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1. In a Δ ABC, D and E are points on the sides AB and AC respectively such that DE || BC.

i) If AD = 6 cm, DB = 9 cm and AE = 8 cm, Find AC.

Solution:

Given: Δ ABC, DE ‖ BC, AD = 6 cm, DB = 9 cm and AE = 8 cm.
Required to find AC.

By using Thales Theorem, [As DE ‖ BC]
AD/BD = AE/CE
Let CE = x.
So then,
6/9 = 8/x
6x = 72 cm
x = 72/6 cm
x = 12 cm
∴ AC = AE + CE = 12 + 8 = 20.

ii) If AD/DB = 3/4 and AC = 15 cm, Find AE.

Solution:

Given: AD/BD = 3/4 and AC = 15 cm [As DE ‖ BC]
Required to find AE.

By using Thales Theorem, [As DE ‖ BC]
AD/BD = AE/CE
Let, AE = x, then CE = 15-x.
⇒ 3/4 = x/ (15–x)
45 – 3x = 4x
-3x – 4x = – 45
7x = 45
x = 45/7
x = 6.43 cm
∴ AE= 6.43cm

iii) If AD/DB = 2/3 and AC = 18 cm, Find AE.

Solution:

Given: AD/BD = 2/3 and AC = 18 cm
Required to find AE.

By using Thales Theorem, [As DE ‖ BC]
AD/BD = AE/CE
Let, AE = x and CE = 18 – x
⇒ 23 = x/ (18–x)

3x = 36 – 2x
5x = 36 cm
x = 36/5 cm
x = 7.2 cm
∴ AE = 7.2 cm

iv) If AD = 4 cm, AE = 8 cm, DB = x – 4 cm and EC = 3x – 19, find x.
Solution:

Given: AD = 4 cm, AE = 8 cm, DB = x – 4 and EC = 3x – 19
Required to find x.

By using Thales Theorem, [As DE ‖ BC]
AD/BD = AE/CE
Then, 4/ (x – 4) = 8/ (3x – 19)
4(3x – 19) = 8(x – 4)
12x – 76 = 8(x – 4)
12x – 8x = – 32 + 76
4x = 44 cm
x = 11 cm

v) If AD = 8 cm, AB = 12 cm and AE = 12 cm, find CE.
Solution:

Given: AD = 8 cm, AB = 12 cm, and AE = 12 cm.
Required to find CE,

By using Thales Theorem, [As DE ‖ BC]
AD/BD = AE/CE
8/4 = 12/CE
8 x CE = 4 x 12 cm
CE = (4 x 12)/8 cm
CE = 48/8 cm
∴ CE = 6 cm

vi) If AD = 4 cm, DB = 4.5 cm and AE = 8 cm, find AC.
Solution:

Given: AD = 4 cm, DB = 4.5 cm, AE = 8 cm
Required to find AC.

By using Thales Theorem, [As DE ‖ BC]
AD/BD = AE/CE
4/4.5 = 8/AC
AC = (4.5 × 8)/4 cm
∴AC = 9 cm

vii) If AD = 2 cm, AB = 6 cm and AC = 9 cm, find AE.
Solution:

Given: AD = 2 cm, AB = 6 cm and AC = 9 cm
Required to find AE.

DB = AB – AD = 6 – 2 = 4 cm
By using Thales Theorem, [As DE ‖ BC]
AD/BD = AE/CE
2/4 = x/ (9–x)
4x = 18 – 2x
6x = 18
x = 3 cm
∴ AE= 3cm

viii) If AD/BD = 4/5 and EC = 2.5 cm, Find AE.
Solution:

Given: AD/BD = 4/5 and EC = 2.5 cm
Required to find AE.

By using Thales Theorem, [As DE ‖ BC]
AD/BD = AE/CE
Then, 4/5 = AE/2.5
∴ AE = 4 × 2.55 = 2 cm

ix) If AD = x cm, DB = x – 2 cm, AE = x + 2 cm, and EC = x – 1 cm, find the value of x.
Solution:

Given: AD = x, DB = x – 2, AE = x + 2 and EC = x – 1
Required to find the value of x.

By using Thales Theorem, [As DE ‖ BC]
AD/BD = AE/CE
So, x/ (x–2) = (x+2)/ (x–1)
x(x – 1) = (x – 2)(x + 2)
$x^2 – x – x^2 + 4 = 0$
x = 4

x) If AD = 8x – 7 cm, DB = 5x – 3 cm, AE = 4x – 3 cm, and EC = (3x – 1) cm, Find the value of x.
Solution:

Given: AD = 8x – 7, DB = 5x – 3, AER = 4x – 3 and EC = 3x -1
Required to find x.

By using Thales Theorem, [As DE ‖ BC]

AD/BD = AE/CE

(8x–7)/ (5x–3) = (4x–3)/ (3x–1)

(8x – 7)(3x – 1) = (5x – 3)(4x – 3)

$24x^2 – 29x + 7 = 20x^2 – 27x + 9$

$4x^2 – 2x – 2 = 0$

$2(2x^2 – x – 1) = 0$

$2x^2 – x – 1 = 0$

$2x^2 – 2x + x – 1 = 0$

2x(x – 1) + 1(x – 1) = 0

(x – 1)(2x + 1) = 0

⇒ x = 1 or x = -1/2

We know that the side of triangle can never be negative. Therefore, we take the positive value.

∴ x = 1.

xi) If AD = 4x – 3, AE = 8x – 7, BD = 3x – 1, and CE = 5x – 3, find the value of x.
Solution:

Given: AD = 4x – 3, BD = 3x – 1, AE = 8x – 7 and EC = 5x – 3

Required to find x.

By using Thales Theorem, [As DE ‖ BC]

AD/BD = AE/CE

So, (4x–3)/ (3x–1) = (8x–7)/ (5x–3)

(4x – 3)(5x – 3) = (3x – 1)(8x – 7)

4x(5x – 3) -3(5x – 3) = 3x(8x – 7) -1(8x – 7)

$20x^2 – 12x – 15x + 9 = 24x^2 – 29x + 7$

$20x^2 -27x + 9 = 24^2 -29x + 7$

⇒ $-4x^2 + 2x + 2 = 0$

$4x^2 – 2x – 2 = 0$

$4x^2 – 4x + 2x – 2 = 0$

4x(x – 1) + 2(x – 1) = 0

(4x + 2)(x – 1) = 0

⇒ x = 1 or x = -2/4

We know that the side of triangle can never be negative. Therefore, we take the positive value.

∴ x = 1

xii) If AD = 2.5 cm, BD = 3.0 cm, and AE = 3.75 cm, find the length of AC.
Solution:

Given: AD = 2.5 cm, AE = 3.75 cm and BD = 3 cm

Required to find AC.

By using Thales Theorem, [As DE ‖ BC]

AD/BD = AE/CE

2.5/ 3 = 3.75/ CE
2.5 x CE = 3.75 x 3
CE = 3.75×32.5
CE = 11.252.5
CE = 4.5

Now, AC = 3.75 + 4.5
∴ AC = 8.25 cm.

2. In a Δ ABC, D and E are points on the sides AB and AC respectively. For each of the following cases show that DE ‖ BC:
i) AB = 12 cm, AD = 8 cm, AE = 12 cm, and AC = 18 cm.
Solution:

Required to prove DE ‖ BC.
We have,
AB = 12 cm, AD = 8 cm, AE = 12 cm, and AC = 18 cm**.** (Given)
So,
BD = AB – AD = 12 – 8 = 4 cm
And,
CE = AC – AE = 18 – 12 = 6 cm

It's seen that,
AD/BD = 8/4 = 1/2
AE/CE = 12/6 = 1/2
Thus,
AD/BD = AE/CE
So, by the converse of Thale's Theorem
We have,
DE ‖ BC.
Hence Proved.

ii) AB = 5.6 cm, AD = 1.4 cm, AC = 7.2 cm, and AE = 1.8 cm.
Solution:

Required to prove DE ‖ BC.
We have,
AB = 5.6 cm, AD = 1.4 cm, AC = 7.2 cm, and AE = 1.8 cm**.** (Given)
So,
BD = AB – AD = 5.6 – 1.4 = 4.2 cm
And,
CE = AC – AE = 7.2 – 1.8 = 5.4 cm

It's seen that,
AD/BD = 1.4/4.2 = 1/3
AE/CE = 1.8/5.4 =1/3

Thus,

AD/BD = AE/CE

So, by the converse of Thale's Theorem

We have,

DE ‖ BC.

Hence Proved.

iii) AB = 10.8 cm, BD = 4.5 cm, AC = 4.8 cm, and AE = 2.8 cm.

Solution:

Required to prove DE ‖ BC.

We have

AB = 10.8 cm, BD = 4.5 cm, AC = 4.8 cm, and AE = 2.8 cm.

So,

AD = AB – DB = 10.8 – 4.5 = 6.3

And,

CE = AC – AE = 4.8 – 2.8 = 2

It's seen that,

AD/BD = 6.3/ 4.5 = 2.8/ 2.0 = AE/CE = 7/5

So, by the converse of Thale's Theorem

We have,

DE ‖ BC.

Hence Proved.

iv) AD = 5.7 cm, BD = 9.5 cm, AE = 3.3 cm, and EC = 5.5 cm.

Solution:

Required to prove DE ‖ BC.

We have

AD = 5.7 cm, BD = 9.5 cm, AE = 3.3 cm, and EC = 5.5 cm

Now,

AD/BD = 5.7/9.5 =3/5

And,

AE/CE = 3.3/5.5 = 3/5

Thus,

AD/BD = AE/CE

So, by the converse of Thale's Theorem

We have,

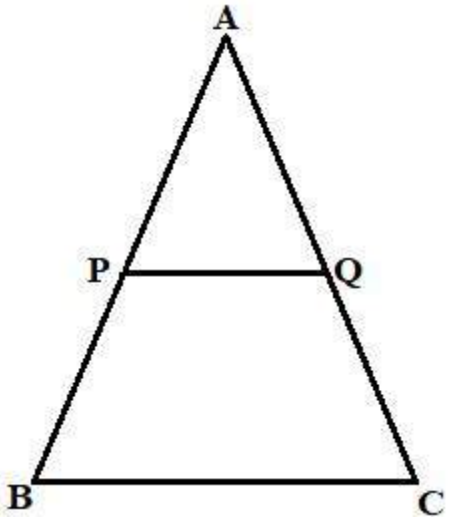
DE ‖ BC.

Hence Proved.

3. In a ∆ ABC, P and Q are the points on sides AB and AC respectively, such that PQ ‖ BC. If AP = 2.4 cm, AQ = 2 cm, QC = 3 cm and BC = 6 cm. Find AB and PQ.

Solution:

Given: Δ ABC, AP = 2.4 cm, AQ = 2 cm, QC = 3 cm, and BC = 6 cm. Also, PQ ‖ BC.
Required to find: AB and PQ.



By using Thales Theorem, we have [As it's given that PQ ‖ BC]

AP/PB = AQ/ QC
2.4/PB = 2/3
2 x PB = 2.4 x 3
PB = (2.4 × 3)/2 cm
⇒ PB = 3.6 cm

Now finding, AB = AP + PB
AB = 2.4 + 3.6
⇒ AB = 6 cm

Now, considering Δ APQ and Δ ABC
We have,

∠A = ∠A
∠APQ = ∠ABC (Corresponding angles are equal, PQ||BC and AB being a transversal)

Thus, Δ APQ and Δ ABC are similar to each other by AA criteria.
Now, we know that
Corresponding parts of similar triangles are propositional.
⇒ AP/AB = PQ/ BC
⇒ PQ = (AP/AB) x BC
= (2.4/6) x 6 = 2.4

∴ PQ = 2.4 cm.

4. In a Δ ABC, D and E are points on AB and AC respectively, such that DE ‖ BC. If AD = 2.4 cm, AE = 3.2 cm, DE = 2 cm and BC = 5 cm. Find BD and CE.
Solution:

Given: Δ ABC such that AD = 2.4 cm, AE = 3.2 cm, DE = 2 cm and BE = 5 cm. Also DE ∥ BC.
Required to find: BD and CE.

As DE ∥ BC, AB is transversal,
∠APQ = ∠ABC (corresponding angles)
As DE ∥ BC, AC is transversal,
∠AED = ∠ACB (corresponding angles)

In Δ ADE and Δ ABC,
∠ADE=∠ABC
∠AED=∠ACB
∴ Δ ADE = Δ ABC (AA similarity criteria)

Now, we know that
Corresponding parts of similar triangles are propositional.
⇒ AD/AB = AE/AC = DE/BC
AD/AB = DE/BC
2.4/ (2.4 + DB) = 2/5 [Since, AB = AD + DB]
2.4 + DB = 6
DB = 6 – 2.4
DB = 3.6 cm

In the same way,
⇒ AE/AC = DE/BC
3.2/ (3.2 + EC) = 2/5 [Since AC = AE + EC]
3.2 + EC = 8
EC = 8 – 3.2
EC = 4.8 cm

∴ BD = 3.6 cm and CE = 4.8 cm.