Exercise 3.8 Page No: 3.88

1. The numerator of a fraction is 4 less than the denominator. If the numerator is decreased by 2 and denominator is increased by 1, then the denominator is eight times the numerator. Find the fraction.

Solution:

Let's assume the numerator of the fraction to be x and the denominator of the fraction to be y.
So, the required fraction is x/y.
From the question it's given as,
The numerator of the fraction is 4 less the denominator.
Thus, the equation so formed is,

$x = y - 4$

$\Rightarrow \quad x - y = -4$ …… (i)

And also it's given in the question as,
If the numerator is decreased by 2 and denominator is increased by 1, then the denominator is 8 times the numerator.
Putting the above condition in an equation, we get

$y + 1 = 8(x-2)$

$\Rightarrow \quad y + 1 = 8x - 16$

$\Rightarrow \quad 8x - y = 1 + 16$

$\Rightarrow \quad 8x - y = 17$ …… (ii)

Solving (i) and (ii),
Subtracting the equation (ii) from (i), we get

$(x - y) - (8x - y) = -4 - 17$

$\Rightarrow \quad x - y - 8x + y = -21$

$\Rightarrow \quad -7x = -21$

$\Rightarrow \quad x = 21/7$

$\Rightarrow \quad x = 3$

Substituting the value of x =3 in the equation (i), we find y

$3 - y = -4$

$\Rightarrow \quad y = 3+4$

$\Rightarrow \quad y = 7$

Therefore, the fraction is 3/7.

2. A fraction becomes 9/ 11 if 2 is added to both numerator and the denominator. If 3 is added to both the numerator and the denominator it becomes 5/ 6. Find the fraction.

Solution:

Let's assume the numerator of the fraction to be x and the denominator of the fraction to be y.
So, the required fraction is x/y.

From the question it's given as,
If 2 is added to both numerator and the denominator, the fraction becomes 9/ 11 .
Thus, the equation so formed is,

$x + 2y + 2 = 9/ 11$

$\Rightarrow \quad 11(x+2) = 9(y+2)$

$\Rightarrow \quad 11x + 22 = 9y+18$

$\Rightarrow \quad 11x – 9y = 18 – 22$

$\Rightarrow \quad 11x – 9y + 4 = 0$ ……. (i)

And also it's given in the question as,
If 3 is added to both numerator and the denominator, the fraction becomes 5/ 6,
Expressing the above condition in an equation, we have

$x + 3y + 3 = 56$

$\Rightarrow \quad 6(x+3) = 5(y+3)$

$\Rightarrow \quad 6x + 18 = 5y + 15$

$\Rightarrow \quad 6x – 5y = 15 – 18$

$\Rightarrow \quad 6x – 5y + 3 = 0$…….. (ii)

Solving (i) and (ii), to find the fraction
By using cross-multiplication method, we have

$$\frac{x}{-9*3-(-5)*4} = \frac{-y}{11*3-6*4} = \frac{1}{11*(-5)-6*(-9)}$$

$$\Rightarrow \frac{x}{-27+20} = \Rightarrow \frac{-y}{33-24} = \frac{1}{-55+54}$$

$$\Rightarrow \frac{x}{-7} = \frac{-y}{9} = \frac{1}{-1}$$

$$\Rightarrow \frac{x}{7} = \frac{y}{9} = 1$$

x = 7, y = 9

Hence, the required fraction is 7/ 9.

3. A fraction becomes 1/ 3 if 1 is subtracted from both its numerator and denominator. If 1 is added to both the numerator and denominator, it becomes 1/ 2. Find the fraction.
Solution:

Let's assume the numerator of the fraction to be x and the denominator of the fraction to be y.
So, the required fraction is x/y.
From the question it's given as,
If 1 is subtracted from both numerator and the denominator, the fraction becomes 1/ 3.
Thus, the equation so formed is,

$(x – 1)/ (y -1) = 1/ 3$

$\Rightarrow \quad 3(x–1) = (y–1)$

$\Rightarrow \quad 3x – 3 = y – 1$

$\Rightarrow \quad 3x – y – 2 = 0$…. (i)

And also it's given in the question as,
If 1 is added to both numerator and the denominator, the fraction becomes 12. Expressing the above condition in an equation, we have

$(x+1)/ (y+1) = 1/ 2$

$\Rightarrow \quad 2(x+1) = (y+1)$

$\Rightarrow \quad 2x + 2 = y + 1$

$\Rightarrow \quad 2x – y + 1 = 0$ …….. (ii)

Solving (i) and (ii), to find the fraction
By using cross-multiplication, we have

$$\frac{x}{(-1)*1-(-1)*(-2)} = \frac{-y}{3*1-2*(-2)} = \frac{1}{3*(-1)-2*(-1)}$$

$$\Rightarrow \frac{x}{-1-2} = \Rightarrow \frac{-y}{3+4} = \frac{1}{-3+2}$$

$$\Rightarrow \frac{x}{-3} = \frac{-y}{7} = \frac{1}{-1}$$

$$\Rightarrow \frac{x}{3} = \frac{y}{7} = 1$$

$\Rightarrow x = 3, y = 7$

Hence, the required fraction is 3/7.

4. If we add 1 to the numerator and subtract 1 from the denominator, a fraction becomes 1. It also becomes 1/2 if we only add 1 to the denominator. What is the fraction?
Solution:

Let's assume the numerator of the fraction to be x and the denominator of the fraction to be y.
So, the required fraction is x/y.
From the question it's given as,
If 1 is added to the numerator and 1 is subtracted from the denominator, the fraction becomes 1.
Thus, the equation so formed is,

$(x +1)/ (y−1) = 1$

$\Rightarrow \quad (x+1) = (y–1)$

$\Rightarrow \quad x + 1 – y + 1 = 0$

$\Rightarrow \quad x – y + 2 = 0$ …….. (i)

And also it's given in the question as,
If 1 is added to the denominator, the fraction becomes 12.
Expressing the above condition in an equation, we have

$x/ (y+1) = 1/ 2$

$\Rightarrow \quad 2x = (y+1)$

$\Rightarrow \quad 2x – y – 1 = 0$ …… (ii)

Solving (i) and (ii), to find the fraction

By using cross-multiplication, we have

$$\frac{x}{(-1)*(-1)-(-1)*2} = \frac{-y}{1*(-1)-2*2} = \frac{1}{1*(-1)-2*(-1)}$$

$$\Rightarrow \frac{x}{1+2} = \Rightarrow \frac{-y}{-1-4} = \frac{1}{-1+2}$$

$$\Rightarrow \frac{x}{3} = \frac{-y}{-5} = \frac{1}{1}$$

$$\Rightarrow \frac{x}{3} = \frac{y}{5} = 1$$

⇒x = 3, y = 5

Hence, the required fraction is 3/ 5.

5. The sum of the numerator and denominator of a fraction is 12. If the denominator is increased by 3, the fraction becomes 12. Find the fraction.
Solution:

Let's assume the numerator of the fraction to be x and the denominator of the fraction to be y.
So, the required fraction is x/y.
From the question it's given as,
The sum of the numerator and denominator of the fraction is 12.
Thus, the equation so formed is,

x + y = 12
⇒ x + y – 12 = 0

And also it's given in the question as,
If the denominator is increased by 3, the fraction becomes 1/2.
Putting this as an equation, we get

x/ (y+3) = 1/2
⇒ 2x = (y+3)
⇒ 2x – y – 3 = 0

The two equations are,
x + y – 12 = 0…… (i)
2x – y – 3 = 0…….. (ii)

Adding (i) and (ii), we get

x + y – 12 + (2x – y – 3) = 0
⇒ 3x -15 = 0
⇒ x = 5

Using x = 5 in (i), we find y
5 + y – 12 = 0

$\Rightarrow \quad y = 7$

Therefore, the required fraction is 5/7.