

Exercise 6.1

Page No: 6.43

Prove the following trigonometric identities:

1. $(1 - \cos^2 A) \operatorname{cosec}^2 A = 1$

Solution:

Taking the L.H.S,

$$\begin{aligned} & (1 - \cos^2 A) \operatorname{cosec}^2 A \\ &= (\sin^2 A) \operatorname{cosec}^2 A \\ &= 1^2 \\ &= 1 = \text{R.H.S} \end{aligned}$$

$$[\because \sin^2 A + \cos^2 A = 1 \Rightarrow 1 - \sin^2 A = \cos^2 A]$$

- Hence Proved

2. $(1 + \cot^2 A) \sin^2 A = 1$

Solution:

By using the identity,

$$\operatorname{cosec}^2 A - \cot^2 A = 1 \Rightarrow \operatorname{cosec}^2 A = \cot^2 A + 1$$

Taking,

$$\begin{aligned} \text{L.H.S} &= (1 + \cot^2 A) \sin^2 A \\ &= \operatorname{cosec}^2 A \sin^2 A \\ &= (\operatorname{cosec} A \sin A)^2 \\ &= ((1/\sin A) \times \sin A)^2 \\ &= (1)^2 \\ &= 1 \\ &= \text{R.H.S} \end{aligned}$$

- Hence Proved

3. $\tan^2 \theta \cos^2 \theta = 1 - \cos^2 \theta$

Solution:

We know that,

$$\sin^2 \theta + \cos^2 \theta = 1$$

Taking,

$$\begin{aligned} \text{L.H.S} &= \tan^2 \theta \cos^2 \theta \\ &= (\tan \theta \times \cos \theta)^2 \\ &= (\sin \theta)^2 \\ &= \sin^2 \theta \\ &= 1 - \cos^2 \theta \\ &= \text{R.H.S} \end{aligned}$$

- Hence Proved

4. $\operatorname{cosec} \theta \sqrt{1 - \cos^2 \theta} = 1$

Solution:

Using identity,

$$\sin^2 \theta + \cos^2 \theta = 1 \quad \Rightarrow \sin^2 \theta = 1 - \cos^2 \theta$$

Taking L.H.S,

$$\begin{aligned} \text{L.H.S} &= \operatorname{cosec} \theta \sqrt{1 - \cos^2 \theta} \\ &= \operatorname{cosec} \theta \sqrt{\sin^2 \theta} \\ &= \operatorname{cosec} \theta \times \sin \theta \\ &= 1 \\ &= \text{R.H.S} \end{aligned}$$

- Hence Proved

5. $(\sec^2 \theta - 1)(\operatorname{cosec}^2 \theta - 1) = 1$

Solution:

Using identities,

$$(\sec^2 \theta - \tan^2 \theta) = 1 \text{ and } (\operatorname{cosec}^2 \theta - \cot^2 \theta) = 1$$

We have,

$$\begin{aligned} \text{L.H.S} &= (\sec^2 \theta - 1)(\operatorname{cosec}^2 \theta - 1) \\ &= \tan^2 \theta \times \cot^2 \theta \\ &= (\tan \theta \times \cot \theta)^2 \\ &= (\tan \theta \times 1/\tan \theta)^2 \\ &= 1^2 \\ &= 1 \\ &= \text{R.H.S} \end{aligned}$$

- Hence Proved

6. $\tan \theta + 1/\tan \theta = \sec \theta \operatorname{cosec} \theta$

Solution:

We have,

$$\begin{aligned} \text{L.H.S} &= \tan \theta + 1/\tan \theta \\ &= (\tan^2 \theta + 1)/\tan \theta \\ &= \sec^2 \theta / \tan \theta \\ &= (1/\cos^2 \theta) \times 1/(\sin \theta/\cos \theta) && [\because \sec^2 \theta - \tan^2 \theta = 1] \\ &= \cos \theta / (\sin \theta \times \cos^2 \theta) && [\because \tan \theta = \sin \theta / \cos \theta] \\ &= 1/\cos \theta \times 1/\sin \theta \\ &= \sec \theta \times \operatorname{cosec} \theta \\ &= \sec \theta \operatorname{cosec} \theta \\ &= \text{R.H.S} \end{aligned}$$

- Hence Proved

7. $\cos \theta / (1 - \sin \theta) = (1 + \sin \theta) / \cos \theta$

Solution:

We know that,

$$\sin^2 \theta + \cos^2 \theta = 1$$

So, by multiplying both the numerator and the denominator by $(1 + \sin \theta)$, we get

$$\begin{aligned}\text{L.H.S} &= \frac{\cos \theta}{1 - \sin \theta} \\ &= \frac{\cos \theta(1 + \sin \theta)}{(1 - \sin \theta)(1 + \sin \theta)} \\ &= \frac{\cos \theta}{(1 + \sin \theta)(1 - \sin^2 \theta)} \\ &= \frac{\cos \theta(1 + \sin \theta)}{\cos^2 \theta} \\ &= \frac{(1 + \sin \theta)}{\cos \theta} \\ &= \text{R.H.S}\end{aligned}$$

- Hence Proved

8. $\cos \theta / (1 + \sin \theta) = (1 - \sin \theta) / \cos \theta$

Solution:

We know that,

$$\sin^2 \theta + \cos^2 \theta = 1$$

So, by multiplying both the numerator and the denominator by $(1 - \sin \theta)$, we get

$$\begin{aligned}
 \text{L.H.S} &= \frac{\cos \theta}{1 + \sin \theta} \\
 &= \frac{\cos \theta(1 - \sin \theta)}{(1 + \sin \theta)(1 - \sin \theta)} \\
 &= \frac{\cos \theta(1 - \sin \theta)}{(1 - \sin^2 \theta)} \\
 &= \frac{\cos \theta(1 - \sin \theta)}{(\cos^2 \theta)} \\
 &= \frac{(1 - \sin \theta)}{\cos \theta} \\
 &= \frac{(1 - \sin \theta)}{\cos \theta} \\
 &= \text{R.H.S}
 \end{aligned}$$

- Hence Proved

9. $\cos^2 \theta + 1/(1 + \cot^2 \theta) = 1$

Solution:

We already know that,

$$\operatorname{cosec}^2 \theta - \cot^2 \theta = 1 \text{ and } \sin^2 \theta + \cos^2 \theta = 1$$

Taking L.H.S,

$$\begin{aligned}
 \text{L.H.S} &= \cos^2 A + \frac{1}{1 + \cot^2 A} \\
 &= \cos^2 A + \frac{1}{\operatorname{cosec}^2 A} \\
 &= \cos^2 A + \left(\frac{1}{\operatorname{cosec} A}\right)^2 \\
 &= \cos^2 A + \sin^2 A \\
 &= 1 \\
 &= \text{R.H.S}
 \end{aligned}$$

- Hence Proved

10. $\sin^2 A + 1/(1 + \tan^2 A) = 1$

Solution:

We already know that,

$$\sec^2 \theta - \tan^2 \theta = 1 \text{ and } \sin^2 \theta + \cos^2 \theta = 1$$

Taking L.H.S,

$$\text{L.H.S} = \sin^2 A + \frac{1}{1 + \tan^2 A}$$

$$= \sin^2 A + \frac{1}{\sec^2 A}$$

$$= \sin^2 A + \left(\frac{1}{\sec A}\right)^2$$

$$= \sin^2 A + \cos^2 A$$

$$= 1$$

$$= \text{R.H.S}$$

- Hence Proved

11. $\sqrt{\frac{1 - \cos \theta}{1 + \cos \theta}} = \operatorname{cosec} \theta - \cot \theta$

Solution:

We know that, $\sin^2 \theta + \cos^2 \theta = 1$

Taking the L.H.S,

$$\begin{aligned}
 \text{L.H.S} &= \sqrt{\frac{1 - \cos \theta}{1 + \cos \theta}} \\
 &= \sqrt{\frac{(1 - \cos \theta)(1 - \cos \theta)}{(1 + \cos \theta)(1 - \cos \theta)}} \\
 &= \sqrt{\frac{(1 - \cos \theta)^2}{1 - \cos^2 \theta}} \\
 &= \frac{(1 - \cos \theta)^2}{\sin^2 \theta} \\
 &= \frac{(1 - \cos \theta)}{\sin \theta} \\
 &= \frac{1}{\sin \theta} - \frac{\cos \theta}{\sin \theta} \\
 &= \frac{1}{\sin \theta} - \frac{\cos \theta}{\sin \theta} \\
 &= \text{R.H.S}
 \end{aligned}$$

- Hence Proved

12. $1 - \cos \theta / \sin \theta = \sin \theta / 1 + \cos \theta$

Solution:

We know that,

$$\sin^2 \theta + \cos^2 \theta = 1$$

So, by multiplying both the numerator and the denominator by $(1 + \cos \theta)$, we get

$$\begin{aligned}
 \text{L.H.S} &= \frac{1 - \cos^2 \theta}{(1 + \cos \theta)(\sin \theta)} \\
 &= \frac{(\sin^2 \theta)}{(1 + \cos \theta)(\sin \theta)} \\
 &= \frac{(\sin \theta)}{(1 + \cos \theta)} \\
 &= \text{R.H.S}
 \end{aligned}$$

- Hence Proved

13. $\sin \theta / (1 - \cos \theta) = \operatorname{cosec} \theta + \cot \theta$

Solution:

Taking L.H.S,

$$\text{L. H. S} = \frac{\sin \theta}{1 - \cos \theta}$$

On multiplying by its conjugates, we have

$$\begin{aligned} &= \frac{\sin \theta}{1 - \cos \theta} \times \frac{1 + \cos \theta}{1 + \cos \theta} \\ &= \frac{\sin \theta(1 + \cos \theta)}{1 - \cos^2 \theta} \end{aligned}$$

Since, $(1 - \cos^2 \theta) = \sin^2 \theta$

$$= \frac{\sin \theta + (\sin \theta \times \cos \theta)}{\sin^2 \theta}$$

$$= \frac{\sin \theta}{\sin^2 \theta} + \frac{\sin \theta \times \cos \theta}{\sin^2 \theta}$$

$$= \frac{1}{\sin \theta} + \frac{\cos \theta}{\sin \theta}$$

$$= \operatorname{cosec} \theta + \cot \theta$$

$$= \text{R.H.S}$$

- Hence Proved

14. $(1 - \sin \theta) / (1 + \sin \theta) = (\sec \theta - \tan \theta)^2$

Solution:

Taking the L.H.S,

$$\text{L. H. S} = \frac{1 - \sin \theta}{1 + \sin \theta}$$

On multiplying by its conjugate, we have

$$= \frac{1 - \sin \theta}{1 + \sin \theta} \times \frac{1 - \sin \theta}{1 - \sin \theta}$$

Since, $1 - \sin^2 \theta = \cos^2 \theta$

$$= \frac{(1 - \sin \theta)^2}{\cos^2 \theta}$$

$$= \left(\frac{1 - \sin \theta}{\cos \theta} \right)^2$$

$$= \left(\frac{1}{\cos \theta} - \frac{\sin \theta}{\cos \theta} \right)^2$$

$$= (\sec \theta - \tan \theta)^2$$

$$= \text{R.H.S}$$

- Hence Proved

$$\frac{(1 + \cot^2 \theta) \tan \theta}{\sec^2 \theta} = \cot \theta$$

15.

Solution:

Taking L.H.S,

$$\text{L. H. S} = \frac{(1 + \cot^2 \theta) \tan \theta}{\sec^2 \theta}$$

Here, $1 + \cot^2 \theta = \operatorname{cosec}^2 \theta$

$$= \frac{\operatorname{cosec}^2 \theta \times \tan \theta}{\sec^2 \theta}$$

$$= \frac{1}{\sin^2 \theta} \times \frac{\cos^2 \theta}{1} \times \frac{\sin \theta}{\cos \theta}$$

$$= \frac{\cos \theta}{\sin \theta}$$

$$= \cot \theta$$

$$= \text{R.H.S}$$

- Hence Proved

16. $\tan^2 \theta - \sin^2 \theta = \tan^2 \theta \sin^2 \theta$

Solution:

Taking L.H.S,

$$\text{L.H.S} = \tan^2 \theta - \sin^2 \theta$$

$$\text{Since, } \tan^2 \theta = \frac{\sin^2 \theta}{\cos^2 \theta}$$

$$= \frac{\sin^2 \theta}{\cos^2 \theta} - \sin^2 \theta$$

$$= \sin^2 \theta \left[\frac{1}{\cos^2 \theta} - 1 \right]$$

$$= \sin^2 \theta \left[\frac{1 - \cos^2 \theta}{\cos^2 \theta} \right]$$

$$= \frac{\sin^2 \theta}{\cos^2 \theta} \times \sin^2 \theta$$

$$= \tan^2 \theta \sin^2 \theta$$

$$= \text{R.H.S}$$

- Hence Proved

17. $(\operatorname{cosec} \theta + \sin \theta)(\operatorname{cosec} \theta - \sin \theta) = \cot^2 \theta + \cos^2 \theta$

Solution:

Taking L.H.S = $(\operatorname{cosec} \theta + \sin \theta)(\operatorname{cosec} \theta - \sin \theta)$

On multiplying we get,

$$= \operatorname{cosec}^2 \theta - \sin^2 \theta$$

$$= (1 + \cot^2 \theta) - (1 - \cos^2 \theta)$$

[Using $\operatorname{cosec}^2 \theta - \cot^2 \theta = 1$ and $\sin^2 \theta + \cos^2 \theta = 1$]

$$= 1 + \cot^2 \theta - 1 + \cos^2 \theta$$

$$= \cot^2 \theta + \cos^2 \theta$$

$$= \text{R.H.S}$$

- Hence Proved

18. $(\sec \theta + \cos \theta)(\sec \theta - \cos \theta) = \tan^2 \theta + \sin^2 \theta$

Solution:

Taking L.H.S = $(\sec \theta + \cos \theta)(\sec \theta - \cos \theta)$

On multiplying we get,

$$\begin{aligned} &= \sec^2 \theta - \cos^2 \theta \\ &= (1 + \tan^2 \theta) - (1 - \sin^2 \theta) && \text{[Using } \sec^2 \theta - \tan^2 \theta = 1 \text{ and } \sin^2 \theta + \cos^2 \theta = 1\text{]} \\ &= 1 + \tan^2 \theta - 1 + \sin^2 \theta \\ &= \tan^2 \theta + \sin^2 \theta \\ &= \text{R.H.S} \end{aligned}$$

- Hence Proved

19. $\sec A(1 - \sin A)(\sec A + \tan A) = 1$

Solution:

Taking L.H.S = $\sec A(1 - \sin A)(\sec A + \tan A)$

Substituting $\sec A = 1/\cos A$ and $\tan A = \sin A/\cos A$ in the above we have,

$$\begin{aligned} \text{L.H.S} &= 1/\cos A (1 - \sin A)(1/\cos A + \sin A/\cos A) \\ &= 1 - \sin^2 A / \cos^2 A && \text{[After taking L.C.M]} \\ &= \cos^2 A / \cos^2 A && [\because 1 - \sin^2 A = \cos^2 A] \\ &= 1 \\ &= \text{R.H.S} \end{aligned}$$

- Hence Proved

20. $(\operatorname{cosec} A - \sin A)(\sec A - \cos A)(\tan A + \cot A) = 1$

Solution:

Taking L.H.S = $(\operatorname{cosec} A - \sin A)(\sec A - \cos A)(\tan A + \cot A)$

Putting, $\operatorname{cosec} A = \frac{1}{\sin A}$, $\sec A = \frac{1}{\cos A}$, $\tan A = \frac{\sin A}{\cos A}$, $\cot A = \frac{\cos A}{\sin A}$

Substituting the above in the L.H.S, we get

$$\begin{aligned} &= \left(\frac{1}{\sin A} - \sin A \right) \left(\frac{1}{\cos A} - \cos A \right) \left(\frac{\sin A}{\cos A} + \frac{\cos A}{\sin A} \right) \\ &= \left(\frac{1 - \sin^2 A}{\sin A} \right) \left(\frac{1 - \cos^2 A}{\cos A} \right) \left(\frac{\sin^2 A + \cos^2 A}{\sin A \cos A} \right) \\ &= (\cos^2 A / \sin A) (\sin^2 A / \cos A) (1 / \sin A \cos A) && [\because \sin^2 \theta + \cos^2 \theta = 1] \\ &= \sin A \times \cos A \times (1 / \cos A \sin A) \\ &= \text{R.H.S} \end{aligned}$$

- Hence Proved

21. $(1 + \tan^2 \theta)(1 - \sin \theta)(1 + \sin \theta) = 1$

Solution:

Taking L.H.S = $(1 + \tan^2\theta)(1 - \sin \theta)(1 + \sin \theta)$
 And, we know $\sin^2 \theta + \cos^2 \theta = 1$ and $\sec^2 \theta - \tan^2 \theta = 1$

So,
 L.H.S = $(1 + \tan^2 \theta)(1 - \sin \theta)(1 + \sin \theta)$
 $= (1 + \tan^2 \theta)\{(1 - \sin \theta)(1 + \sin \theta)\}$
 $= (1 + \tan^2 \theta)(1 - \sin^2 \theta)$
 $= \sec^2 \theta (\cos^2 \theta)$
 $= (1/\cos^2 \theta) \times \cos^2 \theta$
 $= 1$
 $= \text{R.H.S}$

- Hence Proved

22. $\sin^2 A \cot^2 A + \cos^2 A \tan^2 A = 1$

Solution:

We know that,

$$\cot^2 A = \cos^2 A / \sin^2 A \text{ and } \tan^2 A = \sin^2 A / \cos^2 A$$

Substituting the above in L.H.S, we get

$$\begin{aligned} \text{L.H.S} &= \sin^2 A \cot^2 A + \cos^2 A \tan^2 A \\ &= \{\sin^2 A (\cos^2 A / \sin^2 A)\} + \{\cos^2 A (\sin^2 A / \cos^2 A)\} \\ &= \cos^2 A + \sin^2 A \\ &= 1 \end{aligned} \quad [\because \sin^2 \theta + \cos^2 \theta = 1]$$

= R.H.S

- Hence Proved

23. (i) $\cot \theta - \tan \theta = \frac{2 \cos 2\theta - 1}{\sin \theta * \cos \theta}$

(ii) $\tan \theta - \cot \theta = \left(\frac{2 \sin^2 \theta - 1}{\sin \theta * \cos \theta} \right)$

Solution:

(i) Taking the L.H.S and using $\sin^2 \theta + \cos^2 \theta = 1$, we have
 L.H.S = $\cot \theta - \tan \theta$

$$\begin{aligned}
 &= \frac{\cos \theta}{\sin \theta} - \frac{\sin \theta}{\cos \theta} \\
 &= \frac{\cos^2 \theta - \sin^2 \theta}{\sin \theta \times \cos \theta} \\
 &= \frac{\cos^2 \theta - (1 - \cos^2 \theta)}{\sin \theta \times \cos \theta} \\
 &= \frac{\cos^2 \theta - 1 + \cos^2 \theta}{\sin \theta \times \cos \theta} \\
 &= \left(\frac{2 \cos^2 \theta - 1}{\sin \theta \times \cos \theta} \right) \\
 &= \text{R.H.S}
 \end{aligned}$$

- Hence Proved

(ii) Taking the L.H.S and using $\sin^2 \theta + \cos^2 \theta = 1$, we have

$$\begin{aligned}
 \text{L.H.S} &= \tan \theta - \cot \theta \\
 &= \frac{\sin \theta}{\cos \theta} - \frac{\cos \theta}{\sin \theta} \\
 &= \frac{\sin^2 \theta - \cos^2 \theta}{\sin \theta \cos \theta} \\
 &= \frac{\sin^2 \theta - (1 - \sin^2 \theta)}{\sin \theta \cos \theta} \\
 &= \frac{\sin^2 \theta - (1 + \sin^2 \theta)}{\sin \theta \cos \theta} \\
 &= \left(\frac{2 \sin^2 \theta - 1}{\sin \theta \cos \theta} \right) \\
 &= \text{R.H.S}
 \end{aligned}$$

- Hence Proved

24. $(\cos^2 \theta / \sin \theta) - \operatorname{cosec} \theta + \sin \theta = 0$

Solution:

Taking L.H.S and using $\sin^2 \theta + \cos^2 \theta = 1$, we have

$$\begin{aligned}
 \text{L.H.S} &= \frac{\cos^2\theta}{\sin\theta} - \operatorname{cosec}\theta + \sin\theta \\
 &= \left(\frac{\cos^2\theta}{\sin\theta} - \operatorname{cosec}\theta \right) + \sin\theta \\
 &= \left(\frac{\cos^2\theta}{\sin\theta} - \frac{1}{\sin\theta} \right) + \sin\theta \\
 &= \left(\frac{\cos^2\theta - 1}{\sin\theta} \right) + \sin\theta \\
 &= \left(\frac{-\sin^2\theta}{\sin\theta} \right) + \sin\theta \\
 &= -\sin\theta + \sin\theta \\
 &= 0 \\
 &= \text{R.H.S}
 \end{aligned}$$

- Hence proved

25.
$$\frac{1}{1 + \sin A} + \frac{1}{1 - \sin A} = 2 \sec^2 A$$

Solution:

Taking L.H.S,

$$\begin{aligned}
 \text{LHS} &= \frac{1}{1 + \sin A} + \frac{1}{1 - \sin A} \\
 &= \frac{(1 - \sin A) + (1 + \sin A)}{(1 + \sin A)(1 - \sin A)} \\
 &= \frac{1 - \sin A + 1 + \sin A}{1 - \sin^2 A} \quad \because (1 + \sin A)(1 - \sin A) = 1 - \sin^2 A \\
 &= \frac{2}{1 - \sin^2 A} \\
 &= \frac{2}{\cos^2 A} \quad [\because 1 - \sin^2 A = \cos A] \\
 &= 2 \sec^2 A
 \end{aligned}$$

= R.H.S

- Hence proved

$$\frac{1 + \sin \theta}{\cos \theta} + \frac{\cos \theta}{1 + \sin \theta} = 2 \sec \theta$$

26.

Solution:

Taking the LHS and using $\sin^2 \theta + \cos^2 \theta = 1$, we have

$$\begin{aligned} \text{LHS} &= \frac{1 + \sin \theta}{\cos \theta} + \frac{\cos \theta}{1 + \sin \theta} \\ &= \frac{(1 + \sin \theta)^2 + \cos^2 \theta}{\cos \theta(1 + \sin \theta)} \\ &= \frac{1 + \sin^2 \theta + 2 \sin \theta + \cos^2 \theta}{\cos \theta(1 + \sin \theta)} \\ &= \frac{2(1 + \sin \theta)}{\cos \theta(1 + \sin \theta)} \\ &= 2 / \cos \theta \\ &= 2 \sec \theta \\ &= \text{R.H.S} \end{aligned}$$

- Hence proved

$$27. \quad \frac{(1 + \sin \theta)^2 + (1 - \sin \theta)^2}{2 \cos^2 \theta} = \frac{1 + \sin^2 \theta}{1 - \sin^2 \theta}$$

Solution:

Taking the LHS and using $\sin^2 \theta + \cos^2 \theta = 1$, we have

$$\begin{aligned}
 \text{L. H. S} &= \frac{(1 + \sin \theta)^2 + (1 - \sin \theta)^2}{2\cos^2\theta} \\
 &= \frac{(1 + 2\sin\theta + \sin^2\theta) + (1 - 2\sin\theta + \sin^2\theta)}{2\cos^2\theta} \\
 &= \frac{1 + 2\sin\theta + \sin^2\theta + 1 - 2\sin\theta + \sin^2\theta}{2\cos^2\theta} \\
 &= \frac{2 + 2\sin^2\theta}{2\cos^2\theta} \\
 &= \frac{2(1 + \sin^2\theta)}{2(1 - \sin^2\theta)} \\
 &= \frac{(1 + \sin^2\theta)}{(1 - \sin^2\theta)} \\
 &= \text{R.H.S}
 \end{aligned}$$

- Hence proved

28. $\frac{1 + \tan^2\theta}{1 + \cot^2\theta} = \left[\frac{1 - \tan \theta}{\cot \theta} \right]^2 = \tan^2\theta$

Solution:

Taking L.H.S,

$$\frac{1 + \tan^2\theta}{1 + \cot^2\theta}$$

Using $\sec^2 \theta - \tan^2 \theta = 1$ and $\operatorname{cosec}^2 \theta - \cot^2 \theta = 1$

$$\begin{aligned}
 &= \frac{1 + \tan^2\theta}{1 + \cot^2\theta} = \frac{\sec^2\theta}{\operatorname{cosec}^2\theta} \\
 &= \frac{1}{\cos^2\theta} \cdot \sin^2\theta = \tan^2\theta \\
 &= \text{R.H.S}
 \end{aligned}$$

And, taking

$$\begin{aligned} \left[\frac{1 - \tan \theta}{\cot \theta} \right]^2 &= \frac{1 + \tan^2 \theta - 2 \tan \theta}{1 + \cot^2 \theta - 2 \cot \theta} \\ &= \frac{\sec^2 \theta - 2 \tan \theta}{\operatorname{cosec}^2 \theta - 2 \cot \theta} \quad [\text{Using } \sec^2 \theta - \tan^2 \theta = 1 \text{ and } \operatorname{cosec}^2 \theta - \cot^2 \theta = 1] \\ &= \frac{\frac{1}{\cos^2 \theta} - \frac{2 \sin \theta}{\cos \theta}}{\frac{1}{\sin^2 \theta} - \frac{2 \cos \theta}{\sin \theta}} = \frac{\frac{1 - 2 \sin \theta \cos \theta}{\cos^2 \theta}}{\frac{1 - 2 \sin \theta \cos \theta}{\sin^2 \theta}} \\ &= \frac{\sin^2 \theta}{\cos^2 \theta} \\ &= \tan^2 \theta = \text{R.H.S} \end{aligned}$$

- Hence proved

29. $\frac{1 + \sec \theta}{\sec \theta} = \frac{\sin^2 \theta}{1 - \cos \theta}$

Solution:

Taking L.H.S and using $\sin^2 \theta + \cos^2 \theta = 1$, we have

$$\begin{aligned}
 \text{LHS} &= \frac{1 + \sec \theta}{\sec \theta} \\
 &= \frac{1 + \frac{1}{\cos \theta}}{\frac{1}{\cos \theta}} \\
 &= \frac{\cos \theta + 1}{\cos \theta} \cdot \cos \theta \\
 &= 1 + \cos \theta \\
 &= \frac{(1 - \cos \theta)(1 + \cos \theta)}{1 - \cos \theta} \\
 &= \frac{1 - \cos^2 \theta}{1 - \cos \theta} \\
 &= \frac{\sin^2 \theta}{1 - \cos \theta} \\
 &= \text{R.H.S}
 \end{aligned}$$

Multiplying by $(1 - \cos \theta)$ to numerator and denominator

- Hence proved

30.
$$\frac{\tan \theta}{1 - \cot \theta} + \frac{\cot \theta}{1 - \tan \theta} = 1 + \tan \theta + \cot \theta$$

Solution:

Taking LHS, we have

$$\begin{aligned}
 \text{LHS} &= \frac{\tan \theta}{1 - \frac{1}{\tan \theta}} + \frac{\cot \theta}{1 - \tan \theta} \\
 &= \frac{\tan^2 \theta}{\tan \theta - 1} + \frac{\cot \theta}{1 - \tan \theta} \\
 &= \frac{1}{1 - \tan \theta} \left[\frac{1}{\tan \theta} - \tan^2 \theta \right] \\
 &= \frac{1}{1 - \tan \theta} \left[\frac{1 - \tan^3 \theta}{\tan \theta} \right] \\
 &= \frac{1}{1 - \tan \theta} \frac{(1 - \tan \theta)(1 + \tan \theta + \tan^2 \theta)}{\tan \theta} \quad [\text{Since, } a^3 - b^3 = (a - b)(a^2 + ab + b^2)] \\
 &= \frac{1 + \tan \theta + \tan^2 \theta}{\tan \theta} \\
 &= \frac{1}{\tan \theta} + \frac{\tan \theta}{\tan \theta} + \frac{\tan^2 \theta}{\tan \theta} \\
 &= 1 + \tan \theta + \cot \theta \\
 &= \text{R.H.S}
 \end{aligned}$$

- Hence proved

31. $\sec^6 \theta = \tan^6 \theta + 3 \tan^2 \theta \sec^2 \theta + 1$

Solution:

From trig. Identities we have,

$$\sec^2 \theta - \tan^2 \theta = 1$$

On cubing both sides,

$$(\sec^2 \theta - \tan^2 \theta)^3 = 1$$

$$\sec^6 \theta - \tan^6 \theta - 3 \sec^2 \theta \tan^2 \theta (\sec^2 \theta - \tan^2 \theta) = 1$$

$$[\text{Since, } (a - b)^3 = a^3 - b^3 - 3ab(a - b)]$$

$$\sec^6 \theta - \tan^6 \theta - 3 \sec^2 \theta \tan^2 \theta = 1$$

$$\Rightarrow \sec^6 \theta = \tan^6 \theta + 3 \sec^2 \theta \tan^2 \theta + 1$$

Hence, L.H.S = R.H.S

- Hence proved

32. $\operatorname{cosec}^6 \theta = \cot^6 \theta + 3 \cot^2 \theta \operatorname{cosec}^2 \theta + 1$

Solution:

From trig. Identities we have,

$$\operatorname{cosec}^2 \theta - \cot^2 \theta = 1$$

On cubing both sides,

$$(\operatorname{cosec}^2 \theta - \cot^2 \theta)^3 = 1$$

$$\operatorname{cosec}^6 \theta - \cot^6 \theta - 3\operatorname{cosec}^2 \theta \cot^2 \theta (\operatorname{cosec}^2 \theta - \cot^2 \theta) = 1$$

$$[\text{Since, } (a - b)^3 = a^3 - b^3 - 3ab(a - b)]$$

$$\operatorname{cosec}^6 \theta - \cot^6 \theta - 3\operatorname{cosec}^2 \theta \cot^2 \theta = 1$$

$$\Rightarrow \operatorname{cosec}^6 \theta = \cot^6 \theta + 3 \operatorname{cosec}^2 \theta \cot^2 \theta + 1$$

Hence, L.H.S = R.H.S

- Hence proved

33.
$$\frac{(1 + \tan^2 \theta) \cot \theta}{\operatorname{cosec}^2 \theta} = \tan \theta$$

Solution:

Taking L.H.S and using $\sec^2 \theta - \tan^2 \theta = 1 \Rightarrow 1 + \tan^2 \theta = \sec^2 \theta$

$$\begin{aligned} \text{LHS} &= \frac{\sec^2 \theta \cdot \cot \theta}{\operatorname{cosec}^2 \theta} \\ &= \frac{1 \cdot \sin^2 \theta \cdot \cos \theta}{\cos^2 \theta \cdot \sin \theta} \\ &= \frac{\sin \theta}{\cos \theta} = \tan \theta \\ &= \text{R.H.S} \end{aligned}$$

- Hence proved

34.
$$\frac{1 + \cos A}{\sin^2 A} = \frac{1}{1 - \cos A}$$

Solution:

Taking L.H.S and using the identity $\sin^2 A + \cos^2 A = 1$, we get

$$\sin^2 A = 1 - \cos^2 A$$

$$\Rightarrow \sin^2 A = (1 - \cos A)(1 + \cos A)$$

$$\begin{aligned} \text{LHS} &= \frac{1 + \cos A}{(1 - \cos A)(1 + \cos A)} \\ &= \frac{1}{(1 - \cos A)} \end{aligned}$$

- Hence proved

$$35. \quad \frac{\sec A - \tan A}{\sec A + \tan A} = \frac{\cos^2 A}{(1 + \sin A)^2}$$

Solution:

We have,

$$\text{LHS} = \frac{\sec A - \tan A}{\sec A + \tan A}$$

Rationalizing the denominator and numerator with $(\sec A + \tan A)$ and using $\sec^2 \theta - \tan^2 \theta = 1$ we get,

$$\begin{aligned} &= \frac{\sec^2 A - \tan^2 A}{(\sec A + \tan A)^2} \\ &= \frac{1}{(\sec A + \tan A)^2} \\ &= \frac{1}{(\sec^2 A + \tan^2 A + 2 \sec A \tan A)} \\ &= \frac{1}{\left(\frac{1}{\cos^2 A} + \frac{\sin^2 A}{\cos^2 A} + \frac{2 \sin A}{\cos A}\right)} \\ &\Rightarrow \frac{\cos^2 A}{1 + \sin^2 A + 2 \sin A} \\ &= \frac{\cos^2 A}{(1 + \sin A)^2} \\ &= \text{R.H.S} \end{aligned}$$

- Hence proved

$$36. \quad 1 + \frac{\cos A}{\sin A} = \frac{\sin A}{1 - \cos A}$$

Solution:

We have,

$$\text{LHS} = \frac{1 + \cos A}{\sin A}$$

On multiplying numerator and denominator by $(1 - \cos A)$, we get

$$\begin{aligned}
 &= \frac{(1 + \cos A)(1 - \cos A)}{\sin A(1 - \cos A)} \\
 &= \frac{1 - \cos^2 A}{\sin A(1 - \cos A)} \\
 &= \frac{\sin^2 A}{\sin A(1 - \cos A)} \\
 &= \frac{\sin A}{1 - \cos A} \\
 &= \text{R.H.S}
 \end{aligned}$$

- Hence proved

37. (i) $\sqrt{\frac{1 + \sin A}{1 - \sin A}} = \sec A + \tan A$

Solution:

Taking L.H.S and rationalizing the numerator and denominator with $\sqrt{(1 + \sin A)}$, we get

$$\begin{aligned}
 &= \sqrt{\frac{(1 + \sin A)(1 + \sin A)}{(1 - \sin A)(1 + \sin A)}} = \sqrt{\frac{(1 + \sin A)^2}{1 - \sin^2 A}} \\
 &= \sqrt{\frac{(1 + \sin A)^2}{\cos^2 A}} = \sqrt{\frac{(1 + \sin A)}{\cos A}} \\
 &= \frac{1}{\cos A} + \frac{\sin A}{\cos A} = \sec A + \tan A \\
 &= \text{R.H.S}
 \end{aligned}$$

- Hence proved

(ii) $\sqrt{\frac{1 - \cos A}{1 + \cos A}} + \sqrt{\frac{1 + \cos A}{1 - \cos A}} = 2 \operatorname{cosec} A$

Solution:

Taking L.H.S and rationalizing the numerator and denominator with its respective conjugates, we get

$$\begin{aligned}
 &= \sqrt{\frac{(1 - \cos A)(1 - \cos A)}{(1 + \cos A)(1 - \cos A)}} + \sqrt{\frac{(1 + \cos A)(1 + \cos A)}{(1 - \cos A)(1 + \cos A)}} \\
 &= \sqrt{\frac{(1 - \cos A)^2}{(1 - \cos^2 A)}} + \sqrt{\frac{(1 + \cos A)^2}{(1 - \cos^2 A)}} \\
 &= \sqrt{\frac{(1 - \cos A)^2}{(\sin^2 A)}} + \sqrt{\frac{(1 + \cos A)^2}{(\sin^2 A)}} \\
 &= \frac{(1 - \cos A)}{(\sin A)} + \frac{(1 + \cos A)}{(\sin A)} \\
 &= \frac{(1 - \cos A + 1 + \cos A)}{(\sin A)} \\
 &= \frac{(2)}{(\sin A)} \\
 &= 2 \operatorname{cosec} A \\
 &= \text{R.H.S}
 \end{aligned}$$

- Hence proved

38. Prove that:

(i)
$$\sqrt{\frac{(\sec \theta - 1)}{(\sec \theta + 1)}} + \sqrt{\frac{(\sec \theta + 1)}{(\sec \theta - 1)}} = 2 \operatorname{cosec} \theta$$

Solution:

Taking L.H.S and rationalizing the numerator and denominator with its respective conjugates, we get

$$\begin{aligned}
 &= \sqrt{\frac{(\sec \theta - 1)(\sec \theta - 1)}{(\sec \theta + 1)(\sec \theta - 1)}} + \sqrt{\frac{(\sec \theta + 1)(\sec \theta + 1)}{(\sec \theta - 1)(\sec \theta + 1)}} \\
 &= \sqrt{\frac{(\sec \theta - 1)^2}{(\sec^2 \theta - 1)}} + \sqrt{\frac{(\sec \theta + 1)^2}{(\sec^2 \theta - 1)}} \\
 &= \sqrt{\frac{(\sec \theta - 1)^2}{\tan^2 \theta}} + \sqrt{\frac{(\sec \theta + 1)^2}{\tan^2 \theta}} \\
 &= \frac{(\sec \theta - 1)}{\tan \theta} + \frac{(\sec \theta + 1)}{\tan \theta} \\
 &= \frac{(\sec \theta - 1 + \sec \theta + 1)}{\tan \theta} \\
 &= \frac{(2 \cos \theta)}{\cos \theta \sin \theta} \\
 &= \frac{2}{\sin \theta} \\
 &= \text{R.H.S}
 \end{aligned}$$

Hence proved

(ii)
$$\sqrt{\frac{(1 + \sin \theta)}{(1 - \sin \theta)}} + \sqrt{\frac{(1 - \sin \theta)}{(1 + \sin \theta)}} = 2 \sec \theta$$

Solution:

Taking L.H.S and rationalizing the numerator and denominator with its respective conjugates, we get

$$\begin{aligned}
 &= \sqrt{\frac{(1 + \sin \theta)(1 + \sin \theta)}{(1 - \sin \theta)(1 + \sin \theta)}} + \sqrt{\frac{(1 - \sin \theta)(1 - \sin \theta)}{(1 + \sin \theta)(1 - \sin \theta)}} \\
 &= \sqrt{\frac{(1 + \sin \theta)^2}{(1 - \sin^2 \theta)}} + \sqrt{\frac{(1 - \sin \theta)^2}{(1 - \sin^2 \theta)}} \\
 &= \sqrt{\frac{(1 + \sin \theta)^2}{(\cos^2 \theta)}} + \sqrt{\frac{(1 - \sin \theta)^2}{(\cos^2 \theta)}} \\
 &= \frac{(1 + \sin \theta)}{(\cos \theta)} + \frac{(1 - \sin \theta)}{(\cos \theta)} \\
 &= \sqrt{\frac{(1 + \sin \theta + 1 - \sin \theta)}{(\cos \theta)}} \\
 &= \frac{(2)}{(\cos \theta)} = 2 \sec \theta \\
 &= \text{R.H.S}
 \end{aligned}$$

- Hence proved

(iii)
$$\sqrt{\frac{(1 + \cos \theta)}{(1 - \cos \theta)}} + \sqrt{\frac{(1 - \cos \theta)}{(1 + \cos \theta)}} = 2 \operatorname{cosec} \theta$$

Solution:

Taking L.H.S and rationalizing the numerator and denominator with its respective conjugates, we get

$$\begin{aligned}
 &= \sqrt{\frac{(1 - \cos \theta)(1 - \cos \theta)}{(1 + \cos \theta)(1 - \cos \theta)}} + \sqrt{\frac{(1 + \cos \theta)(1 + \cos \theta)}{(1 - \cos \theta)(1 + \cos \theta)}} \\
 &= \sqrt{\frac{(1 - \cos \theta)^2}{(1 - \cos^2 \theta)}} + \sqrt{\frac{(1 + \cos \theta)^2}{(1 - \cos^2 \theta)}} \\
 &= \sqrt{\frac{(1 - \cos \theta)^2}{(\sin^2 \theta)}} + \sqrt{\frac{(1 + \cos \theta)^2}{(\sin^2 \theta)}} \\
 &= \frac{(1 - \cos \theta)}{(\sin \theta)} + \frac{(1 + \cos \theta)}{(\sin \theta)} \\
 &= \frac{(1 - \cos \theta + 1 + \cos \theta)}{(\sin \theta)} \\
 &= \frac{(2)}{(\sin \theta)} \\
 &= 2 \operatorname{cosec} \theta \\
 &= \text{R.H.S}
 \end{aligned}$$

- Hence proved

(iv) $\frac{\sec \theta - 1}{\sec \theta + 1} = \left(\frac{\sin \theta}{1 + \cos \theta} \right)^2$

Solution:

Taking L.H.S, we have

$$= \frac{\sec \theta - 1}{\sec \theta + 1} = \frac{1 - \cos \theta}{1 + \cos \theta}$$

On multiplying numerator and denominator by $1 + \cos \theta$, we get

$$= \frac{(1 - \cos \theta)(1 + \cos \theta)}{(1 + \cos \theta)(1 + \cos \theta)}$$

$$= \frac{(1 - \cos^2 \theta)}{(1 + \cos \theta)^2}$$

$$= \frac{\sin^2 \theta}{(1 + \cos \theta)^2}$$

$$= \left(\frac{\sin \theta}{1 + \cos \theta} \right)^2$$

= R.H.S

- Hence proved

39. $(\sec A - \tan A)^2 = \frac{1 - \sin A}{1 + \sin A}$

Solution:

Taking LHS = $(\sec A - \tan A)^2$, we have

$$= \left[\frac{1}{\cos A} - \frac{\sin A}{\cos A} \right]^2$$

$$= \frac{(1 - \sin A)^2}{\cos^2 A}$$

$$= \frac{(1 - \sin A)^2}{1 - \sin^2 A}$$

$$= \frac{(1 - \sin A)^2}{(1 + \sin A)(1 - \sin A)}$$

$$= \frac{(1 - \sin A)}{(1 + \sin A)}$$

= R.H.S

- Hence proved

40.
$$\frac{1 - \cos A}{1 + \cos A} = (\cot A - \operatorname{cosec} A)^2$$

Solution:

Taking L.H.S and rationalizing the numerator and denominator with $(1 - \cos A)$, we get

$$= \frac{(1 - \cos A)(1 - \cos A)}{(1 + \cos A)(1 - \cos A)}$$

$$= \frac{(1 - \cos A)^2}{(1 - \cos^2 A)}$$

$$= \frac{(1 - \cos A)^2}{(\sin^2 A)}$$

$$= \left(\frac{1}{\sin A} - \frac{\cos A}{\sin A} \right)^2$$

$$= (\operatorname{cosec} A - \cot A)^2$$

$$= (\cot A - \operatorname{cosec} A)^2$$

$$= \text{R.H.S}$$

- Hence proved

41.
$$\frac{1}{\sec A - 1} + \frac{1}{\sec A + 1} = 2 \operatorname{cosec} A \cot A$$

Solution:

Considering L.H.S and taking L.C.M and on simplifying we have,

$$= \frac{\sec A + 1 + \sec A - 1}{(\sec A + 1)(\sec A - 1)}$$

$$= \frac{2 \sec A}{(\sec^2 A - 1)}$$

$$= \frac{2 \sec A}{(\tan^2 A)}$$

$$= \frac{2 \cos^2 A}{(\cos A \sin^2 A)}$$

$$= \frac{2 \cos A}{(\sin^2 A)}$$

$$= \frac{2 \cos A}{(\sin A)(\sin A)}$$

$$= 2 \operatorname{cosec} A \cot A = \text{RHS}$$

- Hence proved

42. $\frac{\cos A}{1 - \tan A} + \frac{\sin A}{1 - \cot A} = \sin A + \cos A$

Solution:

Taking LHS, we have

$$\begin{aligned}
 &= \frac{\cos A}{1 - \tan A} + \frac{\sin A}{1 - \cot A} \\
 &= \frac{\cos A}{1 - \frac{\sin A}{\cos A}} + \frac{\sin A}{1 - \frac{\cos A}{\sin A}} \\
 &= \frac{\cos^2 A}{\cos A - \sin A} - \frac{\sin^2 A}{\cos A - \sin A} \\
 &= \frac{\cos^2 A - \sin^2 A}{\cos A - \sin A} \\
 &= \frac{(\cos A + \sin A)(\cos A - \sin A)}{\cos A - \sin A} \\
 &= \cos A + \sin A \\
 &= \text{RHS}
 \end{aligned}$$

- Hence proved

43.
$$\frac{(\operatorname{cosec} A)}{(\operatorname{cosec} A - 1)} + \frac{(\operatorname{cosec} A)}{(\operatorname{cosec} A + 1)} = 2 \sec^2 A$$

Solution:

Considering L.H.S and taking L.C.M and on simplifying we have,

$$\begin{aligned}
 &= \frac{(\operatorname{cosec} A)(\operatorname{cosec} A + 1 + \operatorname{cosec} A - 1)}{(\operatorname{cosec}^2 A - 1)} \\
 &= \frac{(2 \operatorname{cosec}^2 A)}{\cot^2 A} \\
 &= \frac{2 \sin^2 A}{\sin^2 A \cdot \cos^2 A} \\
 &= \frac{2}{\cos^2 A} \\
 &= 2 \sec^2 A \\
 &= \text{RHS}
 \end{aligned}$$

- Hence proved