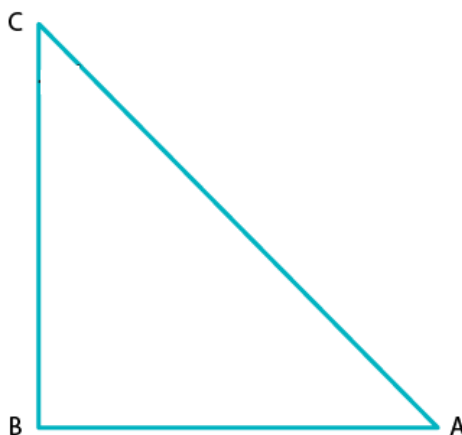


Exercise 5.1

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1. In each of the following, one of the six trigonometric ratios is given. Find the values of the other trigonometric ratios.



(i) $\sin A = 2/3$

Solution:

We have,

$$\sin A = 2/3 \dots\dots\dots (1)$$

As we know, by sin definition;

$$\sin A = \text{Perpendicular/ Hypotenuse} = 2/3 \dots\dots(2)$$

By comparing eq. (1) and (2), we have

Opposite side = 2 and Hypotenuse = 3

Now, on using Pythagoras theorem in ΔABC

$$AC^2 = AB^2 + BC^2$$

Putting the values of perpendicular side (BC) and hypotenuse (AC) and for the base side as (AB), we get

$$\Rightarrow 3^2 = AB^2 + 2^2$$

$$AB^2 = 3^2 - 2^2$$

$$AB^2 = 9 - 4$$

$$AB^2 = 5$$

$$AB = \sqrt{5}$$

Hence, Base = $\sqrt{5}$

By definition,

$$\cos A = \text{Base/Hypotenuse}$$

$$\Rightarrow \cos A = \sqrt{5}/3$$

Since, $\operatorname{cosec} A = 1/\sin A = \text{Hypotenuse/Perpendicular}$

$$\Rightarrow \operatorname{cosec} A = 3/2$$

And, $\sec A = \text{Hypotenuse/Base}$

$$\Rightarrow \sec A = 3/\sqrt{5}$$

And, $\tan A = \text{Perpendicular/Base}$

$$\Rightarrow \tan A = 2/\sqrt{5}$$

And, $\cot A = 1/\tan A = \text{Base/Perpendicular}$
 $\Rightarrow \cot A = \sqrt{5}/2$

(ii) $\cos A = 4/5$

Solution:

We have,
 $\cos A = 4/5$ (1)
As we know, by cos definition
 $\cos A = \text{Base/Hypotenuse}$ (2)
By comparing eq. (1) and (2), we get
Base = 4 and Hypotenuse = 5

Now, using Pythagoras theorem in ΔABC

$$AC^2 = AB^2 + BC^2$$

Putting the value of base (AB) and hypotenuse (AC) and for the perpendicular (BC), we get

$$5^2 = 4^2 + BC^2$$

$$BC^2 = 5^2 - 4^2$$

$$BC^2 = 25 - 16$$

$$BC^2 = 9$$

$$BC = 3$$

Hence, Perpendicular = 3

By definition,

$$\sin A = \text{Perpendicular/Hypotenuse}$$

$$\Rightarrow \sin A = 3/5$$

Then, $\operatorname{cosec} A = 1/\sin A$

$$\Rightarrow \operatorname{cosec} A = 1/(3/5) = 5/3 = \text{Hypotenuse/Perpendicular}$$

And, $\sec A = 1/\cos A$

$$\Rightarrow \sec A = \text{Hypotenuse/Base}$$

$$\sec A = 5/4$$

And, $\tan A = \text{Perpendicular/Base}$

$$\Rightarrow \tan A = 3/4$$

Next, $\cot A = 1/\tan A = \text{Base/Perpendicular}$

$$\therefore \cot A = 4/3$$

(iii) $\tan \theta = 11/1$

Solution:

We have, $\tan \theta = 11$ (1)

By definition,

$$\tan \theta = \text{Perpendicular/ Base} \dots (2)$$

On Comparing eq. (1) and (2), we get;

$$\text{Base} = 1 \text{ and Perpendicular} = 11$$

Now, using Pythagoras theorem in ΔABC .

$$AC^2 = AB^2 + BC^2$$

Putting the value of base (AB) and perpendicular (BC) to get hypotenuse(AC), we get;

$$AC^2 = 1^2 + 11^2$$

$$AC^2 = 1 + 121$$

$$AC^2 = 122$$

$$AC = \sqrt{122}$$

Hence, hypotenuse = $\sqrt{122}$

By definition,

$$\sin = \text{Perpendicular/Hypotenuse}$$

$$\Rightarrow \sin \theta = 11/\sqrt{122}$$

And, $\operatorname{cosec} \theta = 1/\sin \theta$

$$\Rightarrow \operatorname{cosec} \theta = \sqrt{122}/11$$

Next, $\cos \theta = \text{Base/ Hypotenuse}$

$$\Rightarrow \cos \theta = 1/\sqrt{122}$$

And, $\sec \theta = 1/\cos \theta$

$$\Rightarrow \sec \theta = \sqrt{122}/1 = \sqrt{122}$$

And, $\cot \theta = 1/\tan \theta$

$$\therefore \cot \theta = 1/11$$

(iv) $\sin \theta = 11/15$

Solution:

We have, $\sin \theta = 11/15$ (1)

By definition,

$$\sin \theta = \text{Perpendicular/ Hypotenuse} \dots (2)$$

On Comparing eq. (1) and (2), we get;

Perpendicular = 11 and Hypotenuse= 15

Now, using Pythagoras theorem in ΔABC

$$AC^2 = AB^2 + BC^2$$

Putting the value of perpendicular (BC) and hypotenuse (AC) to get the base (AB), we have

$$15^2 = AB^2 + 11^2$$

$$AB^2 = 15^2 - 11^2$$

$$AB^2 = 225 - 121$$

$$AB^2 = 104$$

$$AB = \sqrt{104}$$

$$AB = \sqrt{(2 \times 2 \times 2 \times 13)}$$

$$AB = 2\sqrt{(2 \times 13)}$$

$$AB = 2\sqrt{26}$$

Hence, Base = $2\sqrt{26}$

By definition,

$$\cos \theta = \text{Base/Hypotenuse}$$

$$\therefore \cos \theta = 2\sqrt{26}/15$$

And, $\operatorname{cosec} \theta = 1/\sin \theta$
 $\therefore \operatorname{cosec} \theta = 15/11$
 And, $\sec \theta = \text{Hypotenuse}/\text{Base}$
 $\therefore \sec \theta = 15/2\sqrt{26}$
 And, $\tan \theta = \text{Perpendicular}/\text{Base}$
 $\therefore \tan \theta = 11/2\sqrt{26}$
 And, $\cot \theta = \text{Base}/\text{Perpendicular}$
 $\therefore \cot \theta = 2\sqrt{26}/11$

(v) $\tan \alpha = 5/12$

Solution:

We have, $\tan \alpha = 5/12$ (1)
 By definition,
 $\tan \alpha = \text{Perpendicular}/\text{Base}$ (2)
 On Comparing eq. (1) and (2), we get
 Base = 12 and Perpendicular side = 5

Now, using Pythagoras theorem in ΔABC

$$AC^2 = AB^2 + BC^2$$

Putting the value of base (AB) and the perpendicular (BC) to get hypotenuse (AC), we have

$$AC^2 = 12^2 + 5^2$$

$$AC^2 = 144 + 25$$

$$AC^2 = 169$$

$$AC = 13 \quad [\text{After taking sq root on both sides}]$$

Hence, Hypotenuse = 13

By definition,

$$\sin \alpha = \text{Perpendicular}/\text{Hypotenuse}$$

$$\therefore \sin \alpha = 5/13$$

And, $\operatorname{cosec} \alpha = \text{Hypotenuse}/\text{Perpendicular}$

$$\therefore \operatorname{cosec} \alpha = 13/5$$

And, $\cos \alpha = \text{Base}/\text{Hypotenuse}$

$$\therefore \cos \alpha = 12/13$$

And, $\sec \alpha = 1/\cos \alpha$

$$\therefore \sec \alpha = 13/12$$

And, $\tan \alpha = \sin \alpha / \cos \alpha$

$$\therefore \tan \alpha = 5/12$$

Since, $\cot \alpha = 1/\tan \alpha$

$$\therefore \cot \alpha = 12/5$$

(vi) $\sin \theta = \sqrt{3}/2$

Solution:

We have, $\sin \theta = \sqrt{3}/2$ (1)

By definition,

$$\sin \theta = \text{Perpendicular} / \text{Hypotenuse} \dots (2)$$

On Comparing eq. (1) and (2), we get;

$$\text{Perpendicular} = \sqrt{3} \text{ and Hypotenuse} = 2$$

Now, using Pythagoras theorem in ΔABC

$$AC^2 = AB^2 + BC^2$$

Putting the value of perpendicular (BC) and hypotenuse (AC) and get the base (AB), we get;

$$2^2 = AB^2 + (\sqrt{3})^2$$

$$AB^2 = 2^2 - (\sqrt{3})^2$$

$$AB^2 = 4 - 3$$

$$AB^2 = 1$$

$$AB = 1$$

Thus, Base = 1

By definition,

$$\cos \theta = \text{Base} / \text{Hypotenuse}$$

$$\therefore \cos \theta = 1/2$$

And, $\operatorname{cosec} \theta = 1/\sin \theta$

Or $\operatorname{cosec} \theta = \text{Hypotenuse} / \text{Perpendicular}$

$$\therefore \operatorname{cosec} \theta = 2/\sqrt{3}$$

And, $\sec \theta = \text{Hypotenuse} / \text{Base}$

$$\therefore \sec \theta = 2/1$$

And, $\tan \theta = \text{Perpendicular} / \text{Base}$

$$\therefore \tan \theta = \sqrt{3}/1$$

And, $\cot \theta = \text{Base} / \text{Perpendicular}$

$$\therefore \cot \theta = 1/\sqrt{3}$$

(vii) $\cos \theta = 7/25$

Solution:

We have, $\cos \theta = 7/25 \dots \dots \dots (1)$

By definition,

$$\cos \theta = \text{Base} / \text{Hypotenuse}$$

On Comparing eq. (1) and (2), we get;

$$\text{Base} = 7 \text{ and Hypotenuse} = 25$$

Now, using Pythagoras theorem in ΔABC

$$AC^2 = AB^2 + BC^2$$

Putting the value of base (AB) and hypotenuse (AC) to get the perpendicular (BC)

$$25^2 = 7^2 + BC^2$$

$$BC^2 = 25^2 - 7^2$$

$$BC^2 = 625 - 49$$

$$BC^2 = 576$$

$$BC = \sqrt{576}$$

$$BC = 24$$

Hence, Perpendicular side = 24

By definition,

$$\sin \theta = \text{perpendicular/Hypotenuse}$$

$$\therefore \sin \theta = 24/25$$

Since, $\operatorname{cosec} \theta = 1/\sin \theta$

Also, $\operatorname{cosec} \theta = \text{Hypotenuse/Perpendicular}$

$$\therefore \operatorname{cosec} \theta = 25/24$$

Since, $\sec \theta = 1/\operatorname{cosec} \theta$

Also, $\sec \theta = \text{Hypotenuse/Base}$

$$\therefore \sec \theta = 25/7$$

Since, $\tan \theta = \text{Perpendicular/Base}$

$$\therefore \tan \theta = 24/7$$

Now, $\cot \theta = 1/\tan \theta$

So, $\cot \theta = \text{Base/Perpendicular}$

$$\therefore \cot \theta = 7/24$$

(viii) $\tan \theta = 8/15$

Solution:

We have, $\tan \theta = 8/15$ (1)

By definition,

$\tan \theta = \text{Perpendicular/Base}$ (2)

On Comparing eq. (1) and (2), we get;

Base = 15 and Perpendicular = 8

Now, using Pythagoras theorem in ΔABC

$$AC^2 = 15^2 + 8^2$$

$$AC^2 = 225 + 64$$

$$AC^2 = 289$$

$$AC = \sqrt{289}$$

$$AC = 17$$

Hence, Hypotenuse = 17

By definition,

Since, $\sin \theta = \text{perpendicular/Hypotenuse}$

$$\therefore \sin \theta = 8/17$$

Since, $\operatorname{cosec} \theta = 1/\sin \theta$

Also, $\operatorname{cosec} \theta = \text{Hypotenuse/Perpendicular}$

$$\therefore \operatorname{cosec} \theta = 17/8$$

Since, $\cos \theta = \text{Base/Hypotenuse}$

$$\therefore \cos \theta = 15/17$$

Since, $\sec \theta = 1/\cos \theta$

Also, $\sec \theta = \text{Hypotenuse/Base}$

$$\therefore \sec \theta = 17/15$$

Since, $\cot \theta = 1/\tan \theta$

Also, $\cot \theta = \text{Base/Perpendicular}$
 $\therefore \cot \theta = 15/8$

(ix) $\cot \theta = 12/5$

Solution:

We have, $\cot \theta = 12/5$ (1)

By definition,

$$\cot \theta = 1/\tan \theta$$

$$\cot \theta = \text{Base/Perpendicular} \dots\dots (2)$$

On Comparing eq. (1) and (2), we have

Base = 12 and Perpendicular side = 5

Now, using Pythagoras theorem in ΔABC

$$AC^2 = AB^2 + BC^2$$

Putting the value of base (AB) and perpendicular (BC) to get the hypotenuse (AC);

$$AC^2 = 12^2 + 5^2$$

$$AC^2 = 144 + 25$$

$$AC^2 = 169$$

$$AC = \sqrt{169}$$

$$AC = 13$$

Hence, Hypotenuse = 13

By definition,

Since, $\sin \theta = \text{perpendicular/Hypotenuse}$

$$\therefore \sin \theta = 5/13$$

Since, $\text{cosec } \theta = 1/\sin \theta$

Also, $\text{cosec } \theta = \text{Hypotenuse/Perpendicular}$

$$\therefore \text{cosec } \theta = 13/5$$

Since, $\cos \theta = \text{Base/Hypotenuse}$

$$\therefore \cos \theta = 12/13$$

Since, $\sec \theta = 1/\cos \theta$

Also, $\sec \theta = \text{Hypotenuse/Base}$

$$\therefore \sec \theta = 13/12$$

Since, $\tan \theta = 1/\cot \theta$

Also, $\tan \theta = \text{Perpendicular/Base}$

$$\therefore \tan \theta = 5/12$$

(x) $\sec \theta = 13/5$

Solution:

We have, $\sec \theta = 13/5$ (1)

By definition,

$$\sec \theta = \text{Hypotenuse/Base} \dots\dots\dots (2)$$

On Comparing eq. (1) and (2), we get

Base = 5 and Hypotenuse = 13

Now, using Pythagoras theorem in ΔABC

$$AC^2 = AB^2 + BC^2$$

And, putting the value of base side (AB) and hypotenuse (AC) to get the perpendicular side (BC)

$$13^2 = 5^2 + BC^2$$

$$BC^2 = 13^2 - 5^2$$

$$BC^2 = 169 - 25$$

$$BC^2 = 144$$

$$BC = \sqrt{144}$$

$$BC = 12$$

Hence, Perpendicular = 12

By definition,

Since, $\sin \theta = \text{perpendicular/Hypotenuse}$

$$\therefore \sin \theta = 12/13$$

Since, $\csc \theta = 1/\sin \theta$

Also, $\csc \theta = \text{Hypotenuse/Perpendicular}$

$$\therefore \csc \theta = 13/12$$

Since, $\cos \theta = 1/\sec \theta$

Also, $\cos \theta = \text{Base/Hypotenuse}$

$$\therefore \cos \theta = 5/13$$

Since, $\tan \theta = \text{Perpendicular/Base}$

$$\therefore \tan \theta = 12/5$$

Since, $\cot \theta = 1/\tan \theta$

Also, $\cot \theta = \text{Base/Perpendicular}$

$$\therefore \cot \theta = 5/12$$

(xi) $\csc \theta = \sqrt{10}$

Solution:

We have, $\csc \theta = \sqrt{10}/1$ (1)

By definition,

$\csc \theta = \text{Hypotenuse/ Perpendicular}$ (2)

And, $\csc \theta = 1/\sin \theta$

On comparing eq.(1) and(2), we get

Perpendicular side = 1 and Hypotenuse = $\sqrt{10}$

Now, using Pythagoras theorem in ΔABC

$$AC^2 = AB^2 + BC^2$$

Putting the value of perpendicular (BC) and hypotenuse (AC) to get the base side (AB)

$$(\sqrt{10})^2 = AB^2 + 1^2$$

$$AB^2 = (\sqrt{10})^2 - 1^2$$

$$AB^2 = 10 - 1$$

$$AB = \sqrt{9}$$

$$AB = 3$$

So, Base side = 3

By definition,

Since, $\sin \theta = \text{Perpendicular/Hypotenuse}$

$$\therefore \sin \theta = 1/\sqrt{10}$$

Since, $\cos \theta = \text{Base/Hypotenuse}$

$$\therefore \cos \theta = 3/\sqrt{10}$$

Since, $\sec \theta = 1/\cos \theta$

Also, $\sec \theta = \text{Hypotenuse/Base}$

$$\therefore \sec \theta = \sqrt{10}/3$$

Since, $\tan \theta = \text{Perpendicular/Base}$

$$\therefore \tan \theta = 1/3$$

Since, $\cot \theta = 1/\tan \theta$

$$\therefore \cot \theta = 3/1$$

(xii) $\cos \theta = 12/15$

Solution:

We have; $\cos \theta = 12/15$ (1)

By definition,

$\cos \theta = \text{Base/Hypotenuse}$ (2)

By comparing eq. (1) and (2), we get;

Base = 12 and Hypotenuse = 15

Now, using Pythagoras theorem in ΔABC , we get

$$AC^2 = AB^2 + BC^2$$

Putting the value of base (AB) and hypotenuse (AC) to get the perpendicular (BC);

$$15^2 = 12^2 + BC^2$$

$$BC^2 = 15^2 - 12^2$$

$$BC^2 = 225 - 144$$

$$BC^2 = 81$$

$$BC = \sqrt{81}$$

$$BC = 9$$

So, Perpendicular = 9

By definition,

Since, $\sin \theta = \text{perpendicular/Hypotenuse}$

$$\therefore \sin \theta = 9/15 = 3/5$$

Since, $\text{cosec } \theta = 1/\sin \theta$

Also, $\text{cosec } \theta = \text{Hypotenuse/Perpendicular}$

$$\therefore \text{cosec } \theta = 15/9 = 5/3$$

Since, $\sec \theta = 1/\cos \theta$

Also, $\sec \theta = \text{Hypotenuse/Base}$

$$\therefore \sec \theta = 15/12 = 5/4$$

Since, $\tan \theta = \text{Perpendicular/Base}$

$$\therefore \tan \theta = 9/12 = 3/4$$

Since, $\cot \theta = 1/\tan \theta$

Also, $\cot \theta = \text{Base/Perpendicular}$

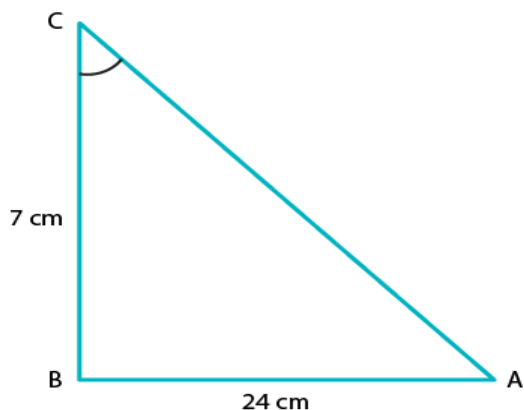
$$\therefore \cot \theta = 12/9 = 4/3$$

2. In a ΔABC , right angled at B, $AB = 24$ cm , $BC = 7$ cm. Determine

(i) $\sin A$, $\cos A$

(ii) $\sin C$, $\cos C$

Solution:



(i) Given: In ΔABC , $AB = 24$ cm, $BC = 7$ cm and $\angle ABC = 90^\circ$

To find: $\sin A$, $\cos A$

By using Pythagoras theorem in ΔABC we have

$$AC^2 = AB^2 + BC^2$$

$$AC^2 = 24^2 + 7^2$$

$$AC^2 = 576 + 49$$

$$AC^2 = 625$$

$$AC = \sqrt{625}$$

$$AC = 25$$

Hence, Hypotenuse = 25

By definition,

$\sin A = \text{Perpendicular side opposite to angle A} / \text{Hypotenuse}$

$$\sin A = BC / AC$$

$$\sin A = 7 / 25$$

And,

$\cos A = \text{Base side adjacent to angle A} / \text{Hypotenuse}$

$$\cos A = AB / AC$$

$$\cos A = 24 / 25$$

(ii) Given: In ΔABC , $AB = 24$ cm and $BC = 7$ cm and $\angle ABC = 90^\circ$

To find: $\sin C$, $\cos C$

By using Pythagoras theorem in ΔABC we have

$$AC^2 = AB^2 + BC^2$$

$$AC^2 = 24^2 + 7^2$$

$$AC^2 = 576 + 49$$

$$AC^2 = 625$$

$$AC = \sqrt{625}$$

$$AC = 25$$

Hence, Hypotenuse = 25

By definition,

$\sin C = \text{Perpendicular side opposite to angle C} / \text{Hypotenuse}$

$$\sin C = AB / AC$$

$$\sin C = 24 / 25$$

And,

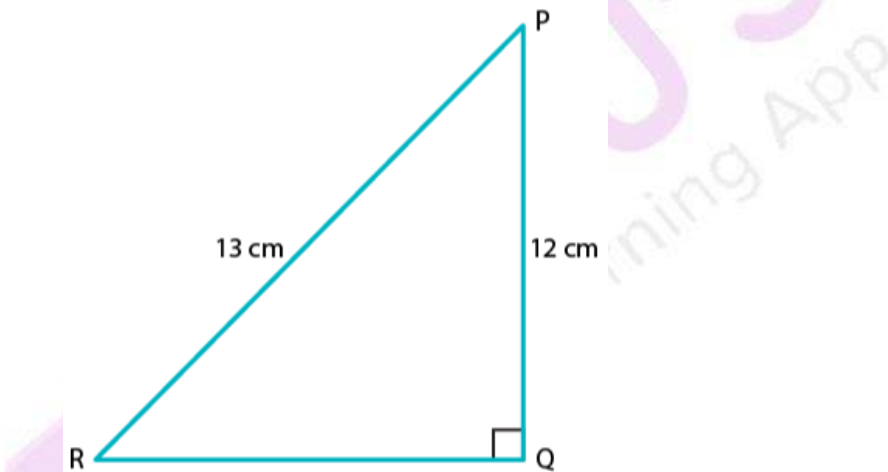
$\cos C = \text{Base side adjacent to angle C} / \text{Hypotenuse}$

$$\cos A = BC / AC$$

$$\cos A = 7 / 25$$

3. In fig. 5.37, find $\tan P$ and $\cot R$. Is $\tan P = \cot R$?

Solution:



By using Pythagoras theorem in ΔPQR , we have

$$PR^2 = PQ^2 + QR^2$$

Putting the length of given side PR and PQ in the above equation

$$13^2 = 12^2 + QR^2$$

$$QR^2 = 13^2 - 12^2$$

$$QR^2 = 169 - 144$$

$$QR^2 = 25$$

$$QR = \sqrt{25} = 5$$

By definition,

$\tan P = \text{Perpendicular side opposite to P} / \text{Base side adjacent to angle P}$

$$\tan P = QR / PQ$$

$$\tan P = 5 / 12 \dots\dots\dots (1)$$

And,

$\cot R = \text{Base} / \text{Perpendicular}$

$$\cot R = QR / PQ$$

$$\cot R = 5 / 12 \dots\dots (2)$$

When comparing equation (1) and (2), we can see that R.H.S of both the equation is equal.

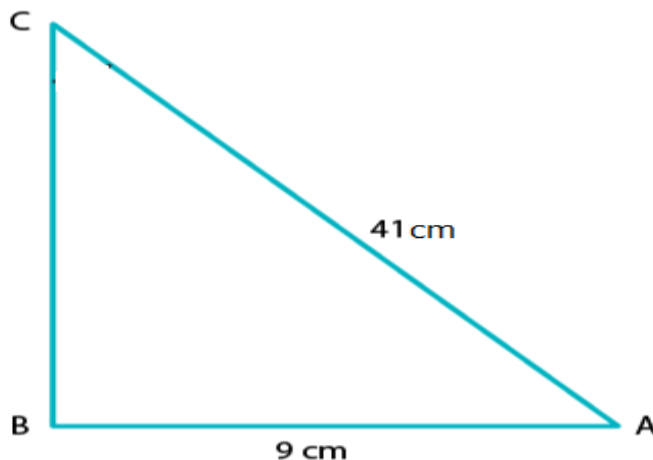
Therefore, L.H.S of both equations should also be equal.

$$\therefore \tan P = \cot R$$

Yes, $\tan P = \cot R = 5/12$

4. If $\sin A = 9/41$, compute $\cos A$ and $\tan A$.

Solution:



Given that, $\sin A = 9/41$ (1)

Required to find: $\cos A$, $\tan A$

By definition, we know that

$\sin A = \text{Perpendicular} / \text{Hypotenuse}$(2)

On Comparing eq. (1) and (2), we get;

Perpendicular side = 9 and Hypotenuse = 41

Let's construct $\triangle ABC$ as shown below,

And, here the length of base AB is unknown.

Thus, by using Pythagoras theorem in $\triangle ABC$, we get;

$$AC^2 = AB^2 + BC^2$$

$$41^2 = AB^2 + 9^2$$

$$AB^2 = 41^2 - 9^2$$

$$AB^2 = 168 - 81$$

$$AB = 1600$$

$$AB = \sqrt{1600}$$

$$AB = 40$$

\Rightarrow Base of triangle ABC, $AB = 40$

We know that,

$$\cos A = \text{Base} / \text{Hypotenuse}$$

$$\cos A = AB / AC$$

$$\cos A = 40/41$$

And,

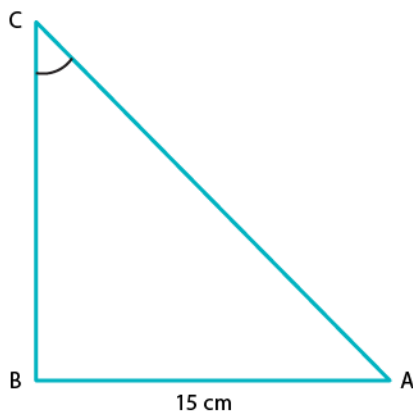
$$\tan A = \text{Perpendicular} / \text{Base}$$

$$\tan A = BC / AB$$

$$\tan A = 9/40$$

5. Given $15\cot A = 8$, find $\sin A$ and $\sec A$.

Solution



We have, $15\cot A = 8$

Required to find: $\sin A$ and $\sec A$

As, $15 \cot A = 8$

$$\Rightarrow \cot A = 8/15 \dots\dots(1)$$

And we know,

$$\cot A = 1/\tan A$$

Also by definition,

$\cot A = \text{Base side adjacent to } \angle A / \text{Perpendicular side opposite to } \angle A \dots (2)$

On comparing equation (1) and (2), we get;

Base side adjacent to $\angle A = 8$

Perpendicular side opposite to $\angle A = 15$

So, by using Pythagoras theorem to $\triangle ABC$, we have

$$AC^2 = AB^2 + BC^2$$

Substituting values for sides from the figure

$$AC^2 = 8^2 + 15^2$$

$$AC^2 = 64 + 225$$

$$AC^2 = 289$$

$$AC = \sqrt{289}$$

$$AC = 17$$

Therefore, hypotenuse = 17

By definition,

$$\sin A = \text{Perpendicular/Hypotenuse}$$

$$\Rightarrow \sin A = BC/AC$$

$$\sin A = 15/17 \text{ (using values from the above)}$$

Also,

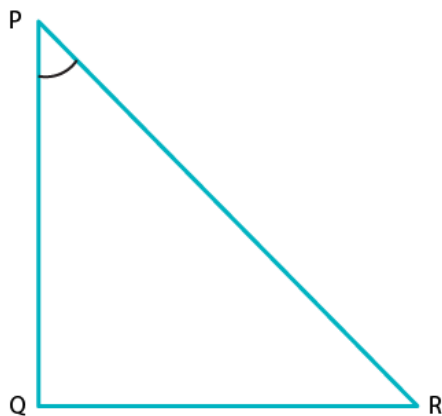
$$\sec A = 1/\cos A$$

$$\Rightarrow \sec A = \text{Hypotenuse/ Base side adjacent to } \angle A$$

$$\therefore \sec A = 17/8$$

6. In ΔPQR , right-angled at Q, $PQ = 4\text{cm}$ and $RQ = 3\text{ cm}$. Find the value of $\sin P$, $\sin R$, $\sec P$ and $\sec R$.

Solution:



Given:

ΔPQR is right-angled at Q.

$PQ = 4\text{cm}$

$RQ = 3\text{cm}$

Required to find: $\sin P$, $\sin R$, $\sec P$, $\sec R$

Given ΔPQR ,

By using Pythagoras theorem to ΔPQR , we get

$$PR^2 = PQ^2 + RQ^2$$

Substituting the respective values,

$$PR^2 = 4^2 + 3^2$$

$$PR^2 = 16 + 9$$

$$PR^2 = 25$$

$$PR = \sqrt{25}$$

$$PR = 5$$

$$\Rightarrow \text{Hypotenuse} = 5$$

By definition,

$\sin P = \text{Perpendicular side opposite to angle P} / \text{Hypotenuse}$

$$\sin P = RQ / PR$$

$$\Rightarrow \sin P = 3/5$$

And,

$\sin R = \text{Perpendicular side opposite to angle R} / \text{Hypotenuse}$

$$\sin R = PQ / PR$$

$$\Rightarrow \sin R = 4/5$$

And,

$$\sec P = 1 / \cos P$$

$\sec P = \text{Hypotenuse} / \text{Base side adjacent to } \angle P$

$$\sec P = PR / PQ$$

$$\Rightarrow \sec P = 5/4$$

Now,

$$\sec R = 1/\cos R$$

$$\sec R = \text{Hypotenuse} / \text{Base side adjacent to } \angle R$$

$$\sec R = PR / RQ$$

$$\Rightarrow \sec R = 5/3$$

7. If $\cot \theta = 7/8$, evaluate

(i) $(1+\sin \theta)(1-\sin \theta) / (1+\cos \theta)(1-\cos \theta)$

(ii) $\cot^2 \theta$

Solution:

(i) Required to evaluate: $\frac{(1 + \sin \theta)(1 - \sin \theta)}{(1 + \cos \theta)(1 - \cos \theta)}$, given $\cot \theta = 7/8$

Taking the numerator, we have

$$(1+\sin \theta)(1-\sin \theta) = 1 - \sin^2 \theta$$

$$[\text{Since, } (a+b)(a-b) = a^2 - b^2]$$

Similarly,

$$(1+\cos \theta)(1-\cos \theta) = 1 - \cos^2 \theta$$

We know that,

$$\sin^2 \theta + \cos^2 \theta = 1$$

$$\Rightarrow 1 - \cos^2 \theta = \sin^2 \theta$$

And,

$$1 - \sin^2 \theta = \cos^2 \theta$$

Thus,

$$(1+\sin \theta)(1-\sin \theta) = 1 - \sin^2 \theta = \cos^2 \theta$$

$$(1+\cos \theta)(1-\cos \theta) = 1 - \cos^2 \theta = \sin^2 \theta$$

$$\begin{aligned} \Rightarrow \frac{(1 + \sin \theta)(1 - \sin \theta)}{(1 + \cos \theta)(1 - \cos \theta)} \\ &= \cos^2 \theta / \sin^2 \theta \\ &= (\cos \theta / \sin \theta)^2 \end{aligned}$$

And, we know that $(\cos \theta / \sin \theta) = \cot \theta$

$$\begin{aligned} \Rightarrow \frac{(1 + \sin \theta)(1 - \sin \theta)}{(1 + \cos \theta)(1 - \cos \theta)} \\ &= (\cot \theta)^2 \\ &= (7/8)^2 \\ &= 49/64 \end{aligned}$$

(ii) Given,

$$\cot \theta = 7/8$$

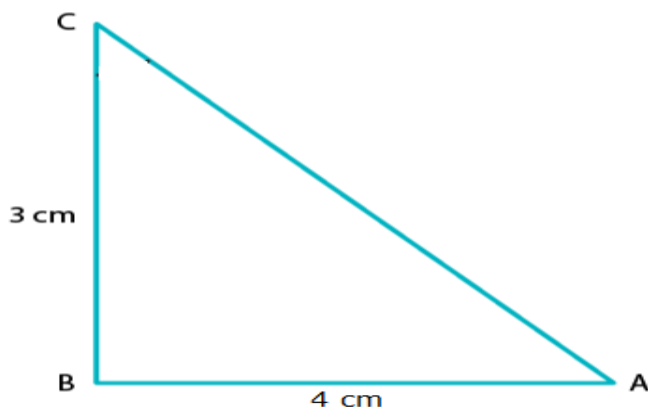
So, by squaring on both sides we get

$$(\cot \theta)^2 = (7/8)^2$$

$$\therefore \cot \theta^2 = 49/64$$

8. If $3\cot A = 4$, check whether $(1-\tan^2 A)/(1+\tan^2 A) = (\cos^2 A - \sin^2 A)$ or not.

Solution:



Given,

$$3\cot A = 4$$

$$\Rightarrow \cot A = 4/3$$

By definition,

$$\tan A = 1/\cot A = 1/(4/3)$$

$$\Rightarrow \tan A = 3/4$$

Thus,

Base side adjacent to $\angle A = 4$

Perpendicular side opposite to $\angle A = 3$

In $\triangle ABC$, Hypotenuse is unknown

Thus, by applying Pythagoras theorem in $\triangle ABC$

We get

$$AC^2 = AB^2 + BC^2$$

$$AC^2 = 4^2 + 3^2$$

$$AC^2 = 16 + 9$$

$$AC^2 = 25$$

$$AC = \sqrt{25}$$

$$AC = 5$$

Hence, hypotenuse = 5

Now, we can find that

$$\sin A = \text{opposite side to } \angle A / \text{Hypotenuse} = 3/5$$

And,

$$\cos A = \text{adjacent side to } \angle A / \text{Hypotenuse} = 4/5$$

Taking the LHS,

$$\text{L.H.S} = \frac{1 - \tan^2 A}{1 + \tan^2 A}$$

Putting value of $\tan A$

We get,

$$\text{L.H.S} = \frac{1 - \left(\frac{3}{4}\right)^2}{1 + \left(\frac{3}{4}\right)^2}$$

$$\frac{1 - \tan^2 A}{1 + \tan^2 A} = \frac{1 - \left(\frac{3}{4}\right)^2}{1 + \left(\frac{3}{4}\right)^2}$$

$$\frac{1 - \tan^2 A}{1 + \tan^2 A} = \frac{1 - \frac{9}{16}}{1 + \frac{9}{16}}$$

Take L.C.M of both numerator and denominator;

$$\frac{1 - \tan^2 A}{1 + \tan^2 A} = \frac{\frac{16 - 9}{16}}{\frac{16 + 9}{16}}$$

$$\frac{1 - \tan^2 A}{1 + \tan^2 A} = \frac{7}{25}$$

Thus, LHS = 7/25

Now, taking RHS

$$\text{R.H.S} = \cos^2 A - \sin^2 A$$

Putting value of sin A and cos A

$$\text{R.H.S} = \left(\frac{4}{5}\right)^2 - \left(\frac{3}{5}\right)^2$$

$$\cos^2 A - \sin^2 A = \left(\frac{4}{5}\right)^2 - \left(\frac{3}{5}\right)^2$$

$$\cos^2 A - \sin^2 A = \frac{16}{25} - \frac{9}{25}$$

$$\cos^2 A - \sin^2 A = \frac{16-9}{25}$$

$$\cos^2 A - \sin^2 A = \frac{7}{25}$$

Therefore,

$$\frac{1-\tan^2 A}{1+\tan^2 A} = \cos^2 A - \sin^2 A$$

Hence Proved

9. If $\tan \theta = a/b$, find the value of $(\cos \theta + \sin \theta) / (\cos \theta - \sin \theta)$

Solution:

Given,

$$\tan \theta = a/b$$

And, we know by definition that

$$\tan \theta = \text{opposite side} / \text{adjacent side}$$

Thus, by comparison

Opposite side = a and adjacent side = b

To find the hypotenuse, we know that by Pythagoras theorem that

$$\text{Hypotenuse}^2 = \text{opposite side}^2 + \text{adjacent side}^2$$

$$\Rightarrow \text{Hypotenuse} = \sqrt{a^2 + b^2}$$

So, by definition

$$\sin \theta = \text{opposite side} / \text{Hypotenuse}$$

$$\sin \theta = a / \sqrt{a^2 + b^2}$$

And,

$$\cos \theta = \text{adjacent side} / \text{Hypotenuse}$$

$$\cos \theta = b / \sqrt{a^2 + b^2}$$

Now,

After substituting for cos θ and sin θ , we have

$$\frac{\cos \theta + \sin \theta}{\cos \theta - \sin \theta} = \frac{(a+b)/\sqrt{(a^2+b^2)}}{(a-b)/\sqrt{(a^2+b^2)}}$$

$$\therefore \frac{\cos \theta + \sin \theta}{\cos \theta - \sin \theta} = \frac{(a+b)}{(a-b)}$$

Hence Proved.

10. If $3 \tan \theta = 4$, find the value of
Solution:

$$\frac{4 \cos \theta - \sin \theta}{2 \cos \theta + \sin \theta}$$

Given, $3 \tan \theta = 4$
 $\Rightarrow \tan \theta = 4/3$

From, $\frac{4 \cos \theta - \sin \theta}{2 \cos \theta + \sin \theta}$ let's divide the numerator and denominator by $\cos \theta$.

We get,

$$\begin{aligned} & (4 - \tan \theta) / (2 + \tan \theta) \\ \Rightarrow & (4 - (4/3)) / (2 + (4/3)) && \text{[using the value of } \tan \theta \text{]} \\ \Rightarrow & (12 - 4) / (6 + 4) && \text{[After taking LCM and cancelling it]} \\ \Rightarrow & 8/10 = 4/5 \end{aligned}$$

$$\therefore \frac{4 \cos \theta - \sin \theta}{2 \cos \theta + \sin \theta} = 4/5$$

11. If $3 \cot \theta = 2$, find the value of

$$\frac{4 \sin \theta - 3 \cos \theta}{2 \sin \theta + 6 \cos \theta}$$

Solution:

Given, $3 \cot \theta = 2$
 $\Rightarrow \cot \theta = 2/3$

From, $\frac{4 \sin \theta - 3 \cos \theta}{2 \sin \theta + 6 \cos \theta}$ let's divide the numerator and denominator by $\sin \theta$.

We get,

$$\begin{aligned} & (4 - 3 \cot \theta) / (2 + 6 \cot \theta) \\ \Rightarrow & (4 - 3(2/3)) / (2 + 6(2/3)) && \text{[using the value of } \tan \theta \text{]} \\ \Rightarrow & (4 - 2) / (2 + 4) && \text{[After taking LCM and simplifying it]} \\ \Rightarrow & 2/6 = 1/3 \end{aligned}$$

$$\therefore \frac{4 \sin \theta - 3 \cos \theta}{2 \sin \theta + 6 \cos \theta} = 1/3$$

12. If $\tan \theta = a/b$, prove that $\frac{a \sin \theta - b \cos \theta}{a \sin \theta + b \cos \theta} = \frac{a^2 - b^2}{a^2 + b^2}$

Solution:

Given, $\tan \theta = a/b$

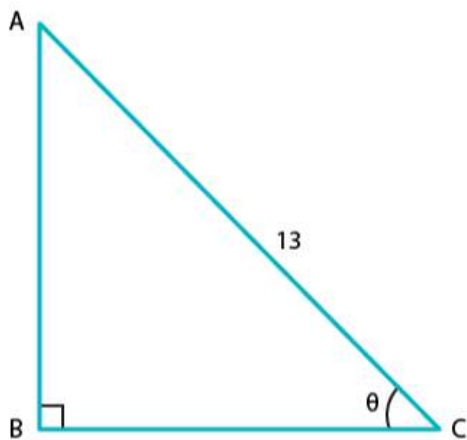
From LHS, $\frac{a \sin \theta - b \cos \theta}{a \sin \theta + b \cos \theta}$ let's divide the numerator and denominator by $\cos \theta$.

And we get,

$$\begin{aligned} & \frac{(a \tan \theta - b)}{(a \tan \theta + b)} \\ \Rightarrow & \frac{(a(a/b) - b)}{(a(a/b) + b)} && \text{[using the value of } \tan \theta \text{]} \\ \Rightarrow & \frac{(a^2 - b^2)/b^2}{(a^2 + b^2)/b^2} && \text{[After taking LCM and simplifying it]} \\ \Rightarrow & \frac{(a^2 - b^2)}{(a^2 + b^2)} \\ & = \text{RHS} \\ & \quad \text{- Hence Proved} \end{aligned}$$

13. If $\sec \theta = 13/5$, show that $\frac{2 \sin \theta - 3 \cos \theta}{4 \sin \theta - 9 \cos \theta} = 3$

Solution:



Given,

$$\sec \theta = 13/5$$

We know that,

$$\sec \theta = 1 / \cos \theta$$

$$\Rightarrow \cos \theta = 1 / \sec \theta = 1 / (13/5)$$

$$\therefore \cos \theta = 5/13 \dots\dots (1)$$

By definition,

$$\cos \theta = \text{adjacent side} / \text{hypotenuse} \dots\dots (2)$$

Comparing (1) and (2), we have

$$\text{Adjacent side} = 5 \text{ and hypotenuse} = 13$$

By Pythagoras theorem,

$$\begin{aligned} \text{Opposite side} &= \sqrt{(\text{hypotenuse})^2 - (\text{adjacent side})^2} \\ &= \sqrt{13^2 - 5^2} \\ &= \sqrt{169 - 25} \\ &= \sqrt{144} \\ &= 12 \end{aligned}$$

Thus, opposite side = 12

By definition,

$$\tan \theta = \text{opposite side} / \text{adjacent side}$$

$$\therefore \tan \theta = 12/5$$

From, $\frac{2 \sin \theta - 3 \cos \theta}{4 \sin \theta - 9 \cos \theta}$ let's divide the numerator and denominator by $\cos \theta$.

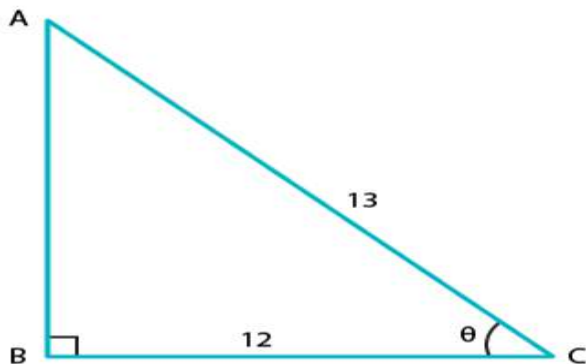
We get,

$$\begin{aligned} &(2 \tan \theta - 3) / (4 \tan \theta - 9) \\ \Rightarrow &(2(12/5) - 3) / (4(12/5) - 9) && [\text{using the value of } \tan \theta] \\ \Rightarrow &(24 - 15) / (48 - 45) && [\text{After taking LCM and cancelling it}] \\ \Rightarrow &9/3 = 3 \end{aligned}$$

$$\therefore \frac{2 \sin \theta - 3 \cos \theta}{4 \sin \theta - 9 \cos \theta} = 3$$

14. If $\cos \theta = 12/13$, show that $\sin \theta(1 - \tan \theta) = 35/156$

Solution:



Given, $\cos \theta = 12/13$ (1)

By definition we know that,

$\cos \theta = \text{Base side adjacent to } \angle \theta / \text{Hypotenuse}$ (2)

When comparing equation (1) and (2), we get

Base side adjacent to $\angle \theta = 12$ and Hypotenuse = 13

From the figure,

Base side BC = 12

Hypotenuse AC = 13

Side AB is unknown here and it can be found by using Pythagoras theorem

Thus by applying Pythagoras theorem,

$$AC^2 = AB^2 + BC^2$$

$$13^2 = AB^2 + 12^2$$

Therefore,

$$AB^2 = 13^2 - 12^2$$

$$AB^2 = 169 - 144$$

$$AB^2 = 25$$

$$AB = \sqrt{25}$$

$$AB = 5 \dots (3)$$

Now, we know that

$\sin \theta = \text{Perpendicular side opposite to } \angle \theta / \text{Hypotenuse}$

Thus, $\sin \theta = AB/AC$ [from figure]

$$\Rightarrow \sin \theta = 5/13 \dots (4)$$

And, $\tan \theta = \sin \theta / \cos \theta = (5/13) / (12/13)$

$$\Rightarrow \tan \theta = 12/13 \dots (5)$$

Taking L.H.S we have

$$\text{L.H.S} = \sin \theta (1 - \tan \theta)$$

Substituting the value of $\sin \theta$ and $\tan \theta$ from equation (4) and (5)

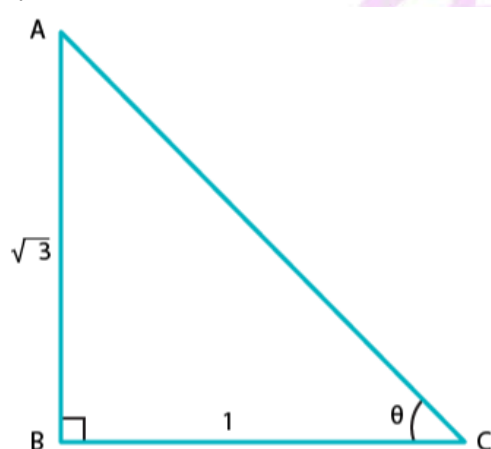
We get,

$$\begin{aligned}
 \Rightarrow \quad \text{L.H.S} &= \frac{5}{13} \left(1 - \frac{5}{12} \right) \\
 \text{L.H.S} &= \frac{5}{13} \left(\frac{1 \times 12}{1 \times 12} - \frac{5}{12} \right) \quad [\text{Taking LCM}] \\
 \text{L.H.S} &= \frac{5}{13} \left(\frac{12-5}{12} \right) \\
 \text{L.H.S} &= \frac{5}{13} \left(\frac{7}{12} \right) \\
 \text{L.H.S} &= \frac{5 \times 7}{13 \times 12} \\
 \text{L.H.S} &= \frac{35}{136}
 \end{aligned}$$

Therefore it's shown that $\sin \theta (1 - \tan \theta) = \frac{35}{136}$

15. If $\cot \theta = \frac{1}{\sqrt{3}}$, show that $\frac{1 - \cos^2 \theta}{2 - \sin^2 \theta} = \frac{3}{5}$

Solution:



Given, $\cot \theta = 1/\sqrt{3}$ (1)

By definition we know that,

$$\cot \theta = 1 / \tan \theta$$

And, since $\tan \theta = \text{perpendicular side opposite to } \angle \theta / \text{Base side adjacent to } \angle \theta$

$$\Rightarrow \quad \cot \theta = \text{Base side adjacent to } \angle \theta / \text{perpendicular side opposite to } \angle \theta \dots\dots (2)$$

[Since they are reciprocal to each other]

On comparing equation (1) and (2), we get

Base side adjacent to $\angle \theta = 1$ and Perpendicular side opposite to $\angle \theta = \sqrt{3}$

Therefore, the triangle formed is,

On substituting the values of known sides as $AB = \sqrt{3}$ and $BC = 1$

$$AC^2 = (\sqrt{3})^2 + 1$$

$$AC^2 = 3 + 1$$

$$AC^2 = 4$$

$$AC = \sqrt{4}$$

Therefore, $AC = 2$... (3)

Now, by definition

$$\sin \theta = \text{Perpendicular side opposite to } \angle \theta / \text{Hypotenuse} = AB / AC$$

$$\Rightarrow \sin \theta = \sqrt{3} / 2 \dots\dots(4)$$

And, $\cos \theta = \text{Base side adjacent to } \angle \theta / \text{Hypotenuse} = BC / AC$

$$\Rightarrow \cos \theta = 1 / 2 \dots\dots(5)$$

Now, taking L.H.S we have

$$\text{L. H. S} = \frac{1 - \cos^2 \theta}{2 - \sin^2 \theta}$$

Substituting the values from equation (4) and (5), we have

$$\text{L. H. S} = \frac{1 - \left(\frac{1}{2}\right)^2}{2 - \left(\frac{\sqrt{3}}{2}\right)^2}$$

$$\text{L. H. S} = \frac{1 - \frac{1}{4}}{2 - \frac{3}{4}}$$

Now by taking L.C.M in numerator and denominator, we get

$$\text{L. H. S} = \frac{\frac{(4 \times 1) - 1}{4}}{\frac{(4 \times 2) - 3}{4}}$$

$$\text{L. H. S} = \frac{\frac{4 - 1}{4}}{\frac{8 - 3}{4}}$$

$$\text{L. H. S} = \frac{3}{4} \times \frac{4}{5}$$

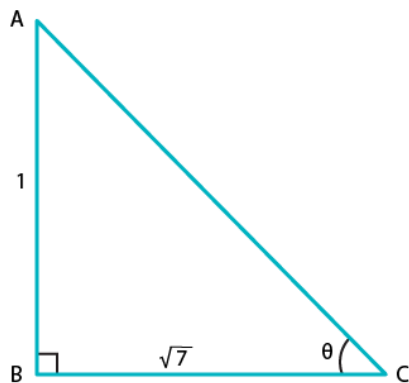
$$\text{L. H. S} = \frac{3}{5} = \text{R. H. S}$$

Therefore, $\frac{1 - \cos^2 \theta}{2 - \sin^2 \theta} = \frac{3}{5}$

16. If $\tan \theta = \frac{1}{\sqrt{7}}$, then show that $\frac{\operatorname{cosec}^2 \theta - \sec^2 \theta}{\operatorname{cosec}^2 \theta + \sec^2 \theta} = \frac{3}{4}$

Solution:

Given, $\tan \theta = 1/\sqrt{7}$ (1)



By definition, we know that

$$\tan \theta = \text{Perpendicular side opposite to } \angle \theta / \text{Base side adjacent to } \angle \theta \text{(2)}$$

On comparing equation (1) and (2), we have

$$\text{Perpendicular side opposite to } \angle \theta = 1$$

$$\text{Base side adjacent to } \angle \theta = \sqrt{7}$$

Thus, the triangle representing $\angle \theta$ is,

Hypotenuse AC is unknown and it can be found by using Pythagoras theorem

By applying Pythagoras theorem, we have

$$AC^2 = AB^2 + BC^2$$

$$AC^2 = 1^2 + (\sqrt{7})^2$$

$$AC^2 = 1 + 7$$

$$AC^2 = 8$$

$$AC = \sqrt{8}$$

$$\Rightarrow AC = 2\sqrt{2}$$

By definition,

$$\sin \theta = \text{Perpendicular side opposite to } \angle \theta / \text{Hypotenuse} = AB / AC$$

$$\Rightarrow \sin \theta = 1 / 2\sqrt{2}$$

$$\text{And, since } \operatorname{cosec} \theta = 1/\sin \theta$$

$$\Rightarrow \operatorname{cosec} \theta = 2\sqrt{2} \text{ (3)}$$

Now,

$$\cos \theta = \text{Base side adjacent to } \angle \theta / \text{Hypotenuse} = BC / AC$$

$$\Rightarrow \cos \theta = \sqrt{7} / 2\sqrt{2}$$

$$\text{And, since } \sec \theta = 1/\cos \theta$$

$$\Rightarrow \sec \theta = 2\sqrt{2} / \sqrt{7} \text{ (4)}$$

Taking the L.H.S of the equation,

$$\text{L. H. S} = \frac{\operatorname{cosec}^2 \theta - \sec^2 \theta}{\operatorname{cosec}^2 \theta + \sec^2 \theta}$$

Substituting the value of cosec θ and sec θ from equation (3) and (4), we get

$$\text{L. H. S} = \frac{[(2\sqrt{2})]^2 - \left(\frac{2\sqrt{2}}{\sqrt{7}}\right)^2}{[(2\sqrt{2})]^2 + \left(\frac{2\sqrt{2}}{\sqrt{7}}\right)^2}$$

$$\text{L. H. S} = \frac{(8) - \left(\frac{8}{7}\right)}{(8) + \left(\frac{8}{7}\right)} = \frac{\frac{56-8}{7}}{\frac{56+8}{7}} \quad [\text{Taking L.C.M and simplifying}]$$

$$\text{L. H. S} = \frac{48}{64}$$

Therefore,

$$\text{L.H.S} = 48/64 = 3/4 = \text{R.H.S}$$

Hence proved that $\frac{\operatorname{cosec}^2 \theta - \sec^2 \theta}{\operatorname{cosec}^2 \theta + \sec^2 \theta} = \frac{3}{4}$

17. If sec $\theta = 5/4$, find the value of
Solution:

$$\frac{\sin \theta - 2\cos \theta}{\tan \theta - \cot \theta}$$

Given,

$$\sec \theta = 5/4$$

We know that,

$$\sec \theta = 1/\cos \theta$$

$$\Rightarrow \cos \theta = 1/(5/4) = 4/5 \dots\dots (1)$$

By definition,

$$\cos \theta = \text{Base side adjacent to } \angle \theta / \text{Hypotenuse} \dots (2)$$

On comparing equation (1) and (2), we have

$$\text{Hypotenuse} = 5$$

$$\text{Base side adjacent to } \angle \theta = 4$$

Thus, the triangle representing $\angle \theta$ is ABC.

Perpendicular side opposite to $\angle\theta$, AB is unknown and it can be found by using Pythagoras theorem

By applying Pythagoras theorem, we have

$$AC^2 = AB^2 + BC^2$$

$$AB^2 = AC^2 - BC^2$$

$$AB^2 = 5^2 - 4^2$$

$$AB^2 = 25 - 16$$

$$AB = \sqrt{9}$$

$$\Rightarrow AB = 3$$

By definition,

$$\sin \theta = \text{Perpendicular side opposite to } \angle\theta / \text{Hypotenuse} = AB / AC$$

$$\Rightarrow \sin \theta = 3/5 \dots\dots(3)$$

Now, $\tan \theta = \text{Perpendicular side opposite to } \angle\theta / \text{Base side adjacent to } \angle\theta$

$$\Rightarrow \tan \theta = 3/4 \dots\dots(4)$$

And, since $\cot \theta = 1/\tan \theta$

$$\Rightarrow \cot \theta = 4/3 \dots\dots(5)$$

Now,

Substituting the value of $\sin \theta$, $\cos \theta$, $\cot \theta$ and $\tan \theta$ from the equations (1), (3), (4) and (5) we have,

$$\frac{\sin \theta - 2 \cos \theta}{\tan \theta - \cot \theta} = \frac{\frac{3}{5} - 2\left(\frac{4}{5}\right)}{\frac{3}{4} - \frac{4}{3}}$$

$$= 12/7$$

Therefore,

$$\frac{\sin \theta - 2 \cos \theta}{\tan \theta - \cot \theta} = \frac{12}{7}$$

18. If $\tan \theta = 12/13$, find the value of
Solution:

$$\frac{2 \sin \theta \cos \theta}{\cos^2 \theta - \sin^2 \theta}$$

Given,

$$\tan \theta = 12/13 \dots\dots\dots (1)$$

We know that by definition,

$$\tan \theta = \text{Perpendicular side opposite to } \angle\theta / \text{Base side adjacent to } \angle\theta \dots\dots\dots (2)$$

On comparing equation (1) and (2), we have

$$\text{Perpendicular side opposite to } \angle\theta = 12$$

$$\text{Base side adjacent to } \angle\theta = 13$$

Thus, in the triangle representing $\angle \theta$ we have,

Hypotenuse AC is the unknown and it can be found by using Pythagoras theorem

So by applying Pythagoras theorem, we have

$$\begin{aligned} AC^2 &= 12^2 + 13^2 \\ AC^2 &= 144 + 169 \\ AC^2 &= 313 \\ \Rightarrow AC &= \sqrt{313} \end{aligned}$$

By definition,

$$\begin{aligned} \sin \theta &= \text{Perpendicular side opposite to } \angle \theta / \text{Hypotenuse} = AB / AC \\ \Rightarrow \sin \theta &= 12 / \sqrt{313} \dots (3) \end{aligned}$$

$$\text{And, } \cos \theta = \text{Base side adjacent to } \angle \theta / \text{Hypotenuse} = BC / AC$$

$$\Rightarrow \cos \theta = 13 / \sqrt{313} \dots (4)$$

Now, substituting the value of $\sin \theta$ and $\cos \theta$ from equation (3) and (4) respectively in the equation below

$$\begin{aligned} \frac{2 \sin \theta \cos \theta}{\cos^2 \theta - \sin^2 \theta} &= \frac{2 \times \frac{13}{\sqrt{313}} \times \frac{12}{\sqrt{313}}}{\left(\frac{13}{\sqrt{313}}\right)^2 - \left(\frac{12}{\sqrt{313}}\right)^2} \\ &= \frac{\frac{312}{313}}{\frac{25}{313}} \\ &= \frac{312}{25} \end{aligned}$$

Therefore,

$$\frac{2 \sin \theta \cos \theta}{\cos^2 \theta - \sin^2 \theta} = \frac{312}{25}$$