 12 = 0
Solution:

Given,
2x + 3y + 5 = 0……. (i)
3x - 2y - 12 = 0……. (ii)

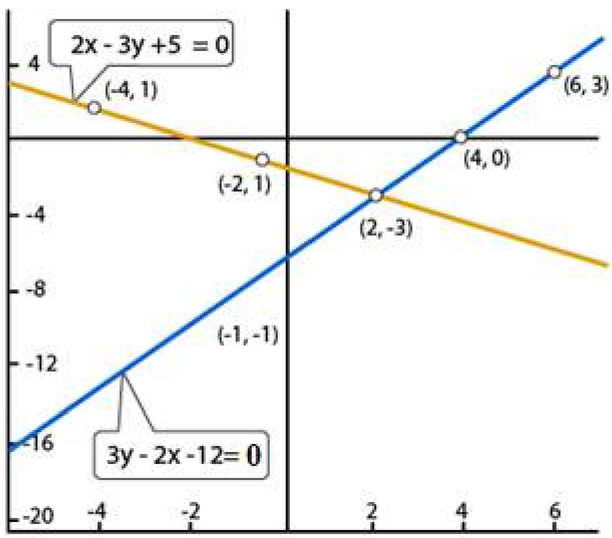
For equation (i),
⇒ y = -(2x + 5) /3
When x = -4, we have y = -(2(-4) + 5))/3 = 1
When x = -2, we have y = -(2(-2) + 5))/3 = -1
Thus we have the following table giving points on the line 2x + 3y + 5 = 0

x	-4	-1
y	1	-1

For equation (ii),
We solve for y:
⇒ y = (3x - 12)/2
So, when x = 4
y = (3(4) - 12)/2 = 0
And, when x = 6
⇒ y = (3(6) - 12)/2 = 3
Thus we have the following table giving points on the line 3x - 2y - 12 = 0

x	4	6
y	0	3

Graph of the equations (i) and (ii) is as below:



Clearly the two lines intersect at a single point P (2, -3)
Hence, x= 2 and y = -3

Show graphically that each one of the following systems of equation has infinitely many solution:
11. 2x + 3y = 6
4x + 6y = 12
Solution:

Given,
$2x + 3y = 6$……. (i)
$4x + 6y = 12$……. (ii)

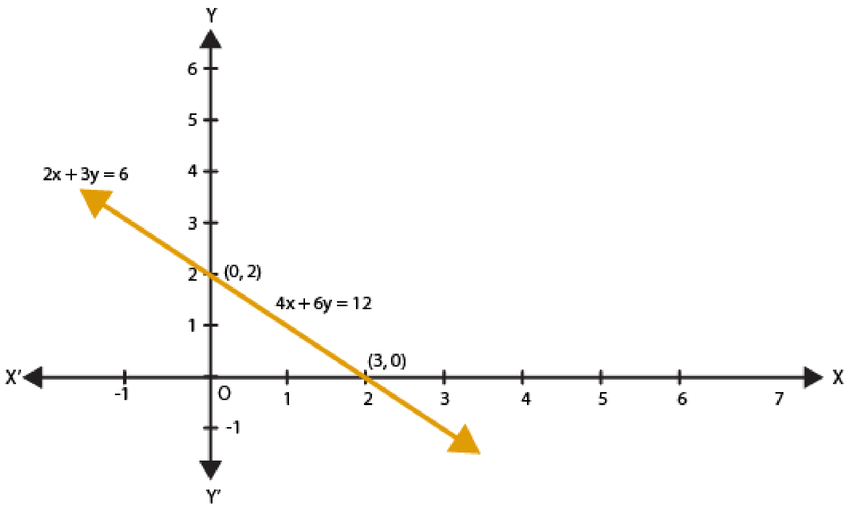
For equation (i),
⇒ $y = (6 - 2x)/3$
When x = 0, we have y = (6 - 2(0))/3 = 2
When x = 3, we have y = (6 - 2(3))/3 = 0
Thus we have the following table giving points on the line $2x + 3y = 6$

x	0	3
y	2	0

For equation (ii),
We solve for y:
⇒ $y = (12 – 4x)/6$
So, when x = 0
y = (12 – 4(0))/6 = 2
And, when x = 3
⇒ y = (12 – 4(3))/6 = 0
Thus we have the following table giving points on the line $4x + 6y = 12$

x	0	3
y	2	0

Graph of the equations (i) and (ii) is as below:



Thus, the graphs of the two equations are coincident.
Hence, the system of equations has infinitely many solutions.

12. **x - 2y = 5**
3x - 6y = 15
Solution:

Given,
x - 2y = 5……. (i)
3x - 6y = 15……. (ii)

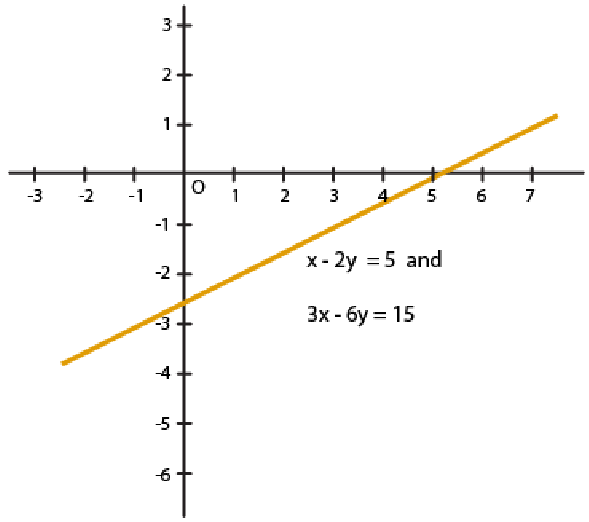
For equation (i),
⇒ y = (x - 5) /2
When x = 3, we have y = (3 - 5) /2 = -1
When x = 5, we have y = (5 - 5) /2 = 0
Thus we have the following table giving points on the line x - 2y = 5

x	3	5
y	-1	0

For equation (ii),
We solve for y:
⇒ y = (3x – 15)/6
So, when x = 3
y = (3(3) – 15)/6= -1
And, when x = 3
⇒ y = (3(5) – 15)/6= 0
Thus we have the following table giving points on the line 3x - 6y = 15

x	3	5
y	-1	0

Graph of the equations (i) and (ii) is as below:



Thus, the graphs of the two equations are coincident.
Hence, the system of equations has infinitely many solutions.

13. **3x + y = 8**
6x + 2y = 16

Solution:

Given,
3x + y = 8……. (i)
6x + 2y = 16……. (ii)

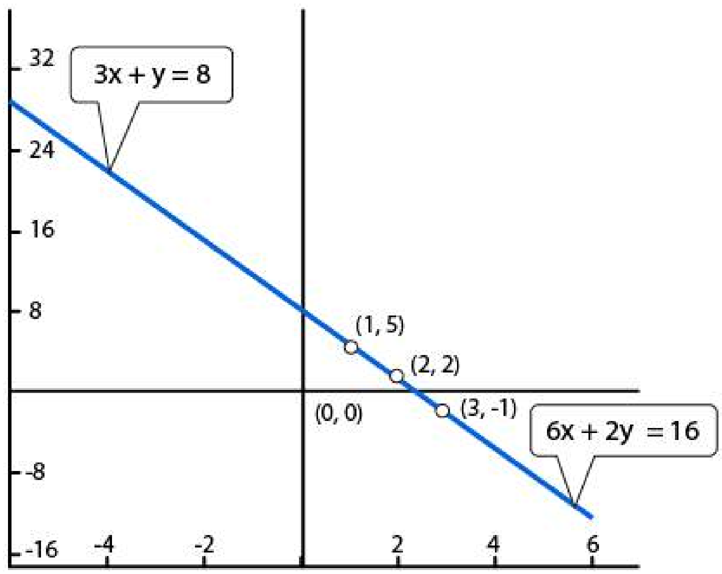
For equation (i),
⇒ y = (8 – 3x)
When x = 2, we have y = (8 – 3(2)) = 2
When x = 3, we have y = (8 – 3(3)) = -1
Thus we have the following table giving points on the line 3x + y = 8

x	2	3
y	2	-1

For equation (ii),
We solve for y:
⇒ y = (16 - 6x)/2
So, when x = 3
y = (16 – 6(3))/2= -1
And, when x = 1
⇒ y = (16 – 6(1))/2= 5
Thus we have the following table giving points on the line 6x + 2y = 16

x	3	1
y	-1	5

Graph of the equations (i) and (ii) is as below:



Thus, the graphs of the two equations are coincident.
Hence, the system of equations has infinitely many solutions.

14. x - 2y + 11 = 0
3x + 6y + 33 = 0
Solution:

Given,
x - 2y + 11 = 0……. (i)
3x - 6y + 33 = 0……. (ii)

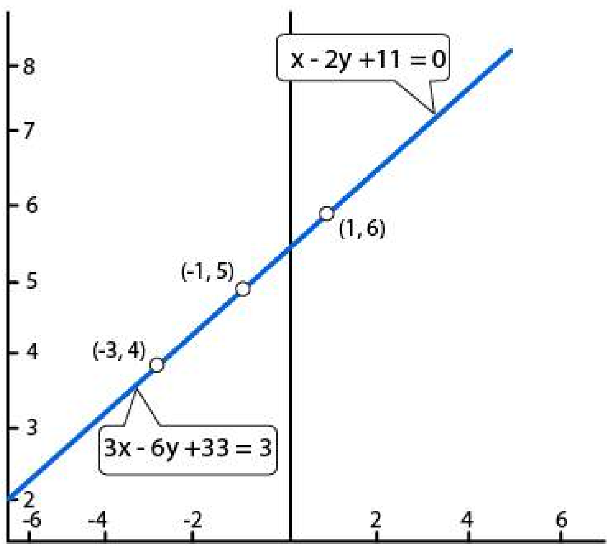
For equation (i),
⇒ y = (x + 11)/2
When x = -1, we have y = (-1 + 11)/2 = 5
When x = -3, we have y = (-3 + 11)/2 = 4
Thus we have the following table giving points on the line x - 2y + 11 = 0

x	-1	-3
y	5	4

For equation (ii),
We solve for y:
⇒ y = (3x + 33)/6
So, when x = 1
y = (3(1) + 33)/6 = 6
And, when x = -1
⇒ y = (3(-1) + 33)/6 = 5
Thus we have the following table giving points on the line 3x - 6y + 33 = 0

x	1	-1
y	6	5

Graph of the equations (i) and (ii) is as below:



Thus, the graphs of the two equations are coincident.
Hence, the system of equations has infinitely many solutions.

Show graphically that each one of the following systems of equations is in-consistent (i.e has no solution):
15. 3x – 5y = 20
6x – 10y = - 40
Solution:

Given,
3x – 5y = 20……. (i)
6x – 10y = - 40……. (ii)

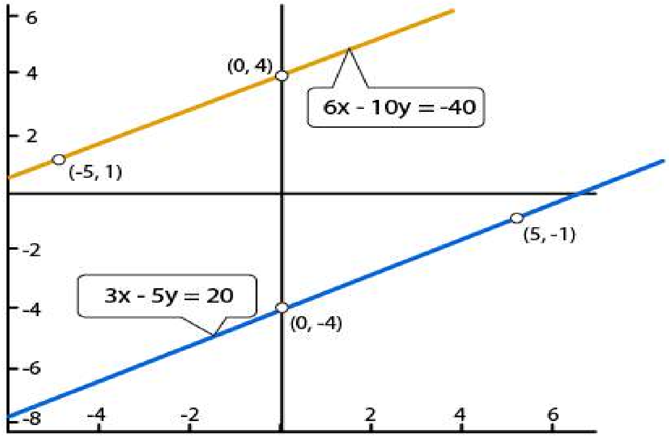
For equation (i),
⇒ y = (3x - 20)/5
When x = 5, we have y = (3(5) - 20)/5 = -1
When x = 0, we have y = (3(0) - 20)/5 = -4
Thus we have the following table giving points on the line 3x – 5y = 20

x	5	0
y	-1	-4

For equation (ii),
We solve for y:
⇒ y = (6x + 40)/10
So, when x = 0
y = (6(0) + 40)/10 = 4
And, when x = -5
⇒ y = (6(-5) + 40)/10 = 1
Thus we have the following table giving points on the line 6x – 10y = - 40

x	0	-5
y	4	1

Graph of the equations (i) and (ii) is as below:



It is clearly seen that, there is no common point between these two lines.
Hence, the given systems of equations is in-consistent.

16. x - 2y = 6
3x - 6y = 0
Solution:

Given,
x - 2y = 6……. (i)
3x - 6y = 0……. (ii)

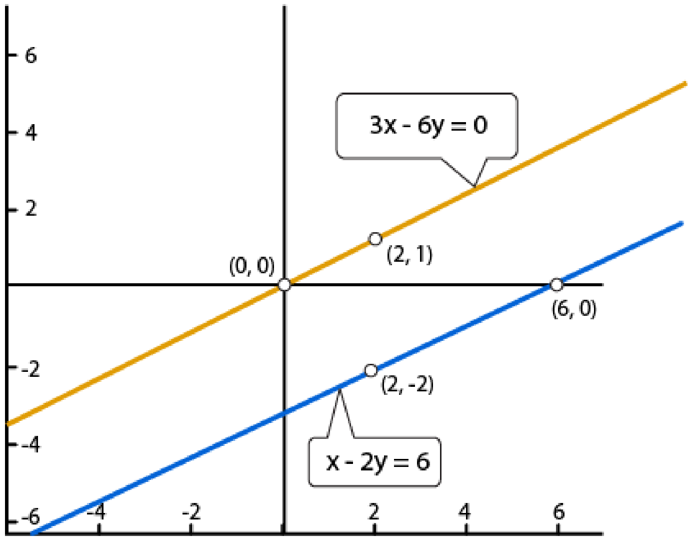
For equation (i),
⇒ y = (x - 6)/2
When x = 6, we have y = (6 - 6)/2 = 0
When x = 2 we have y = (2 - 6)/2 = -2
Thus we have the following table giving points on the line x - 2y = 6

x	6	2
y	0	-2

For equation (ii),
We solve for y:
⇒ y = x/2
So, when x = 0
y = 0/2 = 0
And, when x = 2
⇒ y = 2/2 = 1
Thus we have the following table giving points on the line 3x - 6y = 0

x	0	2
y	0	1

Graph of the equations (i) and (ii) is as below:



It is clearly seen that, there is no common point between these two lines.
Hence, the given systems of equations is in-consistent.

17. 2y – x = 9
6y - 3x = 21
Solution:

Given,
2y – x = 9……. (i)
6y - 3x = 21……. (ii)

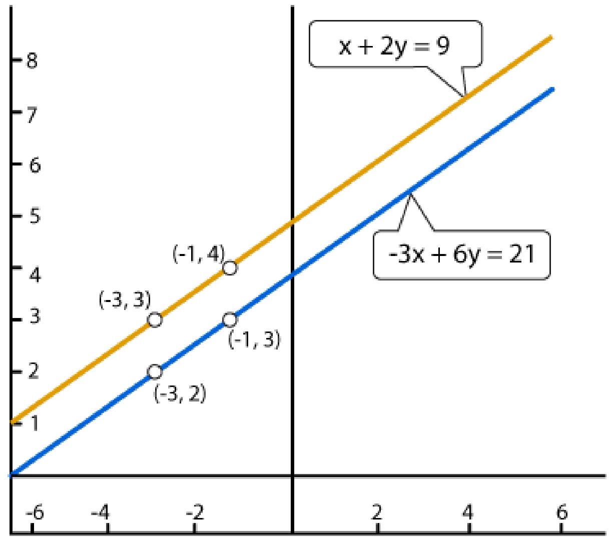
For equation (i),
⇒ y = (x + 9)/2
When x = -3, we have y = (-3 + 9)/2= 3
When x = -1, we have y = (-1 + 9)/2= 4
Thus we have the following table giving points on the line 2y – x = 9

x	-3	-1
y	3	4

For equation (ii),
We solve for y:
⇒ y = (21 + 3x)/6
So, when x = -3
y = (21 + 3(-3))/6 = 2
And, when x = -1
⇒ y = (21 + 3(-1))/6 = 3
Thus we have the following table giving points on the line 6y - 3x = 21

x	-3	-1
y	2	3

Graph of the equations (i) and (ii) is as below:



It is clearly seen that, there is no common point between these two lines.
Hence, the given systems of equations is in-consistent.

18. 3x - 4y - 1 = 0
2x – (8/3)y + 5 = 0
Solution:

Given,
3x - 4y - 1 = 0……. (i)
2x – (8/3)y + 5 = 0……. (ii)

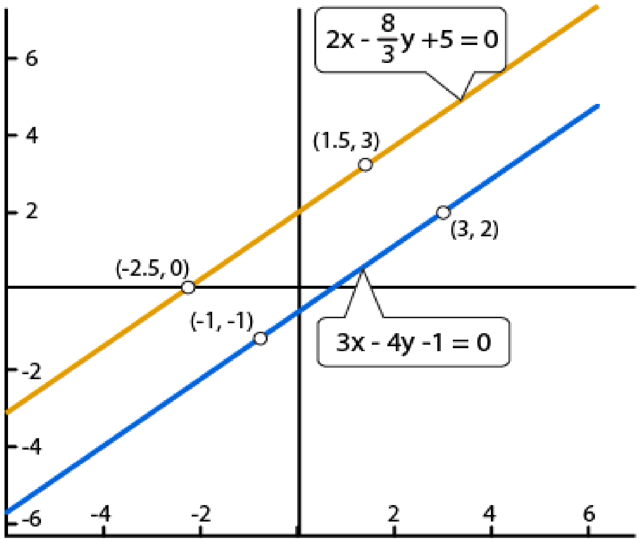
For equation (i),
⇒ y = (3x - 1)/4
When x = -1, we have y = (3(-1) - 1)/4= -1
When x = 3, we have y = (3(3) - 1)/4= 2
Thus we have the following table giving points on the line 3x - 4y - 1 = 0

x	-1	3
y	-1	2

For equation (ii),
We solve for y:
⇒ y = (6x + 15)/8
So, when x = -2.5
y = (6(-2.5) + 15)/8 = 0
And, when x = 1.5
⇒ y = (6(1.5) + 15)/8 = 3
Thus we have the following table giving points on the line 2x – (8/3)y + 5 = 0

x	-2.5	1.5
y	0	3

Graph of the equations (i) and (ii) is as below:



It is clearly seen that, there is no common point between these two lines.
Hence, the given systems of equations is in-consistent.

19. Determine graphically the vertices of the triangle, the equations of whose sides are given below:
(i) 2y – x = 8, 5y – x = 14 and y – 2x = 1
Solution:

Given,
$2y - x = 8$……. (i)
$5y - x = 14$……. (ii)
$y - 2x = 1$……… (iii)

For equation (i),
$\Rightarrow y = (x + 8)/2$
When $x = -4$, we have $y = (-4 + 8)/2 = 2$
When $x = 0$, we have $y = (0 + 8)/2 = 4$
Thus we have the following table giving points on the line $2y - x = 8$

x	-4	0
y	2	4

For equation (ii),
We solve for y:
$\Rightarrow y = (x + 14)/5$

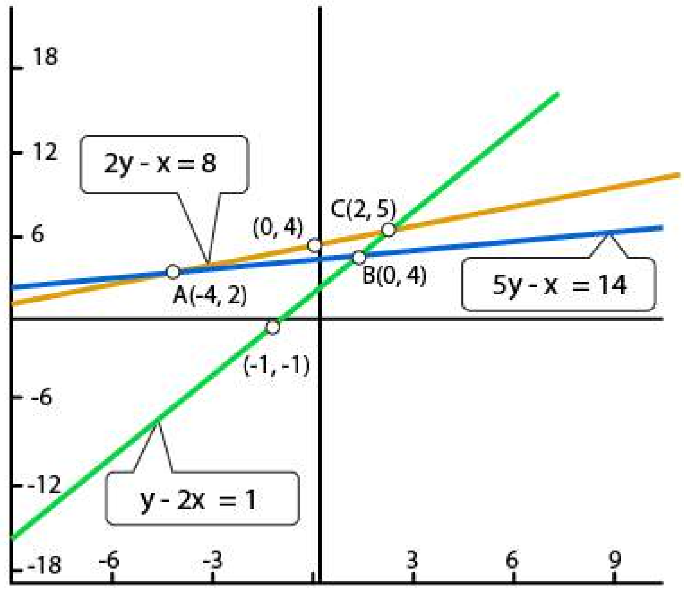
So, when $x = -4$
$y = ((-4) + 14)/5 = 2$
And, when $x = 1$
$\Rightarrow y = (1 + 14)/5 = 3$
Thus we have the following table giving points on the line $5y - x = 14$

x	-4	1
y	2	3

Finally, for equation (iii),
$\Rightarrow y = (2x + 1)$
When $x = -1$, we have $y = (2(-1) + 1) = -1$
When $x = 1$, we have $y = (2(1) + 1) = 3$
Thus we have the following table giving points on the line $y - 2x = 1$

x	-1	1
y	1	3

Graph of the equations (i), (ii) and (iii) is as below:



From the above graph, we observe that the lines taken in pairs intersect at points A(-4,2), B(1,3) and C(2,5)

Hence the vertices of the triangle are A(-4, 2), B(1, 3) and C(2,5)

(ii) y = x, y = 0 and 3x + 3y = 10
Solution:

Given,
y = x ……. (i)
y = 0 ……. (ii)
3x + 3y = 10……… (iii)

For equation (i),
When x = 1, we have y = 1
When x = -2, we have y = -2
Thus we have the following table giving points on the line y = x

x	1	-2
y	1	-2

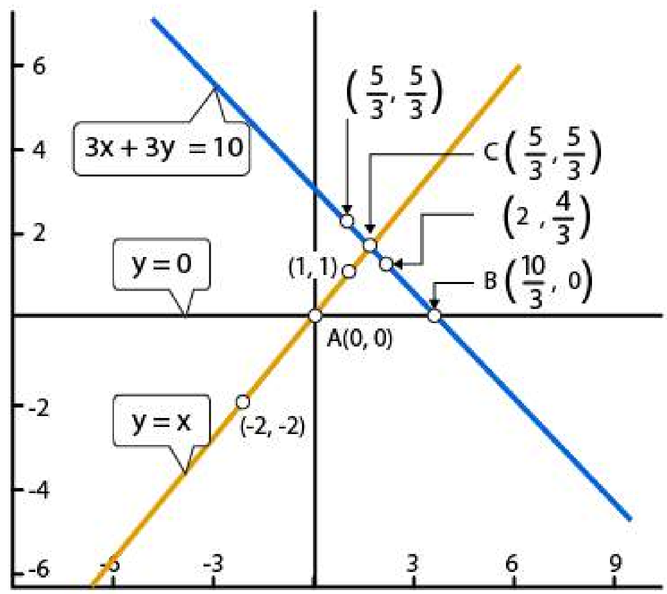
For equation (ii),
When x = 0
y = 0
And, when x = 10/3
⇒ y = 0
Thus we have the following table giving points on the line y = 0

x	0	10/3
y	0	10/3

Finally, for equation (iii),
⇒ y = (10 – 3x)/3
When x = 1, we have y = (10 – 3(1))/3) = 7/3
When x = 2, we have y = (10 – 3(2))/3 = 4/3
Thus we have the following table giving points on the line 3x + 3y = 10

x	1	2
y	7/3	4/3

Graph of the equations (i), (ii) and (iii) is as below:



From the above graph, we observe that the lines taken in pairs intersect at points A(0,0) B(10/3,0) and C(5/3, 5/3)

Hence the vertices of the triangle are A(0,0) B(10/3,0) and C(5/3, 5/3).

20. Determine graphically whether the system of equations x - 2y = 2, 4x - 2y = 5 is consistent or in-consistent.
Solution:

Given,
x - 2y = 2……. (i)
4x - 2y = 5……. (ii)

For equation (i),
⇒ y = (x - 2)/2
When x = 2, we have y = (2 - 2)/2 = 0
When x = 0, we have y = (0 - 2)/2 = -1

Thus we have the following table giving points on the line x - 2y = 2

x	2	0
y	0	-1

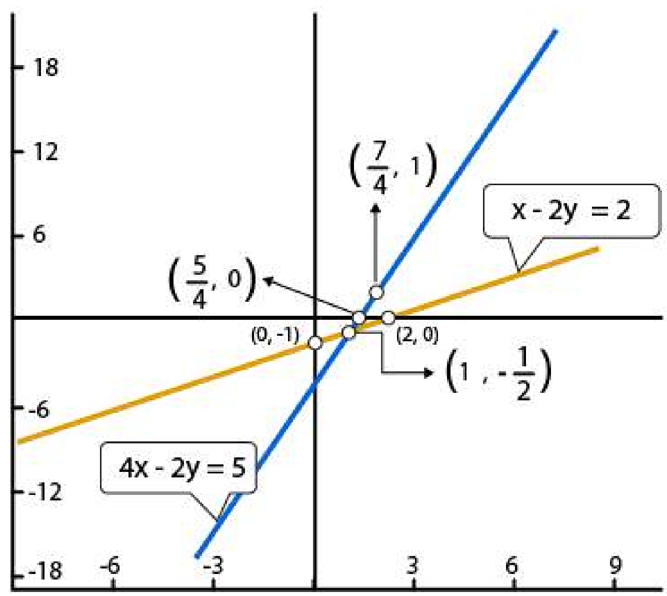
For equation (ii),
We solve for x:
⇒ x = (5 + 2y)/4

So, when y = 0
x = (5 + 2(0))/4 = 5/4
And, when y = 1.5
⇒ x = (5 + 2(1))/4 = 7/4

Thus we have the following table giving points on the line 4x - 2y = 5

x	5/4	7/4
y	0	1

Graph of the equations (i) and (ii) is as below:



It is clearly seen that the two lines intersect at (1,0)
Hence, the system of equations is consistent.

21. Determine by drawing graphs, whether the following system of linear equation has a unique solution or not:

(i) 2x - 3y = 6 and x + y = 1
Solution:

Given,

2x - 3y = 6 ……. (i)
x + y = 1……. (ii)

For equation (i),
⇒ y = (2x - 6)/3
When x = 3, we have y = (2(3) - 6)/3= 0
When x = 0, we have y = (2(0) - 6)/3= -2
Thus we have the following table giving points on the line **2x - 3y = 6**

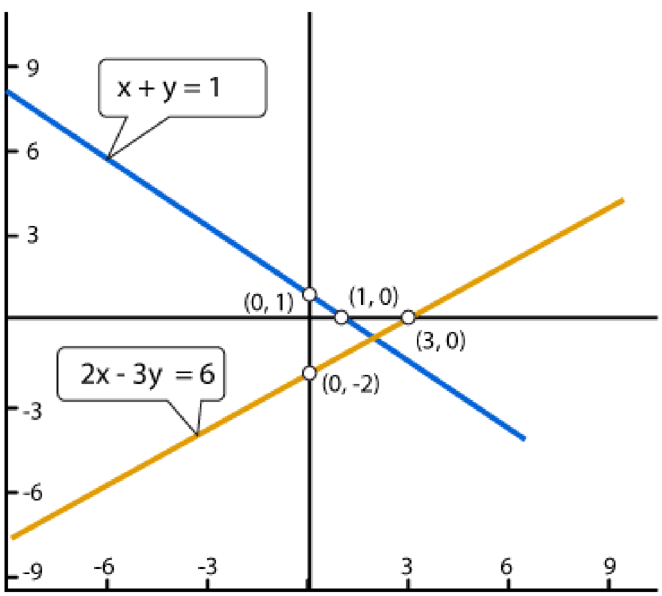
x	3	0
y	0	-2

For equation (ii),
We solve for y:
⇒ y = (1 - x)

So, when x = 0
y = (1 - 0) = 1
And, when x = 1
⇒ y = (1 - 1) = 0
Thus we have the following table giving points on the line **x + y = 1**

x	0	1
y	1	0

Graph of the equations (i) and (ii) is as below:



It’s seen clearly that the two lines intersect at one.
Thus, we can conclude that the system of equations has a unique solution.

(ii) 2y = 4x - 6 and 2x = y + 3
Solution:

Given,
2y = 4x - 6……. (i)
2x = y + 3……. (ii)

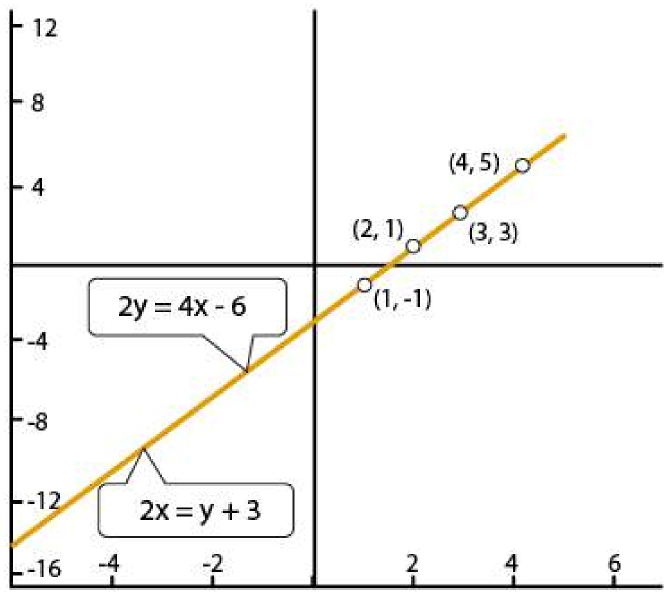
For equation (i),
⇒ y = (4x - 6)/2
When x = 1, we have y = (4(1) - 6)/2 = -1
When x = 4, we have y = (4(4) - 6)/2= 5
Thus we have the following table giving points on the line 2y = 4x - 6

x	1	4
y	-1	5

For equation (ii),
We solve for y:
⇒ y = 2x - 3
So, when x = 2
y = 2(2) – 3 = 1
And, when x = 3
⇒ y = 2(3) – 3 = 3
Thus we have the following table giving points on the line 2x = y + 3

x	2	3
y	1	3

Graph of the equations (i) and (ii) is as below:



We see that the two lines are coincident. And, hence it has infinitely many solutions.
Therefore, the system of equations does not have a unique solution.