

## Exercise 3.5

Page No: 3.73

In each of the following systems of equation determine whether the system has a unique solution, no solution or infinite solutions. In case there is a unique solution, find it from 1 to 4:

1.  $x - 3y = 3$

$$3x - 9y = 2$$

**Solution:**

The given system of equations is:

$$x - 3y - 3 = 0$$

$$3x - 9y - 2 = 0$$

The above equations are of the form

$$a_1 x + b_1 y - c_1 = 0$$

$$a_2 x + b_2 y - c_2 = 0$$

Here,  $a_1 = 1, b_1 = -3, c_1 = -3$

$$a_2 = 3, b_2 = -9, c_2 = -2$$

So according to the question, we get

$$a_1 / a_2 = 1/3$$

$$b_1 / b_2 = -3 / -9 = 1/3 \text{ and,}$$

$$c_1 / c_2 = -3 / -2 = 3/2$$

$$\Rightarrow a_1 / a_2 = b_1 / b_2 \neq c_1 / c_2$$

Hence, we can conclude that the given system of equation has no solution.

2.  $2x + y = 5$

$$4x + 2y = 10$$

**Solution:**

The given system of equations is:

$$2x + y - 5 = 0$$

$$4x + 2y - 10 = 0$$

The above equations are of the form

$$a_1 x + b_1 y - c_1 = 0$$

$$a_2 x + b_2 y - c_2 = 0$$

Here,  $a_1 = 2, b_1 = 1, c_1 = -5$

$$a_2 = 4, b_2 = 2, c_2 = -10$$

So according to the question, we get

$$a_1 / a_2 = 2/4 = 1/2$$

$$b_1 / b_2 = 1/2 \text{ and,}$$

$$c_1 / c_2 = -5 / -10 = 1/2$$

$$\Rightarrow a_1 / a_2 = b_1 / b_2 = c_1 / c_2$$

Hence, we can conclude that the given system of equation has infinity many solutions.

$$3. \quad 3x - 5y = 20$$
$$6x - 10y = 40$$

**Solution:**

The given system of equations is:

$$3x - 5y - 20 = 0$$

$$6x - 10y - 40 = 0$$

The above equations are of the form

$$a_1 x + b_1 y - c_1 = 0$$

$$a_2 x + b_2 y - c_2 = 0$$

Here,  $a_1 = 3, b_1 = -5, c_1 = -20$

$$a_2 = 6, b_2 = -10, c_2 = -40$$

So according to the question, we get

$$a_1 / a_2 = 3/6 = 1/2$$

$$b_1 / b_2 = -5/-10 = 1/2 \text{ and,}$$

$$c_1 / c_2 = -20/-40 = 1/2$$

$$\Rightarrow a_1 / a_2 = b_1 / b_2 = c_1 / c_2$$

Hence, we can conclude that the given system of equation has infinity many solutions.

$$4. \quad x - 2y = 8$$
$$5x - 10y = 10$$

**Solution:**

The given system of equations is:

$$x - 2y - 8 = 0$$

$$5x - 10y - 10 = 0$$

The above equations are of the form

$$a_1 x + b_1 y - c_1 = 0$$

$$a_2 x + b_2 y - c_2 = 0$$

Here,  $a_1 = 1, b_1 = -2, c_1 = -8$

$$a_2 = 5, b_2 = -10, c_2 = -10$$

So according to the question, we get

$$a_1 / a_2 = 1/5$$

$$b_1 / b_2 = -2/-10 = 1/5 \text{ and,}$$

$$c_1 / c_2 = -8/-10 = 4/5$$

$$\Rightarrow a_1 / a_2 = b_1 / b_2 \neq c_1 / c_2$$

Hence, we can conclude that the given system of equation has no solution.

**Find the value of k for which the following system of equations has a unique solution: (5-8)**

5.  $kx + 2y = 5$

$3x + y = 1$

**Solution:**

The given system of equations is:

$$kx + 2y - 5 = 0$$

$$3x + y - 1 = 0$$

The above equations are of the form

$$a_1 x + b_1 y - c_1 = 0$$

$$a_2 x + b_2 y - c_2 = 0$$

Here,  $a_1 = k$ ,  $b_1 = 2$ ,  $c_1 = -5$

$a_2 = 3$ ,  $b_2 = 1$ ,  $c_2 = -1$

So according to the question,

For unique solution, the condition is

$$a_1 / a_2 \neq b_1 / b_2$$

$$k/3 \neq 2/1$$

$$\Rightarrow k \neq 6$$

Hence, the given system of equations will have unique solution for all real values of  $k$  other than 6.

6.  $4x + ky + 8 = 0$

$2x + 2y + 2 = 0$

**Solution:**

The given system of equations is:

$$4x + ky + 8 = 0$$

$$2x + 2y + 2 = 0$$

The above equations are of the form

$$a_1 x + b_1 y - c_1 = 0$$

$$a_2 x + b_2 y - c_2 = 0$$

Here,  $a_1 = 4$ ,  $b_1 = k$ ,  $c_1 = 8$

$a_2 = 2$ ,  $b_2 = 2$ ,  $c_2 = 2$

So according to the question,

For unique solution, the condition is

$$a_1 / a_2 \neq b_1 / b_2$$

$$4/2 \neq k/2$$

$$\Rightarrow k \neq 4$$

Hence, the given system of equations will have unique solution for all real values of  $k$  other than 4.

7.  $4x - 5y = k$

$2x - 3y = 12$

**Solution**

The given system of equations is:

$$4x - 5y - k = 0$$

$$2x - 3y - 12 = 0$$

The above equations are of the form

$$a_1 x + b_1 y - c_1 = 0$$

$$a_2 x + b_2 y - c_2 = 0$$

Here,  $a_1 = 4$ ,  $b_1 = 5$ ,  $c_1 = -k$

$$a_2 = 2, b_2 = 3, c_2 = 12$$

So according to the question,

For unique solution, the condition is

$$a_1 / a_2 \neq b_1 / b_2$$

$$4/2 \neq 5/3$$

$$\Rightarrow k \text{ can have any real values.}$$

Hence, the given system of equations will have unique solution for all real values of  $k$ .

**8.  $x + 2y = 3$**

**$5x + ky + 7 = 0$**

**Solution:**

The given system of equations is:

$$x + 2y - 3 = 0$$

$$5x + ky + 7 = 0$$

The above equations are of the form

$$a_1 x + b_1 y - c_1 = 0$$

$$a_2 x + b_2 y - c_2 = 0$$

Here,  $a_1 = 1$ ,  $b_1 = 2$ ,  $c_1 = -3$

$$a_2 = 5, b_2 = k, c_2 = 7$$

So according to the question,

For unique solution, the condition is

$$a_1 / a_2 \neq b_1 / b_2$$

$$1/5 \neq 2/k$$

$$\Rightarrow k \neq 10$$

Hence, the given system of equations will have unique solution for all real values of  $k$  other than 10.

**Find the value of  $k$  for which each of the following system of equations having infinitely many solution: (9-19)**

**9.  $2x + 3y - 5 = 0$**

**$6x + ky - 15 = 0$**

**Solution:**

The given system of equations is:

$$2x + 3y - 5 = 0$$

$$6x + ky - 15 = 0$$

The above equations are of the form

$$a_1 x + b_1 y - c_1 = 0$$

$$a_2 x + b_2 y - c_2 = 0$$

Here,  $a_1 = 2$ ,  $b_1 = 3$ ,  $c_1 = -5$

$$a_2 = 6, b_2 = k, c_2 = -15$$

So according to the question,

For unique solution, the condition is

$$a_1 / a_2 = b_1 / b_2 = c_1 / c_2$$

$$2/6 = 3/k$$

$$\Rightarrow k = 9$$

Hence, the given system of equations will have infinitely many solutions, if  $k = 9$ .

**10.  $4x + 5y = 3$**

**$kx + 15y = 9$**

**Solution:**

The given system of equations is:

$$4x + 5y - 3 = 0$$

$$kx + 15y - 9 = 0$$

The above equations are of the form

$$a_1 x + b_1 y - c_1 = 0$$

$$a_2 x + b_2 y - c_2 = 0$$

Here,  $a_1 = 4$ ,  $b_1 = 5$ ,  $c_1 = -3$

$$a_2 = k, b_2 = 15, c_2 = -9$$

So according to the question,

For unique solution, the condition is

$$a_1 / a_2 = b_1 / b_2 = c_1 / c_2$$

$$4/k = 5/15 = -3/-9$$

$$4/k = 1/3$$

$$\Rightarrow k = 12$$

Hence, the given system of equations will have infinitely many solutions, if  $k = 12$ .

**11.  $kx - 2y + 6 = 0$**

**$4x - 3y + 9 = 0$**

**Solution:**

The given system of equations is:

$$kx - 2y + 6 = 0$$

$$4x - 3y + 9 = 0$$

The above equations are of the form

$$a_1 x + b_1 y - c_1 = 0$$

$$a_2 x + b_2 y - c_2 = 0$$

Here,  $a_1 = k$ ,  $b_1 = -2$ ,  $c_1 = 6$

$$a_2 = 4, b_2 = -3, c_2 = 9$$

So according to the question,

For unique solution, the condition is

$$a_1 / a_2 = b_1 / b_2 = c_1 / c_2$$

$$k / 4 = -2 / -3 = 2 / 3$$

$$\Rightarrow k = 8 / 3$$

Hence, the given system of equations will have infinitely many solutions, if  $k = 8/3$ .

**12.  $8x + 5y = 9$**

**$kx + 10y = 18$**

**Solution:**

The given system of equations is:

$$8x + 5y - 9 = 0$$

$$kx + 10y - 18 = 0$$

The above equations are of the form

$$a_1 x + b_1 y - c_1 = 0$$

$$a_2 x + b_2 y - c_2 = 0$$

Here,  $a_1 = 8$ ,  $b_1 = 5$ ,  $c_1 = -9$

$$a_2 = k, b_2 = 10, c_2 = -18$$

So according to the question,

For unique solution, the condition is

$$a_1 / a_2 = b_1 / b_2 = c_1 / c_2$$

$$8/k = 5/10 = -9/-18 = 1/2$$

$$\Rightarrow k=16$$

Hence, the given system of equations will have infinitely many solutions, if  $k = 16$ .

**13.  $2x - 3y = 7$**

**$(k+2)x - (2k+1)y = 3(2k-1)$**

**Solution:**

The given system of equations is:

$$2x - 3y - 7 = 0$$

$$(k+2)x - (2k+1)y - 3(2k-1) = 0$$

The above equations are of the form

$$a_1 x + b_1 y - c_1 = 0$$

$$a_2 x + b_2 y - c_2 = 0$$

Here,  $a_1 = 2$ ,  $b_1 = -3$ ,  $c_1 = -7$

$$a_2 = (k+2), b_2 = -(2k+1), c_2 = -3(2k-1)$$

So according to the question,

For unique solution, the condition is

$$a_1 / a_2 = b_1 / b_2 = c_1 / c_2$$

$$2 / (k+2) = -3 / -(2k+1) = -7 / -3(2k-1)$$

$$2 / (k+2) = -3 / -(2k+1) \text{ and } -3 / -(2k+1) = -7 / -3(2k-1)$$

$$\Rightarrow 2(2k+1) = 3(k+2) \text{ and } 3 \times 3(2k-1) = 7(2k+1)$$

$$\Rightarrow 4k+2 = 3k+6 \text{ and } 18k-9 = 14k+7$$

$$\Rightarrow k=4 \text{ and } 4k = 16 \Rightarrow k=4$$

Hence, the given system of equations will have infinitely many solutions, if  $k = 4$ .

### 14. $2x + 3y = 2$

$$(k+2)x + (2k+1)y = 2(k-1)$$

**Solution:**

The given system of equations is:

$$2x + 3y - 2 = 0$$

$$(k+2)x + (2k+1)y - 2(k-1) = 0$$

The above equations are of the form

$$a_1 x + b_1 y - c_1 = 0$$

$$a_2 x + b_2 y - c_2 = 0$$

Here,  $a_1 = 2$ ,  $b_1 = 3$ ,  $c_1 = -5$

$$a_2 = (k+2), b_2 = (2k+1), c_2 = -2(k-1)$$

So according to the question,

For unique solution, the condition is

$$a_1 / a_2 = b_1 / b_2 = c_1 / c_2$$

$$2 / (k+2) = 3 / (2k+1) = -2 / -2(k-1)$$

$$2 / (k+2) = 3 / (2k+1) \text{ and } 3(2k+1) = 22(k-1)$$

$$\Rightarrow 2(2k+1) = 3(k+2) \text{ and } 3(k-1) = (2k+1)$$

$$\Rightarrow 4k+2 = 3k+6 \text{ and } 3k-3 = 2k+1$$

$$\Rightarrow k = 4 \text{ and } k = 4$$

Hence, the given system of equations will have infinitely many solutions, if  $k = 4$ .

### 15. $x + (k+1)y = 4$

$$(k+1)x + 9y = 5k + 2$$

**Solution:**

The given system of equations is:

$$x + (k+1)y - 4 = 0$$

$$(k+1)x + 9y - (5k + 2) = 0$$

The above equations are of the form

$$a_1 x + b_1 y - c_1 = 0$$

$$a_2 x + b_2 y - c_2 = 0$$

Here,  $a_1 = 1$ ,  $b_1 = (k+1)$ ,  $c_1 = -4$   
 $a_2 = (k+1)$ ,  $b_2 = 9$ ,  $c_2 = -(5k+2)$

So according to the question,

For unique solution, the condition is

$$a_1 / a_2 = b_1 / b_2 = c_1 / c_2$$

$$1 / k+1 = (k+1) / 9 = -4 / -(5k+2)$$

$$\begin{aligned} 1 / k+1 &= k+1 / 9 & \text{and} & \quad k+1 / 9 = 4 / 5k+2 \\ \Rightarrow 9 &= (k+1)^2 & \text{and} & \quad (k+1)(5k+2) = 36 \\ \Rightarrow 9 &= k^2 + 2k + 1 & \text{and} & \quad 5k^2 + 2k + 5k + 2 = 36 \\ \Rightarrow k^2 + 2k - 8 &= 0 & \text{and} & \quad 5k^2 + 7k - 34 = 0 \\ \Rightarrow k^2 + 4k - 2k - 8 &= 0 & \text{and} & \quad 5k^2 + 17k - 10k - 34 = 0 \\ \Rightarrow k(k+4) - 2(k+4) &= 0 & \text{and} & \quad (5k+17) - 2(5k+17) = 0 \\ \Rightarrow (k+4)(k-2) &= 0 & \text{and} & \quad (5k+17)(k-2) = 0 \\ \Rightarrow k = -4 \text{ or } k = 2 & & \text{and} & \quad k = -17/5 \text{ or } k = 2 \end{aligned}$$

Its seen that  $k=2$  satisfies both the condition.

Hence, the given system of equations will have infinitely many solutions, if  $k = 9$ .

**16.  $kx + 3y = 2k + 1$**

**$2(k+1)x + 9y = 7k + 1$**

**Solution:**

The given system of equations is:

$$kx + 3y - (2k + 1) = 0$$

$$2(k+1)x + 9y - (7k + 1) = 0$$

The above equations are of the form

$$a_1 x + b_1 y - c_1 = 0$$

$$a_2 x + b_2 y - c_2 = 0$$

Here,  $a_1 = k$ ,  $b_1 = 3$ ,  $c_1 = -(2k+1)$

$$a_2 = 2(k+1), b_2 = 9, c_2 = -(7k+1)$$

So according to the question,

For unique solution, the condition is

$$a_1 / a_2 = b_1 / b_2 = c_1 / c_2$$

$$k / 2(k+1) = 3 / 9 \text{ and } 3 / 9 = -(2k+1) / -(7k+1)$$

$$3k = 2k + 2 \quad \text{and} \quad 7k+1 = 3(2k+1) = 6k + 3$$

$$k = 2 \quad \text{and} \quad k = 2$$

Hence, the given system of equations will have infinitely many solutions, if  $k = 2$ .

**17.  $2x + (k-2)y = k$**

**$6x + (2k-1)y = 2k + 5$**

**Solution:**

The given system of equations is:



$$2x + (k-2)y - k = 0$$

$$6x + (2k-1)y - (2k+5) = 0$$

The above equations are of the form

$$a_1 x + b_1 y - c_1 = 0$$

$$a_2 x + b_2 y - c_2 = 0$$

Here,  $a_1 = 2$ ,  $b_1 = k-2$ ,  $c_1 = -k$

$$a_2 = 6, b_2 = 2k-1, c_2 = -2k-5$$

So according to the question,

For unique solution, the condition is

$$a_1 / a_2 = b_1 / b_2 = c_1 / c_2$$

$$2/6 = (k-2)/(2k-1) \text{ and } (k-2)/(2k-1) = -k/-2k-5$$

$$4k - 2 = 6k - 12 \text{ and } (k-2)(2k+5) = k(2k-1)$$

$$2k = 10 \text{ and } 2k^2 - 4k + 5k - 10 = 2k^2 - k$$

$$\Rightarrow k = 5 \text{ and } 2k = 10 \Rightarrow k = 5$$

Hence, the given system of equations will have infinitely many solutions, if  $k = 5$ .

**18.  $2x + 3y = 7$**

$$(k+1)x + (2k-1)y = 4k+1$$

**Solution:**

The given system of equations is:

$$2x + 3y - 7 = 0$$

$$(k+1)x + (2k-1)y - (4k+1) = 0$$

The above equations are of the form

$$a_1 x + b_1 y - c_1 = 0$$

$$a_2 x + b_2 y - c_2 = 0$$

Here,  $a_1 = 2$ ,  $b_1 = 3$ ,  $c_1 = -7$

$$a_2 = (k+1), b_2 = 2k-1, c_2 = -(4k+1)$$

So according to the question,

For unique solution, the condition is

$$a_1 / a_2 = b_1 / b_2 = c_1 / c_2$$

$$2/(k+1) = 3/(2k-1) = -7/-(4k+1)$$

$$2/(k+1) = 3/(2k-1) \text{ and } 3/(2k-1) = 7/(4k+1)$$

$$2(2k-1) = 3(k+1) \text{ and } 3(4k+1) = 7(2k-1)$$

$$\Rightarrow 4k-2 = 3k+3 \text{ and } 12k+3 = 14k-7$$

$$\Rightarrow k = 5 \text{ and } 2k = 10$$

$$\Rightarrow k = 5 \text{ and } k = 5$$

Hence, the given system of equations will have infinitely many solutions, if  $k = 5$ .