Exercise 3.4 Page No: 3.57

Solve each of the following systems of equations by the method of cross-multiplication:

1. x + 2y + 1 = 0
2x - 3y – 12 = 0

Solution:

The given system of equations is
x + 2y + 1 = 0
2x - 3y – 12 = 0

For cross multiplication we use,

$$\frac{x}{b_1c_2 - b_2c_1} = \frac{-y}{a_1c_2 - a_2c_1} = \frac{1}{a_1b_2 - a_2b_1}$$

Comparing the above two equations with the general form, we get

$a_1 = 1$, $b_1 = 2$, $c_1 = 1$

$a_2 = 2$, $b_2 = -3$, $c_2 = -12$

By cross multiplication method,

$$\frac{x}{-24+3} = \frac{-y}{-12-2} = \frac{1}{-3-4}$$

$$\frac{x}{-21} = \frac{-y}{-14} = \frac{1}{-7}$$

Now,

$$\frac{x}{-21} = \frac{1}{-7}$$

=x= 3

And,

$$\frac{-y}{-14} = \frac{1}{-7}$$

=y=-2

Hence, the solution for the given system of equations is x = 3 and y = -2.

2. 3x + 2y + 25 = 0
2x + y + 10 = 0

Solution:

The given system of equations is
3x + 2y + 25 = 0
2x + y + 10 = 0

For cross multiplication we use,

$$\frac{x}{b_1c_2 - b_2c_1} = \frac{-y}{a_1c_2 - a_2c_1} = \frac{1}{a_1b_2 - a_2b_1}$$

Comparing the above two equations with the general form, we get

$a_1 = 3$, $b_1 = 2$, $c_1 = 25$

$a_2 = 2$, $b_2 = 1$, $c_2 = 10$

By cross multiplication method,

$$\frac{x}{20-25} = \frac{-y}{30-50} = \frac{1}{3-4}$$

$$\frac{x}{-5} = \frac{-y}{-20} = \frac{1}{-1}$$

Now,

$$\frac{x}{-5} = \frac{1}{-1}$$

=x= 5

And,

$$\frac{-y}{-20} = \frac{1}{-1}$$

=y=-20

Hence, the solution for the given system of equations is x = 5 and y = -20.

3. 2x + y = 35, 3x + 4y = 65
Solution:

The given system of equations can be written as
2x + y – 35 = 0
3x + 4y – 65 = 0

For cross multiplication we use,

$$\frac{x}{b_1c_2 - b_2c_1} = \frac{-y}{a_1c_2 - a_2c_1} = \frac{1}{a_1b_2 - a_2b_1}$$

Comparing the above two equations with the general form, we get

$a_1 = 2$, $b_1 = 1$, $c_1 = -35$

$a_2 = 3$, $b_2 = 4$, $c_2 = -65$

By cross multiplication method,

$$\frac{x}{-65+140} = \frac{-y}{-130+105} = \frac{1}{8-3}$$

$$\frac{x}{75} = \frac{-y}{-25} = \frac{1}{5}$$

Now,

$$\frac{x}{75} = \frac{1}{5}$$

=x= 15

And,

$$\frac{-y}{-25} = \frac{1}{5}$$

=y=5

Hence, the solution for the given system of equations is x = 15 and y = 5.

4. **2x – y = 6, x – y = 2**
Solution:

The given system of equations can be written as
2x – y – 6 = 0
x – y – 2 = 0

For cross multiplication we use,

$$\frac{x}{b_1c_2 - b_2c_1} = \frac{-y}{a_1c_2 - a_2c_1} = \frac{1}{a_1b_2 - a_2b_1}$$

Comparing the above two equations with the general form, we get

$a_1 = 2$, $b_1 = -1$, $c_1 = -6$

$a_2 = 1$, $b_2 = -1$, $c_2 = -2$

By cross multiplication method,

$$\frac{x}{2-6} = \frac{-y}{-4+6} = \frac{1}{-2+1}$$

$$\frac{x}{-4} = \frac{-y}{2} = \frac{1}{-1}$$

Now,

$$\frac{x}{-4} = \frac{1}{-1}$$

=x= 4

And,

$$\frac{-y}{2} = \frac{1}{-1}$$

=y=2

Hence, the solution for the given system of equations is x = 4 and y = 2.

5. (x + y)/ xy = 2
(x - y)/ xy = 6
Solution:

The given system of equations is
(x + y)/ xy = 2 ⇒ 1/y + 1/x = 2…….. (i)
(x - y)/ xy = 6 ⇒ 1/y – 1/x = 6…….. (ii)

Let 1/x = u and 1/y = v, so the equation becomes
u + y = 2….. (iii)
u – y = 6……(iv)

For cross multiplication we use,

$$\frac{x}{b_1c_2 - b_2c_1} = \frac{-y}{a_1c_2 - a_2c_1} = \frac{1}{a_1b_2 - a_2b_1}$$

Comparing the above two equations (iii) & (iv) with the general form, we get

$a_1 = 1$, $b_1 = 1$, $c_1 = -2$

$a_2 = 1$, $b_2 = -1$, $c_2 = -6$

By cross multiplication method,

$\frac{u}{6-2} = \frac{-v}{6+2} = \frac{1}{-1-1}$

$\frac{u}{4} = \frac{-v}{8} = \frac{1}{-2}$

Now,

$\frac{u}{4} = \frac{1}{-2}$

=u= -2

And,

$\frac{-v}{-8} = \frac{1}{-2}$

=v=4

$\frac{1}{u}$ =x=$\frac{-1}{2}$

$\frac{1}{v}$ =y=$\frac{1}{4}$

Hence, the solution for the given system of equations is x = -1/2 and y = 1/4.

6. ax + by = a-b
bx – ay = a+b

Solution:

The given system of equations can be written as
ax + by – (a-b) = 0
bx – ay – (a+b) = 0

For cross multiplication we use,

$$\frac{x}{b_1c_2 - b_2c_1} = \frac{-y}{a_1c_2 - a_2c_1} = \frac{1}{a_1b_2 - a_2b_1}$$

Comparing the above two equations with the general form, we get

$a_1 = a$, $b_1 = b$, $c_1 = -(a-b)$

$a_2 = b$, $b_2 = -a$, $c_2 = -(a+b)$

By cross multiplication method,

$$\frac{x}{-ab-b^2+ab-a^2} = \frac{-y}{-a^2-ab-b^2+ab} = \frac{1}{-a^2-b^2}$$

$$\frac{x}{-b^2-a^2} = \frac{-y}{-a^2-b^2} = \frac{1}{-a^2-b^2}$$

Now,

$$\frac{x}{-ab-b^2+ab-a^2} = \frac{1}{-a^2-b^2}$$

=x= 1

And,

$$\frac{-y}{-a^2-ab-b^2+ab} = \frac{1}{-a^2-b^2}$$

=y=-1

Hence, the solution for the given system of equations is x = 1 and y = -1.

7. x + ay = b
ax + by = c
Solution:

The given system of equations can be written as
x + ay – b = 0
ax + by – c = 0

For cross multiplication we use,

$$\frac{x}{b_1c_2 - b_2c_1} = \frac{-y}{a_1c_2 - a_2c_1} = \frac{1}{a_1b_2 - a_2b_1}$$

Comparing the above two equations with the general form, we get

$a_1 = 1$, $b_1 = a$, $c_1 = -b$

$a_2 = a$, $b_2 = -b$, $c_2 = -c$

By cross multiplication method,

$$\frac{x}{-ac-b^2} = \frac{-y}{-c+ab} = \frac{1}{-a^2-b}$$

Now,

$$\frac{x}{-ac-b^2} = \frac{1}{-a^2-b}$$

$$=x= \frac{b^2+ac}{a^2+b}$$

And,

$$\frac{-y}{-c+ab} = \frac{1}{-a^2-b}$$

$$=y= \frac{-c+ab}{a^2+b}$$

Hence, the solution for the given system of equations is $x = (b^2 + ac)/(a^2 + b^2)$ and $y = (-c^2 + ab)/(a^2 + b^2)$.

8. $\mathbf{ax + by = a^2}$
$\mathbf{bx + ay = b^2}$

Solution:

The given system of equations can be written as
$ax + by - (a^2) = 0$
$bx + ay - (b^2) = 0$

For cross multiplication we use,

$$\frac{x}{b_1c_2 - b_2c_1} = \frac{-y}{a_1c_2 - a_2c_1} = \frac{1}{a_1b_2 - a_2b_1}$$

Comparing the above two equations with the general form, we get

$a_1x+b_1y+c_1=0$

$a_2x+b_2y+c_2=0$

$$\Rightarrow \frac{x}{b_1c_2-b_2c_1} = \frac{-y}{a_1c_2-a_2c_1} = \frac{1}{a_1b_2-a_2b_1}$$

Here, According to the question,

$a_1 = a$, $b_1 = b$, $c_1 = a^2$

$a_2 = b$, $b_2 = a$, $c_2 = b^2$

By cross multiplication method,

$$\frac{x}{-b^2+a^2} = \frac{-y}{-ab^2-a^2b} = \frac{1}{a^2-b^2}$$

Now,

$$\frac{x}{-b^2+a^2} = \frac{1}{a^2-b^2}$$

$$=x= \frac{a^2+ab+b^2}{a+b}$$

And,

$$\frac{-y}{-ab^2-a^2b} = \frac{1}{a^2-b^2}$$

$$=y=-\frac{-ab(a-b)}{(a-b)(a+b)}$$

Hence, the solution for the given system of equations is $x = (a^2 + ab + b^2)/(a + b)$ and $y = -ab / (a+ b)$.

9. 5/(x + y) – 2/(x -y) = -1
15/(x + y) + 7/(x - y) = 10
Solution:

Let's substitute 1/(x + y) = u and 1/(x - y) = v, so the given equations becomes
5u - 2v = -1
15u + 7v = 10

For cross multiplication we use,

$$\frac{x}{b_1c_2-b_2c_1} = \frac{-y}{a_1c_2-a_2c_1} = \frac{1}{a_1b_2-a_2b_1}$$

Comparing the above two equations with the general form, we get

$a_1 = 5$, $b_1 = -2$, $c_1 = 1$

$a_2 = 15$, $b_2 = 7$, $c_2 = -10$

By cross multiplication method,

$\frac{u}{20-7} = \frac{-v}{-50-15} = \frac{1}{35+30}$

$\frac{u}{13} = \frac{-v}{-65} = \frac{1}{65}$

Now,

$\frac{u}{13} = \frac{1}{-65}$

$= u = \frac{1}{5}$

$\frac{1}{u} = =x+y$

$=x+y=5$ ....(i)

And,

$\frac{-v}{-65} = \frac{1}{-65}$

$=v=1$

$\frac{1}{v} = =x-y$

$=x-y=1$ .... (ii)

Adding equation (i) and (ii)

$2x = 6$

$=x=3$

Substituting the value of x in equation (i)

$3+y=5$

$=y=2$

Hence, the solution for the given system of equations is x = 3 and y = 2.

10. 2/x + 3/y = 13
5/x – 4/y = -2
Solution:

Let 1/x = u and 1/y = v, so the equation becomes
2u + 3y = 13 ⇒ 2u + 3y – 13 = 0
5u – 4y = -2 ⇒ 5u – 4y + 2 = 0

For cross multiplication we use,

$$\frac{x}{b_1c_2 - b_2c_1} = \frac{-y}{a_1c_2 - a_2c_1} = \frac{1}{a_1b_2 - a_2b_1}$$

Comparing the above two equations with the general form, we get

$a_1 = 2$, $b_1 = 3$, $c_1 = -13$

$a_2 = 5$, $b_2 = -4$, $c_2 = 2$

By cross multiplication method,

$$\frac{u}{6-52} = \frac{-v}{4+65} = \frac{1}{-8-15}$$

$$\frac{u}{-46} = \frac{-v}{69} = \frac{1}{-23}$$

Now,

$$\frac{u}{-46} = \frac{1}{-23}$$

$=u= 2$

$$\frac{1}{u} = x$$

$=x= \frac{1}{2}$

And,

$$\frac{-v}{69} = \frac{1}{-23}$$

$=v=3$

$$\frac{1}{v} = y$$

$=y = \frac{1}{3}$

Hence, the solution for the given system of equations is x = 1/2 and y = 1/3.

11. 57/(x + y) + 6/(x - y) = 5
38/(x + y) + 21/(x - y) = 9
Solution:

Let's substitute 1/(x + y) = u and 1/(x - y) = v, so the given equations becomes
57u + 6v = 5 ⇒ 57u + 6v – 5 = 0
38u + 21v = 9 ⇒ 38u + 21v – 9 = 0

For cross multiplication we use,

$$\frac{x}{b_1c_2 - b_2c_1} = \frac{-y}{a_1c_2 - a_2c_1} = \frac{1}{a_1b_2 - a_2b_1}$$

Comparing the above two equations with the general form, we get

$a_1 = 57, b_1 = 6, c_1 = -5$

$a_2 = 38, b_2 = 21, c_2 = -9$

By cross multiplication method,

$$\frac{u}{-54+105} = \frac{-v}{-513+190} = \frac{1}{1193-228}$$

$$\frac{u}{51} = \frac{-v}{-323} = \frac{1}{969}$$

Now,

$$\frac{u}{51} = \frac{1}{969}$$

$=u= \frac{1}{19}$

$\frac{1}{u}$ =x+y

=x+y=19 ....(i)

And,

$$\frac{-v}{-323} = \frac{1}{969}$$

$=v= \frac{1}{3}$

$\frac{1}{v}$ = =x-y

=x-y =3 .... (ii)

Adding equation (i) and (ii)

2x= 22

=x=11

Substituting the value of x in equation (i)

11+y=19

=y=8

Hence, the solution for the given system of equations is x = 11 and y = 8.

12. xa – yb = 2
ax – by = a^2-b^2
Solution:

The given system of equations can be written as
xa – yb – 2 = 0
ax – by – (a^2-b^2) = 0

For cross multiplication we use,

$$\frac{x}{b_1c_2 - b_2c_1} = \frac{-y}{a_1c_2 - a_2c_1} = \frac{1}{a_1b_2 - a_2b_1}$$

Comparing the above two equations with the general form, we get

$a_1 = \frac{1}{a}$, Let $b_1 = \frac{1}{b}$, Let $c_1 = -2$

$a_2 = a$, $b_2 = -b$, $c_2 = b^2 - a^2$

By cross multiplication method

$$= \frac{x}{\frac{b^2-a^2}{b} - 2b} = \frac{-y}{\frac{b^2-a^2}{b} + 2b} = \frac{1}{\frac{-b}{a} - \frac{a}{b}}$$

$$= \frac{x}{\frac{b^2-a^2-2b^2}{b}} = \frac{-y}{\frac{b^2-a^2+2b^2}{b}} = \frac{1}{\frac{-b^2-a^2}{ab}}$$

Now, $\frac{x}{\frac{b^2-a^2-2b^2}{b}} = \frac{1}{\frac{-b^2-a^2}{ab}}$

$x = a$

and, $\frac{-y}{\frac{b^2-a^2+2b^2}{b}} = \frac{1}{\frac{-b^2-a^2}{ab}}$

$= y = b$

Hence, the solution for the given system of equations is x = a and y = b.

13. **$x/a + y/b = a + b$**

$x/a^2 + y/b^2 = 2$

Solution:

The given system of equations can be written as

$x/a + y/b - (a + b) = 0$

$x/a^2 + y/b^2 - 2 = 0$

For cross multiplication we use,

$$\frac{x}{b_1c_2 - b_2c_1} = \frac{-y}{a_1c_2 - a_2c_1} = \frac{1}{a_1b_2 - a_2b_1}$$

Comparing the above two equations with the general form, we get

$a_1=\frac{1}{a}$, Let $b_1=\frac{1}{b}$, Let $c_1=-(a+b)$

$a_2=\frac{1}{a^2}$, $b_2=\frac{1}{b^2}$, $c_2=-2$

By cross multiplication method

$$=\frac{x}{\frac{-2}{b}+\frac{a}{b^2}+\frac{1}{b}}=\frac{-y}{\frac{-2}{a}+\frac{1}{a}+\frac{b}{a^2}}=\frac{1}{\frac{-1}{ab^2}-\frac{-1}{a^2b}}$$

$$=\frac{x}{\frac{a-b}{b^2}}=\frac{-y}{\frac{-a-b}{a^2}+\frac{1}{a}+\frac{b}{a^2}}=\frac{1}{\frac{-1}{ab^2}-\frac{-1}{a^2b}}$$

Now, $\frac{x}{\frac{a-b}{b^2}}=\frac{1}{\frac{-1}{ab^2}-\frac{-1}{a^2b}}$

$=x=a^2$

$$\frac{-y}{\frac{-a-b}{a^2}+\frac{1}{a}+\frac{b}{a^2}}=\frac{1}{\frac{-1}{ab^2}-\frac{-1}{a^2b}}$$

$=y=b^2$

Hence, the solution for the given system of equations is $x = a^2$ and $y = b^2$.

14. x/a = y/b
ax + by = $a^2 + b^2$
Solution:

The given system of equations can be written as
$x/a - y/b = 0$
$ax + by – (a^2 + b^2) = 0$

For cross multiplication we use,

$$\frac{x}{b_1c_2-b_2c_1}=\frac{-y}{a_1c_2-a_2c_1}=\frac{1}{a_1b_2-a_2b_1}$$

Comparing the above two equations with the general form, we get

$a_1=\frac{1}{a}$, Let $b_1=\frac{1}{b}$, $c_1=0$

Hence, $a_1=a$, $b_2=b$, Let $c_1=-(a^2+b^2)$

By cross multiplication method

$$\frac{x}{\frac{a^2+b^2}{b}} = \frac{y}{\frac{a^2+b^2}{a}} = \frac{1}{\frac{a}{b}+\frac{b}{a}}$$

Now, $\frac{x}{\frac{a^2+b^2}{b}} = \frac{1}{\frac{a}{b}+\frac{b}{a}}$

$=x=a$

And $\frac{y}{\frac{a^2+b^2}{a}} = \frac{1}{\frac{a}{b}+\frac{b}{a}}$

$=y=b$

Hence, the solution for the given system of equations is x = a and y = b.