Exercise 6.2

1. If cos θ = 4/5, find all other trigonometric ratios of angle θ.
Solution:

We have,
$\cos\theta = 4/5$
And we know that,
$\sin\theta = \sqrt{(1 - \cos^2\theta)}$
$\Rightarrow \sin\theta = \sqrt{(1 - (4/5)^2)}$
$= \sqrt{(1 - (16/25))}$
$= \sqrt{[(25 - 16)/25]}$
$= \sqrt{(9/25)}$
$= 3/5$
$\therefore \sin\theta = 3/5$

Since, $\text{cosec}\,\theta = 1/\sin\theta$
$= 1/(3/5)$
$\Rightarrow \text{cosec}\,\theta = 5/3$

And, $\sec\theta = 1/\cos\theta$
$= 1/(4/5)$
$\Rightarrow \text{cosec}\,\theta = 5/4$

Now,
$\tan\theta = \sin\theta/\cos\theta$
$= (3/5)/(4/5)$
$\Rightarrow \tan\theta = 3/4$

And, $\cot\theta = 1/\tan\theta$
$= 1/(3/4)$
$\Rightarrow \cot\theta = 4/3$

2. If sin θ = 1/√2, find all other trigonometric ratios of angle θ.
Solution:

We have,
$\sin\theta = 1/\sqrt{2}$
And we know that,
$\cos\theta = \sqrt{(1 - \sin^2\theta)}$
$\Rightarrow \cos\theta = \sqrt{(1 - (1/\sqrt{2})^2)}$
$= \sqrt{(1 - (1/2))}$
$= \sqrt{[(2 - 1)/2]}$
$= \sqrt{(1/2)}$
$= 1/\sqrt{2}$
$\therefore \cos\theta = 1/\sqrt{2}$

Since, cosec θ = 1/ sin θ
= 1/ (1/√2)
⇒ cosec θ = √2

And, sec θ = 1/ cos θ
= 1/ (1/√2)
⇒ cosec θ = √2

Now,
tan θ = sin θ/ cos θ
= (1/√2)/ (1/√2)
⇒ tan θ = 1

And, cot θ = 1/ tan θ
= 1/ (1)
⇒ cot θ = 1

3. If $\tan\theta = \frac{1}{\sqrt{2}}$, find the value of $\frac{\operatorname{cosec}^2\theta - \sec^2\theta}{\operatorname{cosec}^2\theta + \cot^2\theta}$.

Solution:

Given,
tan θ = 1/√2
By using $\sec^2\theta - \tan^2\theta = 1$,

$$\sec\theta = \sqrt{1+\tan^2\theta} \quad = \sqrt{1+\left(\frac{1}{\sqrt{2}}\right)^2} \quad = \sqrt{1+\frac{1}{2}} \quad = \sqrt{\frac{3}{2}}$$

And,

$$\cot\theta = \frac{1}{\tan\theta} \quad = \frac{1}{\frac{1}{\sqrt{2}}} \quad = \sqrt{2}$$

From identity, we have

$$\operatorname{cosec}\theta = \sqrt{1+\cot^2\theta} \;= \sqrt{1+2} \;= \sqrt{3}$$

Substituting the values, we get

$$\frac{\operatorname{cosec}^2\theta - \sec^2\theta}{\operatorname{cosec}^2\theta + \cot^2\theta} \quad = \frac{(\sqrt{3})^2 - \left(\sqrt{\frac{3}{2}}\right)^2}{(\sqrt{3})^2 + (\sqrt{2})^2}$$

$$= \frac{3-\frac{3}{2}}{3+2} = \frac{\frac{3}{2}}{5} = \frac{3}{10}$$

4. If $\tan\theta = \frac{3}{4}$, find the value of $\frac{1-\cos\theta}{1+\cos\theta}$

Solution:

Given,

$\tan\theta = 3/4$

By using $\sec^2\theta - \tan^2\theta = 1$,

$$\sec\theta = \sqrt{1+\tan^2\theta} \quad = \sqrt{1+\left(\frac{3}{4}\right)^2} \quad = \sqrt{1+\frac{9}{16}} \quad = \sqrt{\frac{16+9}{16}} \quad = \sqrt{\frac{25}{16}}$$

sec $\theta = 5/4$

Since, $\sec\theta = 1/\cos\theta$

$\Rightarrow \cos\theta = 1/\sec\theta$

$= 1/(5/4)$

$= 4/5$

So, $\dfrac{\text{cosec}^2\theta - \sec^2\theta}{\text{cosec}^2\theta + \cot^2\theta} = \dfrac{1-\frac{4}{5}}{1+\frac{4}{5}} = \dfrac{\frac{1}{5}}{\frac{9}{5}} = \dfrac{1}{9}$

5. If $\tan\theta = \dfrac{12}{5}$, find the value of $\dfrac{1+\sin\theta}{1-\sin\theta}$

Solution:

Given, $\tan\theta = 12/5$

Since, $\cot\theta = 1/\tan\theta = 1/(12/5) = 5/12$

Now, by using $\text{cosec}^2\theta - \cot^2\theta = 1$

$\text{cosec}\,\theta = \sqrt{(1 + \cot^2\theta)}$

$= \sqrt{(1 + (5/12)^2)}$

$= \sqrt{(1 + 25/144)}$

$= \sqrt{(169/25)}$

$\Rightarrow \text{cosec}\,\theta = 13/5$

Now, we know that

$\sin\theta = 1/\text{cosec}\,\theta$

$= 1/(13/5)$

$\Rightarrow \sin\theta = 5/13$

Putting value of $\sin\theta$ in the expression we have,

$$= \dfrac{1+\frac{12}{13}}{1-\frac{12}{13}} = \dfrac{\frac{13+12}{18}}{\frac{13-12}{18}}$$

$= 25/1$

$= 25$

6. If $\cot\theta = \dfrac{1}{\sqrt{3}}$, find the value of $\dfrac{1-\cos^2\theta}{2-\sin^2\theta}$

Solution:

Given,

$\cot\theta = 1/\sqrt{3}$

Using $\text{cosec}^2\theta - \cot^2\theta = 1$, we can find $\text{cosec}\,\theta$

$\text{cosec}\,\theta = \sqrt{(1 + \cot^2\theta)}$

$= \sqrt{(1 + (1/\sqrt{3})^2)}$

$= \sqrt{(1 + (1/3))} = \sqrt{((3 + 1)/3)}$

$= \sqrt{(4/3)}$

⇒ cosec θ = 2/√3

So, sin θ = 1/ cosec θ = 1/ (2/√3)

⇒ sin θ = √3/2

And, we know that

$\cos\theta = \sqrt{(1 - \sin^2\theta)}$

$= \sqrt{(1 - (\sqrt{3}/2)^2)}$

$= \sqrt{(1 - (3/4))}$

$= \sqrt{((4 - 3)/4)}$

$= \sqrt{(1/4)}$

⇒ cos θ = 1/2

Now, using cos θ and sin θ in the expression, we have

$$= \frac{1 - \left(\frac{1}{2}\right)^2}{2 - \left(\frac{\sqrt{3}}{2}\right)^2}$$

$$= \frac{1 - \frac{1}{4}}{2 - \frac{3}{4}} = \frac{\frac{3}{4}}{\frac{5}{4}}$$

= 3/5

7. If $\operatorname{cosec} A = \sqrt{2}$, find the value of $\frac{2\sin^2 A + 3\cot^2 A}{4(\tan^2 A - \cos^2 A)}$.

Solution:

Given,

cosec A = √2

Using $\operatorname{cosec}^2 A - \cot^2 A = 1$, we find cot A

$$\cot A = \sqrt{\operatorname{cosec}^2 A - 1} \quad = \sqrt{(\sqrt{2})^2 - 1} \quad = \sqrt{2 - 1} \quad = 1$$

So, tan A = 1/ cot A
= 1/ 1 = 1

And, sin A = 1/ cosec A = 1/ $\sqrt{2}$

$$\cos A = \sqrt{1 - \sin^2 A} \quad = \sqrt{1 - \left(\frac{1}{\sqrt{2}}\right)^2} = \sqrt{\frac{1}{\sqrt{2}}} = \frac{1}{\sqrt{2}}$$

On substituting we get,

$$= \frac{2\left[\frac{1}{\sqrt{2}}\right]^2 + 3[1]^2}{4\left[[1] - \left[\frac{1}{\sqrt{2}}\right]^2\right]}$$

$$= \frac{2 \times \frac{1}{2} + 3}{4\left[1 - \frac{1}{2}\right]} \Rightarrow \frac{1 + 3}{4.\frac{1}{2}}$$